Recommendation System: Basic Methods

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RecSys Drives The World

- Recommendation systems are ubiquitous
  - **E-commerce:** Products to buyers
  - **Job applications:** Jobs to applicants, Applicants to recruiters
  - **Dating:** Partners
  - **Services:** Doctors, lawyers, restaurants
  - **Social networks:** News that you read, posts that you see
- **Most of what you do is powered or influenced by some recommender system!**

Types of Recommendations

- **Editorial and hand curated**
  - List of favorites
  - Lists of “essential” items
- **Simple aggregates**
  - Top 10, Most Popular, Recent Uploads
- **Personalized to individual users**
  - Amazon, Netflix, …
  - **Main focus of this class**

General Setting

- **Input:**
  - U = Set of users
  - I = Set of items
  - R = Set of observed ratings between subset of users and subset of items
- **Output:**
  - Predict ratings for any (user, item) pair

	Avatar	LOTR	Matrix	Pirates
Alice	1		5	
Bob		3		1
Carol				1
David			4	

Recommendations: Key Challenges

- **(1) Gathering “known” ratings for matrix**
  - How to collect the data in the utility matrix

- **(2) Extrapolate unknown ratings from the known ones**
  - Mainly interested in high unknown ratings
    - We are not interested in knowing what you don’t like but what you like

- **(3) Evaluating extrapolation methods**
  - How to measure success/performance of recommendation methods

Challenge 1: Gathering Ratings

- **Explicit**
  - Ask people to rate items
  - Doesn't work well in practice – people can't be bothered
- **Implicit**
  - Learn ratings from user actions
    - E.g., purchase implies high rating
  - What about low ratings?

Challenge 2: Extrapolating Utilities

- **Key problem:** Utility matrix ***U*** is **sparse**
  - Most people have not rated most items
  - **Cold start:**
    - New items have no ratings
    - New users have no history

- **Four approaches:**
  - **1)** Content-based
  - **2)** Collaborative
  - **3)** Latent factor based
  - **4)** Deep learning based

Our focus

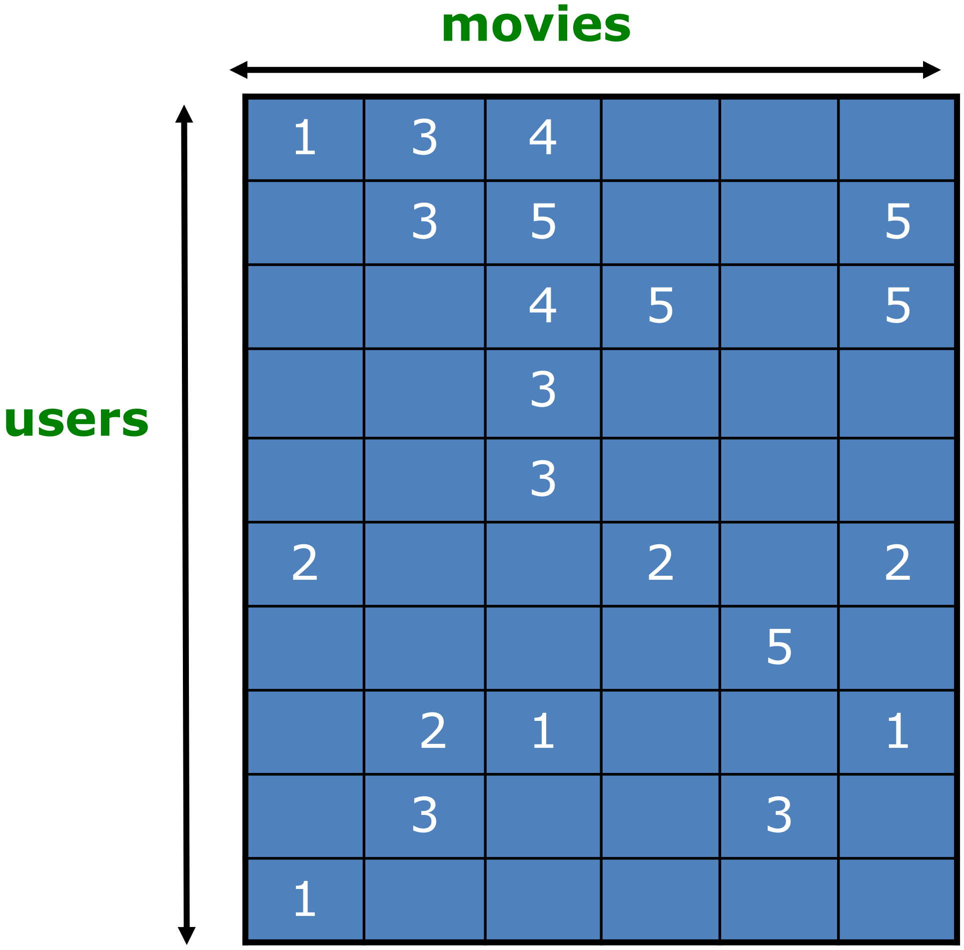
Challenge 3: Evaluations

- Metrics to evaluate recommendations and ranking lists
- **Online setting:** Purchase rate, Click-through rate
- **Offline setting:** which metrics to use?

- **How do we evaluate recommender systems?**
  - Need evaluation setting and metric

Evaluation Setting

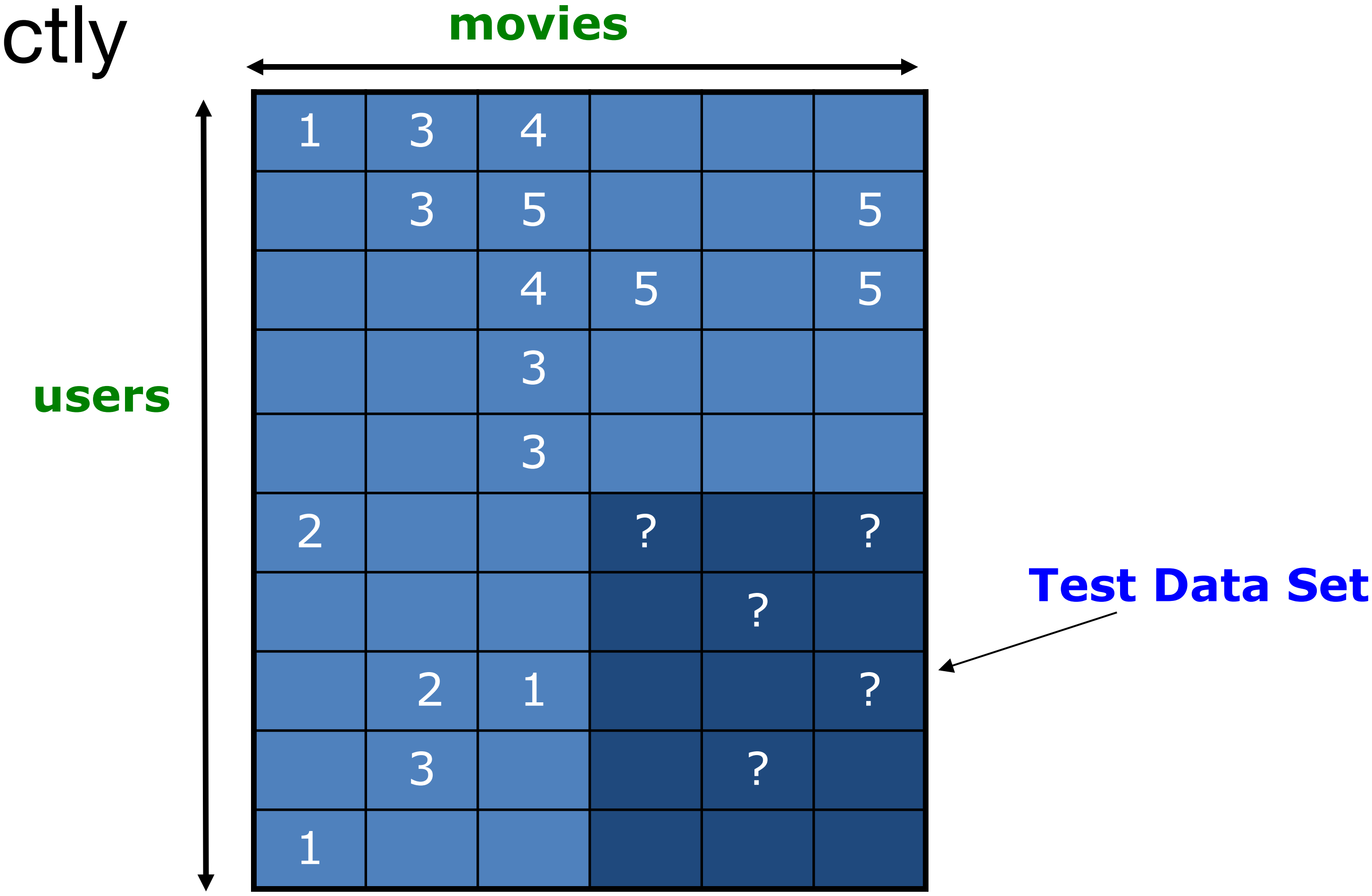
- Input: rating matrix



1	3	4			
	3	5			5
		4	5		5
		3			
		3			
2			2		2
				5	
	2	1			1
	3			3	
1					

Evaluation Setting

- Hide some known ratings and try to predict it correctly



movies

users

Test Data Set

1	3	4			
	3	5			5
		4	5		5
		3			
		3			
2			?		?
				?	
	2	1			?
	3			?	
1					

Evaluation Metrics

- **Compare predictions with known ratings**
  - **Root-mean-square error (RMSE)**
    - $\sqrt{\sum_{xi}\left(r_{xi} - r_{xi}^{*}\right)^{2}}$ where $\boldsymbol{r_{xi}}$ is predicted, $\boldsymbol{r_{xi}^{*}}$ is the true rating of ***x*** on ***i***
    - Lower is better
- **Another approach: 0/1 model**
  - **Mean Reciprocal Rank (MRR)**
    - The 1/rank of the true item in the ranked list of items
  - **Recall@10**
    - Fraction of times the true item is in the top 10 in the ranked list of items

Problems with Evaluation Measures

- **Narrow focus on accuracy sometimes misses the point**
  - Prediction Diversity
  - Prediction Context
  - Order of predictions
- **In practice, we care only to predict high ratings:**
  - RMSE might penalize a method that does well for high ratings and badly for others

RecSys: Approach Styles

1. **Content-based:** which items have properties similar to the items you already like?
2. **Collaborative filtering:** what do similar users like?
3. **Latent factor based:** learn latent representation of all users' behavior
4. **Deep learning based:** combine everything together in a unified, deep framework

Recommendation System Methods

- Content-based Models
- Collaborative Filtering
- Latent Factor Models
- Deep Learning Method: NCF

