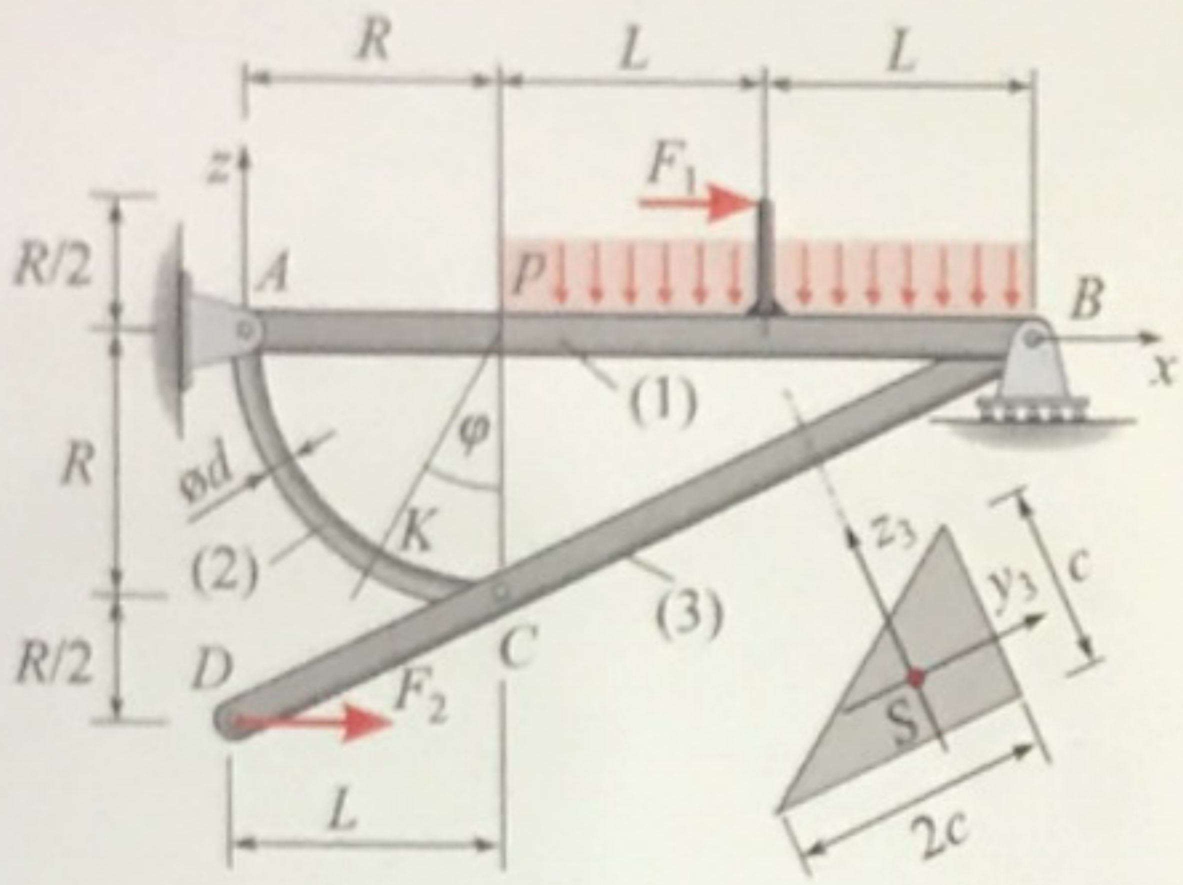
BME Gépészmérnöki Kar	SZILÁRDSÁGTAN	Név: Szigecsán Máté
Műszaki Mechanikai Tanszék	1. HÁZI FELADAT	Neptun kód: `QIA950`
2019/20 II.	Határidő: április 6. 14:00	Késedelmes beadás: ☐ Javítás: ☐
Nyilatkozat: Aláírásommal igazolom, hogy a házi feladatot saját magam készítettem el, az abban leírtak saját megértésemet tükrözik.		Aláírás:

