Investments
FIN-405

Project Report

Adrien Aït Lalim (SCIPER 326588)
Matthias Wyss (SCIPER 329884)
Liess Gröli (SCIPER 327521)
Massimo Berardi (SCIPER 345943)

Financial Engineering, MA2
Group 20

Prof. Pierre Collin Dufresne

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1 Introduction

This project aims to construct a portfolio combining international equity indices and currency exposures to assess the benefits of international diversification. In addition, it evaluates the value of timing allocations across markets and currencies using a limited set of predictive signals: carry, dollar, reversal, and momentum.[1]

2 Data

We construct a monthly panel dataset covering the period from May 2002 to December 2024. The initial data download begins in April 2002, which corresponds to the earliest available observation for Japan's 3-month interbank rate. However, since country-level stock market returns are reported at the end of each month and subsequently shifted forward to align with other macro-financial indicators, the effective starting point of our dataset becomes May 2002. To ensure temporal consistency across all series, we uniformly align all datasets to span from May 2002 to December 2024.
Our dataset includes total return indices (with dividends) for Australia, France, Germany, Japan, Switzerland, the United Kingdom, and the United States (CRSP's value-weighted return index), downloaded from WRDS Monthly World Indices. We also include 1-month U.S. T-Bill returns from the same source. For exchange rate data, we use monthly spot rates against the U.S. dollar for AUD, EUR, JPY, CHF, and GBP. Moreover, we collect 3-month interbank rates for all countries from the FRED database. In addition, we collect the Fama-French 5 research factors for the same time period to enable a performance attribution analysis of the final strategy.
Finally, to handle data quality issues, we impute the single missing value in the 3-month U.S. interbank rate on April 1st, 2020, using the average of its adjacent months, ensuring a smooth and continuous time series.

3 The International Diversification Strategy (DIV)

3.1 Returns in USD

To assess international diversification in a common currency, we convert all local currency stock market returns into USD returns. We use total return indices (including dividends) and monthly exchange rates (USD per unit of local currency) for each country.
Following the formula introduced in the lecture, the USD return at time $t+1$ is calculated as the non-hedged return:

$$\begin{aligned} R_{t+1}^{\text{USD}} &= \frac{P_{t+1}S_{t+1}}{P_t S_t} - 1 \\ &= \left(\frac{P_{t+1}-P_t}{P_t} + 1\right)\left(\frac{S_{t+1}-S_t}{S_t} + 1\right) - 1 \\ &= (R_{t+1}^{\text{local}} + 1)(R_{t+1}^{\text{FX}} + 1) - 1 \end{aligned}$$

where R_{t+1}^{local} is the return in local currency (including dividends), and R_{t+1}^{FX} the exchange rate return with S_t denoting the exchange rate (USD per unit of foreign currency) at time t.
Because this transformation requires data from both t and $t+1$, the USD return series begins in June 2002 and ends in December 2024, even though local and FX data start in May 2002.
Figure 1 shows the evolution of cumulative returns for each market. Over most of the sample, the Australian equity market leads in USD terms, although it is closely overtaken by the US market toward the end of the period. Japan follows as the third-best performer, while France and Germany deliver moderate returns. In contrast, Switzerland and the United Kingdom exhibit the weakest performance in cumulative USD terms. These disparities highlight the combined impact of equity performance and currency movements, underscoring the importance of accounting for exchange rate exposure in international portfolio evaluation.

[1] All code and data used in this project are available at https://github.com/matthias-wyss/Investments-project.

