Pandemic Flu Spread: Mask Up!

ISyE 6644 - Fall 2020 Project Report

by Yi-Hsuan Lu and Matthew Wayne Holland;



Image from https://riseandshine.childrensnational.org/

Abstract

This paper centers around the spread of a disease (such as influenza) in an elementary school classroom with 21 students as the susceptible population. We show how long an outbreak lasts if one student enters a classroom on day one infected with influenza. The base scenario concerns a normal, pre-pandemic society where no one takes preventive measures like wearing face masks. The other scenario contrasts with students wearing face masks and adhering to preventive measures. We find measures such as these to be crucial in preventing the spread of disease. We show this by comparing the average number of students infected each day in the two scenarios. The results indicate that there will be a significant 70% reduction in the infection rate with masks in place.

Introduction

Slowly, but surely, the world is starting to understand the importance of wearing masks to curb the spread of Covid-19. Among the people learning to wear them are children. Children encounter problems with masks. They don't make enough different sizes for them. It's hard to see while wearing them. Their parents sometimes forget to provide them. They don't always know that their nose is also part of their respiratory system. Despite these hiccups, they have proven to be a helpful way to slow the spread of diseases in classrooms all around the world.

Disease propagation in a classroom is of paramount concern to teachers, parents, students, and governments throughout the world. The study of disease spread is instrumental in designing effective measures to control disease spread and mitigate social and economic impact. Of course, the recent COVID-19 outbreak across the globe has emphasized the importance of such study.

A basic disease propagation model is a SIR model, where individuals are either susceptible to disease (S), infected (I) or removed (R) (recovered or dead). The SIR parameters are determined using a transition rate according to preassigned probability distributions or parameters. The SIR model, typically based on differential equations, while succinct, neither accounts for randomness in the system, nor does it accommodate distinct individual-level behaviors or contact rates^[1]. To complement the SIR model, researchers have deployed stochastic simulation modeling to study how diseases propagate through different populations, age groups, and societies, as well as the effects of various preventive measures.

The following sections of this report are organized as follows. In the first part, we outline the initial problem and formulate the probability distribution used in the simulation. Then, we design the simulation model and discuss relevant input parameters. In the second part, we consider the variables we input into our model. In the third part, we present the model results and analyze the output. Finally, in the fourth part, we offer some conclusions for this project.

Given Information

In this problem, we have a class of 21 students, one of whom is infected with the flu on day one. The flu spread from each infected student follows independent, identically distributed Bernoulli trials with p = 0.02. The infected students will be sick and infectious for three days.

We will answer four questions about our model. The answers to the questions are as follows:

- 1. What is the distribution of the number of kids that Tommy infects on Day 1?
 - $\stackrel{4}{\rightleftharpoons}$ It follows 20 iid Bernoulli trials. Therefore, it is a Binomial distribution with n = 20 and p = 0.02.
- 2. What is the expected number of kids that Tommy infects on Day 1?
 - $\stackrel{\text{def}}{=}$ The binomial distribution has expected mean E(X) = n * p = 0.4.
- 3. What is the expected number of kids that are infected by Day 2 (counting Tommy)?
 - Day 2 infections should depend on the number of infected persons on Day 1.
 - They will depend on this expected value formula: E(E(Day 2 infections | Day 1 infections)) = E(Day 2 infections).
 - For example, the probability of having two students infected on Day 1 is 0.2725 (Tommy plus one additional person infected on day one based on the binomial distribution). The infection probability with 2 infectious students on Day 2 is 1- (1-p)** 2 = 0.0396. This means while the expected infected students on Day 2 given 2 infectious is 0.0396 * 19 +2 = 2.75, it will only happen with probability 0.2725.
 - We can construct a table to calculate the conditional expected infections on Day 2. Although there are 21 infection scenarios in Day 1 (from no one being infected to all being infected by Tommy), we will look at up to six infections. We do this because the probability of more infections becomes negligible after six infections.
 - Based on the data in column E, the expected number of infected students on day two is **1.9374**.

Column A	Column B	Column C	Column D	Column E
Number of infectious students on day 2	Infectious Probability (p)	Probability that number infected will be value from column A	Remaining susceptible students on day 2	Expected Value of being infected on day 2 given being infected on day 1
	1-q^k			E(Day 2 Infected Day 1 Infected) (B*D+A)*C
1	0.02	0.6676	20	0.9347
2	0.0396	0.2725	19	0.75
3	0.0588	0.0528	18	0.2144
4	0.0776	0.0065	17	0.0344
5	0.0961	0.0006	16	0.0037
6	0.1142	0	15	0.0003
Sum		1		1.9374

- 4. Simulate the number of kids that are infected on Days 1,2,... Do this many times.

 What are the (estimated) expected numbers of kids that are infected by Day i, i = 1; 2; : :? Produce a histogram detailing how long the "epidemic" will last.
 - The answer to this question can be found in the output analysis section. On average, the outbreak lasts for 15 days with masks and for 10 days without masks.

