CS103 Midterm Exam 1 (PDF)

Matthew Phonchay Vilaysack

TOTAL POINTS

26 / 30

QUESTION 1

Question 2: First Order Logic Translations 8 pts

1.1 Part i 2 / 2

✓ + **2 pts** **Correct**

+ **0 pts** Incorrect

1.2 Part ii 2 / 2

✓ + **2 pts** **Correct**

+ **0 pts** Incorrect

1.3 Part iii 2 / 2

✓ + **2 pts** **Correct**

+ **0 pts** Incorrect

1.4 Part iv 2 / 2

✓ + **2 pts** **Correct**

+ **0 pts** Incorrect

QUESTION 2

Question 3: Intervals 22 pts

2.1 Set builder notation 3 / 3

✓ + **3 pts** **Correct**

+ **1 pts** Partial credit: set builder that scopes the set builder variable to $$\mathbb{R}$$

+ **1 pts** Partial credit: correct four comparisons joined by and (lose this point if uses interval notation)

+ **1 pts** Partial credit: curly braces, midline, other misc syntax correct

+ **0 pts** Incorrect

2.2 Proof (Case 1) 3 / 3

✓ + **3 pts** **Correct**

+ **1.5 pts** Partial credit: gives concrete, correct example (must give actual numbers)

+ **1.5 pts** Partial credit: Justifies given example

+ **0 pts** Incorrect

2.3 Proof (second half of Case 2) 6 / 6

✓ + **6 pts** **Correct**

+ **2 pts** Partial credit: correctly finishes assume for the sake of contradiction sentence.

+ **2 pts** Partial credit: Identifies two cases.

+ **2 pts** Partial credit: Argues two cases.

+ **0 pts** Incorrect

2.4 First-order logic negation 4 / 4

✓ + **4 pts** **Correct**

+ **1 pts** Partial credit: Negates $$\forall$$

+ **1 pts** Partial credit: Negates $$\exists$$

+ **1 pts** Partial credit: Negates =

+ **1 pts** Partial credit: Good syntax

+ **0 pts** Incorrect

2.5 Disproof 2 / 6

+ **6 pts** Correct

✓ + **2 pts** **Partial credit: concrete, correct counterexample a,a',b,b'**

+ **2 pts** Partial credit: identifies constraints on c, c' by relationships to a, a', b, b'

+ **2 pts** Partial credit: uses concrete example value (ie in the middle) and speaks to its relationships to the constraints to identify contradiction/impossibility

+ **0 pts** Incorrect

+ **0 pts** GRADER NOTE: REVIEW

1 your proof must be for any possible value of c, c', not just one

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2 First Order Logic Translations

Here are some predicates for describing people and things:

- $Prince(x)$, which states that x is a prince;
- $Essential(x)$, which states that x is essential;
- $Visible(x)$, which states that x is visible;
- (to express the idea that x is invisible, write $\neg Visible(x)$);
- $Responsible(x, y)$, which states that x is responsible for y.

Remember, you should ***not assume*** you know anything about these predicates apart from how they are defined above and what the ***rules of first-order logic*** dictate. For example, don't make assumptions about whether categories can overlap or not based on your own understanding of *The Little Prince* (1943).

For each of the following pairs of statements (one in English, one in first-order-logic), state whether the statements are equivalent to each other, the negation of each other, or neither equivalent nor the negation of each other. For each question, write ***one*** of "Equivalent," "Negation," or "Neither" in the shaded box.

i. (2 Points) Are these two propositions equivalent to each other, negations of each other, or neither?

- $\exists x.(Essential(x) \land \neg Visible(x))$
- "Anything that is essential is invisible."

Neither.

ii. (2 Points) Are these two propositions equivalent to each other, negations of each other, or neither?

- "There is at least one prince who is responsible for at least two distinct essential things, at least one of which is invisible."
- $\exists x.(Prince(x) \land \exists y.\exists z.(Essential(y) \land Essential(z) \land y \neq z \land \neg Visible(z) \land Responsible(x, y) \land Responsible(x, z)))$

Equivalent.

iii. (2 Points) Are these two propositions equivalent to each other, negations of each other, or neither?

- "All princes are responsible for all things that are essential and all things that are visible."
- $\forall x.(Prince(x) \to \forall y.(Visible(y) \land Essential(y) \to Responsible(x, y)))$

Neither.