Reference material: http://mmds.org/

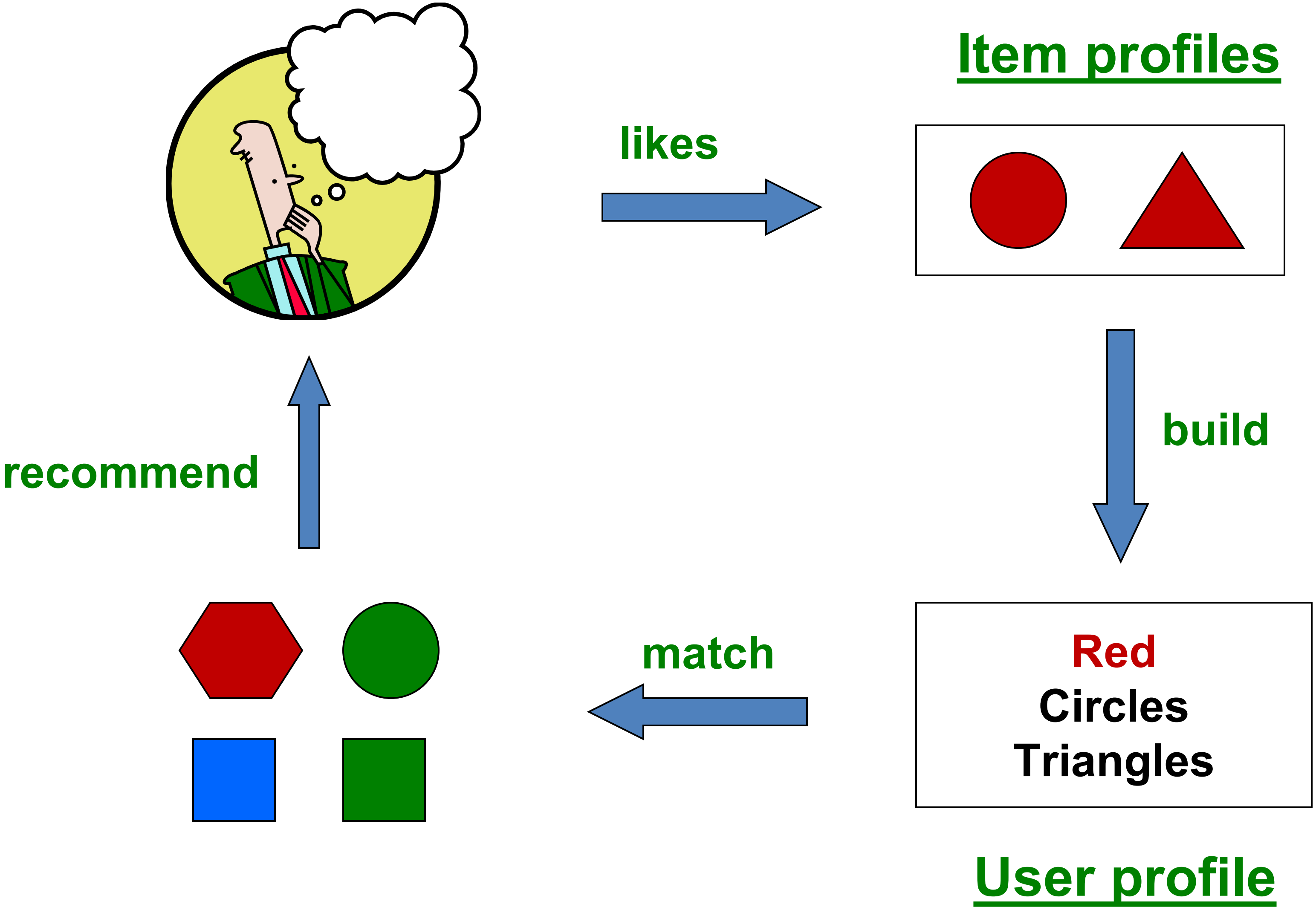
Content-based Recommendations

- **Main idea:** Recommend items to customer ***x*** similar to previous items rated highly by ***x***

Example:

- **Movie recommendations**
  - – Recommend movies with same actor(s), director, genre, …
- **Websites, blogs, news**
  - – Recommend other sites with “similar” content

Workflow



Item profiles

likes

build

Red
Circles
Triangles

User profile

match

recommend

Item Profiles

- For each item, create an **item profile** i

- **Profile is a set (vector) of features**
  - **Movies:** author, title, actor, director,…
  - **Text Embeddings**

User Profiles and Prediction

- **User profile possibilities:**
  - Weighted average of rated item profiles

- **Prediction heuristic:**
  - Given user profile ***x*** and item profile ***i***, estimate $u(\boldsymbol{x}, \boldsymbol{i}) = \cos(\boldsymbol{x}, \boldsymbol{i}) = \frac{\boldsymbol{x} \cdot \boldsymbol{i}}{||\boldsymbol{x}|| \cdot ||\boldsymbol{i}||}$

Pros: Content-based Approach

- **No need for data on other users**
  - No cold-start or sparsity problems
- **Able to recommend to users with unique tastes**
- **Able to recommend new & unpopular items**
  - No first-rater problem
- **Able to provide explanations**
  - Can provide explanations of recommended items by listing content-features that caused an item to be recommended

Cons: Content-based Approach

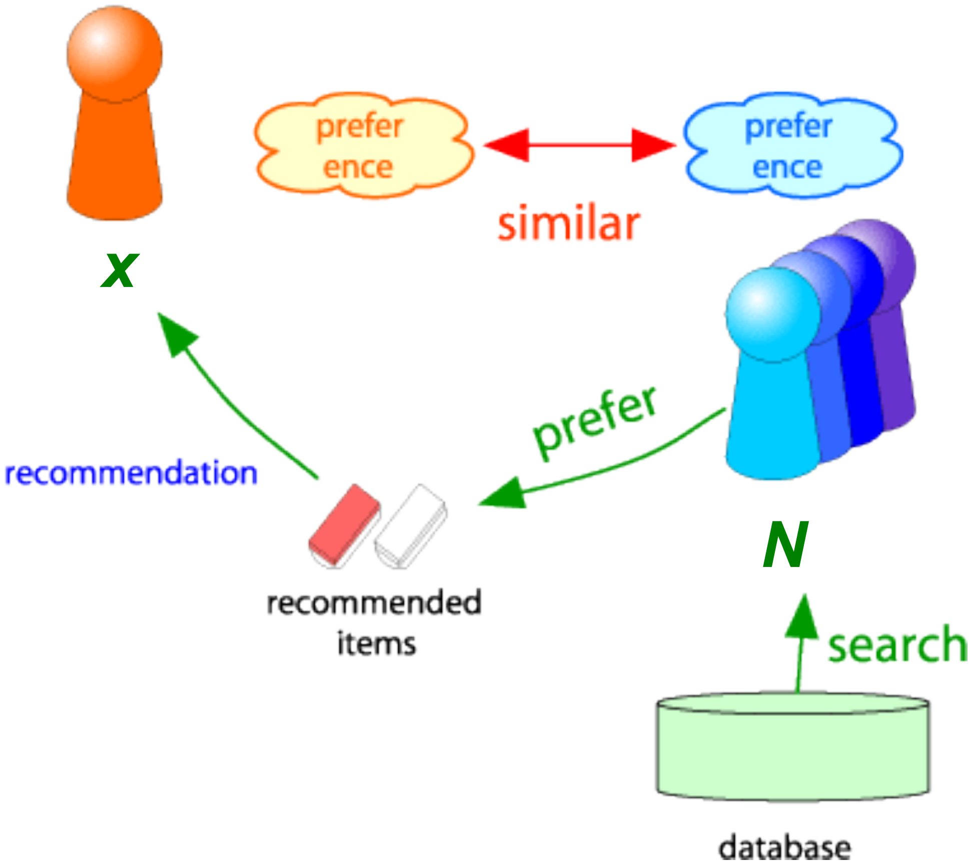
- **Finding the appropriate features is hard**
  - E.g., images, movies, music
- **Recommendations for new users**
  - How to build a user profile?
- **Overspecialization**
  - Never recommends items outside user’s content profile
  - People might have multiple interests
  - **Unable to exploit quality judgments of other users**

Recommendation Systems

- ✓ Content-based
- ☞ Collaborative Filtering
- Latent Factor Models
- Deep Learning Model: NCF

Collaborative Filtering

- **Harnessing quality judgments of other users**
- Consider user x
- Find set ***N*** of other users whose ratings are "**similar**" to ***x***'s ratings
- Estimate ***x***'s ratings based on ratings of users in ***N***



Finding “Similar” Users

- How to find similarity between users?
- r_x = vector of user ***x***’s ratings; r_y = ***y***’s ratings

r_x = [*, _, _, *, ***]
r_y = [*, _, **, **, _]

- **Metric 1: Jaccard similarity measure**
  - Jaccard = (Number of common items rated by both) / (Number of items rated by either)
  - **Problem:** Ignores the value of the rating

Finding “Similar” Users

- **Metric 2: Cosine similarity measure**
  - Treats r_x and r_y as vectors
  - sim(***x***, ***y***) = cos($\boldsymbol{r_x}$, $\boldsymbol{r_y}$) = $\frac{r_x \cdot r_y}{||r_x|| \cdot ||r_y||}$
  - **Problem:** Treats missing ratings as “negative”

r_x, r_y as vectors:

$\boldsymbol{r_x}$ = {1, 0, 0, 1, 3}

$\boldsymbol{r_y}$ = {1, 0, 2, 2, 0}

Similarity Metric: Solution

	HP1	HP2	HP3	TW	SW1	SW2	SW3
A	4			5	1		
B	5	5	4				
C				2	4	5	
D		3					3

- Intuitively we want: sim(A, B) > sim(A, C)
- But, Jaccard similarity: 1/5 < 2/4
- Cosine similarity: 0.386 > 0.322
  - – Problem: Considers missing ratings as "negative"
  - – Solution: Normalize by subtracting the row mean. Then calculate the cosine similarity.
  - – Note: Cosine similarity is correlation when the data is centered at 0

Similarity Metric: Solution

	HP1	HP2	HP3	TW	SW1	SW2	SW3
A	4			5	1		
B	5	5	4				
C				2	4	5	
D		3					3

sim(A,B) vs sim(A,C):
0.386 > 0.322

Subtract the row mean

	HP1	HP2	HP3	TW	SW1	SW2	SW3
A	$2/3$			$5/3$	$-7/3$		
B	$1/3$	$1/3$	$-2/3$				
C				$-5/3$	$1/3$	$4/3$	
D		0					0

sim(A,B) vs sim(A,C):
0.092 > -0.559

Rating Predictions

From similarity metric to recommendations:

- Let $\boldsymbol{r_x}$ be the vector of user ***x***’s ratings
- Let ***N*** be the set of ***k*** users most similar to ***x*** who have rated item ***i***
- **Prediction for item *s* of *user x*:**
  - $r_{xi} = \frac{1}{k} \sum_{y \in N} r_{yi}$
  - $r_{xi} = \frac{\sum_{y \in N} s_{xy} \cdot r_{yi}}{\sum_{y \in N} s_{xy}}$

