CS103 Problem Set 6 (Written)

Matthew Phonchay Vilaysack, Maria Shan Wang

TOTAL POINTS

22 / 23

QUESTION 1

1 $$\wp(\Sigma^\star)$$ **1 / 2**

+ **2 pts** Correct
- ✓ + **1 pts** Mostly Correct
+ **0 pts** Incorrect

1 set of string = language

QUESTION 2

Much Ado About Nothing, Part II 6 pts

2.1 $$\varepsilon \in L$$? **1 / 1**

- ✓ + **1 pts** Correct
+ **0 pts** Incorrect/missing

2.2 $$\varepsilon \not\in L$$? **1 / 1**

- ✓ + **1 pts** Correct
+ **0 pts** Incorrect/missing

2.3 $$\varepsilon \subseteq L$$? **1 / 1**

- ✓ + **1 pts** Correct
+ **0 pts** Incorrect/missing

2.4 $$\varepsilon \not\subseteq L$$? **1 / 1**

- ✓ + **1 pts** Correct
+ **0 pts** Incorrect/missing

2.5 $$\phi = \varepsilon$$? **1 / 1**

- ✓ + **1 pts** Correct
+ **0 pts** Incorrect/missing

2.6 $$\phi \neq \varepsilon$$? **1 / 1**

- ✓ + **1 pts** Correct
+ **0 pts** Incorrect/missing

QUESTION 3

3 Hard Resets **5 / 5**

- ✓ + **5 pts** all correct
+ **0 pts** missing

Automata

+ **3 pts** Correct
+ **2 pts** Mostly correct
+ **1 pts** Partially correct
+ **0 pts** Incorrect

Reset string

+ **2 pts** Correct
+ **1 pts** Partially correct
+ **0 pts** Incorrect

QUESTION 4

Concatenation, Kleene Stars, and Complements 10 pts

4.1 Cardinalities of Powers **5 / 5**

Intuition Scale

- ✓ + **2 pts** Correct
+ **1 pts** Partially correct intuition
+ **0 pts** Incorrect

Execution Scale

- ✓ + **3 pts** Correct
+ **2 pts** Minor execution error
+ **1 pts** Major execution error
+ **0 pts** Incorrect

4.2 Stars and Stripes **5 / 5**

- ✓ + **5 pts** Fully correct

Intuition Scale

+ **2 pts** Correct
+ **1 pts** Partially correct intuition
+ **0 pts** Incorrect

Execution Scale

+ **3 pts** Correct

+ **2 pts** Minor execution error

+ **1 pts** Major execution error

+ **0 pts** Incorrect

+ **0 pts** Incorrect

QUESTION 5

5 Optional Fun Problem: Why Finite? **0 / 0**

+ **0 pts** Correct! Congratulations!!

✓ + **0 pts** **Incorrect / Not attempted**

Problem Three: $\wp(\Sigma^*)$

The powerset of the set of all strings composed from characters in Σ.

1 $$\wp(\Sigma^\star)$$ **1 / 2**

+ **2 pts** Correct

✓ + **1 pts** **Mostly Correct**

+ **0 pts** Incorrect

1 set of string = language

Problem Four: Much Ado About Nothing, Part II

i.

Yes, for example: $L = \{w \in \{a\}^* | \ w$ contains less than five a's$\}$

ii.

Yes, for example: $L = \{w \in \{a\}^* | \ w$ contains aa as a substring $\}$

iii.

No. ε is a transition that does not consume any input but is part of a language L. It represents the empty string. It is not a set and therefore, it cannot be a subset of L.

iv.

Yes, for example: $L = \{w \in \{a, b\}^* | \ w$ contains ab as a substring $\}$

v.

No. ε is not a set and therefore cannot be the empty set.

vi.

No. ε represents the empty string. The empty set does not equal the set with the empty string. The empty string itself is an element, so the set with an empty string inside of it is not empty.

2.1 $$\varepsilon \in L$$? 1 / 1

✓ + **1 pts** **Correct**

+ **0 pts** Incorrect/missing

Problem Four: Much Ado About Nothing, Part II

i.

Yes, for example: $L = \{w \in \{a\}^* |\ w$ contains less than five a's$\}$

ii.

Yes, for example: $L = \{w \in \{a\}^* |\ w$ contains aa as a substring $\}$

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Yes, for example: $L = \{w \in \{a, b\}^* |\ w$ contains ab as a substring $\}$

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No. ε is not a set and therefore cannot be the empty set.

vi.

No. ε represents the empty string. The empty set does not equal the set with the empty string. The empty string itself is an element, so the set with an empty string inside of it is not empty.

2.2 $$\varepsilon \not\in L$$? 1 / 1

✓ + **1 pts** Correct

+ **0 pts** Incorrect/missing

Problem Four: Much Ado About Nothing, Part II

i.