Input Considerations

SIR Model Flowchart

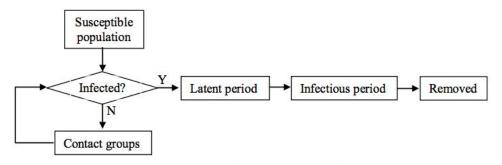


Figure 1: Flowchart of the epidemiological SIR model

Our model of disease spread is based on a binomial distribution. One child enters the classroom on day one already infected. Therefore, the number of students infected on day one is 1.0. The p value of the binomial distribution is 0.02. That p value increases as more children become infected.

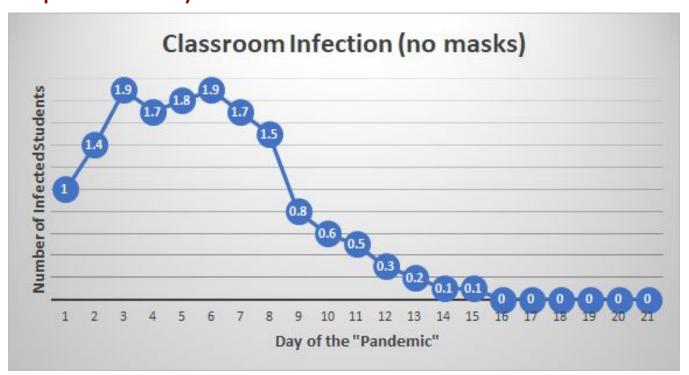
The model is the same for when the children wear masks, except for one difference. Instead of the p value being 0.02, it is 0.008 (0.02 *0.4). According to research results, wearing masks can reduce risk of transmission by 35% to 50% [^{3&4]}. The 0.4 is the mid-point of the range and meant to represent a 40% reduction in transmission of the disease when masks are worn. We will also test for values of 0.3, 0.35, 0.45, and 0.5.

We constructed a simulation program using Python. The .ipynb file has been uploaded along with this report. Taking advantage of a pseudo random number generator in the numpy package, we conducted 2000 replications of pandemic flu spread. The average infected students sans face masks is 5.7035, while it is 1.822 with face masks. The approximate 95% confidence interval for the difference of means is [2.92, 5.25], indicating that wearing masks can indeed lessen the severity of flu spread.

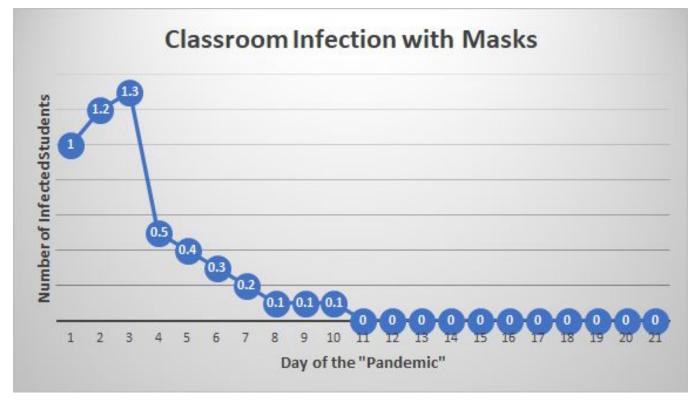
p-value	p-value equation	average number of infections	% reduction *5.7035 infection rate with no masks
0.006	0.02 * 0.30	1.515	73.4%
0.007	0.02 * 0.35	1.706	70.1%
0.008	0.02 * 0.40	1.822	68.1%
0.009	0.02 * 0.45	1.962	65.6%
0.010	0.02 * 0.50	2.212	61.2%

Based on the above chart, the new p-value for the model with masks makes a slight difference in the percent reduction of disease spread. Based on these findings, we are comfortable with a value of 0.4 being used to show the use of masks in the model.

Output Data Analysis



In this histogram covering the 21 day pandemic without masks, the highest average number of infected students was 1.9. The pandemic lasted for a total of 15 days.



In this histogram covering the 21 day pandemic with masks worn (p=0.2 * 0.4), the highest average number of infected students was 1.3. The pandemic lasted for a total of 10 days.

In our models, wearing masks showed about a 70% drop in the infection rate (from 5.7035 to 1.822 people infected). This was more optimistic than we had originally thought.

Conclusion

Outbreaks in a classroom are concerning, but they can be slowed down considerably with a preventative measure such as mask wearing. When combined with measures like social distancing, hand washing, and the use of hand sanitizer, pandemics can be quickly curbed.

Our analysis could be improved by testing other stochastic elements. For example, what type of a child is patient zero? Is he outgoing? Introverted? The way that he interacts with his classmates would change the length of the outbreak. In addition, this report only concerns the mitigating effect of face masks. We can use similar simulation programming to study the effect of other preventive measures such as hand washing and social distancing, and use system comparison methods to pick the most effective combinations.

References

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