Shorthand:
$s_{xy} = sim(x, y)$

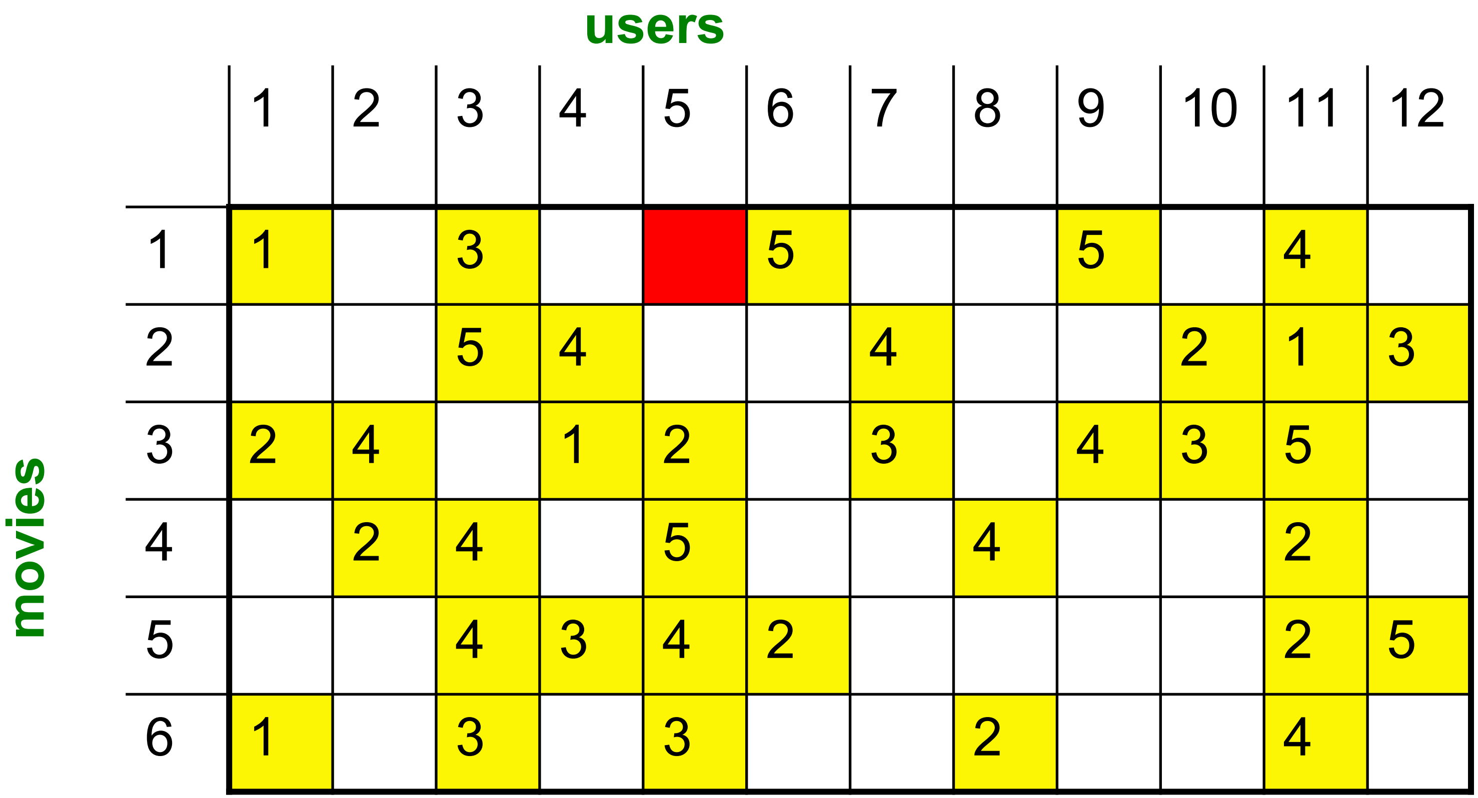
Item-Item Collaborative Filtering

- **So far: User-user collaborative filtering**
- **Another view: Item-item**
  - For item ***i***, find other similar items
  - Estimate rating for item ***i*** based on ratings for similar items
  - Can use same similarity metrics and prediction functions as in user-user model

$$\boxed{r_{xi} = \frac{\sum_{j \in N(i;x)} s_{ij} \cdot r_{xj}}{\sum_{j \in N(i;x)} s_{ij}}}$$

s_{ij} = similarity of items ***i*** and ***j***
r_{xj} = rating of user ***u*** on item ***j***
N(i;x)= set items rated by ***x*** similar to ***i***

Item-Item CF (|N|=2)



users

movies	1	2	3	4	5	6	7	8	9	10	11	12
1	1		3		🟥	5			5		4	
2			5	4			4			2	1	3
3	2	4		1	2		3		4	3	5	
4		2	4		5			4			2	
5			4	3	4	2					2	5
6	1		3		3			2			4	

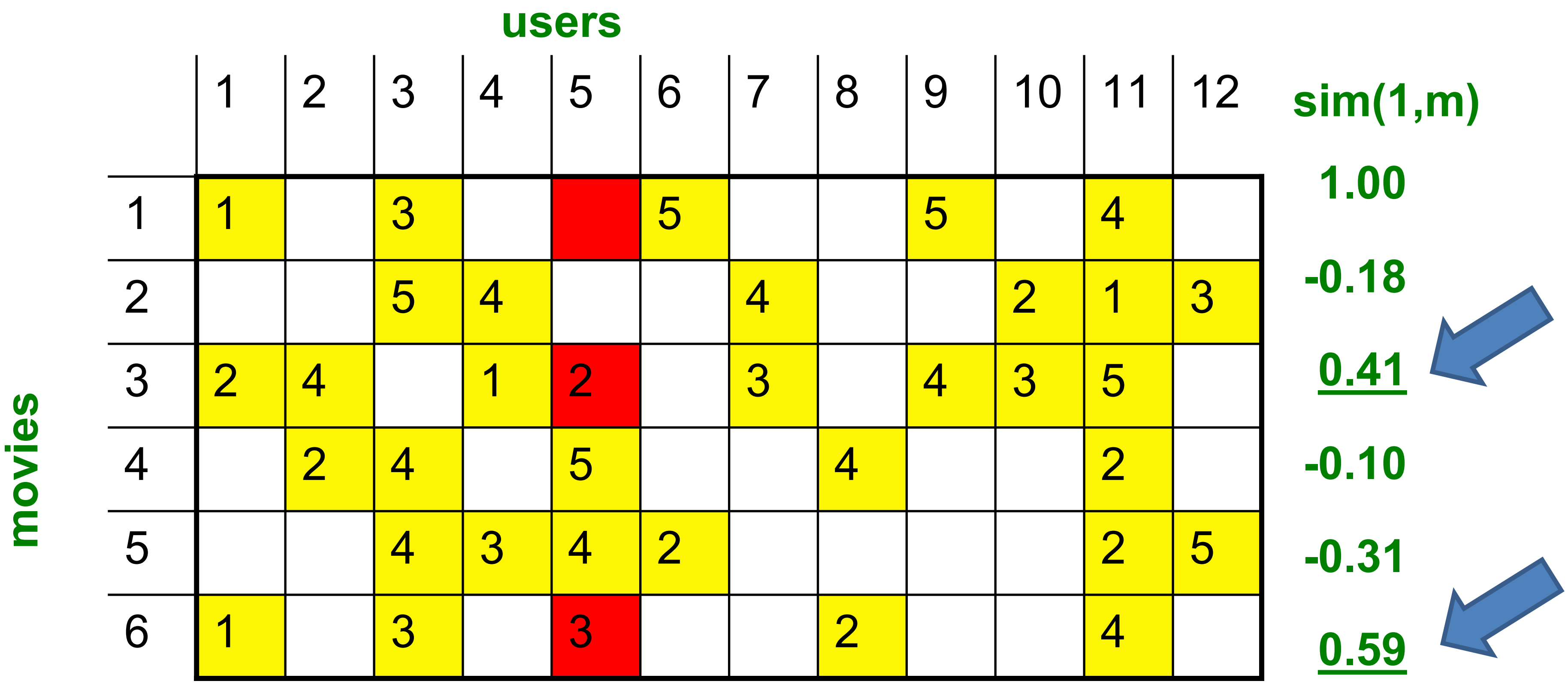
☐ unknown rating

🟨 rating between 1 to 5

🟥 estimate rating of movie **1** by user **5**

Neighbor selection: Identify movies similar to movie **1**, **rated by user 5**

Item-Item CF (|N|=2)



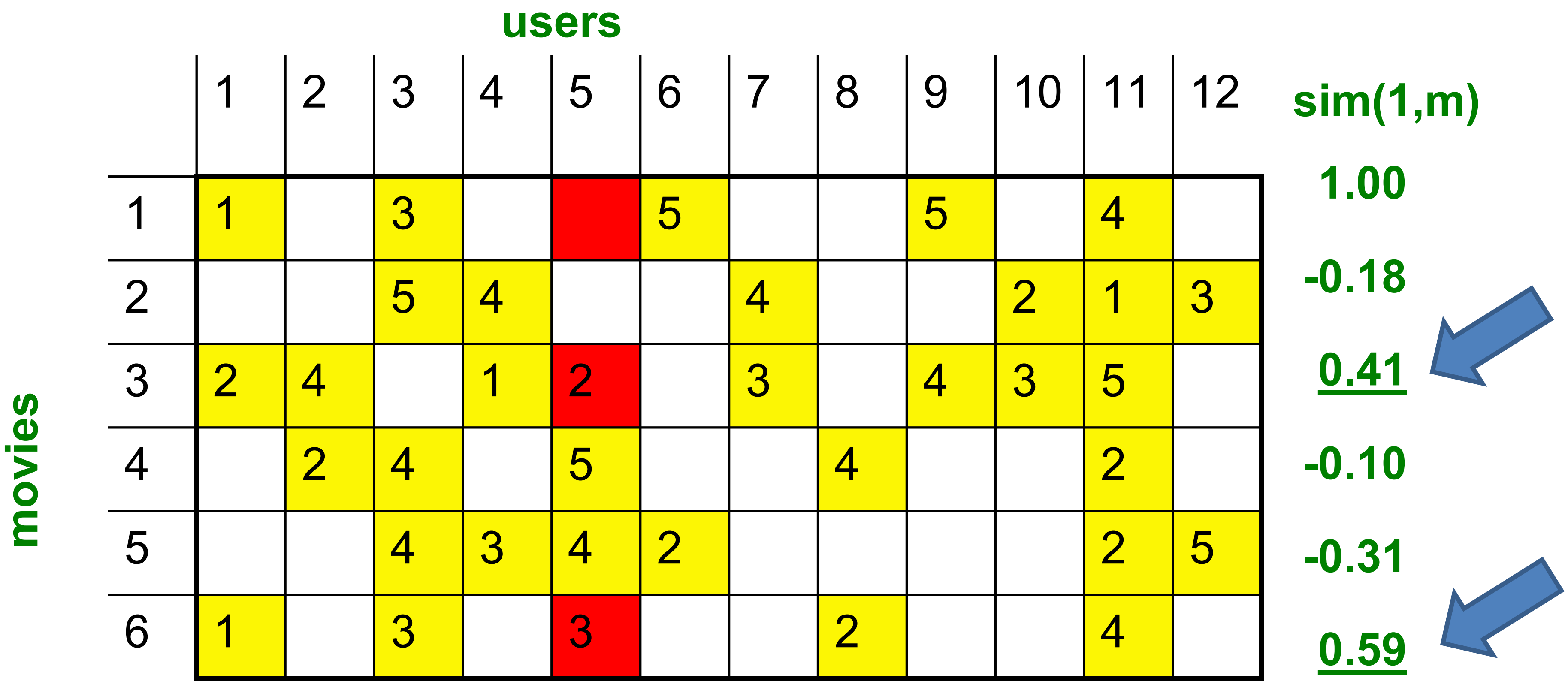
	users												
movies	1	2	3	4	5	6	7	8	9	10	11	12	sim(1,m)
1	1		3		🟥	5			5		4		1.00
2			5	4			4			2	1	3	-0.18
3	2	4		1	2		3		4	3	5		0.41
4		2	4		5			4			2		-0.10
5			4	3	4	2					2	5	-0.31
6	1		3		3			2			4		0.59

Use correlation as similarity: Subtract mean rating m_i from each movie *i*. Mean $m_1 = (1+3+5+5+4)/5 = \mathbf{3.6}$

row 1: *[-2.6, 0, -0.6, 0, 0, 1.4, 0, 0, 1.4, 0, 0.4, 0]*

Then, compute cosine similarities between rows.