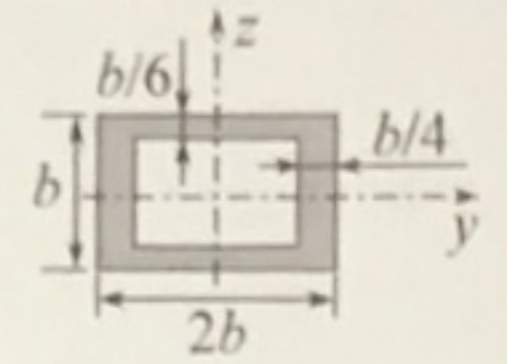
Csak a formai követelményeknek megfelelő feladatokat értékeljük! `http://www.mm.bme.hu/targyak/bsc/sziltan`

Feladatkitűzés

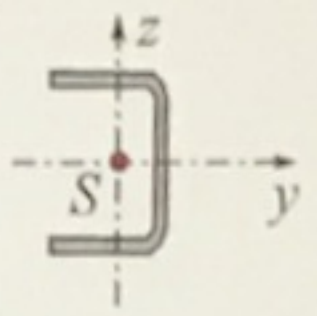
Az ábrán vázolt szerkezet mindhárom rúdja csuklósan kapcsolódik, anyaguk homogén, izotrop, lineárisan rugalmas. Az (1)-es rúd keresztmetszete az ábrán látható téglalap alakú zárt szelvény, a negyedkörív alakú (2)-es rúdé kör, míg a (3)-as rúdé háromszög. Az (1)-es rúd anyagára megengedett feszültség σ_{meg}.



1-es rúd:



U-szelvény:



Adatok

R [m]	L [m]	d [mm]	c [mm]	F_1 [kN]	F_2 [kN]	p [kN/m]	σ_{meg} [MPa]
0.25	0.35	50	36	3	1	3.50	110

(Rész)eredmények

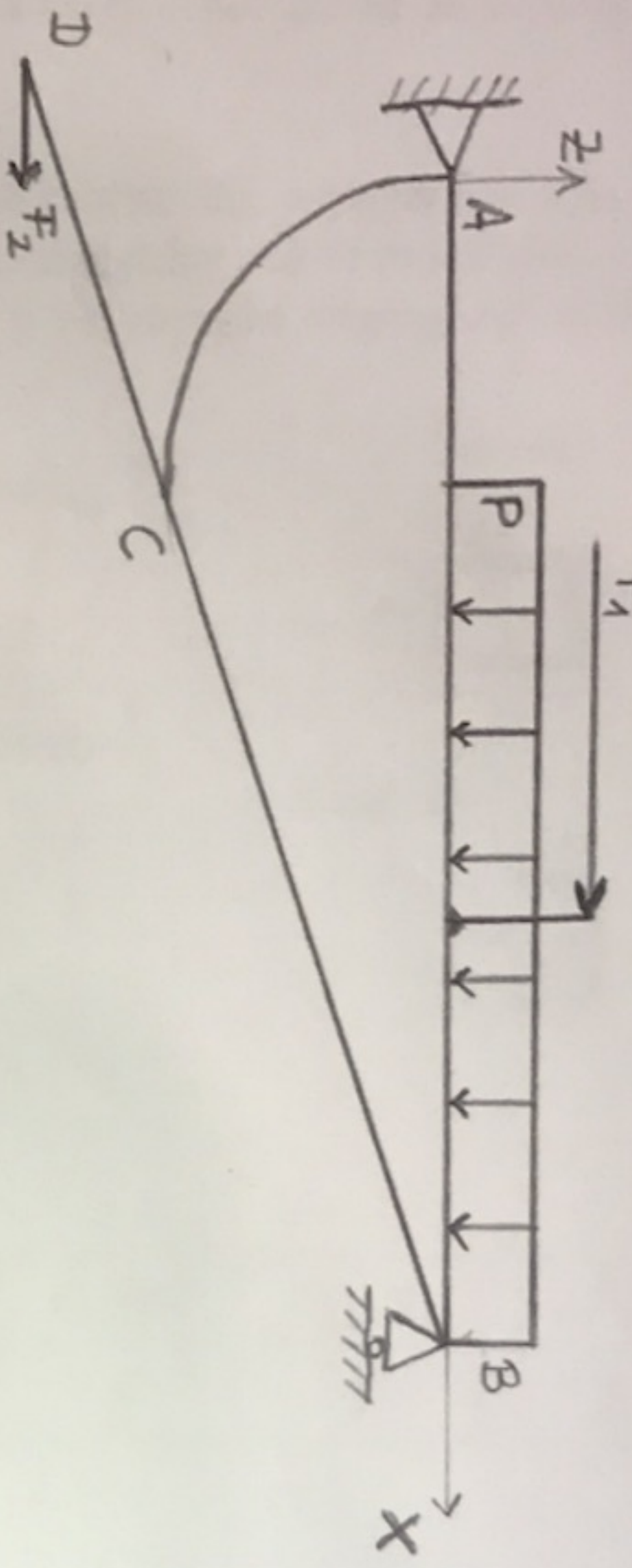
$\|A\|$ [kN]	$\|B\|$ [kN]	$M^{(1)}_{h,max}$ [kNm]	$K_{y,min}$ [cm^3]	b [mm]	Szelv.sorszám
4,1	1,547	-0,465	4,231	26	70
$\sigma^{(1)}_{max}$ [MPa]	$V^{(1)}_{max}$ [kN]	$\|\tau^{(1)}_{max}\|$ [MPa]	$\sigma^{(2)}_{K,max}$ [MPa]	$\sigma^{(3)}_{C,max}$ [MPa]	β_{zerus} [°]
103,023	1,942	7,38	3,47	56,47	-87,19

