# Reinforcement Meta Learning

A thesis submitted by

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A model of some target function is always constrained by available function evaluations (data). Trying to learn under the constraint where there are few example function evaluations is known as few shot learning. One paradigm applied to manage this constraint is meta-learning or "learning to learn". This thesis contributes a new meta-learning algorithm called reinforcement meta-learning (REML). REML casts learning to learn as a markov decision process or reinforcement learning problem. It proposes a heirarchical system with an outer network that constructs inner networks from a pool of layers. The supervisory system is implemented as a recurrent policy gradient method and the subordinate models are neural networks built by the agent for the regression and classificatin tasks. This is to my knowledge the first work to use reinforcement learning as a meta-learner that can learns both parameters and hyperparameters. REML is evaluated on sinuosoidal curves REML is shown to perform transfer learning as well as k=5 and k=10 few shot.

Robust to unrelated tasks? Probably not because the layer pool is set ahead of time

One of these may be robustness to learning on unrelated tasks, relative to offline trained tasks. I hypothesize that because layers in REML are composed individually, they have a more expressive quality in their availability to be sequenced in different combinations. This is as opposed to a single initial set of policy parameters adapted for each task, as is in MAML. This is not needed, but common.

# Chapter 1

# Background

## 1.1 Artificial Neural Networks

Artificial neural networks (ANNs) are non-linear computational models that approximate a target function $f : \mathbb{R}^n \to \mathbb{R}^m$, where $n$ and $m$ are integers [1]. Given a set $X = \{(\mathbf{x}_1, \mathbf{y}_1), \ldots, (\mathbf{x}_N, \mathbf{y}_N)\} \subset \mathbb{R}^n \times \mathbb{R}^m$ of input-output pairs of size $N$ the model is trained to approximate $f$ such that $f(\mathbf{x}_i) = \mathbf{y}_i \forall i \in \{1, 2, \ldots, N\}$. By the Universal Approximation theorem, a neural network's approximation of a continuous function is theoretically guaranteed to be as precise as it needs to be given the network has at least one hidden layer with some finite number of nodes [2].

### 1.1.1 Starting from linear regression

Consider the problem of predicting the value of one or more continuous target variables $\mathbf{t} \subset \mathbb{R}^m$ provided a $D$-dimensional vector $\mathbf{x}_n$ of input variables, or what is called regression. Given a set consisting of $N$ observation and value pairs $\{ (\mathbf{x}_n, \mathbf{t}_n) \}_{n=1}^{N}$, the objective is to predict the value for any input vector $\mathbf{x}_n$ such that it is as as close as possible to the provided target value $\mathbf{t}_n$.

One approach is linear regression, or a linear combination over the components of an input pattern $\mathbf{x}_n$

$$y(\mathbf{x}_n, \mathbf{w}) = w_0 + w_1 x_1 + \ldots + w_D x_D \tag{1.1}$$

where $\mathbf{x}_n$ has $D$ dimensions, $\mathbf{x}_n = (x_1, \ldots, x_D)^T$ and $w \in \mathbb{R}^{D+1}$ represents the parameters of the function, $w = (w_0, \ldots, w_D)$ and $D$ is extended to $D+1$

for the bias weight $w_0$.

As is, this regression function is limited to being a linear function over the input vector $\mathbf{x}_n$. Non-linear basis functions $\phi$ on the input variables make the function $y(\mathbf{x}_n, \mathbf{w})$ non-linear for an input $\mathbf{x}_n$:

$$y(\mathbf{x}_n, \mathbf{w}) = w_0 + \sum_{i=1}^{D} w_i \phi_i (x_i) \tag{1.2}$$

This equation can be simplified further if we define a useful basis function for the bias $\phi_0(\mathbf{x}) = 1$ such that

$$y(\mathbf{x}_n, \mathbf{w}) = \sum_{i=0}^{D} w_i \phi_i (x_i) \tag{1.3}$$

Despite producing non linear outputs over the input $\mathbf{x}$ this is *linear regression* because it is linear with respect to $\mathbf{w}$.