Item-Item CF (|N|=2)



users

movies	1	2	3	4	5	6	7	8	9	10	11	12	**sim(1,m)**
1	1		3			5			5		4		**1.00**
2			5	4			4			2	1	3	**-0.18**
3	2	4		1	2		3		4	3	5		**0.41**
4		2	4		5			4			2		**-0.10**
5			4	3	4	2					2	5	**-0.31**
6	1		3		3			2			4		**0.59**

Predict by taking weighted average:

$r_{1,5}$ = (0.41*2 + 0.59*3) / (0.41+0.59) = 2.6

$$r_{ix} = \frac{\sum_{j \in N(i;x)} s_{ij} \cdot r_{jx}}{\sum s_{ij}}$$

Item-Item vs. User-User

- In practice, it has been observed that **item-item often works better than user-user**
- **Why?** Items are simpler, users have multiple tastes

Pros of Collaborative Filtering

- **Works for any kind of item**
  - No feature selection needed
- **Leverages other users' behavior**
  - Incorporates community actions

Cons of Collaborative Filtering

- **Cold Start:**
  - Need enough users in the system to find a match
- **Sparsity:**
  - The user/ratings matrix is sparse
  - Hard to find users that have rated the same items
- **First rater:**
  - Cannot recommend an item that has not been previously rated. What about new items, esoteric items?
- **Popularity bias:**
  - Cannot recommend items to someone with unique taste
  - Tends to recommend popular items

Recommendation Systems

- ✓ Content-based
- ✓ Collaborative Filtering
- ☞ Latent Factor Models
- Deep Learning Models

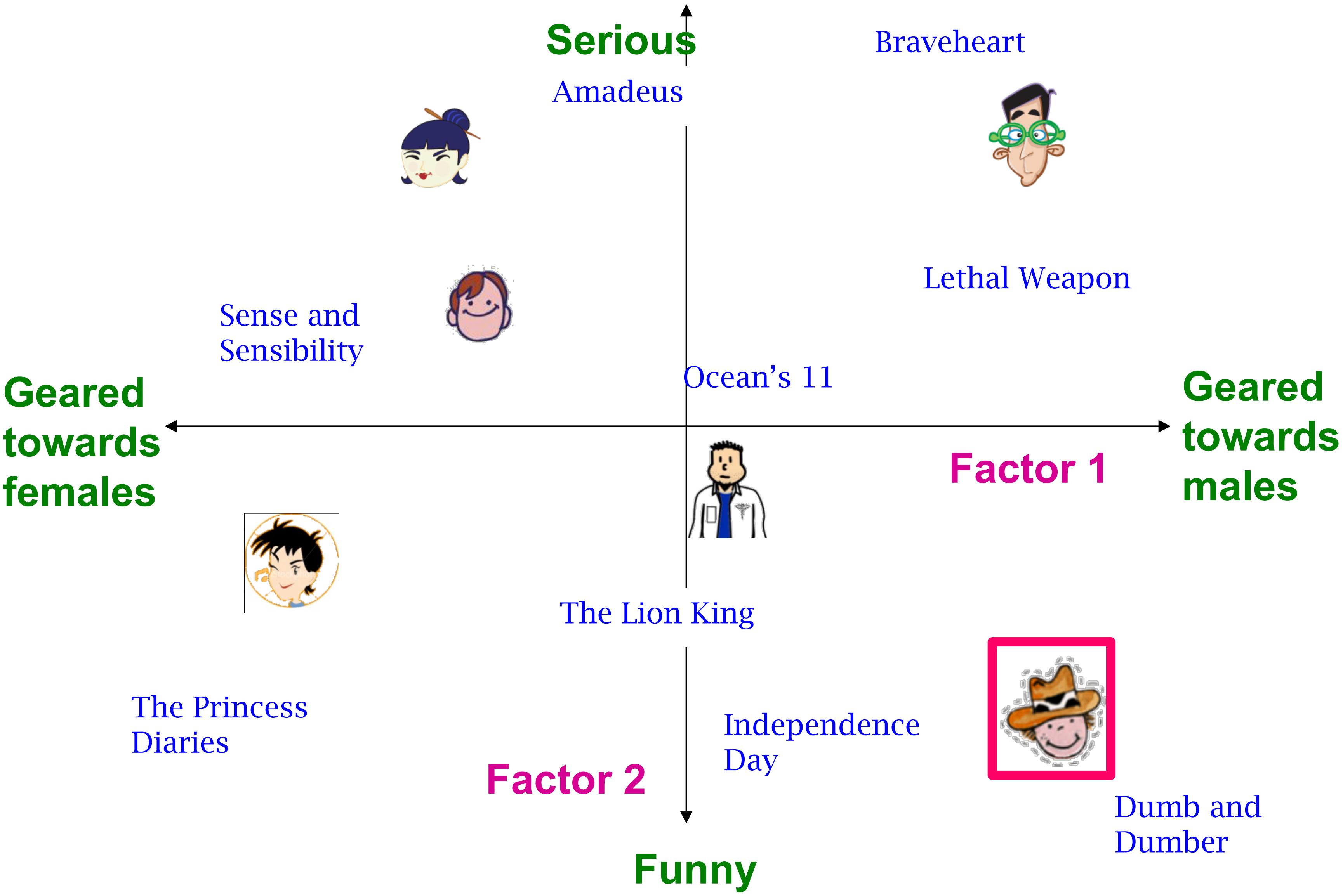
Latent Factor Models

- These models learn **latent factors** to represent users and items from the rating matrix
  - Latent factors are not directly observable
  - These are derived from the data
- **Recall:** Network embeddings
- **Methods:**
  - Singular value decomposition (SVD)
  - Principal Component Analysis (PCA)
  - Eigendecompositon

Latent Factors: Example

- **Embedding axes** are a type of latent factors
- **In a user-movie rating matrix:**
- **Movie latent factors** can represent axes:
  - Comedy vs drama
  - Degree of action
  - Appropriateness to children
- **User latent factors** will measure a user's affinity towards corresponding movie factors

Latent Factors: Example



Serious

Braveheart

Amadeus

Lethal Weapon

Sense and Sensibility

Ocean's 11

Geared towards females

Geared towards males

Factor 1

The Lion King

The Princess Diaries

Independence Day

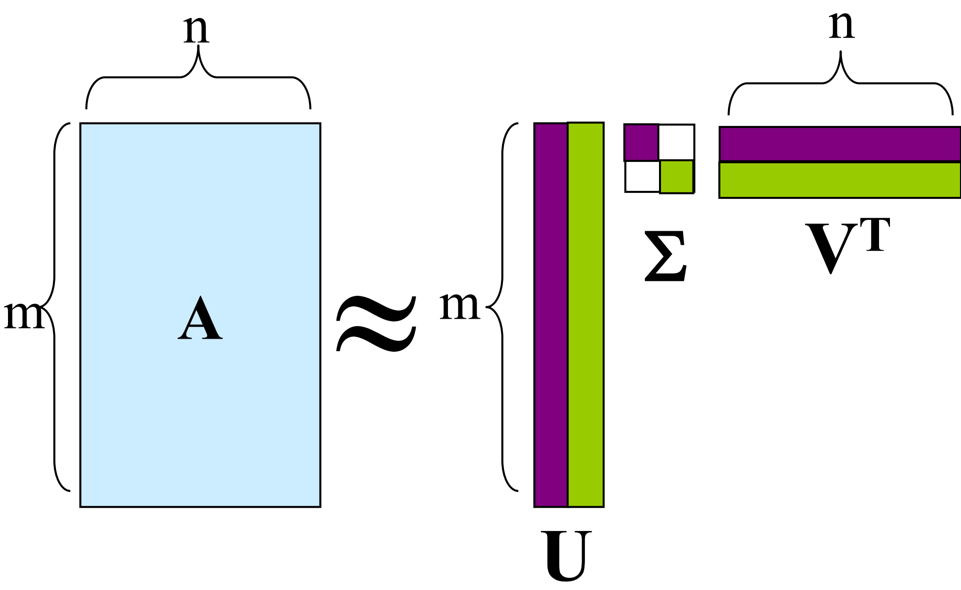
Dumb and Dumber

Factor 2

Funny

SVD

- **SVD:** SVD decomposes an input matrix into multiple factor matrices
  - $A = U \Sigma V^T$
  - Where,
  - **A**: Input data matrix
  - **U**: Left singular vecs
  - **V**: Right singular vecs
  - Σ: Singular values



SVD

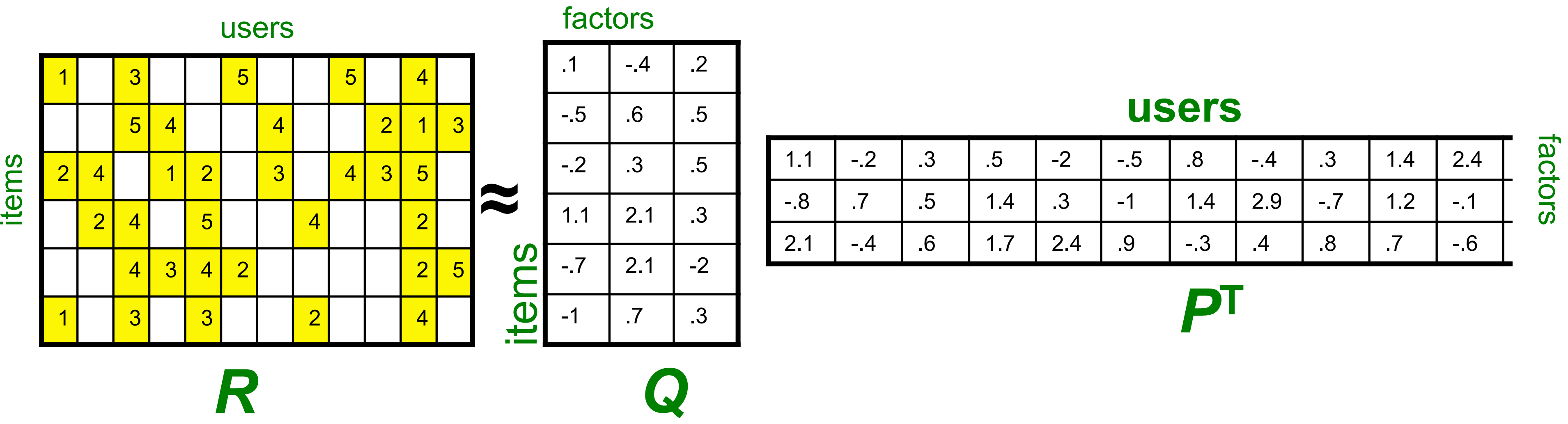
- SVD gives **minimum reconstruction error** (Sum of Squared Errors):

$$\min_{U,\mathrm{V},\Sigma} \sum_{ij\in A} \left(A_{ij} - [U\Sigma V^{\mathrm{T}}]_{ij}\right)^2$$

- **SSE** and **RMSE** are monotonically related:
  - $RMSE = \frac{1}{c}\sqrt{SSE}$ ➔ SVD is minimizing RMSE
- **Complication:** The sum in SVD error term is over all entries. But our ***R*** has missing entries.
  - **Solution:** no-rating in interpreted as zero-rating.