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1.1 Part i 2 / 2

✓ + **2 pts** Correct

+ **0 pts** Incorrect

2 First Order Logic Translations

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- $Visible(x)$, which states that x is visible;
- (to express the idea that x is invisible, write $\neg Visible(x)$);
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- "There is at least one prince who is responsible for at least two distinct essential things, at least one of which is invisible."
- $\exists x.(Prince(x) \land \exists y.\exists z.(Essential(y) \land Essential(z) \land y \neq z \land \neg Visible(z) \land Responsible(x, y) \land Responsible(x, z)))$

Equivalent.

iii. (2 Points) Are these two propositions equivalent to each other, negations of each other, or neither?

- "All princes are responsible for all things that are essential and all things that are visible."
- $\forall x.(Prince(x) \to \forall y.(Visible(y) \land Essential(y) \to Responsible(x, y)))$

Neither.

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1.2 Part ii 2 / 2

✓ + **2 pts** Correct

+ **0 pts** Incorrect

gradescope

2 First Order Logic Translations

Here are some predicates for describing people and things:

- $Prince(x)$, which states that x is a prince;
- $Essential(x)$, which states that x is essential;
- $Visible(x)$, which states that x is visible;
- (to express the idea that x is invisible, write $\neg Visible(x)$);
- $Responsible(x, y)$, which states that x is responsible for y.

Remember, you should ***not assume*** you know anything about these predicates apart from how they are defined above and what the ***rules of first-order logic*** dictate. For example, don't make assumptions about whether categories can overlap or not based on your own understanding of *The Little Prince* (1943).

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- "Anything that is essential is invisible."

Neither.

ii. (2 Points) Are these two propositions equivalent to each other, negations of each other, or neither?

- "There is at least one prince who is responsible for at least two distinct essential things, at least one of which is invisible."
- $\exists x.(Prince(x) \land \exists y.\exists z.(Essential(y) \land Essential(z) \land y \neq z \land \neg Visible(z) \land Responsible(x, y) \land Responsible(x, z)))$

Equivalent.

iii. (2 Points) Are these two propositions equivalent to each other, negations of each other, or neither?

- "All princes are responsible for all things that are essential and all things that are visible."
- $\forall x.(Prince(x) \to \forall y.(Visible(y) \land Essential(y) \to Responsible(x, y)))$

Neither.

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1.3 Part iii 2 / 2

✓ **+ 2 pts** Correct

+ 0 pts Incorrect

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iv. (2 Points) Are these two propositions equivalent to each other, negations of each other, or neither?

- "There exists an invisible essential thing for which no prince is responsible."
- $\forall x.(\neg Essential(x) \lor Visible(x) \lor \exists y.(Prince(y) \land Responsible(y, x)))$

Negation.

It is a violation of the Stanford Honor Code to communicate with other humans about this exam. You must not solicit or share answers to the questions contained here.

1.4 Part iv **2 / 2**

✓ **+ 2 pts** Correct

+ 0 pts Incorrect

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3 Intervals

We now introduce a new* definition. An *interval* $[x, y]$ for any real numbers x and y is the set of real numbers z such $x \leq z$ and $z \leq y$. In other words, $[x, y] = \{z \in \mathbb{R} \mid x \leq z \wedge z \leq y\}$. Note that, depending on the values of x and y, there may be no such z, in which case the range would be equal to $\emptyset$.

Let $a, a', b, b' \in \mathbb{R}$. Consider the following theorem:

For all intervals $[a, a']$ and $[b, b']$, there exist real numbers c and c' such that $[a, a'] \cap [b, b'] = [c, c']$.

In part (i), you'll prepare to prove this theorem, and in parts (ii) and (iii), you'll prove it. Some of the proof is written for you, and you will write two sections that are missing. **Before you begin part (ii)**, you may find it helpful to read ahead to the next page for part (iii), to see how the proof is structured overall.

For parts (iv) and (v), you'll consider another claim related to intervals.

i. (3 Points) Write a set builder expression for the intersection of intervals $[a, a'] \cap [b, b']$. Do ***not*** use interval notation (i.e., the brackets [ ]) in your solution.