### 1.1.2 Constructing neural networks

Basic ANNs can be seen as an extension to linear regression where the basis functions become parameterized. The basis functions continue to be non-linear functions over the linear combination of the input, but now the output of the basis function is dependent on the learned coefficients $\{ w_j \}$. In this construction, basis functions are known as *activation* functions $h$ in the context of neural networks.

We start by rewriting equation 1.2 as a linear combinations over the input variables to produce $a$ or the *activation*.

$$a = \sum_{i=1}^{D} w_i x_i + w_0 \phi (x_i) \tag{1.4}$$

The value $a$ is transformed using a non-linear activation function $h$. This transformation produces $z$ and is referred to as a *hidden unit*.

$$z = h (a) \tag{1.5}$$

The coefficients $\{ w_j \}$ parameterizing this non-linear transformation are referred to as a *layer*.

An ANN has a minimum of two layers - an input layer and output layer. However, ANNs are not limited to 2 layers. ANNs can have $l$ many layers where $l \in [2, +\infty)$. Networks with $> 2$ layers are referred to as *deep neural networks*. For the purposes of the background, we will continue with the simple 2-layer case to establish preliminaries.

The input layer operates on an input $(x_1, \ldots, x_D)$ to produce *activations* $a_j = (a_1, \ldots, a_M)$, where $M$ denotes the number of parameters $\{ w_j \}$ in the input layer. The parameters for the input layer are represented with a superscript (1) and the parameters for the output layer will be represented with a superscript (2).

$$a_j = \sum_{i=1}^{D} w_{ji}^{(1)} x_i + w_{j0}^{(1)} \tag{1.6}$$

The activations are passed through a non-linear activation $h$

$$z_j = h\left(a_j\right) \tag{1.7}$$

The output layer then transforms the hidden units $z_j$ to produce output unit activations $a_k$ where $k \in (1, \ldots, K)$ and $K$ is the number of outputs expected for this problem (i.e., appropriate to the target variable $\mathbf{t}_i$ for $\mathbf{x}_i$).

$$a_k = \sum_{j=1}^{M} w_{kj}^{(2)} z_j + w_{k0}^{(2)} \tag{1.8}$$

The activation $a_k$ is transformed by a different non-linear activation function that is appropriate for $K$. Here this activation function is represented as $\sigma$. A common choice of activation function $h$ for non-output layers is the rectified linear unit $h(a) = \min(0, a)$. A common choice of activation function for $\sigma$ is the sigmoid function $\sigma(a) = \frac{1}{1+e^{-a}}$ for classification problems and the identity $y_k = a_k$ for simple regression problems. We now present the equation for a *feed-forward* pass through a 2-layer ANN.

$$y_k(\mathbf{x}_n, \mathbf{w}) = \sigma\left(\sum_{j=1}^{M} w_{kj}^{(2)} h\left(\sum_{i=1}^{D} w_{ji}^{(1)} + w_{j0}^{(1)} \phi\left(x_i\right)\right) + w_{k0}^{(2)}\right) \tag{1.9}$$

A neural network then is a non-linear function over an input $\mathbf{x_n}$ to an output $y_k$ that seeks to approximate $\mathbf{t_n}$ and is controlled by a set of adaptable parameters $\mathbf{w}$.

### 1.1.3 Training a neural network

The goal of learning for a neural network is to optimize the parameters of the network such that the loss function $E(X, \mathbf{w})$ takes the lowest value. Continuing with the previous example for regression, we look at the sum-of-squares error function

$$E(X, \mathbf{w}) = \frac{1}{2} \sum_{n=1}^{N} \| y\left(\mathbf{x}_n, \mathbf{w}\right) - \mathbf{t}_n \|^2 \tag{1.10}$$