SVD on Rating Matrix

- "SVD" on rating data: $\mathbf{R} \approx \mathbf{Q} \cdot \boldsymbol{P}^T$
- Each row of Q represents an item
- Each column of P represents a user



R

1		3			5			5		4	
		5	4			4			2	1	3
2	4		1	2		3		4	3	5	
	2	4		5			4			2	
		4	3	4	2					2	5
1		3		3			2			4	

Q

.1	-.4	.2
-.5	.6	.5
-.2	.3	.5
1.1	2.1	.3
-.7	2.1	-2
-1	.7	.3

P^T

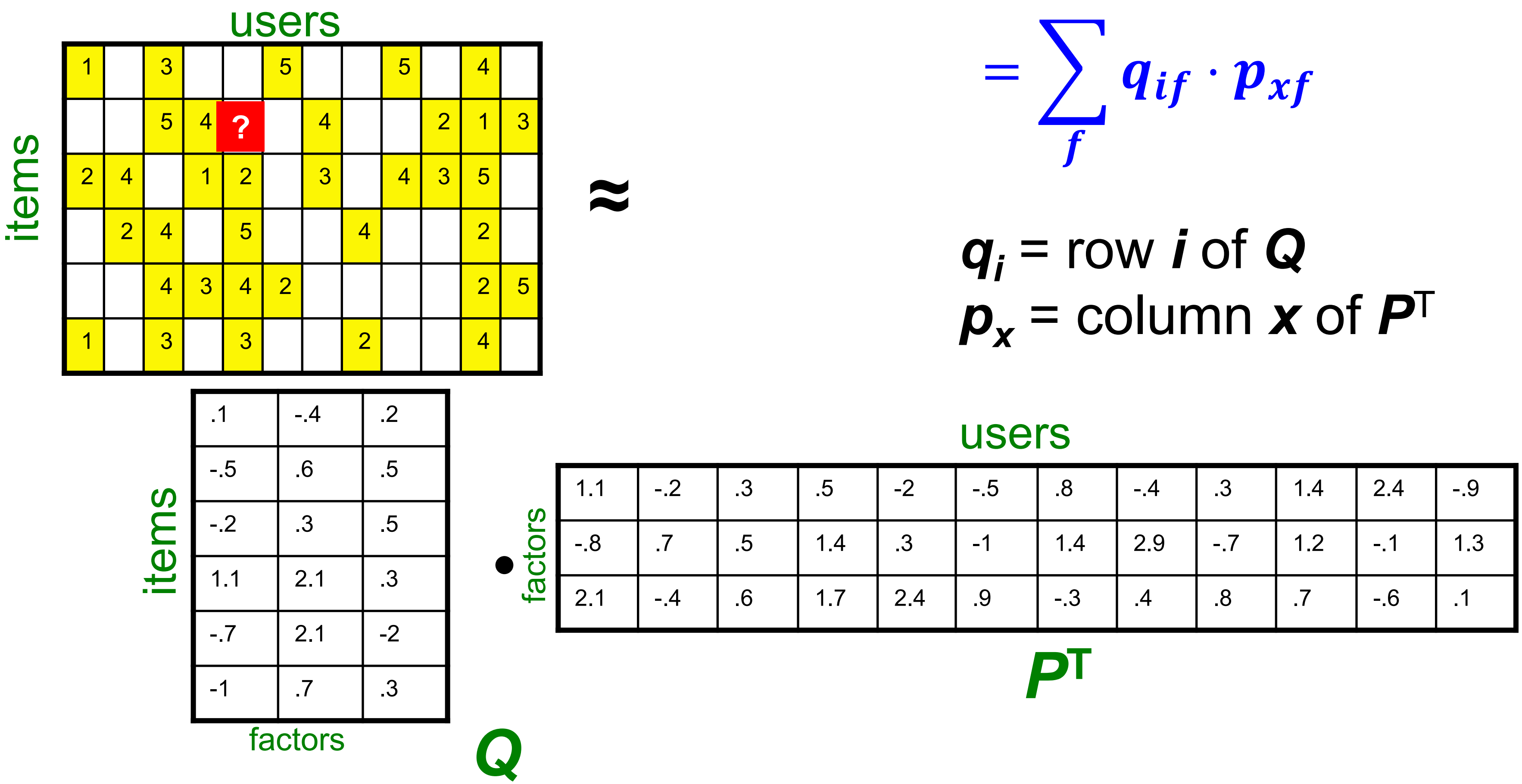
1.1	-.2	.3	.5	-2	-.5	.8	-.4	.3	1.4	2.4
-.8	.7	.5	1.4	.3	-1	1.4	2.9	-.7	1.2	-.1
2.1	-.4	.6	1.7	2.4	.9	-.3	.4	.8	.7	-.6

Ratings as Products of Factors

- **How to estimate the missing rating of user *x* for item *i*?**

$$\hat{r}_{xi} = q_i \cdot p_x$$
$$= \sum_f q_{if} \cdot p_{xf}$$

q_i = row i of Q
p_x = column x of P^T



users

items

1		3			5			5		4	
		5	4	?		4			2	1	3
2	4		1	2		3		4	3	5	
	2	4		5			4			2	
		4	3	4	2					2	5
1		3		3			2			4	

≈

items

.1	-.4	.2
-.5	.6	.5
-.2	.3	.5
1.1	2.1	.3
-.7	2.1	-2
-1	.7	.3

factors

Q

•

users

factors

1.1	-.2	.3	.5	-2	-.5	.8	-.4	.3	1.4	2.4	-.9
-.8	.7	.5	1.4	.3	-1	1.4	2.9	-.7	1.2	-.1	1.3
2.1	-.4	.6	1.7	2.4	.9	-.3	.4	.8	.7	-.6	.1

P^T

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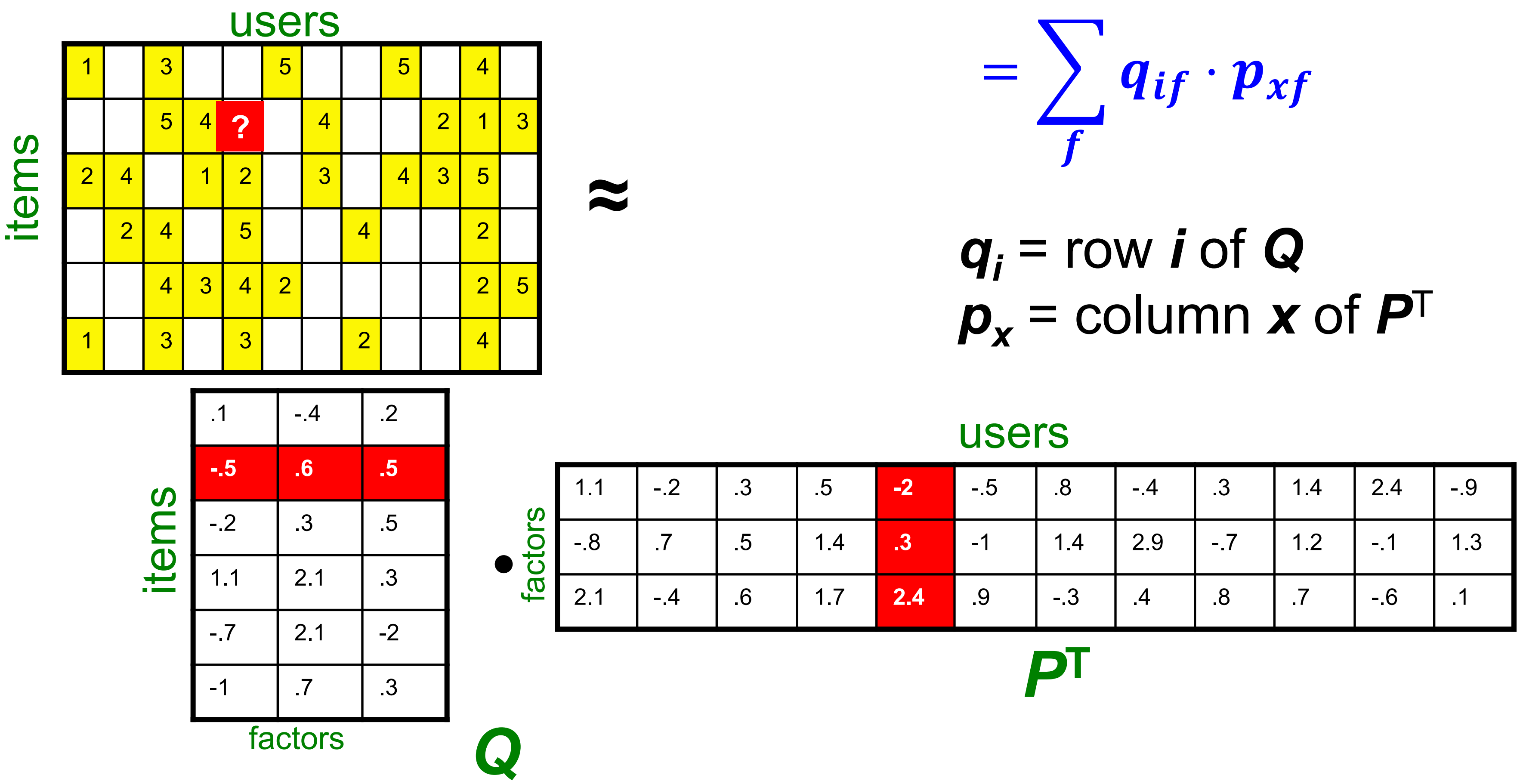

1		3			5			5		4	
		5	4	?		4			2	1	3
2	4		1	2		3		4	3	5	
	2	4		5			4			2	
		4	3	4	2					2	5
1		3		3			2			4	

≈

.1	-.4	.2
-.5	.6	.5
-.2	.3	.5
1.1	2.1	.3
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•