Pontozás

Minimumfeladat	Feladatok						Dokumentáció	Összesen
	4.	5.	6.	7.	8.	9.		
	/4	/2	/3	/4	/3	/4	/5	**/25**

SZIGECSÁN MÁTÉ
Q1A950

1,



$1 mm \triangleq 1 cm$
$1 kN \triangleq 1 cm$

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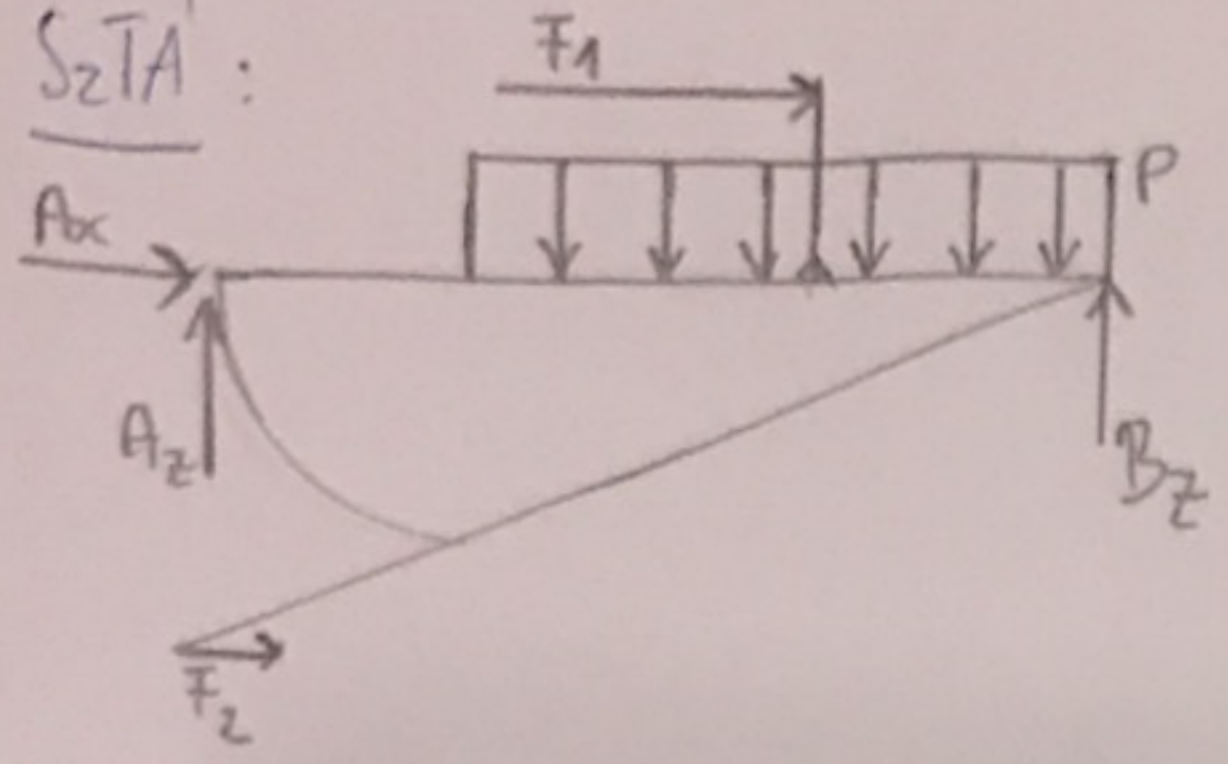
Q1A 950