{ z $\in \mathbb{R} \mid a \leq z \wedge z \leq a' \wedge b \leq z \wedge z \leq b'\}$

ii. (3 Points) Write the section of the proof that argues Case 1, as indicated below.

Theorem: For all intervals $[a, a']$ and $[b, b']$, there exist real numbers c and c' such that $[a, a'] \cap [b, b'] = [c, c']$.

Proof: Let $[a, a']$ and $[b, b']$ be any two intervals. We want to show that there are real numbers c and c' such that $[a, a'] \cap [b, b'] = [c, c']$. As a shorthand, we define $S = [a, a'] \cap [b, b']$. We consider two cases, when $S = \emptyset$ and when $S \neq \emptyset$.

Case 1: $S = \emptyset$. ...

Then S has no values. Let $c = 10$ and $c' = 5$. From the definition of an interval, there exists no real numbers in the interval [10, 5]. Therefore there exists c and c' where $S = [a, a'] \cap [b, b'] = [c, c'] = \emptyset$, as required.

(proof is continued on the next page)

** Intervals were mentioned in a lecture that is past the weeks that you are responsible for on this exam, so for the purposes of this exam we consider this a new definition. That lecture is not necessary for this problem.*

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2.1 Set builder notation 3 / 3

✓ **+ 3 pts** **Correct**

+ **1 pts** Partial credit: set builder that scopes the set builder variable to $$\mathbb{R}$$
+ **1 pts** Partial credit: correct four comparisons joined by and (lose this point if uses interval notation)
+ **1 pts** Partial credit: curly braces, midline, other misc syntax correct
+ **0 pts** Incorrect

gradescope

3 Intervals

We now introduce a new* definition. An *interval* $[x, y]$ for any real numbers x and y is the set of real numbers z such $x \leq z$ and $z \leq y$. In other words, $[x, y] = \{z \in \mathbb{R} \mid x \leq z \land z \leq y\}$. Note that, depending on the values of x and y, there may be no such z, in which case the range would be equal to Ø.

Let $a, a', b, b' \in \mathbb{R}$. Consider the following theorem:

For all intervals $[a, a']$ and $[b, b']$, there exist real numbers c and c' such that $[a, a'] \cap [b, b'] = [c, c']$.

In part (i), you'll prepare to prove this theorem, and in parts (ii) and (iii), you'll prove it. Some of the proof is written for you, and you will write two sections that are missing. **Before you begin part (ii)**, you may find it helpful to read ahead to the next page for part (iii), to see how the proof is structured overall.

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i. (3 Points) Write a set builder expression for the intersection of intervals $[a, a'] \cap [b, b']$. Do ***not*** use interval notation (i.e., the brackets [ ]) in your solution.

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Theorem: For all intervals $[a, a']$ and $[b, b']$, there exist real numbers c and c' such that $[a, a'] \cap [b, b'] = [c, c']$.

Proof: Let $[a, a']$ and $[b, b']$ be any two intervals. We want to show that there are real numbers c and c' such that $[a, a'] \cap [b, b'] = [c, c']$. As a shorthand, we define $S = [a, a'] \cap [b, b']$. We consider two cases, when $S = Ø$ and when $S \neq Ø$.

Case 1: $S = Ø$. ...

Then S has no values. Let $c = 10$ and $c' = 5$. From the definition of an interval, there exists no real numbers in the interval [10, 5]. Therefore there exists c and c' where $S = [a, a'] \cap [b, b'] = [c, c'] = Ø$, as required.

(proof is continued on the next page)

** Intervals were mentioned in a lecture that is past the weeks that you are responsible for on this exam, so for the purposes of this exam we consider this a new definition. That lecture is not necessary for this problem.*

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2.2 Proof (Case 1) **3 / 3**

- ✓ **+ 3 pts** **Correct**
- **+ 1.5 pts** Partial credit: gives concrete, correct example (must give actual numbers)
- **+ 1.5 pts** Partial credit: Justifies given example
- **+ 0 pts** Incorrect

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iii. (6 Points) Complete the proof by contradiction argument that $S \subseteq [c, c']$, as indicated below.

(continued from previous page)

Case 2: $S \neq \emptyset$. Then S has minimum and maximum values, which we call c and c', respectively. This implies that $c \leq c'$ and so the interval $[c, c']$ is non-empty. We will now show that $[c, c'] = S$, by showing that $[c, c'] \subseteq S$ and $S \subseteq [c, c']$.