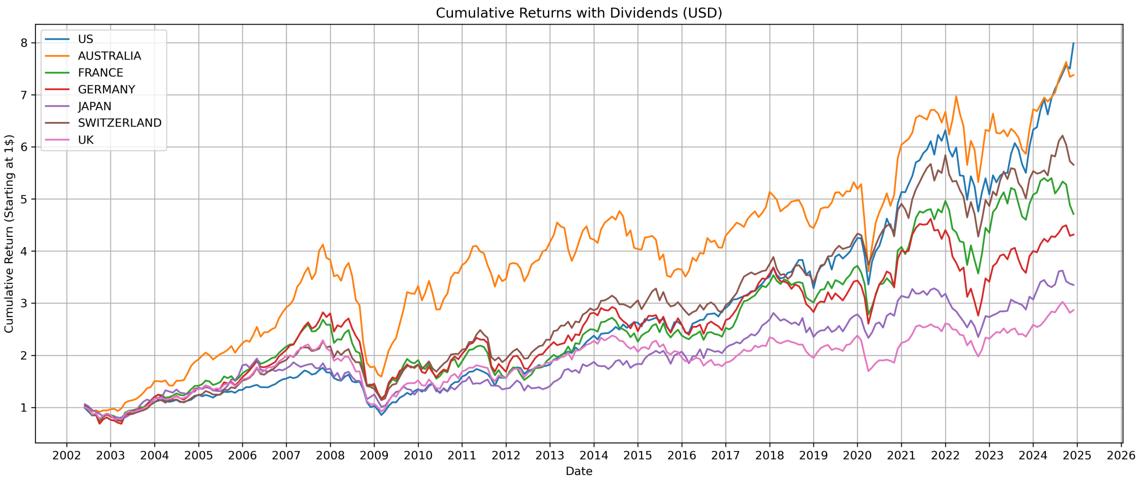

Figure 1: Cumulative returns in USD for $1 invested, June 2002 to December 2024.

3.2 Currency-Hedged Index Returns

To evaluate the performance of international equity markets without the influence of currency fluctuations, we compute currency-hedged returns. A currency-hedged return represents the USD return of an investment in a foreign equity index where the exchange rate risk is neutralized by shorting the corresponding foreign currency.

Formally, the currency excess return to holding one unit of foreign currency (e.g., EUR) is defined as:

$$X_{t+1}^{\mathrm{EUR}} = \frac{S_{t+1}}{S_t}(1 + r_t^{\mathrm{EUR}}) - (1 + r_t^{\mathrm{USD}}),$$

where S_t is the spot exchange rate (USD per EUR) at time t, r_t^{EUR} is the 3-month euro interbank rate, and r_t^{USD} is the 3-month USD interbank rate. This represents the excess return in USD from borrowing in USD, converting to euros, investing at the euro interest rate, and converting back to USD.

Given the unhedged USD return R_{t+1}^{USD} on a foreign equity index (which includes both equity and FX effects), we define the hedged return by subtracting the currency excess return:

$$R_{t+1}^{\mathrm{hedged}} = R_{t+1}^{\mathrm{USD}} - X_{t+1}^{\mathrm{currency}}.$$

Figure 2 displays the cumulative currency-hedged returns in USD for each market over the period from June 2002 to December 2024. By removing the effects of exchange rate fluctuations through currency hedging, the figure isolates the underlying equity market performance. After hedging, Switzerland and Japan emerge by far as the top-performing markets, followed by France, Germany and the United States. The United Kingdom delivers a slightly positive performance, while Australia ends the sample with losses. This contrasts with the unhedged case, where Australia initially dominated due to currency strength. For the United States, no hedging adjustment was required since the returns (sourced directly from CRSP) are already in USD and do not entail foreign exchange exposure for a USD-based investor.

3.3 Currency Hedging and International Portfolio Diversification

We now examine the impact of currency hedging on international portfolio diversification from the perspective of a US-based investor. Specifically, we compare three strategies for combining international equity exposures: (i) equal-weighting, (ii) risk-parity based on a rolling estimate of volatilities, and (iii) mean-variance optimization using rolling estimates of means and covariances. For each strategy, we compute the annualized mean return, annualized standard deviation (volatility), and Sharpe ratio, and compare results between unhedged and currency-hedged cases.

Table 1 summarizes the annualized mean return, annualized volatility, and Sharpe ratio for the three portfolio construction methods with and without currency hedging.