There is typically not an analytical solution and iterative procedures are used to minimize the loss function $E$. The steps taken are

$$\mathbf{w}^{(\tau+1)} = \mathbf{w}^{(\tau)} + \Delta \mathbf{w}^{(\tau)} \tag{1.11}$$

where $\tau$ is the iteration step. An approach for the weight update step with $\Delta \mathbf{w}^{(\tau)}$ is to use the gradient of $E(X, \mathbf{w})$ with respect to the parameters $\mathbf{w}$. The weights are updated in the direction of steepest error function decrease or in the $-\nabla E(X, \mathbf{w})$ direction.

$$\mathbf{w}^{(\tau+1)} = \mathbf{w}^{(\tau)} + \alpha \nabla E\left(X, \mathbf{w}^{(\tau)}\right) \tag{1.12}$$

where $\alpha > 0$ is the learning rate controlling the size of update step taken. This iterative procedure is called *gradient descent optimization* [3].

### 1.1.4 Error function derivatives

The gradient $\nabla E(X, \mathbf{w})$ is calculated with respect to every $w \in \mathbf{w}$, for all $\mathbf{x}_n \in X$.

We start with one input pattern $\mathbf{x_n}$ and rewrite the error function as

$$\begin{aligned} E_n &= \frac{1}{2} \left( y(\mathbf{x_n}, \mathbf{w}) - \mathbf{t_n} \right)^2 \\ &= \frac{1}{2} \sum_{j=1} (w_{kj} z_j - t_{nk})^2 \end{aligned} \tag{1.13}$$

The calculation starts with the gradient of $E_n$ with respect to each $w_{kj}$ in the output layer (2) then continues backwards to layer (1) for $w_{ji}$. This method can extend to $l$-layer networks where $l = (1, \ldots, L)$ and $L \subseteq \mathbb{R}$.

Observe that

$$\frac{\partial E_n}{\partial w_{kj}} = \frac{\partial E_n}{\partial a_k} \frac{\partial a_k}{\partial w_{kj}} \tag{1.14}$$

We start by calculating the partial derivative of $E_n$ with respect to the activation $a_k$. Recall that $a_k = \sum_k w_{kj} z_j$. By the chain rule:

$$\begin{aligned} \frac{\partial E_n}{\partial a_k} &= (h(a_k) - t_{nk}) h'(a_k) \\ &= h'(a_k)(\hat{y}_n - t_{nk}) \end{aligned} \tag{1.15}$$

We introduce a new notation to call this partial derivative an *error*

$$\delta_k \equiv \frac{\partial E_n}{\partial a_k} \tag{1.16}$$

Next we calculate the partial derivate of $a_k$ with respect to $w_{kj}$

$$\begin{aligned} \frac{\partial a_k}{\partial w_{kj}} &= \frac{\partial}{\partial w_{kj}} \left( \sum_k w_{kj} z_k \right) \\ &= z_k \end{aligned} \tag{1.17}$$

With which we can write

$$\frac{\partial E_n}{\partial w_{kj}} = \delta_k z_k \tag{1.18}$$

The procedure will continue in the same way for the remainder of the layers and their units, where we calculate the errors $\delta$ for the units in the layer and multiply error of that unit by its activation $z$. For layer (1) (input layer) we need to calculate

$$\frac{\partial E_n}{\partial w_{ji}} = \frac{\partial E_n}{\partial a_j} \frac{\partial a_j}{\partial w_{ji}} \tag{1.19}$$

starting with $\delta_j$ or $\frac{\partial E_n}{\partial a_j}$. Observe that

$$\frac{\partial E_n}{\partial a_j} = \sum_k \frac{\partial E_n}{\partial a_k} \frac{\partial a_k}{\partial a_j} \tag{1.20}$$

where k is the number of outputs (here, k=1 for the continued regression example).