To show that $[c, c'] \subseteq S$, let x be an arbitrary element of $[c, c']$. We want to show that $x \in S$. From the definition of an interval, we know that $c \leq x \leq c'$. Now because $c \in S$, we know that $c \in [a, a']$ and $c \in [b, b']$. From the definition of an interval it follows that $a \leq c$, $c \leq a'$, $b \leq c$ and $c \leq b'$. A similar argument shows that $a \leq c'$, $c' \leq a'$, $b \leq c'$ and $c' \leq b'$. Combining inequalities, we have $a \leq c \leq x \leq c' \leq a'$ and $b \leq c \leq x \leq c' \leq b'$, showing that x is an element of both $[a, a']$ and $[b, b']$ and therefore that $x \in [a, a'] \cap [b, b']$. Since x was chosen arbitrarily from $[c, c']$, we have $[c, c'] \subseteq [a, a'] \cap [b, b'] = S$.

To show that $S \subseteq [c, c']$, assume for the sake of a contradiction that there is some element $y \in S$, but...

$y \notin [c, c']$. We know that S is non-empty, so $y < c$ or $y > c'$ by definition of the interval $[c, c']$. Because S has the minimum and maximum values c and c', it must be the case that $y \geq c$ and $y \leq c'$. We have reached a contradiction, so our assumption must have been wrong. Therefore, for any $y \in [c, c']$ and $y \in S$.

Since $S \subseteq [c, c']$ and $[c, c'] \subseteq S$, we know that $S = [a, a'] \cap [b, b'] = [c, c']$, as required for Case 2.

In both cases, we have shown $[a, a'] \cap [b, b'] = [c, c']$, which is what we wanted to show. ■

Hints: This proof uses the conventional way to prove two sets are equal, which is to show they are subsets of each other. That is, for all sets S and T, $S = T$ iff $S \subseteq T$ and $T \subseteq S$. The Guide to Subset Proofs has more detail on element analysis technique for proving a subset relationship, which is used in the given part of the proof above. For your answer to part (iii), you'll use a proof by contradiction argument.

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2.3 Proof (second half of Case 2) **6 / 6**

- ✓ **+ 6 pts** **Correct**
- **+ 2 pts** Partial credit: correctly finishes assume for the sake of contradiction sentence.
- **+ 2 pts** Partial credit: Identifies two cases.
- **+ 2 pts** Partial credit: Argues two cases.
- **+ 0 pts** Incorrect

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Again, let $a, a', b, b' \in \mathbb{R}$. Now consider the following claim, and note its similarity to the previous one, but with set union instead of intersection:

For all intervals $[a, a']$ and $[b, b']$, there exist real numbers c and c' such that $[a, a'] \cup [b, b'] = [c, c']$.

In the next two parts, you will show that this claim is false.

iv. (4 Points) Consider this first order logic translation of the above claim:

$$\forall a, a', b, b' \in \mathbb{R}.\ \exists c, c' \in \mathbb{R}.\ [a, a'] \cup [b, b'] = [c, c']$$

Write a **step-by-step** negation of this statement in first order logic, pushing negations inside the formula as much as possible. Show each step of applying a logical rule as a separate line; don't combine or leave out any steps.

$$\neg(\forall a, a', b, b' \in \mathbb{R}.\ \exists c, c' \in \mathbb{R}.\ [a, a'] \cup [b, b'] = [c, c'])$$
$$\exists a, a', b, b' \in \mathbb{R}.\ \neg(\exists c, c' \in \mathbb{R}.\ [a, a'] \cup [b, b'] = [c, c'])$$
$$\exists a, a', b, b' \in \mathbb{R}.\ \forall c, c' \in \mathbb{R}.\ \neg([a, a'] \cup [b, b'] = [c, c'])$$
$$\exists a, a', b, b' \in \mathbb{R}.\ \forall c, c' \in \mathbb{R}.\ [a, a'] \cup [b, b'] \neq [c, c']$$

v. (6 Points) Write a disproof of the claim.

Claim: For all intervals [a,a'] and [b,b'], there exist real numbers c and c' such that $[a, a'] \cup [b, b'] = [c, c']$.
Disproof: We will show that the negation of this statement is true, namely, that there exists real numbers a, a', b, b', such that $[a, a'] \cup [b, b'] \neq [c, c']$ where c and c' are any real numbers.