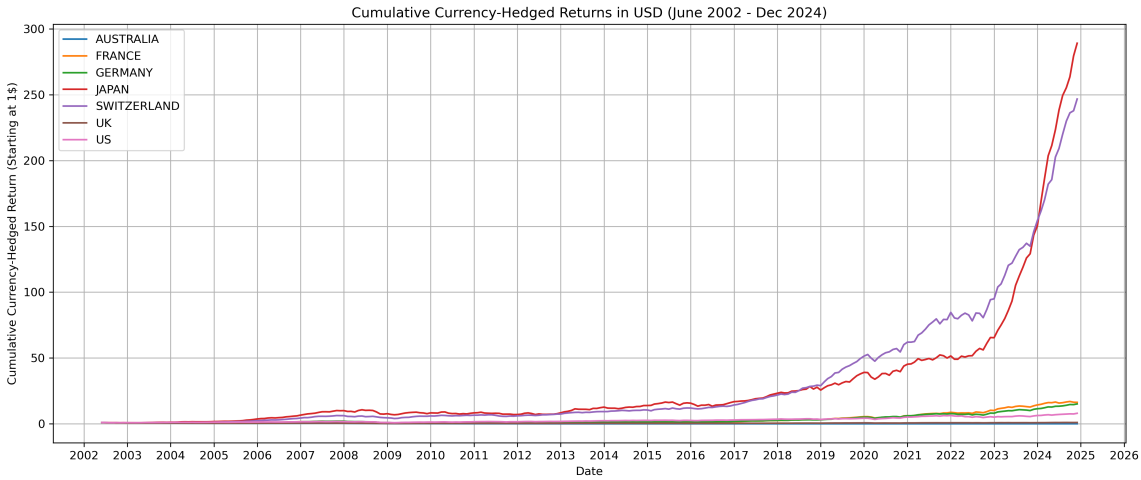

Figure 2: Cumulative currency-hedged returns in USD for $1 invested, June 2002 - December 2024.

Strategy	**Hedged Portfolio** Mean (%)	Std. Dev. (%)	Sharpe Ratio	**Unhedged Portfolio** Mean (%)	Std. Dev. (%)	Sharpe Ratio
Equal Weight	11.51	13.91	0.72	8.56	15.28	0.46
Risk Parity	10.72	13.99	0.66	6.01	15.52	0.29
Mean-Variance Optimal	2205.18	739.60	2.98	60.88	199.59	0.30

Table 1: Annualized mean return, annualized standard deviation, and Sharpe ratio for different international diversification strategies applied to currency-hedged and unhedged portfolios.

The equal-weighted portfolio allocates an identical weight to each country index. Under this naive diversification approach, the currency-hedged portfolio achieves a mean annual return of 11.5% with an annualized standard deviation of 13.9%, resulting in a Sharpe ratio of 0.72. In contrast, the unhedged portfolio delivers a lower mean return of 8.6% with a higher annualized volatility of 15.3%, leading to a Sharpe ratio of only 0.46. This suggests that currency hedging improves both return and risk-adjusted performance in the equal-weighted context.

The risk-parity approach assigns portfolio weights inversely proportional to the estimated rolling volatilities of the individual country returns over a 60-month rolling window. With currency hedging, this strategy achieves a mean return of 10.7% and an annualized standard deviation of 14.0%, yielding a Sharpe ratio of 0.66. Without hedging, the performance deteriorates, with a lower mean return of 6.0%, similar annualized volatility, and a Sharpe ratio of 0.29. Again, currency hedging significantly enhances performance.

The mean-variance optimal portfolio is constructed using rolling estimates of expected returns and the covariance matrix, optimizing weights to maximize the ex-ante Sharpe ratio. These estimates are also based on a 60-month rolling window. The resulting currency-hedged portfolio delivers an exceptionally high annualized return of 2205% with an annualized standard deviation of 739%, corresponding to a Sharpe ratio of 2.98. The unhedged version, while still superior to the other strategies, achieves only a 60% return with 199% annualized volatility and a Sharpe ratio of 0.30. The difference highlights how currency risk can obscure underlying return patterns and how its removal via hedging enables more precise allocation in optimal portfolio construction.

Overall, the results illustrate that currency hedging consistently improves portfolio performance across all diversification approaches. It reduces volatility, increases mean returns, and leads to substantially better Sharpe ratios. For a US investor seeking international exposure, these findings confirm the importance of accounting for exchange rate risk in portfolio design.

3.4 The DIV Strategy

Going forward, we define the DIV strategy as the return series of the currency-hedged risk-parity portfolio introduced earlier. This strategy will serve as the baseline for comparing a range of more sophisticated, dynamic portfolio strategies aimed at improving upon the simple DIV approach. The DIV return series spans from June 2007 to December 2024.

4 Equity Index Momentum Strategy (MOM)

4.1 Construction of Long-Short Momentum Portfolio

We construct a monthly long-short equity momentum strategy using currency-hedged equity index returns from seven countries (Australia, France, Germany, Japan, Switzerland, UK, US). At each time t, indexes are ranked by their cumulative return over the period $[t-12, t-1]$, denoted Rank_t^i. The portfolio allocates weights as

$$w_t^i = Z\left(\text{Rank}_t^i - \frac{N+1}{2}\right), \quad i = 1, \ldots, N,$$

For $N = 7$, this yields ranks from 1 to 7 and corresponding raw weights $\{-3, -2, -1, 0, 1, 2, 3\}$.
To ensure the portfolio is dollar-neutral with gross exposure equal to 1 for long (-1 for shorts), we rescale the raw weights by a normalization factor Z:

$$Z = \frac{1}{\sum_{i:\tilde{w}_{i,t}>0} \tilde{w}_{i,t}} = \frac{1}{6},$$

so that the final weight is $w_{i,t} = Z \cdot \tilde{w}_{i,t}$.
Monthly returns are computed by weighting each index's return with w_t^i, and we also decompose the strategy into its long and short components.

