

$$w_1 x_1 + w_2 x_2 + w_3 x_3 = x$$

$$w_1 y_1 + w_2 y_2 + w_3 y_3 = y$$

$$w_1 + w_2 + w_3 = 1$$

$$(A \ b) = \left(\begin{array}{ccc|c} x_1 & x_2 & x_3 & x \\ y_1 & y_2 & y_3 & y \\ 1 & 1 & 1 & 1 \end{array} \right)$$

$$w_1 = \frac{\det(A_1)}{\det(A)} =$$

$$\det(A_1) = \det \begin{pmatrix} x & x_2 & x_3 \\ y & y_2 & y_3 \\ 1 & 1 & 1 \end{pmatrix}$$

$$= x y_2 + x_2 y_3 + x_3 y - x_3 y_2 - x_2 y - x y_3$$

$$\det(A) = \det \begin{pmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \\ 1 & 1 & 1 \end{pmatrix}$$

$$= x_1 y_2 + x_2 y_3 + x_3 y_1 - x_3 y_2 - x_2 y_1 - x_1 y_3$$

$$w_1 = \frac{x y_2 + x_2 y_3 + x_3 y - x_3 y_2 - x_2 y - x y_3}{x_1 y_2 + x_2 y_3 + x_3 y_1 - x_3 y_2 - x_2 y_1 - x_1 y_3}$$

$$w_2 = \det(A_2) = \det \begin{pmatrix} x_1 & x & x_3 \\ y_1 & y & y_3 \\ 1 & 1 & 1 \end{pmatrix} = x_1 y + x y_3 + x_3 y_1 - x_3 y - x y_1 - x_1 y_3$$

$$w_2 = \frac{\det(A_2)}{\det(A)} = \frac{x_1 y + x y_3 + x_3 y_1 - x_3 y - x y_1 - x_1 y_3}{x_1 y_2 + x_2 y_3 + x_3 y_1 - x_3 y_2 - x_2 y_1 - x_1 y_3}$$

$$\det(A_3) = \det \begin{pmatrix} x_1 & x_2 & x \\ y_1 & y_2 & y \\ 1 & 1 & 1 \end{pmatrix}$$

$$= x_1 x_2 + x_2 y + x y_1 - x y_2 - x_2 y_1 - x_1 y$$

$$w_3 = \frac{\det(A_3)}{\det(A)} = \frac{x_1 x_2 + x_2 y + x y_1 - x y_2 - x_2 y_1 - x_1 y}{x_1 y_2 + x_2 y_3 + x_3 y_1 - x_1 y_2 - x_2 y_1 - x_1 y_3}$$