- reakciókról:

<u>SZTÁ</u>:



$$\sum F_x = 0 \quad F_1 + F_2 + A_x = 0 \Rightarrow F_1 + F_2 = -A_x$$

$$A_x = -4 \; [kN]$$

$$\sum F_z = 0 \quad A_z + B_z - p \cdot 2L = 0 \Rightarrow A_z + B_z = p \cdot 2L$$

$$A_z + B_z = 2{,}45$$

$$A_z = 2{,}45 - B_z = 2{,}45 - 1{,}547$$

$$A_z = 0{,}903 \; [kN]$$

$$\sum M_A = 0 \quad -\frac{3R}{2} F_2 + \frac{R}{2} \cdot F_1 + p \cdot 2L\,(R+L) - B_z\,(R+2L) = 0$$

$$-0{,}375 + 0{,}575 + 1{,}47 = 0{,}95 B_z$$

$$B_z = 1{,}547 \; [kN]$$

$$|\underline{A}| = \sqrt{A_x^2 + A_z^2} = 4{,}1 \; [kN]$$

$$|\underline{B}| = |B_z| = 1{,}547 \; [kN]$$

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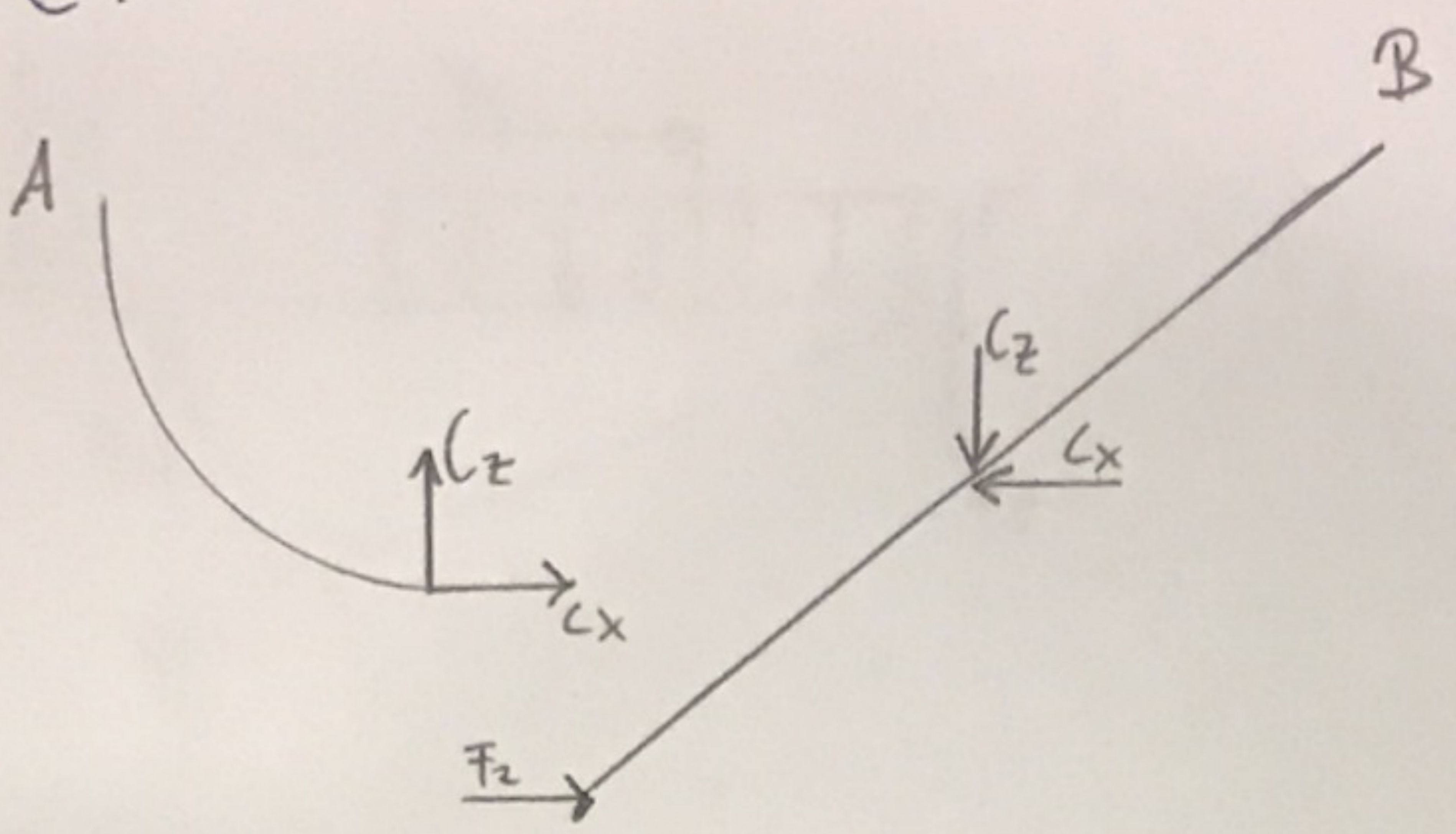
QIA950

2.)

C:



$\Sigma M_A = 0 \quad -C_x \cdot R - C_z \cdot R = 0$

$C_x = -C_z$

$\Sigma M_B = 0 \quad -F_z \cdot \frac{3R}{2} - C_z \cdot 2L + C_x \cdot R = 0$

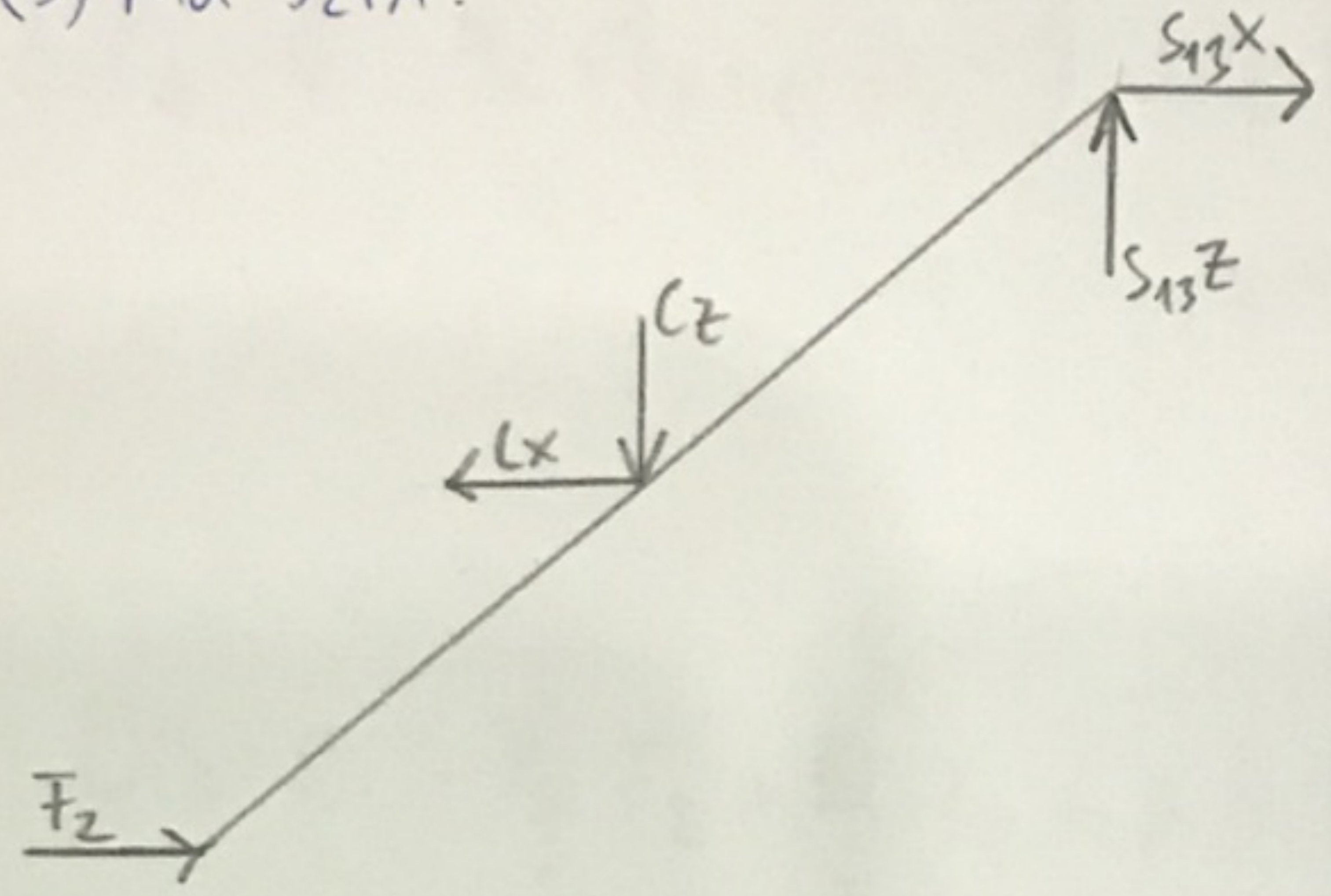
$-0{,}375 - 0{,}7\, C_z + 0{,}25\, C_x = 0$

$-0{,}375 = 0{,}95\, C_z$

$C_z = -0{,}395\,[kN] = -395\,[N]$