$$r_t^{\text{MOM}} = \sum_{i=1}^{N} w_{i,t} \cdot r_{i,t}.$$

We also track the contributions from the long and short legs separately:

$$r_t^{\text{LONG}} = \sum_{i:w_{i,t}>0} w_{i,t} \cdot r_{i,t}, \quad r_t^{\text{SHORT}} = \sum_{i:w_{i,t}<0} w_{i,t} \cdot r_{i,t}.$$

4.2 Performance Statistics and Statistical Significance Tests

We report below the annualized return, volatility, and Sharpe ratio for the overall MOM strategy, as well as its long and short legs:

Portfolio	**Annualized Return**	**Annualized Volatility**	**Sharpe Ratio**
MOM (Overall)	27.58%	8.14%	3.39
MOM Long Leg	26.26%	13.98%	1.77
MOM Short Leg	1.32%	13.67%	-0.01

Table 2: Annualized performance statistics of the MOM strategy and its components.

- The strategy delivers an annualized mean return of 27.6% with an annualized volatility of 8.1%, yielding a Sharpe ratio of 3.39. A Sharpe ratio above 3 is exceptionally high in asset-pricing practice, indicating that the risk-adjusted performance of the long–short momentum portfolio is very strong.

- The long side alone earns 26.3% annually with a Sharpe of 1.77, whereas the short side is roughly flat (1.3% return, Sharpe ratio ≈ -0.01). Hence almost the entirety of the momentum premium comes from buying past winners; shorting past losers neither helps nor hurts on average. The overall portfolio's volatility (8.1%) is much lower than the volatility of each leg ($\approx 14\%$) because the long and short positions partially offset one another, providing natural hedging benefits.

A one-sample t-test of the monthly momentum returns against the null $H_0\colon \mathbb{E}[r_t^{\text{MOM}}] = 0$ gives

$$t = 15.751, \qquad p\text{-value} = 0.000.$$

With such a large t-statistic (far above the 1.96 critical value at the 5% level) we reject H_0 decisively. The momentum strategy's mean return is statistically different from zero at any conventional significance level.

4.3 Regression of MOM Returns on DIV Returns and Interpretation

We regress MOM strategy returns on the DIV factor:

$$r_t^{\text{MOM}} = \alpha + \beta \cdot r_t^{\text{DIV}} + \varepsilon_t.$$

The regression results are:

Coefficient	**Estimate**	**Std. Error**	**p-value**
α (Intercept)	0.0193	0.002	0.000
β (DIV)	-0.0343	0.039	0.378

Table 3: OLS regression of MOM returns on DIV returns.

- The intercept is $\alpha = 0.0193$ per month ($\approx 1.9\%$), with a Newey–West standard error of 0.002 and a p-value < 0.001. This highly significant alpha means the momentum strategy earns a sizable return that is not explained by exposure to the DIV factor.

- The slope is $\beta = -0.0343$ with a p-value of 0.378, so the loading on DIV is statistically indistinguishable from zero. In economic terms, momentum returns are orthogonal to the DIV factor; there is no evidence that the strategy is merely capturing the same risk premium.

The long–short momentum portfolio delivers a large and statistically robust excess return with modest volatility and virtually no exposure to the DIV factor. Because the positive performance is driven almost exclusively by the long positions, the data suggest that international equity index momentum is mainly a "winner continuation" phenomenon rather than a "loser reversal." To conclude, the MOM strategy captures a return dimension orthogonal to DIV, making it a potentially attractive diversification instrument for investors exposed to DIV strategies.

5 Equity Index Long Term Reversal Strategy (REV)

5.1 Construction of Long-Short Reversal Portfolio

We construct a long-short reversal (REV) strategy based on the past performance of currency-hedged equity indexes. Specifically, at each month t, we compute the cumulative return of each index over the 5-year window ending 12 months ago, i.e., over the period $[t-12, t-1]$. Indexes are then ranked in ascending order based on these 5-year lagged returns, and their ranks are denoted by $\text{Rank}_{i,t}$. The portfolio weights are defined as:

$$w_{i,t} = Z \cdot \left(\frac{N+1}{2} - \text{Rank}_{i,t} \right)$$

where $N = 7$ and $Z = \frac{1}{6}$ (Z is computed in the same manner as in part 4.1).

