Investments Project (Spring 2025)

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Guidelines

- You have to work in groups of up to 4 students.
- You have to submit ONE project per group on Moodle. You have to register in a group on Moodle in advance.
- The project is due on **June 4 at 23:59**. Late submissions will be accepted until **June 15 at 23:59**, but be aware that your student WRDS access will be shut-down by June 4, so all your data needs to be downloaded prior to June 4. Late submissions **after June 15 will not be accepted.**
- **You have to return i) a report of no more than 10 pages (excluding tables/figures) in pdf format, where you explain and justify all your results, and, ii) a separate file with your code (either a Jupyter Notebook or preferably a ".py" file). Explanations need to be concise and precise.**
- **Every answer has to be reported and justified in the pdf report. The report must be self-explanatory.**
- **We must be able to run your code and reproduce all your results. The code must be written in Python.**

1 Introduction

This project will make you construct a portfolio that combines international stock market indices and currency exposures to analyze the benefits to international diversification, as well as the benefits to timing your allocation to international stock markets and currencies based on a small set of predictive signals including: carry, dollar, reversal, and momentum.

2 The Data

Download monthly stock returns from WRDS Monthly World Indices from 2002-01 to 2024-12 for the following stock market indices of the following country: Australia, France, Germany, Japan, Switzerland, the United Kingdom, and the CRSP's value-weighted return index for the US. Download also the 1-month T-Bill.

Further, download the exchange rate vis-a-vis the USD for the relevant currencies: AUD, EUR, JPY, CHF, GBP. Also, download the 3-month interbank rate corresponding to each country. These data can be found on FRED website.

3 The international diversification strategy (DIV)

(a) Compute the returns to each of the stock market indexes in USD by converting the local currency returns into USD using the appropriate exchange rate.

(b) A currency-hedged index return is defined as the return in USD to a $1 investment in a foreign stock market index hedged by shorting $1 of the corresponding foreign currency. To be specific, the *excess return* in USD to a $1 investment in the Euro-currency for example (with time t spot exchange rate S_t USD per EU) is:

$$X_{t+1}^{\mathrm{EU}} = \frac{S_{t+1}}{S_t}(1 + r^{\mathrm{EU}}) - (1 + r^{US}) \tag{1}$$

It is the return to borrowing $1 at the US 3-month rate r^{US}, converting into $\frac{1}{S_t}$ euros, investing at the 3-month euro risk-free rate r^{EU}, and converting the proceeds back into dollar after one month at the then prevailing spot exchange rate S_{t+1}.

If the return in USD of the French index is R_t^{FR}, then we define the currency hedged return on the French Index in USD simply as $F_t^{FR,US} = R_t^{FR} - X_t^{EU}$.[1]

Compute currency hedged stock index returns.

(c) Consider three different approaches for a US investor to diversify internationally:

- Equal weight the indexes.

[1] The idea is that of the " currency hedged" return is to decompose our $-investment return on the French stock index into the component we would have obtained investing purely in the Euro currency from the additional component of the return coming from the French equity exposure.

- Risk-Parity based on a 60 months rolling window estimate of the strategy returns volatilities. Going forward, use a rolling window of 60 months each time you are asked to use a rolling window.
- Mean-variance optimal combination based on the rolling window mean and covariance matrix of the strategy returns.

For each of the three approaches to combining the strategies compute the overall mean, standard deviation, and Sharpe ratio of the resulting 'optimal' portfolio. Compare the case where you use unhedged returns and currency hedged returns. What is your conclusion regarding the benefits of international diversification for a US investor?

(d) Going forward we define the DIV strategy as the return to the currency hedged, risk-parity strategy. We will now consider a number of dynamic portfolio strategies and investigate whether they can improve on the simple DIV strategy.

4 Equity Index Momentum Strategy (MOM)

(a) Construct the return to a long-short momentum strategy portfolio. To that effect every month sort currency hedged stock indexes based on their 1-month lagged 11-month return (that is in month t rank stocks based on their $t-12$ to $t-1$ cumulative return). Call Rank_t^i the corresponding rank of index i at time t (e.g., $\text{Rank}_t^i = 2$ if country i has the second lowest Currency-hedged return over the last year.)

Then compute monthly returns to a portfolio that invests in index i the weight

$$w_t^i = Z\,(Rank_i - \frac{N+1}{2})$$

for all $i = 1, \ldots, N$, and where N is the total number of stock indexes traded and Z is a factor that insures that the the sum of the long positions is +\$1 and the sum of the short positons is −\$1.

(b) Compute and compare the mean, standard deviation, and Sharpe ratios of the long and short legs of the strategy as well as of the strategy itself. Test if the strategy has an average return that is statistically significantly different from zero.

(c) Regress the MOM strategy return on the DIV return. Interpret the regression results. In particular, do you think that it is interesting for a DIV-investor to also invest in the MOM strategy?

5 Equity Index Long Term Reversal strategy (REV)

(a) Construct the return to a long-short reversal strategy portfolio. To that effect every month sort currency hedged stock indexes based on their 12-month lagged 5-year past return (that is in month t rank stocks based on their $t-60$ to $t-12$ cumulative return). Call Rank_t^i the corresponding rank of index i at time t.