1.1	-.2	.3	.5	-2	-.5	.8	-.4	.3	1.4	2.4	-.9
-.8	.7	.5	1.4	.3	-1	1.4	2.9	-.7	1.2	-.1	1.3
2.1	-.4	.6	1.7	2.4	.9	-.3	.4	.8	.7	-.6	.1

q_i = row ***i*** of ***Q***

p_x = column ***x*** of P^T

Ratings as Products of Factors

- **How to estimate the missing rating of user *x* for item *i*?**

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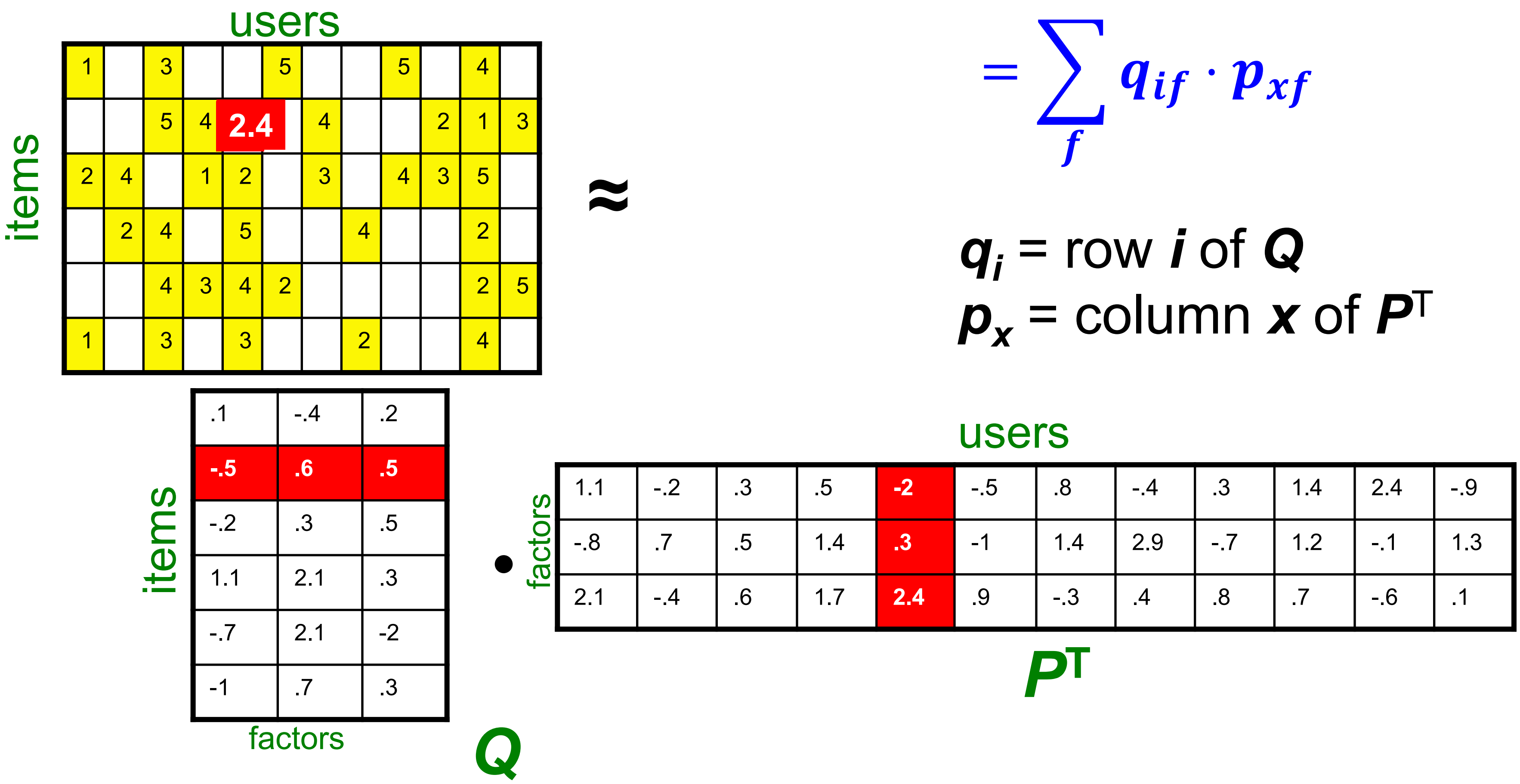

users

items

1		3			5			5		4	
		5	4	**2.4**		4			2	1	3
2	4		1	2		3		4	3	5	
	2	4		5			4			2	
		4	3	4	2					2	5
1		3		3			2			4	

≈

q_i = row ***i*** of ***Q***
p_x = column ***x*** of P^T

items

.1	-.4	.2
-.5	**.6**	**.5**
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factors

Q

•

users

factors

1.1	-.2	.3	.5	**-2**	-.5	.8	-.4	.3	1.4	2.4	-.9
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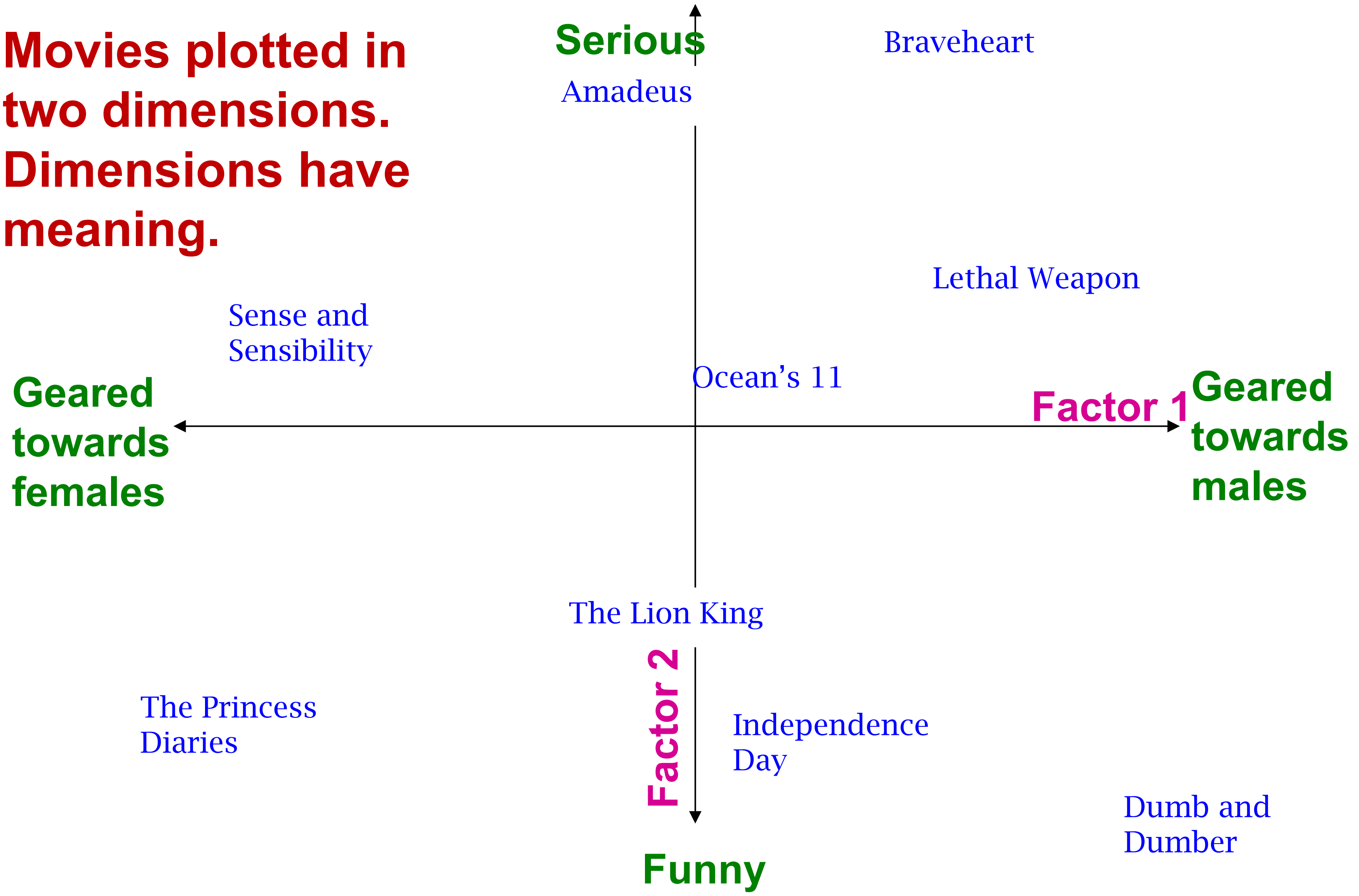
2.1	-.4	.6	1.7	**2.4**	.9	-.3	.4	.8	.7	-.6	.1

P^T

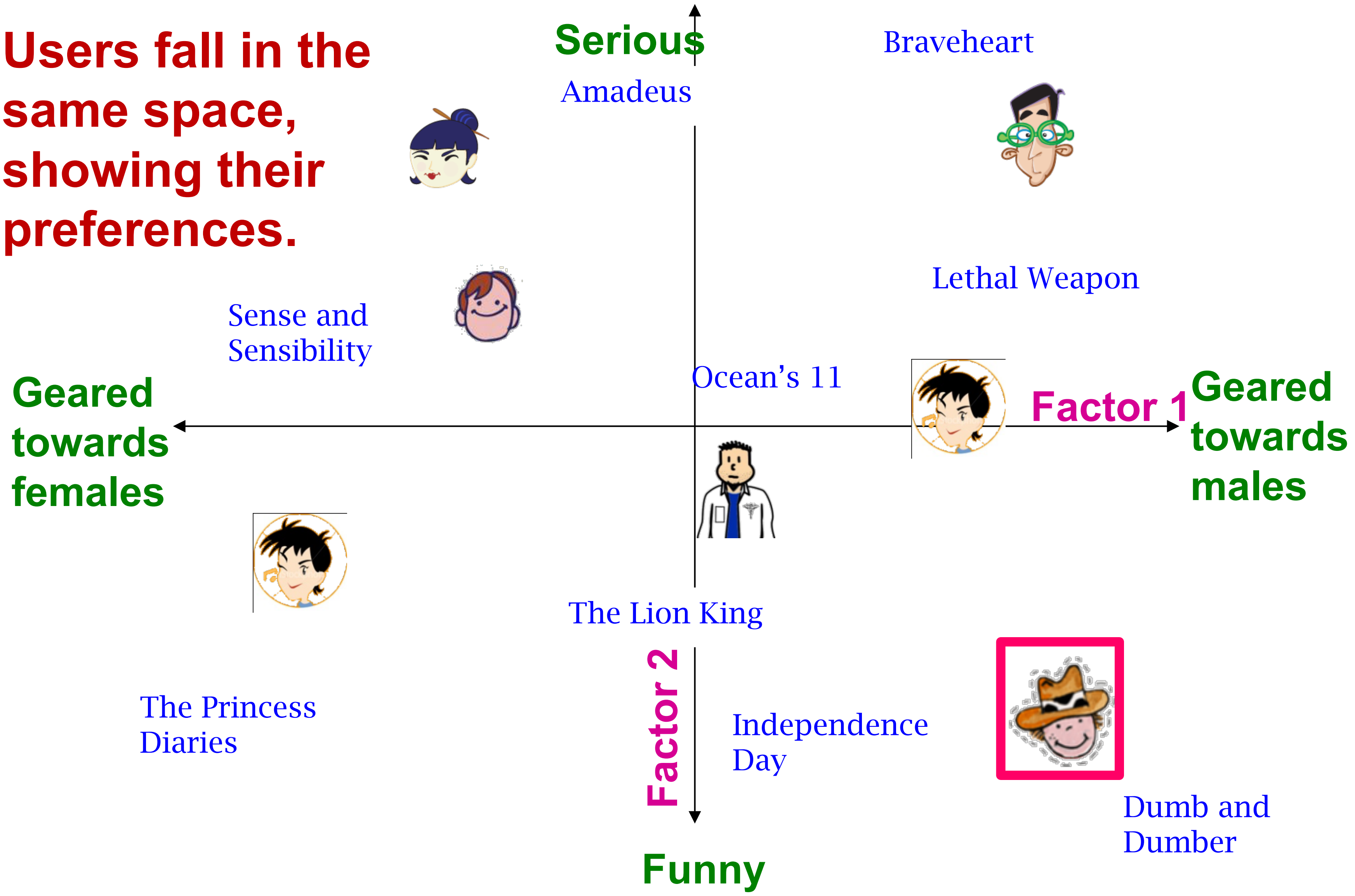
Latent Factor Models: Example

Movies plotted in two dimensions. Dimensions have meaning.