The REV portfolio return at time t is given by:

$$r_t^{\text{REV}} = \sum_{i=1}^{N} w_{i,t} \cdot r_{i,t}$$

where $r_{i,t}$ denotes the monthly return of index i.

5.2 Performance Statistics and Statistical Significance Tests

We compute the annualized return, volatility, and Sharpe ratio for the overall strategy, as well as for its long and short legs:

Portfolio	**Annualized Return**	**Annualized Volatility**	**Sharpe Ratio**
REV (Total)	-20.55%	7.45%	-2.76
REV Long Leg	-1.27%	14.37%	-0.17
REV Short Leg	-19.28%	14.04%	-1.46

Table 4: Annualized performance statistics of the REV strategy and its components.

- The total reversal portfolio loses roughly 20% per year with a Sharpe ratio of -2.8, indicating a large negative risk-adjusted payoff.
- Both legs underperform, but most of the loss originates from the long positions in past losers (the "contrarian" buy-side), while the short book is only mildly negative. Hence betting on a 12–1-month mean-reversion signal in international indices appears to be a poor stand-alone idea over the sample.

A one-sample t-test of monthly REV returns under $H_0 : \mathbb{E}[r_t^{\text{REV}}] = 0$ yields

$$t = -11.567, \qquad p\text{-value} < 0.001$$

rejecting the null at any conventional level. The negative mean return is highly significant.

5.3 Regression of REV Returns on DIV Returns and Interpretation

To assess whether the REV strategy captures distinct sources of return relative to the dividend yield (DIV) strategy, we regress REV returns on DIV returns using OLS:

$$r_t^{\text{REV}} = \alpha + \beta \cdot r_t^{\text{DIV}} + \varepsilon_t$$

The regression results are:

Coefficient	**Estimate**	**Std. Error**	**p-value**
α (Intercept)	-0.0174	0.002	0.000
β (DIV)	0.0252	0.037	0.494

Table 5: OLS regression of REV returns on DIV returns.

- The intercept, $\alpha = -1.74\%$ per month, is statistically significant. The strategy earns a negative abnormal return that is not explained by the DIV factor.
- The slope, $\beta = 0.0252$, is economically small and statistically indistinguishable from zero ($p \approx 0.49$). Thus REV returns are essentially uncorrelated with DIV.

- The regression R^2 is only 0.2%, confirming that DIV carries virtually no explanatory power for reversal profits or losses.

The international reversal signal produces large, statistically significant losses. These losses cannot be attributed to dividend-yield exposure, implying that the contrarian tilt represents an independent and unfavourable source of return. While REV's low correlation with DIV may offer diversification when paired with other factors, its strongly negative alpha means that holding such a strategy unhedged would likely erode portfolio performance.

6 Currency Carry Strategy (CARRY)

6.1 Construction of Long-Short Currency Carry Portfolio

We construct a long-short currency carry strategy using the monthly 3-month interbank rates of five currencies (AUD, CHF, EUR, GBP, JPY) relative to the USD. For each month t, we compute the carry for each currency i as:

$$\text{carry}_i^t = r_i^t - r_{\text{USD}}^t$$

We then rank the currencies according to their interest rate differential (carry), and compute the weights:

$$w_i^t = Z \cdot \left(\text{Rank}_i^t - \frac{N+1}{2}\right)$$

For $N = 5$, this yields ranks from 1 to 5 and corresponding raw weights $\{-2, -1, 0, 1, 2\}$.
To ensure the portfolio is dollar-neutral with gross exposure equal to 1 for long (-1 for shorts), we rescale the raw weights by a normalization factor Z:

$$Z = \frac{1}{\sum_{i:\tilde{w}_{i,t}>0} \tilde{w}_{i,t}} = \frac{1}{3},$$

The portfolio return at $t+1$ is given by:

$$R_{t+1}^{\text{CARRY}} = \sum_{i=1}^{N} w_i^t X_i^{t+1}$$

where X_i^{t+1} is the excess return of currency i over the USD from t to $t+1$.

6.2 Performance Statistics and Statistical Significance Tests

We report below the annualized return, volatility, and Sharpe ratio for the overall CARRY strategy, as well as its long and short legs:

Portfolio	Annualized Return	Annualized Volatility	Sharpe Ratio
CARRY (Total)	36.69%	8.91%	4.12
CARRY Long Leg	16.97%	9.78%	1.58
CARRY Short Leg	19.73%	9.00%	2.03

Table 6: Annualized performance statistics of the CARRY strategy and its components.