We calculated $\frac{\partial E_n}{\partial a_k}$ above. Continue with

$$\begin{aligned} \frac{\partial a_k}{\partial a_j} &= \frac{\partial}{\partial a_j}\left(\sum_k w_{kj} h(a_j)\right) \\ &= h'(a_j) w_{kj} \end{aligned} \tag{1.21}$$

We can finish calculating the error $\delta_j$ for equation 1.18

$$\begin{aligned} \frac{\partial E_n}{\partial a_k}\frac{\partial a_k}{\partial a_j} &= h'(a_k)(\hat{y}_n - t_{nk}) h'(a_j) w_{kj} \\ &= \frac{\partial E_n}{\partial a_j} \\ &= \delta_j \end{aligned} \tag{1.22}$$

Thus we obtain the *backpropagation* formula

$$\delta_j = h'(a_j) \sum_k w_{kj} \delta_k \tag{1.23}$$

where the error for a unit $j$ is the result of backpropagating the errors in the units later in the network.

Calculating the gradient is backpropgating the errors. As we have seen, this procedure begins with a forward propagation of the input vectors $x_n$ to calculate the activations of all units. Then, it involves calculating the errors $\delta_k$ in the output layer. Using $\delta_k$ we can calculate $\delta_j$ for the hidden units in previous layers. With all errors $\delta$, the gradient is calculated by multiplying the error by the activations $a$ transformed by their non-linear function $h$ where $h(a) = z$.

### 1.1.5 Recurrent Neural Networks

ANNs can be constructed as *directed graphs*, formally defined as $G = (V, E)$ where $V$ is the set of vertices $\{ v_1, \ldots, v_n \}$ and $E$ is the set of edges $\{ (u, v) \mid u, v \in V \}$. We show neural networks are directed becuause the edges are a set of ordered

pairs. In comparison, an undirected graph would have edges $\{ \{u, v\} \mid u, v \in V \}$. In terms appropriate to neural networks, $V$ corresponds to our hidden units $\{ z \}$ and output units and $E$ corresponds to the parameters $\{ w \}$.

The 2-layer network we constructed above was a *directed acyclic graph*. $G$ is acyclic if $\forall v \in V$, there does not exist a cycle containing $v$. This means that for $\forall (u, v) \in E$, $u \neq v$.

ANNs can contain cycles however. A type of ANN that contains cycles is a *recurrent neural network* (RNN) [3]. An RNN is recurrent in that information persists in the network by being passed from one forward propagation step to the next. This ability to incorporate past network data makes RNNs useful for simulating dynamical systems.

RNNs model past network data as $\mathbf{h}_t$ or the *hidden state*

$$\mathbf{h}_t = f_h(\mathbf{x}_t, \mathbf{h}_{t-1}) \tag{1.24}$$

$$\mathbf{y}_t = f_o(\mathbf{h}_t) \tag{1.25}$$

where $f_h$ is a transition function parameterized by $\theta_h$ and $f_o$ is an output function parameterized by $\theta_o$ [4]. The transition function can be a non-linear function such as the rectified linear unit or the sigmoid function.

Datasets used with RNNs may include $T_n$ many input patterns $\mathbf{x}^{(n)}$ where $T_n \in \mathbb{R}$ is the number of timesteps for which there is data for the datapoint

$$\{ (\mathbf{x}_1^{(n)}, \mathbf{y}_1^{(n)}), \ldots, (\mathbf{x}_{Tn}^{(n)}, \mathbf{y}_{Tn}^{(n)}) \}_{n=1}^{N} \tag{1.26}$$

The cost function is

$$E(\theta) = \frac{1}{N} \sum_{n=1}^{N} \sum_{t=1}^{T} d(\mathbf{y}_t^{(n)}, f_o(\mathbf{h}_t^{(n)})) \tag{1.27}$$

where $\theta$ is the parameters of the network, and $d(\mathbf{a}, \mathbf{b})$ is the divergence measure such as Euclidean distance used in the above sum of squares error.

The parameters $\theta$ are updated with a variant of backpropagation that works with sequential data called *backpropagation through time (BPTT)* [5]. This method "unrolls" the RNN into each a computational graph one time step

at a time. This unrolled RNN is equivalent to a deep neural network where the same parameters re-appear throughout the network per timestep. Backpropagation through time sums the gradient with respect to each parameter for all times the parameter appears in the network.