Latent Factor Models

Users fall in the same space, showing their preferences.



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Recommendation Systems

- ✓ Content-based
- ✓ Collaborative Filtering
- ✓ Latent Factor Models
- ☞ Deep Learning Models: NCF

- **Reference Paper:** **Neural Collaborative Filtering.** He Xiangnan, Liao Lizi, Zhang Hanwang, Nie Liqiang, Hu Xia, Tat-Seng Chua. WWW 2017

Matrix Factorization

- MF uses an **inner product as the interaction function**
  - – Latent factors are **independent** with each other
- **Limitations:** The simple choice of **inner product** function can limit the expressiveness of a MF model.
- **Potential solution:** increase the number of factors. However,
  - – This increases the complexity of the model
  - – Leads to overfitting

Improving Matrix Factorization

- **Key question:** How can we improve matrix factorization?
- **Answer:** Learn the relation between factors from the data, rather than fixing it to be the simple, fixed inner product
  - Does not increase the complexity
  - Does not lead to overfitting
- **One solution:** Neural Collaborative Filtering

Neural Collaborative Filtering

- **Neural Collaborative Filtering (NCF)** is a deep learning version of the traditional recommender system
- **Learns the interaction function** with a deep neural network
  - Non-linear functions, e.g., multi-layer perceptrons, to learn the interaction function
  - Models well when **latent factors are not independent** with each other, especially true in large real datasets

50

Neural Collaborative Filtering

- Neural extensions of traditional recommender system
- **Input:** rating matrix, user profile and item features (optional)
  - If user/item features are unavailable, we can use one-hot vectors
- **Output:** User and item embeddings, prediction scores
- Traditional matrix factorization is a special case of NCF

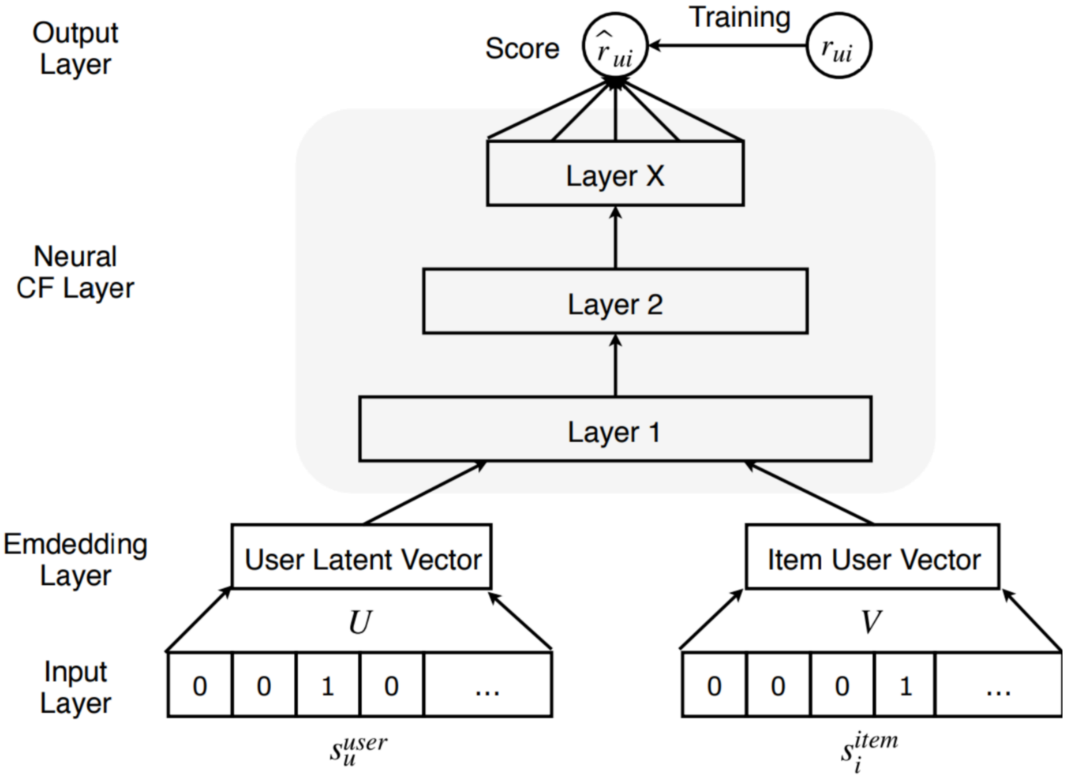
NCF Setup

- User feature vector: s_u^{user}
- Item feature vector: s_i^{item}
- User embedding matrix: U
- Item embedding matrix: I
- Neural network: f
- Neural network parameters: Θ
- Predicted rating:

$$\hat{r}_{ui} = f(U^T \cdot s_u^{user}, V^T \cdot s_i^{item} | U, V, \theta)$$

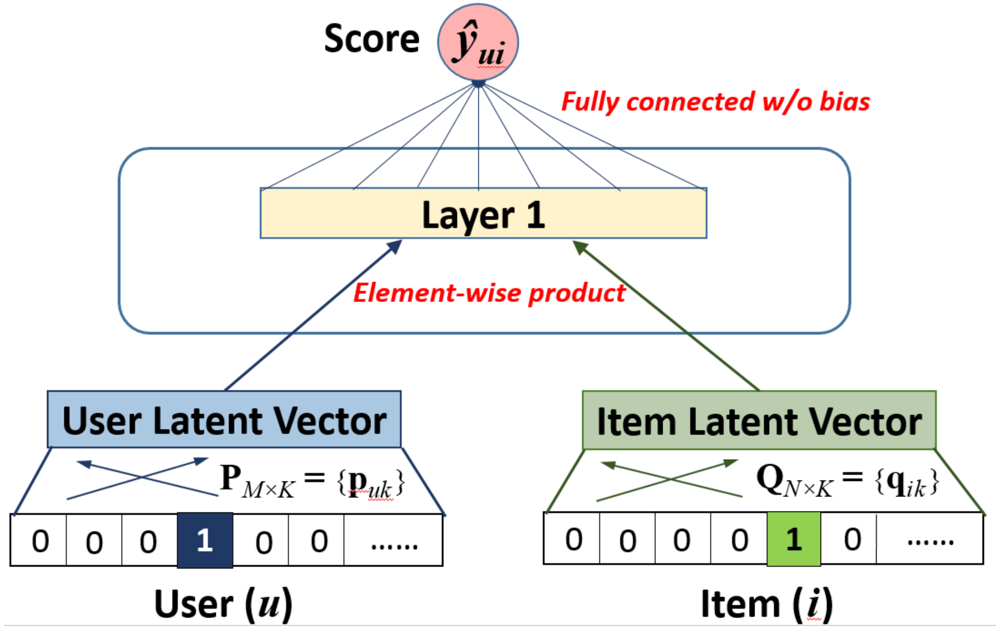
NCF Model Architecture

- **Multiple layers of fully connected layers** form the Neural CF layer.
- **Output is a rating score** $\hat{r}_{ui}$
- **Real rating score** is r_{ui}



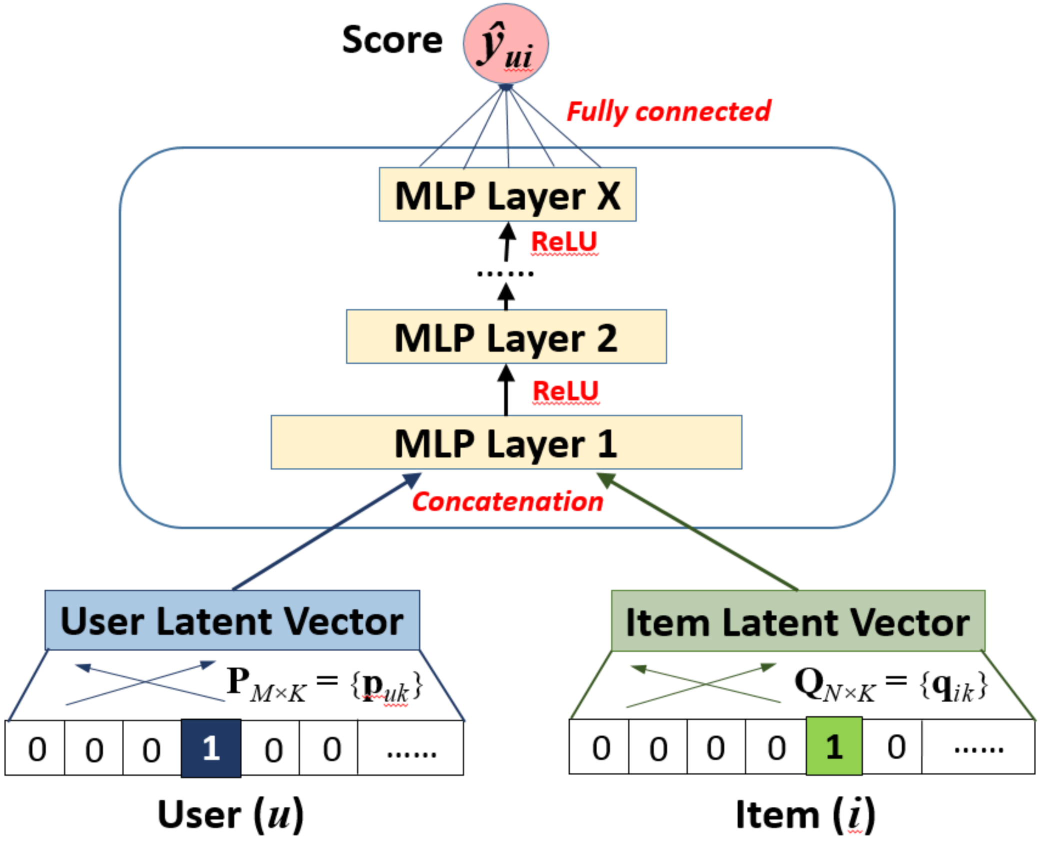
1-Layer NCF

- **Layer 1** an element-wise product
- **Output Layer** as a fully connected layer without bias



Multi-Layer NCF

- Each layer is a **multi-layer perceptron**, with non-linearity on the top
- Final score is used to **calculate the loss and train** the layers



NCF model: Loss function

- **Train** on the difference between predicted rating and the real rating
- **Use negative sampling** to reduce the negative data points
- **Loss = cross-entropy loss**

$$\mathcal{L} = -\sum_{(u,i)\in\mathcal{O}\cup\mathcal{O}^-} r_{ui}\log\hat{r}_{ui} + (1-r_{ui})\log(1-\hat{r}_{ui})$$

Experimental Setup

- **Two public datasets:** MovieLens, Pinterest
  - Transform MovieLens ratings to 0/1 implicit case

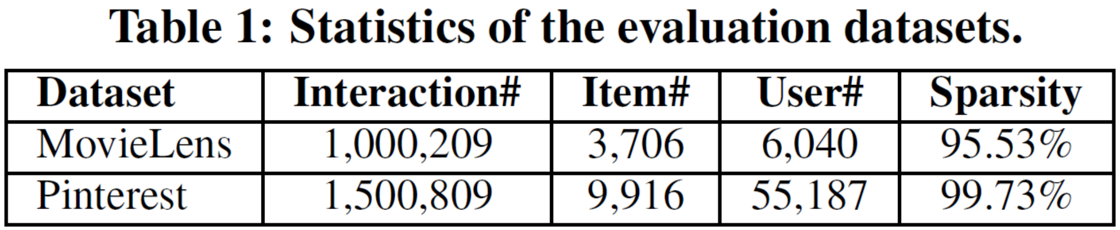


Table 1: Statistics of the evaluation datasets.