- The total portfolio earns an exceptional 36% per year with only 8.9% volatility, producing a Sharpe ratio above 4, a level rarely observed in liquid asset-pricing strategies.
- Both sides of the trade add value. The short book (short low-carry indices) contributes roughly 20 % annually with a Sharpe of 2.03, while the long book (long high-carry indices) adds 17% with a Sharpe of 1.58. Hence the premium is well balanced across legs.

A one-sample t-test of monthly CARRY returns under $H_0 : \mathbb{E}[r_t^{\text{CARRY}}] = 0$ yields

$$t = 19.579, \qquad p < 0.001$$

decisively rejecting the null hypothesis of zero mean. The strategy's positive return is not only large but also statistically very robust.

6.3 Regression of CARRY Returns on DIV Returns and Interpretation

We regress CARRY strategy returns on DIV returns using OLS:

$$r_t^{\text{CARRY}} = \alpha + \beta \cdot r_t^{\text{DIV}} + \varepsilon_t$$

The regression results are:

Coefficient	**Estimate**	**Std. Error**	**p-value**
α (Intercept)	0.0238	0.002	0.000
β (DIV)	0.0666	0.042	0.112

Table 7: OLS regression of CARRY returns on DIV returns.

- The intercept, $\alpha = 2.38\%$ per month, is highly significant, confirming that the carry premium is not subsumed by the DIV factor.
- The slope, $\beta = 0.07$, is positive but statistically insignificant at the 10 % level ($p = 0.11$). Thus the strategy's payoff is largely orthogonal to dividend yield shocks.
- The regression explains very little variation in returns ($R^2 \approx 1\%$), reinforcing the conclusion that CARRY taps a distinct source of risk/return.

The cross-sectional equity carry trade delivers a large, statistically significant return with low volatility and minimal exposure to the DIV factor. Its high Sharpe ratio and near-zero correlation with DIV suggest that it constitutes a powerful, independent factor. Consequently, an investor focused on dividend yield could improve risk-adjusted performance by adding the CARRY strategy, which appears to provide uncorrelated alpha.

7 Currency Dollar Strategy (DOLLAR)

7.1 Construction of Long-Dollar vs Rest of World Portfolio

We construct a long-dollar strategy by shorting an equally weighted basket of foreign currencies (AUD, CHF, EUR, GBP, JPY) against the USD. The return of the DOLLAR strategy from t to $t+1$ is given by:

$$R_{t+1}^{\text{DOLLAR}} = -\frac{1}{N} \sum_{i=1}^{N} X_i^{t+1}$$

where X_i^{t+1} is the excess return on currency i over the USD during period t to $t+1$, and $N = 5$ is the number of foreign currencies traded. The minus sign in the DOLLAR strategy weights reflects that we hold an equally sized short position in every foreign currency, so that a negative excess return X_{t+1} (foreign currency depreciation) translates into a positive portfolio payoff.

7.2 Performance Statistics and Statistical Significance Tests

The table below reports the annualized performance statistics for the DOLLAR strategy:

Portfolio	Annualized Return	Annualized Volatility	Sharpe Ratio
DOLLAR	3.08%	7.74%	0.40

Table 8: Annualized performance statistics of the DOLLAR strategy.

To assess statistical significance, we conduct a one-sample t-test under the null hypothesis $H_0 : \mathbb{E}[r^{\text{DOLLAR}}] = 0$. The results are:

$$t = 1.892, \quad p\text{-value} = 0.060$$

The null hypothesis cannot be rejected at the 5% significance level. This suggests that the DOLLAR strategy's mean return is not statistically significantly different from zero at conventional levels, despite being positive on average.

7.3 Regression of DOLLAR Returns on DIV Returns and Interpretation

We examine the relationship between the DOLLAR and DIV strategies by regressing the DOLLAR returns on DIV returns using OLS:

$$r_t^{\text{DOLLAR}} = \alpha + \beta \cdot r_t^{\text{DIV}} + \varepsilon_t$$

The estimated regression coefficients are reported below:

Coefficient	Estimate	Std. Error	p-value
α (Intercept)	0.0046	0.001	0.002
β (DIV)	-0.0238	0.036	0.510

Table 9: OLS regression of DOLLAR returns on DIV returns.

The regression reveals no statistically significant relationship between the DOLLAR and DIV strategies ($p = 0.510$ for β), and the R^2 is close to zero. This suggests that the DOLLAR strategy captures a different risk premium from the DIV strategy. Therefore, from a diversification standpoint, a DIV investor could benefit from including the DOLLAR strategy in their portfolio, as it potentially offers uncorrelated alpha.