RNNs are prone to challenges during training including *exploding gradient* and *vanishing gradient*. During training with BPTT the gradients can become very large (i.e., exploding) or very small (i.e., vanishing). Calculating the errors involves multiplying the errors from later layers by the activations in earlier layers as defined above. RNNs can have long sequences in the unrolled network, meaning many multiplication operations over the gradients. Multiplying large or small numbers many times will lead to very large numbers and very small numbers, respectively.

A large gradient will cause large weight updates in the gradient update step, such as in gradient descent optimization, which will make training unstable. A small gradient will cause negligent or no weight updates such that no learning happens and hidden unit activation trend to 0. These activations are called *dead neurons* where "neuron" is another word for a hidden unit.

### 1.1.6 Long Short-Term Memory Networks

An extension of the RNN is the Long Short-Term Memory Network (LSTM), intended to address the exploding and vanishing gradient problems or "error back-flow problems"[6]. LSTMs introduce additional calculations called "gates" within the cells of an RNN. These gates control how much information is retained or discarded in each timestep. Each cell has state $C_t$ and the gates responsible for modifying $C_t$ across $T_n$ for $(x_{1n}, \ldots, x_{Tn})$.

The *forget gate* controls the amount of information retained from the previous unit $h_{t-1}$ and the input $x_t$ to include in this state $C_t$

$$f_t = \sigma(W_f \cdot [h_{t-1}, x_t] + b_f) \tag{1.28}$$

where $W_f$ is a weight matrix and $b_f$ is the bias term. The sigmoid is used as it outputs a value in $[0, 1]$, with 0 meaning to discard all previous network data and 1 meaning to keep all previous network data.

The *input gate* controls the amount of information to be included from the input $x_t$

$$i_t = \sigma(W_i \cdot [h_{t-1}, x_t] + b_i) \tag{1.29}$$

where $W_i$ is the weight matrix and $b_i$ is the bias term for this gate, respectively.

The output of the forget gate and input gate are composed as a proposal vector that would be added element-wise to $C_t$.

$$\widetilde{C}_t = \tanh\left(W_C \cdot [h_{t-1}, x_t] + b_C\right) \tag{1.30}$$

where $\widetilde{C}_t$ holds the amount of information to include from $x_t$ and $h_{t-1}$. The tanh function is the hyperbolic tangent function $\frac{e^{2x}-1}{e^{2x}+1}$. It is used to transform $x$ to a value within $[-1, 1]$ that results in more stable gradient calculations.

The cell state $C_t$ is the sum of the two values we have constructed: some amount of the previous state $C_{t-1}$ and some amount of the proposed $\widetilde{C}_t$.

$$C_t = f_t * C_{t-1} + i_t * \widetilde{C}_t \tag{1.31}$$

The final calculation what to output - the new hidden state $h_t$. The cell calculates how much of the new cell state $C_t$ to output for this timesteep.

$$o_t = \sigma(W_o \cdot [h_{t-1}, x_t] + b_o) \tag{1.32}$$

$$h_t = o_t * \tanh(C_t) \tag{1.33}$$

## 1.2 Reinforcement Learning

An agent exhibiting reinforcement learning (RL) learns from actions it takes in an environment that gives the agent a numerical reward signal for the agent's actions. The environment is represented in terms of states it can take on based on the agent's actions. The action taken in a state yields a new state together with some reward. The goal of the agent then is to learn a *policy*, a map from states to actions for the environment, that maximizes the cumulative reward over time for the agent. As the agent learns this policy, the agent needs to decide when to take an action it knows (i.e, it has taken and knows the value of) and an action it doesn't know [7].