Let $a = -6$, $a' = -5$. $b = 5$, $b' = 6$ This means the following is true:

$$[a, a'] = [-6, -5]$$
$$[b, b'] = [5, 6]$$
$$[a, a'] \cup [b, b'] = [-6, -5] \cup [5, 6]$$

Now, assume for the sake of contradiction that there exists $[a, a'] \cup [b, b'] = [c, c']$. For the union $[a, a'] \cup [b, b']$, c must be the minimum value of the union and c' must be the maximum value of the union, so $c = -6$, and $c' = 6$. By the definition of an interval, every real number in $[c, c'] \in [-6, -5] \cup [5, 6]$.

However, we have reached a contradiction. There exists a real number $0 \in [-6, 6]$ but $0 \notin [-6, -5] \cup [5, 6]$.

As shown here, there are not any real numbers c and c' such that $[c, c'] = [-6, -5] \cup [5, 6]$, so it is the case that $[a, a'] \cup [b, b'] \neq [c, c']$, as required.

■

Hints: For the disproof, you should use a disproof by counterexample approach. Then, for the argument that your counterexample meets the necessary criteria, one approach we recommend is to use a proof by contradiction (i.e., a proof by contradiction within a disproof by counterexample).

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2.4 First-order logic negation 4 / 4

✓ + **4 pts** **Correct**

+ **1 pts** Partial credit: Negates $$\forall$$

+ **1 pts** Partial credit: Negates $$\exists$$

+ **1 pts** Partial credit: Negates =

+ **1 pts** Partial credit: Good syntax

+ **0 pts** Incorrect

gradescope

Again, let $a, a', b, b' \in \mathbb{R}$. Now consider the following claim, and note its similarity to the previous one, but with set union instead of intersection:

For all intervals $[a, a']$ and $[b, b']$, there exist real numbers c and c' such that $[a, a'] \cup [b, b'] = [c, c']$.

In the next two parts, you will show that this claim is false.

iv. (4 Points) Consider this first order logic translation of the above claim:

$$\forall a, a', b, b' \in \mathbb{R}.\ \exists c, c' \in \mathbb{R}.\ [a, a'] \cup [b, b'] = [c, c']$$

Write a **step-by-step** negation of this statement in first order logic, pushing negations inside the formula as much as possible. Show each step of applying a logical rule as a separate line; don't combine or leave out any steps.

$$\neg(\forall a, a', b, b' \in \mathbb{R}.\ \exists c, c' \in \mathbb{R}.\ [a, a'] \cup [b, b'] = [c, c'])$$
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v. (6 Points) Write a disproof of the claim.

Claim: For all intervals [a,a'] and [b,b'], there exist real numbers c and c' such that $[a, a'] \cup [b, b'] = [c, c']$.
Disproof: We will show that the negation of this statement is true, namely, that there exists real numbers a, a', b, b', such that $[a, a'] \cup [b, b'] \neq [c, c']$ where c and c' are any real numbers.

Let $a = -6$, $a' = -5$. $b = 5$, $b' = 6$ This means the following is true:

$$[a, a'] = [-6, -5]$$
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Now, assume for the sake of contradiction that there exists $[a, a'] \cup [b, b'] = [c, c']$. For the union $[a, a'] \cup [b, b']$, c must be the minimum value of the union and c' must be the maximum value of the union, so $c = -6$, and $c' = 6$. By the definition of an interval, every real number in $[c, c'] \in [-6, -5] \cup [5, 6]$.

However, we have reached a contradiction. There exists a real number $0 \in [-6, 6]$ but $0 \notin [-6, -5] \cup [5, 6]$.

As shown here, there are not any real numbers c and c' such that $[c, c'] = [-6, -5] \cup [5, 6]$, so it is the case that $[a, a'] \cup [b, b'] \neq [c, c']$, as required.

■

Hints: For the disproof, you should use a disproof by counterexample approach. Then, for the argument that your counterexample meets the necessary criteria, one approach we recommend is to use a proof by contradiction (i.e., a proof by contradiction within a disproof by counterexample).