$C_x = 0{,}395\,[kN] = 395\,[N]$

(3) rúd SzTÁ:



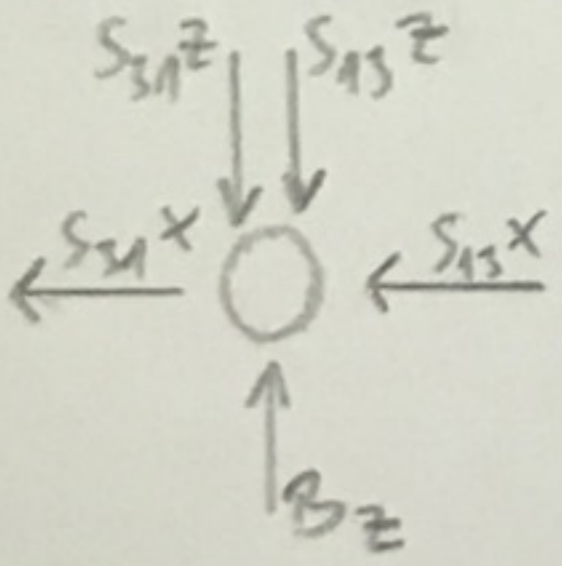
$\Sigma F_x = 0 \quad S_{13}x + F_z - C_x = 0 \Rightarrow 1000 - 395 = -S_{13}x$

$-605\,[N] = S_{13}x$

$\Sigma F_z = 0 \quad S_{13}z - C_z = 0 \Rightarrow S_{13}z = C_z$

$S_{13}z = -395\,[N]$

B csukló:



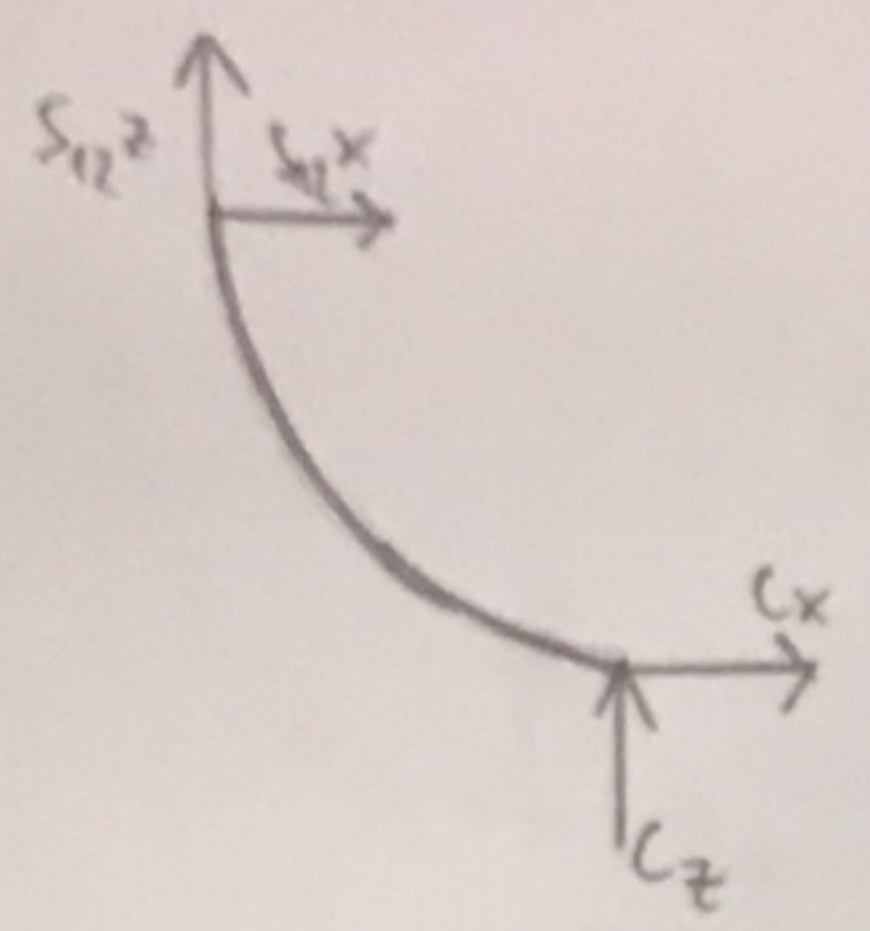
$\Sigma F_x = 0 \quad -S_{31}x - S_{13}x = 0 \Rightarrow S_{31}x = -S_{13}x$

$S_{31}x = 605\,[N]$

$\Sigma F_z = 0 \quad B_z - S_{13}z - S_{31}z = 0 \Rightarrow 1547 + 395 = S_{31}z$

$S_{31}z = 1942\,[N]$

(2) rúd SzTÁ:



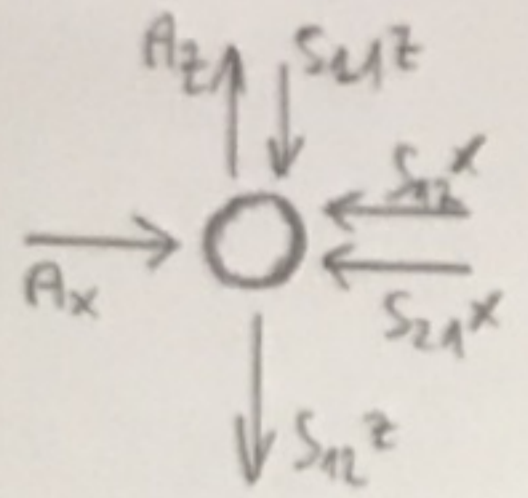
$\sum F_x=0 \quad C_x+S_{12}x=0$

$S_{12}x=-395$ [N]

$\sum F_z=0 \quad C_z+S_{12}z=0$

$S_{12}z=395$ [N]