Reinforcement learning is considered a distinct type of learning to *supervised learning* and *unsupervised learning*. The learning considered thus far has been *supervised learning* or learning from labeled examples where the ground truth is known. In this supervised context, the agent or model is given the answer after it acts. The model's task is instead to learn from labeled data such that it can generalize or approximate to unseen examples where a label does not exist. RL is not supervied learning because no answer is ever provided to the agent; the agent needs to discover its' own answer. RL is also not *unsupervised learning*, where the objective is to find patterns in unlabeled data; while the agent may build a model of the environment it interacts with as a kind of pattern recognition, the objective of RL is to maximize the numerical reward signal rather than to discover hidden structure.

A reason reinforcement learning is used over supervised learning is the answer may not be known for a sufficiently complex task. Another reason is its often impractical to provided a full set of representative examples of all states the agent may experience.

### 1.2.1 Markov Decision Processes

The problems solved by RL are often formalized as a markov decision process (MDP) [8]. A markov decision process (MDP) is often used to formalize and model the problems and is represented as a 4-tuple $(S, A, P_a, R_a)$ where

- $S$ is the set of states or *state space*
- $A = \{ A_s \mid s \in S \}$ is the set of actions or *action space*
- $P$ is $P_a(s, s') = P(s_{t+1} = s' | s, a)$ or *transition function*
- $R_a(s, s') = \{ r \mid r \in \mathbb{R} \}$ is the reward upon transition from state s to state s' or *reward function*

Consider an agent that interacts with the environment over timesteps $t \in \mathbb{R}_{\geq 0}$. For each timestep $t = 0, 1, 2, \ldots$ the agent is in a state $s \in S$ where it can take an action $a \in A_s$ and recieve the reward signal $r_{t+1}$ as it transitions from state $s = s_t$ to state $s' = s_{t+1}$. This sequences would look something like

$$S_0, A_0, R_1, S_1, A_1, R_2, \ldots \tag{1.34}$$

In a finite MDP where there is a finite number of states in $S$, actions in $A$, and rewards in $R$, the random variables for $S$ and $R$ have some probability of occuring in the environment.

Consider that transition function has complete information on the dynamics of the environment. For this to be true, $\forall s \in S$, $s$ has a history of all seqeuences possible from that state. The probability of the next state $s_{t+1}$ only depends on $s_t$ and $a_s$. The past history of states and action transitions is not needed. This condition is the *Markov property*.

### 1.2.2 Policy Gradient Methods

### 1.2.3 PPO

### 1.2.4 Recurrent Policy

## 1.3 Meta Learning

### 1.3.1 Few shot learning

# Chapter 2

# Title chapter 2

## 2.1 Title section 2.1

### 2.1.1 If needed

### 2.1.2 If needed

## 2.2 Title section 2.2

### 2.2.1 If needed

### 2.2.2 If needed

## 2.3 Title section 2.3

### 2.3.1 If needed

### 2.3.2 If needed

# Appendix A

# Title of the Appendix

# Bibliography


[1] C. M. Bishop and N. M. Nasrabadi, *Pattern recognition and machine learning*. Springer, 2006, vol. 4.

[2] K. Hornik, M. Stinchcombe, and H. White, "Multilayer feedforward networks are universal approximators," *Neural networks*, vol. 2, no. 5, pp. 359–366, 1989.

[3] D. E. Rumelhart, G. E. Hinton, and R. J. Williams, "Learning representations by back-propagating errors," *nature*, vol. 323, no. 6088, pp. 533–536, 1986.

[4] R. Pascanu, C. Gulcehre, K. Cho, and Y. Bengio, "How to construct deep recurrent neural networks," *arXiv preprint arXiv:1312.6026*, 2013.

[5] P. J. Werbos, "Backpropagation through time: What it does and how to do it," *Proceedings of the IEEE*, vol. 78, no. 10, pp. 1550–1560, 1990.

[6] S. Hochreiter and J. Schmidhuber, "Long short-term memory," *Neural computation*, vol. 9, no. 8, pp. 1735–1780, 1997.

[7] R. S. Sutton and A. G. Barto, *Reinforcement learning: An introduction*. MIT press, 2018.

[8] R. Bellman, "A markovian decision process," *Journal of mathematics and mechanics*, pp. 679–684, 1957.