Then compute monthly returns to a portfolio that invests in index i the weight:[2]

$$w_t^i = Z\,(\frac{N+1}{2} - Rank_i)$$

for all $i = 1, \ldots, N$, and where N is the total number of stock indexes traded and Z is a factor that insures that the the sum of the long positions is +\$1 and the sum of the short positons is −\$1.

(b) Compute and compare the mean, standard deviation, and Sharpe ratios of the long and short legs of the strategy as well as of the strategy itself. Test if the strategy has an average return that is statistically significantly different from zero.

(c) Regress the REV strategy return on the DIV return. Interpret the regression results. In particular, do you think that it is interesting for a DIV-investor to also invest in the REV strategy?

6 Currency Carry Strategy (CARRY)

(a) Construct the return to a long-short currency carry strategy portfolio. To that effect every month sort currency returns based on their interest rate differential (i.e., the 'carry' is defined as the foreign risk-free 3-month interest rate minus the 3-month USD risk-free rate).

[2] Note that we go long stocks with low long-term past returns. This is a reversal strategy akin to a 'value' signal. It would be preferable to sort on average valuation ratios such as the Shiller PE ratio. But to simplify the data-collection we focus on this reversal proxy.

Call Rank_t^i the corresponding rank of index i at time t.

Then compute monthly returns to a portfolio that invests in index i the weight

$$w_t^i = Z\,(Rank_i - \frac{N+1}{2})$$

for all $i = 1, \ldots, N$, and where N is the total number of currencies traded and Z is a factor that insures that the the sum of the long positions is $+\$1$ and the sum of the short positions is $-\$1$. Its return is then:

$$R_{t+1}^{CARRY} = \sum_{i=1}^{N} w_t^i X_{t+1}^i$$

where X_{t+1}^i is given by equation (1).

(b) Compute and compare the mean, standard deviation, and Sharpe ratios of the long and short legs of the strategy as well as of the strategy itself. Test if the strategy has an average return that is statistically significantly different from zero.

(c) Regress the CARRY strategy return on the DIV return. Interpret the regression results. In particular, do you think that it is interesting for a DIV-investor to also invest in the CARRY strategy?

7 Currency dollar Strategy (DOLLAR)

(a) Construct the return to a long-dollar versus rest of the world strategy portfolio. That portfolio simply goes short an equal weighted basket of all the foreign currencies against the dollar. Its return is then simply:

$$R_{t+1}^{DOLLAR} = \sum_{i=1}^{N} \frac{1}{N} X_{t+1}^i$$

where X_{t+1}^i is the excess return on currency i (as computed in equation (1) above), and N denotes the total number of foreign currency traded.

(b) Compute and compare the mean, standard deviation, and Sharpe ratios of the strategy. Test if the strategy has an average return that is statistically significantly different from zero.

(c) Regress the DOLLAR strategy return on the DIV return. Interpret the regression results. In particular, do you think that it is interesting for a DIV-investor to also invest in the DOLLAR strategy?

8 Optimal Fund Portfolio Return (STRAT)

1. We now assume that you are running a fund that is currently investing in the 1-month T-Bill and the diversified stock-index strategy (DIV). That is your current (FUND) strategy has return $R_{FUND} = R_{T-Bill} + a(R_{DIV} - R_{T-Bill)}$ where a is chosen so that the annualized volatility of you fund return is 15% (over the past history available). Find a.

   You are considering adding an 'overlay' investment in the dynamic stock index strategies (MOM, REV) and the currency strategies (CARRY, DOLLAR) targeting an average annual volatility for your fund of 15%.

   Specifically, consider the return to the fund to be $R_{FUND} = R_{T-Bill} + b * (R_{DIV} - R_{T-Bill}) + c * R_{STRAT}$, where R_{STRAT} is the return to a strategy that combines MOM, REV, CARRY, and DOLLAR and where b and c are constants that you will choose so that the average annual volatility remains at $Vol(R_{fund}) = 15\%$.

2. Use risk-parity based on rolling window estimate of the strategy returns volatilities to combine the four strategies, MOM, REV, CARRY, DOLLAR to generate R_{STRAT}.

3. Then solve for b and c so as to obtain the mean-variance optimal combination based on the rolling window mean and covariance matrix of $R_{DIV} - R_{T-Bill}$ and R_{STRAT} and so as to achieve the annual volatility target of 15% for your fund.

   Give the resulting weights your fund would invest in each substrategy (T-Bill, DIV, MOM, REV, CARRY, and DOLLAR).

4. Plot the graph of the cumulative performance of $1 invested in your proposed strategy with the overlay on the same graph as the performance if you only invest as in (a) above in only T-Bill and DIV (with the same target annual volatility of 15%). Also compare the Mean, Standard Deviation, and Sharpe ratios of both.

9 Performance and risk analysis for the Fund strategy

(a) Regress the time series of your new strategy returns on the the Fama-French 5 research factors (that can be downloaded from Ken French's webpage). Based on the magnitude of the beta estimates and their t-statistic, which factors seem to be significant drivers of the strategy returns **unconditionally**? Do these purely US-based risk factors explain the international equity and currency strategy performance well (look at the R^2, the βs, and the α from the regression)?

(b) How can we explain the performance of the strategy that you just built? Is it consistent with efficient markets, with the CAPM, and with the APT?