8 Optimal Fund Portfolio Return (STRAT)

8.1 FUND portfolio combining DIV and T-Bill

We first construct a simple two-asset portfolio combining the diversified stock-index strategy (DIV) and the 1-month U.S. T-Bill. The goal is to determine the weight a on DIV such that the annualized volatility of the overall fund reaches the target of 15%. Since T-Bills are considered risk-free and the volatility of DIV is known from Part 3.3, we have:

$$\sigma(R_{FUND}) = a\sigma(R_{DIV}) = 15\% \implies a = \frac{0.15}{\sigma(R_{DIV})}$$

Calculations performed in the notebook give $a \approx 1.07$.

8.2 Combining MOM, REV, CARRY, DOLLAR via Risk-Parity

To enhance the portfolio, we introduce an overlay component combining four dynamic strategies: MOM, REV, CARRY, and DOLLAR. We compute their weights using a risk-parity allocation method based on rolling volatility estimates, as in Part 3.3. The resulting aggregated return is denoted R_{STRAT} and will serve as a new building block in the optimized portfolio.
The monthly returns and corresponding weights are stored for later use.

8.3 Mean-Variance Optimization for Fund: Solving for b and c

We now compute the optimal combination of the baseline and overlay components using a mean-variance framework. The objective is to find weights b and c such that the fund return

$$R_{\text{FUND}} = R_{\text{TBill}} + b(R_{\text{DIV}} - R_{\text{TBill}}) + cR_{\text{STRAT}}$$

meets the volatility target of 15% while maximizing expected return.
The optimization algorithm, similar to that in Part 3.3, yields a portfolio with initial standard deviation σ, which we scale to match the 15% target via $a = \frac{\sigma}{0.15}$, resulting in $a \approx 28.94$.
This gives $b \approx 0.41$ and $c \approx 4.56$ on average. Using these values, we reconstruct the final portfolio. The average weights assigned to each component are summarized below:

Strategy	Mean Weight
MOM	0.9608
REV	0.9867
CARRY	1.2764
DOLLAR	1.3343
TBILL	0.5882
DIV	0.4118

Table 10: Mean weight of the different strategies for the final R_{FUND}

8.4 Performance Comparison and Cumulative Return Graph

We now compare the performance of the simple (T-Bill + DIV) strategy from Part 8.1 with the enhanced strategy incorporating STRAT from Part 8.2 and 8.3. The figure below shows the cumulative return of 1$ invested in each approach:

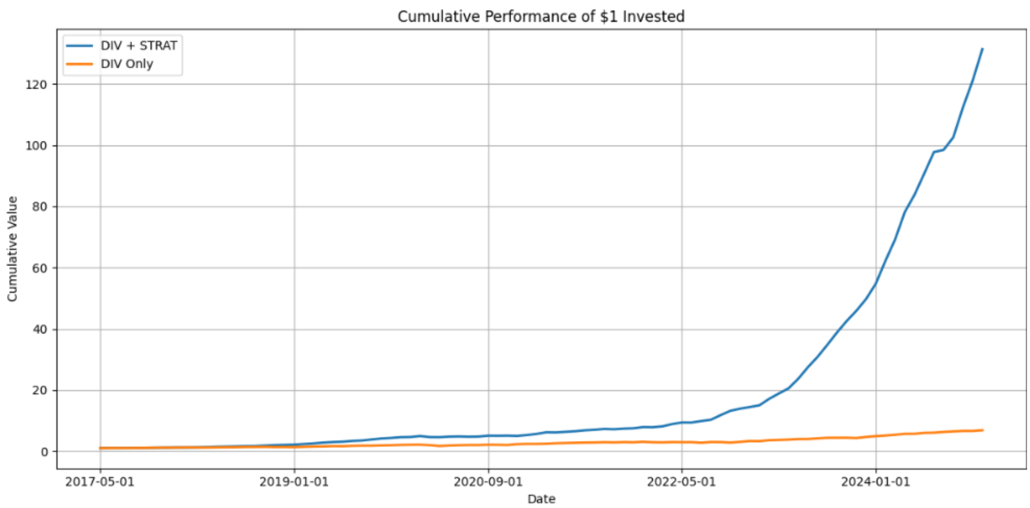

Figure 3: Cumulative performance for 1$ invested for both strategies

The table below compares the key performance metrics between the simple DIV-only strategy and the optimized portfolio that incorporates the STRAT overlay:

Strategy	Annualized Return	Annualized Volatility	Sharpe Ratio
DIV-only	11.41%	15.00%	0.66
DIV + STRAT	64.13%	15.33%	4.18

Table 11: Performance comparison between baseline and optimized strategies

This comparison clearly shows that incorporating the dynamic STRAT overlay significantly enhances the fund's performance, delivering substantially higher returns and Sharpe ratio while maintaining a similar level of volatility. This improvement highlights the benefits of diversifying across multiple risk factors and strategies beyond the basic DIV exposure.
Note that the slight deviation from the 15% volatility target arises from assuming T-Bill returns are perfectly risk-free. In practice, even T-Bills exhibit minor volatility, which explains this small discrepancy.