Yes, for example: $L = \{w \in \{a\}^* |\ w \text{ contains less than five } a\text{'s}\}$

ii.

Yes, for example: $L = \{w \in \{a\}^* |\ w$ contains aa as a substring $\}$

iii.

No. ε is a transition that does not consume any input but is part of a language L. It represents the empty string. It is not a set and therefore, it cannot be a subset of L.

iv.

Yes, for example: $L = \{w \in \{a, b\}^* |\ w$ contains ab as a substring $\}$

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vi.

No. ε represents the empty string. The empty set does not equal the set with the empty string. The empty string itself is an element, so the set with an empty string inside of it is not empty.

2.3 $$\varepsilon \subseteq L$$? **1 / 1**

✓ **+ 1 pts** Correct

+ 0 pts Incorrect/missing

Problem Four: Much Ado About Nothing, Part II

i.

Yes, for example: $L = \{w \in \{a\}^* |\ w$ contains less than five a's$\}$

ii.

Yes, for example: $L = \{w \in \{a\}^* |\ w$ contains aa as a substring $\}$

iii.

No. ε is a transition that does not consume any input but is part of a language L. It represents the empty string. It is not a set and therefore, it cannot be a subset of L.

iv.

Yes, for example: $L = \{w \in \{a, b\}^* |\ w$ contains ab as a substring $\}$

v.

No. ε is not a set and therefore cannot be the empty set.

vi.

No. ε represents the empty string. The empty set does not equal the set with the empty string. The empty string itself is an element, so the set with an empty string inside of it is not empty.

2.4 $$\varepsilon \not\subseteq L$$? 1 / 1

✓ + 1 pts Correct

+ 0 pts Incorrect/missing

Problem Four: Much Ado About Nothing, Part II

i.

Yes, for example: $L = \{w \in \{a\}^* |\ w$ contains less than five a's$\}$

ii.

Yes, for example: $L = \{w \in \{a\}^* |\ w$ contains aa as a substring $\}$

iii.

No. ε is a transition that does not consume any input but is part of a language L. It represents the empty string. It is not a set and therefore, it cannot be a subset of L.

iv.

Yes, for example: $L = \{w \in \{a, b\}^* |\ w$ contains ab as a substring $\}$

v.

No. ε is not a set and therefore cannot be the empty set.

vi.

No. ε represents the empty string. The empty set does not equal the set with the empty string. The empty string itself is an element, so the set with an empty string inside of it is not empty.

2.5 $$\phi = \varepsilon$$? 1 / 1

✓ + 1 pts Correct

+ 0 pts Incorrect/missing

Problem Four: Much Ado About Nothing, Part II

i.

Yes, for example: $L = \{w \in \{a\}^* |\ w$ contains less than five a's$\}$

ii.

Yes, for example: $L = \{w \in \{a\}^* |\ w$ contains aa as a substring $\}$

iii.

No. ε is a transition that does not consume any input but is part of a language L. It represents the empty string. It is not a set and therefore, it cannot be a subset of L.

iv.

Yes, for example: $L = \{w \in \{a, b\}^* |\ w$ contains ab as a substring $\}$

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No. ε is not a set and therefore cannot be the empty set.

vi.

No. ε represents the empty string. The empty set does not equal the set with the empty string. The empty string itself is an element, so the set with an empty string inside of it is not empty.

2.6 $$\phi \neq \varepsilon$$? 1 / 1

✓ **+ 1 pts** Correct

+ 0 pts Incorrect/missing

Problem Five: Hard Resets

Fill in the table and blank to Problem Five below.

state	a	b
$\{q_s, q_0, q_1, q_2\}$	$\{q_1, q_2\}$	$\{q_0, q_1, q_2\}$
$\{q_1, q_2\}$	$\{q_1, q_2\}$	$\{q_0, q_2\}$
$\{q_0, q_1, q_2\}$	$\{q_1, q_2\}$	$\{q_0, q_1, q_2\}$
$\{q_0, q_2\}$	$\{q_1, q_2\}$	$\{q_0, q_1\}$
$\{q_0, q_1\}$	$\{q_1\}$	$\{q_1, q_2\}$
$\{q_1\}$	$\{q_1\}$	$\{q_2\}$
$\{q_2\}$	$\{q_2\}$	$\{q_0\}$
$\{q_0\}$	$\{q_1\}$	$\{q_1\}$

Sample hard reset string: "abbabb".