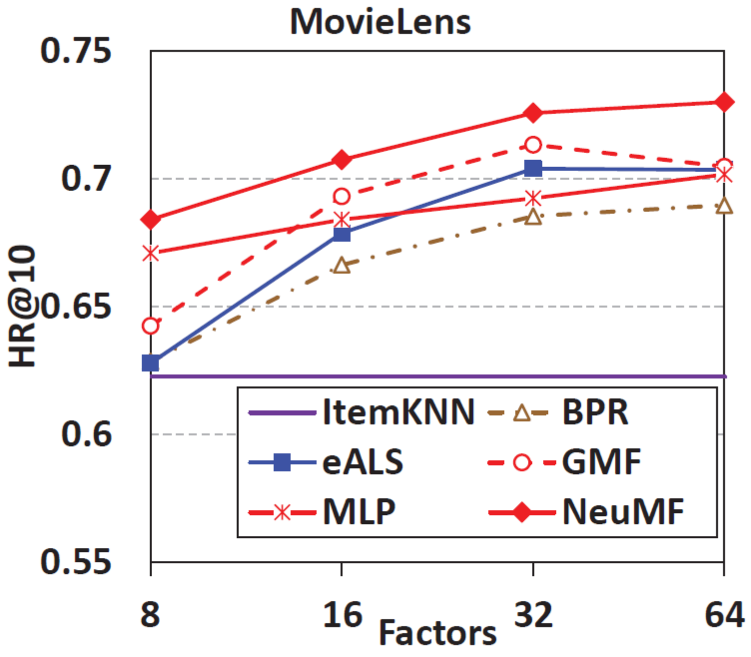
Dataset	Interaction#	Item#	User#	Sparsity
MovieLens	1,000,209	3,706	6,040	95.53%
Pinterest	1,500,809	9,916	55,187	99.73%

- **Evaluation protocols:**
  - **Leave-one-out setting:** hold-out the latest rating of each user as the test
  - **Top-k evaluation:** create a ranked list of items
  - **Evaluation metrics:**
    - **Hit Ratio:** does the correct item appear in top 10

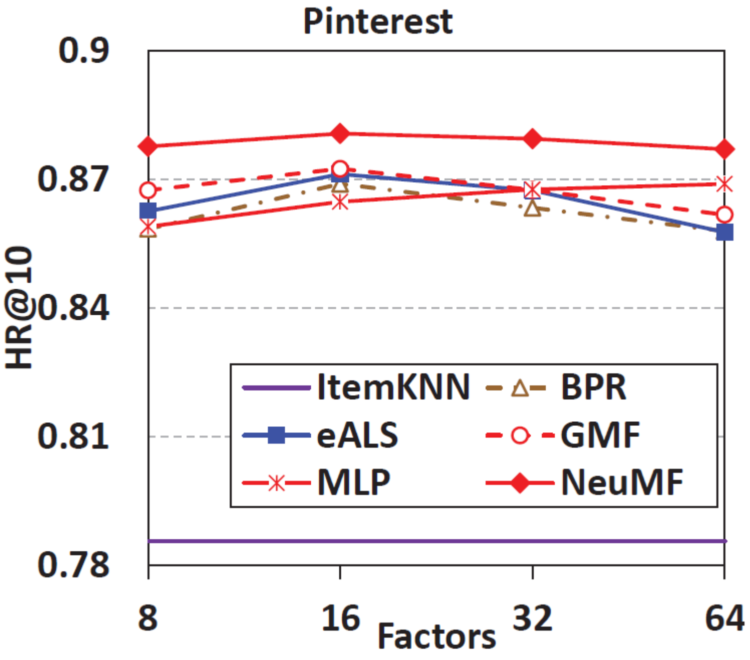
Baselines

- **Item Popularity**
  - Items are ranked by their popularity
- **ItemKNN** [Sarwar et al, WWW'01]
  - The standard item-based CF method
- **BPR** [Rendle et al, UAI'09]
  - Bayesian Personalized Ranking optimizes MF model with a pairwise ranking loss
- **eALS** [He et al, SIGIR'16]
  - The state-of-the-art CF method for implicit data. It optimizes MF model with a varying-weighted regression loss.

Performance vs. Embedding Size



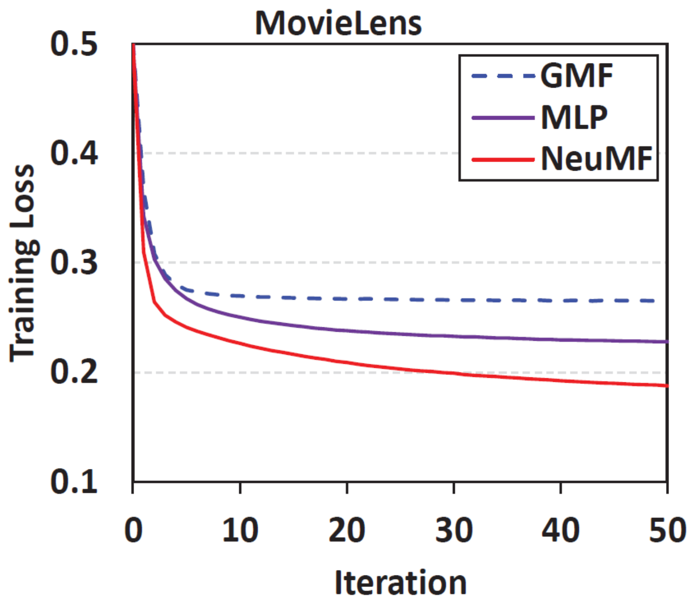
(a) MovieLens — HR@10



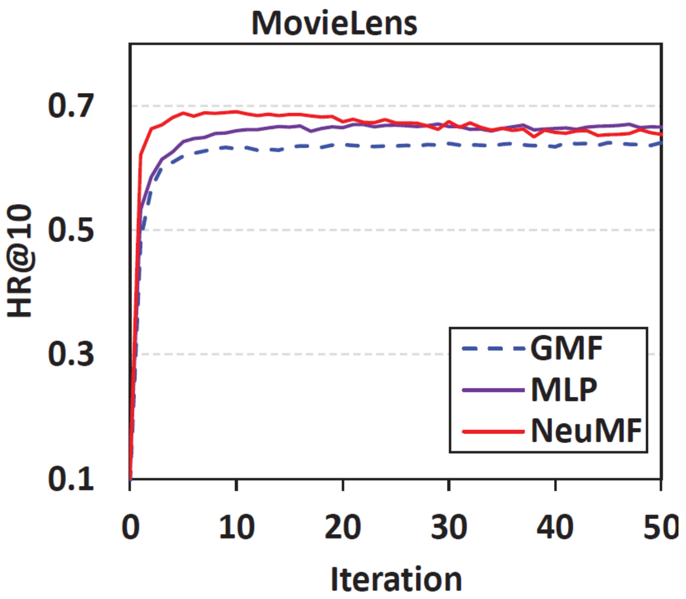
(c) Pinterest — HR@10

- **NeuMF > eALS and BPR** (5% improvement)
- **NeuMF > MLP** (MLP has lower training loss but higher test loss)

Convergence Behavior



(a) Training Loss



(b) HR@10

- Most effective updates in the **first 10 iterations**
- More iterations make **NeuMF overfit**
- **Trade-off** between representation ability and generalization ability of a model.

Is Deeper Helpful?

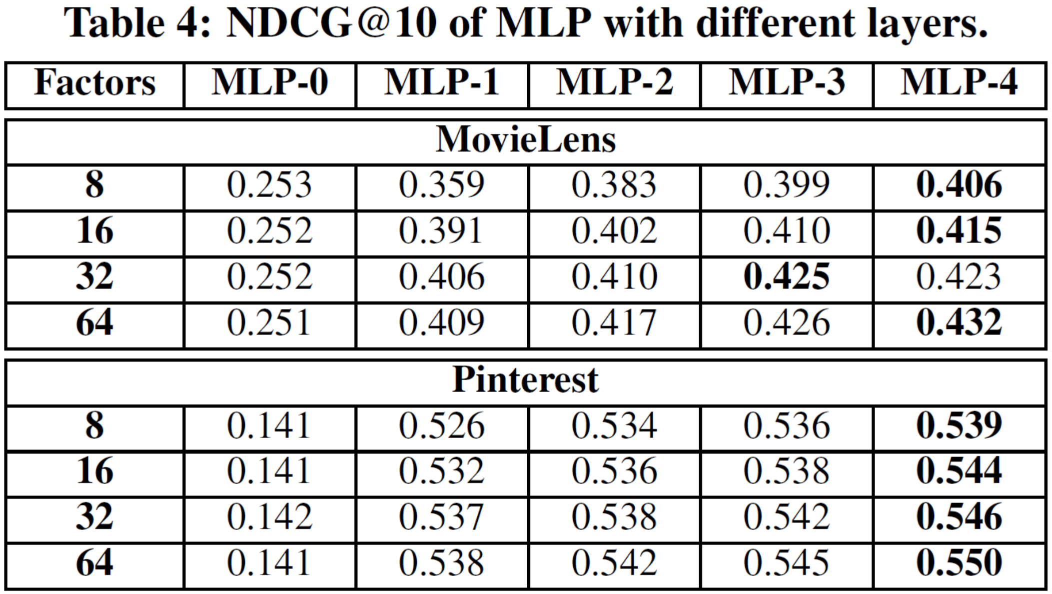


Table 4: NDCG@10 of MLP with different layers.

Factors	**MLP-0**	**MLP-1**	**MLP-2**	**MLP-3**	**MLP-4**
MovieLens					
8	0.253	0.359	0.383	0.399	**0.406**
16	0.252	0.391	0.402	0.410	**0.415**
32	0.252	0.406	0.410	**0.425**	0.423
64	0.251	0.409	0.417	0.426	**0.432**
Pinterest					
8	0.141	0.526	0.534	0.536	**0.539**
16	0.141	0.532	0.536	0.538	**0.544**
32	0.142	0.537	0.538	0.542	**0.546**
64	0.141	0.538	0.542	0.545	**0.550**

- Same number of factors, but more nonlinear layers improves the performance.
- Linear layers degrades the performance
- Improvement diminishes for more layers

NCF: Shortcomings

- Architecture is **limited**
- NCF **does not model the temporal behavior** of users or items
  - Recall: users and items exhibit temporal bias
  - NCF has the same input for user
- **Non-inductive:** new users and new items, on which training was not done, can not be processed