9 Performance and risk analysis for the Fund strategy

9.1 Regression on Fama-French 5 Factors: Beta Estimates and Significance

To assess how much of the strategy's returns can be explained by standard U.S. equity risk premia, we run an OLS regression of the fund's excess returns on the Fama-French 5 research factors (Mkt-RF, SMB, HML, RMW, CMA):

$$r_t^{\text{Fund}} = \alpha + \beta_{\text{Mkt-RF}} \cdot \text{Mkt-RF}_t + \beta_{\text{SMB}} \cdot \text{SMB}_t + \beta_{\text{HML}} \cdot \text{HML}_t + \beta_{\text{RMW}} \cdot \text{RMW}_t + \beta_{\text{CMA}} \cdot \text{CMA}_t + \varepsilon_t$$

The regression output is summarized in the Table below:

Coefficient	Estimate	Std. Error	p-value
Constant (α)	+0.0551	0.0045	<0.001
$\beta_{\text{Mkt-RF}}$	−0.1193	0.0989	0.231
β_{SMB}	−0.1963	0.1884	0.300
β_{HML}	+0.5609	0.1663	0.001
β_{RMW}	+0.0549	0.2276	0.810
β_{CMA}	−0.7146	0.2414	0.004

Table 12: OLS regression of strategy excess returns on the Fama-French 5 factors.

The results show that the intercept is large and highly significant, indicating a strong positive abnormal return that is not explained by the five standard equity factors. Among the factor loadings, HML (value) and CMA (investment) are statistically significant, with positive exposure to value stocks and negative exposure to the conservative-investment factor (i.e., the strategy favors firms with aggressive investment profiles). Other factor exposures, including market (Mkt-RF), size (SMB), and profitability (RMW), are not statistically different from zero, suggesting no meaningful tilt in these dimensions. The model explains only a small fraction of the return variation, with an R^2 of 0.141 and an adjusted R^2 of 0.090, confirming that the U.S. Fama-French 5-factor model captures only a limited share of the strategy's dynamics.

9.2 Interpretation: Consistency with Efficiency, CAPM, and APT

To better understand the nature of the strategy's performance, we evaluate whether it aligns with the predictions of three major asset pricing paradigms: the Capital Asset Pricing Model (CAPM), the Arbitrage Pricing Theory (APT), and the Efficient Markets Hypothesis (EMH).

The CAPM posits that expected excess returns are fully explained by exposure to the market factor, such that the regression intercept (α) should be zero. In our CAPM regression, however, we find a highly significant and positive alpha ($\alpha = 0.054$, $t = 11.6$, $p < 0.001$) and a statistically insignificant market beta ($\beta_{\text{Mkt}} = -0.056$, $p = 0.55$). This result contradicts the CAPM's central implication and indicates that the strategy's returns are not explained by systematic market risk alone.

The APT generalizes the CAPM by allowing for multiple priced sources of systematic risk. In equilibrium, if all relevant risk factors are included, most assets should exhibit no abnormal return (i.e., $\alpha \approx 0$). The significant alpha we observe suggests that important risk factors are missing from the regression. These could include exposures to global equity risk, FX carry, international momentum, liquidity shocks, or tail-risk compensation. While our results are inconsistent with a fully specified APT model, they do not contradict the theory itself, rather they point to an incomplete factor specification.

According to the Efficient Markets Hypothesis (EMH), once all known sources of risk are properly accounted for, no strategy should consistently earn statistically significant abnormal returns. The persistent and significant alpha observed here could thus reflect either (i) compensation for omitted but legitimate sources of risk, in line with EMH, or (ii) the presence of true market inefficiencies that persist due to frictions such as transaction costs, funding constraints, or behavioral biases. The EMH does not rule out abnormal returns, but predicts that they should disappear when all relevant risks are correctly modeled.

Overall, the strategy's performance appears inconsistent with the CAPM, potentially consistent with the APT if global and currency risk factors were included, and inconclusive with respect to EMH. The combination of a negative market beta and a large positive alpha also suggests attractive diversification properties, reinforcing the international and multi-asset insights discussed in Lectures 2–4.