3 Hard Resets **5 / 5**

- ✓ **+ 5 pts** **all correct**
- **+ 0 pts** missing

Automata

- **+ 3 pts** Correct
- **+ 2 pts** Mostly correct
- **+ 1 pts** Partially correct
- **+ 0 pts** Incorrect

Reset string

- **+ 2 pts** Correct
- **+ 1 pts** Partially correct
- **+ 0 pts** Incorrect

Problem Seven: Concatenation, Kleene Stars, and Complements

i.

Claim: If L is a nonempty, finite language and k is a positive natural number, then $|L|^k = |L^k|$.
Disproof: We will prove the negation of this claim, specifically: there exists a nonempty, finite language L and a positive natural number k such that $|L|^k \neq |L^k|$.
To see this, let $L = \{a, aa\}$ where $|L| = 2\}$ and let $k = 2$. We see that $|L| = 2$ and $|L|^2 = 4$. However $L^2 = \{aa, aaa, aaaa\}$ and $|L^2| = 3$. We have found a nonempty, finite language L and a positive natural number k such that $|L|^k \neq |L^k|$, as required. ■

ii.

Claim: There is a language L where $\overline{(L^*)} = (\overline{L})^*$
Disproof: We will prove the negation of this claim, specifically: for any language L, $\overline{(L^*)} \neq (\overline{L})^*$
To see this, pick any language L. We know that $\varepsilon \in L^*$ by definition of the Kleene Closure. Therefore $\varepsilon \notin \overline{(L^*)}$ by definition of the complement of a language. However the Kleene Closure of $\overline{L}$ will contain ε by definition of the Kleene Closure, that is, $\varepsilon \in (\overline{L})^*$. We have found an element, namely ε, where $\varepsilon \notin \overline{(L^*)}$ and $\varepsilon \in (\overline{L})^*$, therefore $\overline{(L^*)} \neq (\overline{L})^*$, as required. ■

4.1 Cardinalities of Powers **5 / 5**

Intuition Scale

- ✓ **+ 2 pts** **Correct**
- **+ 1 pts** Partially correct intuition
- **+ 0 pts** Incorrect

Execution Scale

- ✓ **+ 3 pts** **Correct**
- **+ 2 pts** Minor execution error
- **+ 1 pts** Major execution error
- **+ 0 pts** Incorrect

Problem Seven: Concatenation, Kleene Stars, and Complements

i.

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To see this, let $L = \{a, aa\}$ where $|L| = 2\}$ and let $k = 2$. We see that $|L| = 2$ and $|L|^2 = 4$. However $L^2 = \{aa, aaa, aaaa\}$ and $|L^2| = 3$. We have found a nonempty, finite language L and a positive natural number k such that $|L|^k \neq |L^k|$, as required. ■

ii.

Claim: There is a language L where $\overline{(L^*)} = (\overline{L})^*$
Disproof: We will prove the negation of this claim, specifically: for any language L, $\overline{(L^*)} \neq (\overline{L})^*$
To see this, pick any language L. We know that $\varepsilon \in L^*$ by definition of the Kleene Closure. Therefore $\varepsilon \notin \overline{(L^*)}$ by definition of the complement of a language. However the Kleene Closure of $\overline{L}$ will contain ε by definition of the Kleene Closure, that is, $\varepsilon \in (\overline{L})^*$. We have found an element, namely ε, where $\varepsilon \notin \overline{(L^*)}$ and $\varepsilon \in (\overline{L})^*$, therefore $\overline{(L^*)} \neq (\overline{L})^*$, as required. ■

4.2 Stars and Stripes **5 / 5**

✓ **+ 5 pts** **Fully correct**

Intuition Scale

+ **2 pts** Correct
+ **1 pts** Partially correct intuition
+ **0 pts** Incorrect

Execution Scale

+ **3 pts** Correct
+ **2 pts** Minor execution error
+ **1 pts** Major execution error
+ **0 pts** Incorrect

+ **0 pts** Incorrect

5 Optional Fun Problem: Why Finite? **0 / 0**

+ **0 pts** Correct! Congratulations!!

✓ + **0 pts** **Incorrect / Not attempted**

CS 103: Mathematical Foundations of Computing
Problem Set #6

Maria Wang and Matthew Vilaysack

May 13, 2022

Due Friday, November 5 at 2:30 pm Pacific

Do not put your answers to Problems 1, 2, and 7 in this file. You'll submit those separately on Gradescope. Here's a quick reference of symbols you may want to use in this problem set.

- Alphabets are written as Σ.
- The set of all strings over Σ is denoted $\Sigma^\star$
- The empty string is written as ε. The "var" here refers to a "variant" of the letter epsilon; that's the one we use in this class.
- Subscripts are done as is q_{137}; superscripts are done as a^{137}.
- You can make text render `like a typewriter` in text mode or in $\mathtt{math}$ mode.
- The Greek letter ρ appears in Q6.
- The Greek letter δ appears in Q7.