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2.5 Disproof 2 / 6

+ **6 pts** Correct

✓ + **2 pts** **Partial credit: concrete, correct counterexample a,a',b,b'**

+ **2 pts** Partial credit: identifies constraints on c, c' by relationships to a, a', b, b'

+ **2 pts** Partial credit: uses concrete example value (ie in the middle) and speaks to its relationships to the constraints to identify contradiction/impossibility

+ **0 pts** Incorrect

+ **0 pts** GRADER NOTE: REVIEW

1 your proof must be for any possible value of c, c', not just one

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CS103: Mathematical Foundations of Computing
Midterm Exam 1

Instructors: Alex Aiken & Cynthia Bailey Lee

April 24, 2022

1 Truth Tables

In this problem, we will examine how truth tables can be used to define new, "made-up" logical connectives. You'll be asked to discover ways to translate between our traditional connectives and the made-up ones.

Example: Consider the following truth table for a new propositional logic connective, $\odot$:

p	q	r	$\odot(p,q,r)$
T	T	T	T
T	T	F	T
T	F	T	F
T	F	F	F
F	T	T	F
F	T	F	F
F	F	T	F
F	F	F	F

In this example, we can rewrite any proposition $\odot(p,q,r)$ in terms of our traditional propositional connectives as: $\odot(p,q,r) \equiv (p \wedge q \wedge r) \vee (p \wedge q \wedge \neg r)$ (a simpler translation is possible, but this one is easier to map onto how we derived it from looking at the input combinations that give us T output rows in the truth table). And we can rewrite the traditional connective $\wedge$ as: $p \wedge q \equiv \odot(p,q,\top)$. Another valid translation is: $p \wedge q \equiv \odot(p,q,p)$.

One trick to keep in mind is that it is possible to nest uses of the $\odot$ connective. For example, we can reuse the $p \wedge q \equiv \odot(p,q,\top)$ equivalence we found above to help us translate this more complex expression: $p \wedge (q \wedge r) \equiv \odot(p, \odot(q,r,\top), \top)$.

For the rest of this problem, you will perform similar translations between our traditional connectives and the new connective $\bowtie$, defined as shown in this truth table:

p	q	r	$\bowtie(p,q,r)$
T	T	T	F
T	T	F	F
T	F	T	F
T	F	F	F
F	T	T	T
F	T	F	F
F	F	T	T
F	F	F	T

This question is autograded. Download the starter files for Midterm 1 and extract them somewhere convenient on your computer. Open the project in QT Creator. To answer this question, edit the file `res/TruthTables.proplogic`. To check that your solution can be understood by our parser, run the project, and then click the "View Formulas" button. In the window, you should then see either your correctly parsed formulas, or error messages from the parser. Note that this parsing test does not check that you followed the limitations specific to each problem (for example, the instructions for part (ii) say, "your response should use only p, $\bowtie$, $\top$, $\bot$, and parentheses"). It is your responsibility to ensure that you follow those rules. To submit, upload the file `res/TruthTables.proplogic` to Gradescope. Note: because this is an exam, your autograding results will not be visible until after the deadline. You will need to manually check your work.

Answer the following questions:

i. (2 Points) Rewrite the proposition $\bowtie(p, q, r)$ in terms of our traditional connectives (i.e., your response should use only $p, q, r, \wedge, \vee, \neg, \to, \top, \bot$, and parentheses).

ii. (2 Points) Rewrite the proposition $\neg p$ in terms of the new connective $\bowtie$ (i.e., your response should use only $p, \bowtie, \top, \bot$, and parentheses).

iii. (2 Points) Rewrite the proposition $p \vee q$ in terms of the new connective $\bowtie$ (i.e., your response should use only $p, q, \bowtie, \top, \bot$, and parentheses).

iv. (2 Points) Rewrite the proposition $p \to q$ in terms of the new connective $\bowtie$ (i.e., your response should use only $p, q, \bowtie, \top, \bot$, and parentheses).

Hints: This problem is similar (but not the same) as problems used on a couple of recent exams. There is a help video on Canvas: go to Panopto Course Videos > Practice Midterm 1 Advice: Q1. Among other things, the help video describes some ways that the truth table tool on the course website can be leveraged for this problem, even though the tool doesn't recognize the $\bowtie$ connective.

It is a violation of the Stanford Honor Code to communicate with other humans about this exam. You must not solicit or share answers to the questions contained here.