A csukló:



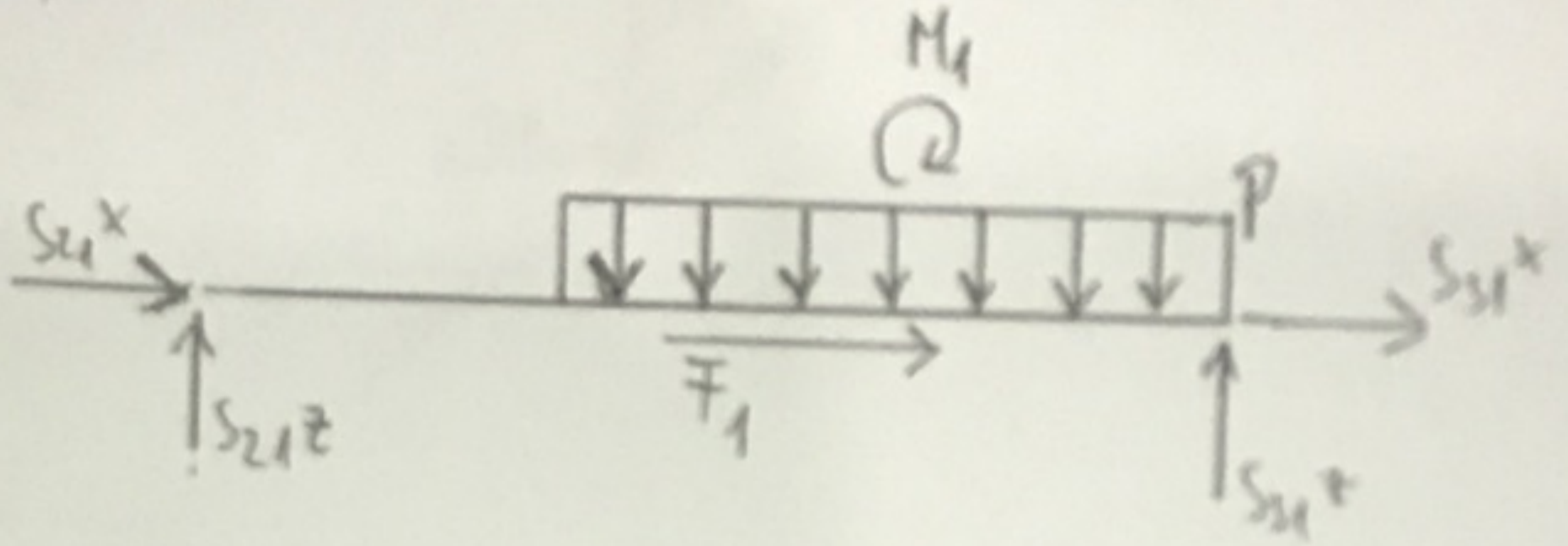
$\sum F_x=0 \quad A_x-S_{12}x-S_{21}x=0 \Rightarrow A_x-S_{12}x=S_{21}x$

$S_{21}x=-3605$ [N]

$\sum F_z=0 \quad A_z-S_{12}z-S_{21}z=0 \Rightarrow A_z-S_{12}z=S_{21}z$

$S_{21}z=508$ [N]

(1) rúd SzTÁ:



$\sum F_x=0 \quad F_1+S_{21}x+S_{31}x=0 \Rightarrow -F_1-S_{21}x=S_{31}x$

$-3000+3605=S_{31}x$

$S_{31}x=605$ [N]

$\sum F_z=0 \quad S_{21}z+S_{31}z-p\cdot 2L=0 \Rightarrow p\cdot 2L-S_{21}z=S_{31}z$

$2450-508=S_{31}z$

$S_{31}z=1942$ [N]

$$\sum M_A=0 \quad \underbrace{F_1\cdot\frac{R}{2}}_{M_1}+p\cdot 2L\cdot(R+L)-(2L\cdot R)S_{31}z=0$$

$$M_1=F_1\cdot\frac{R}{2}=3000\cdot\frac{0{,}25}{2}=375\ [Nm]$$

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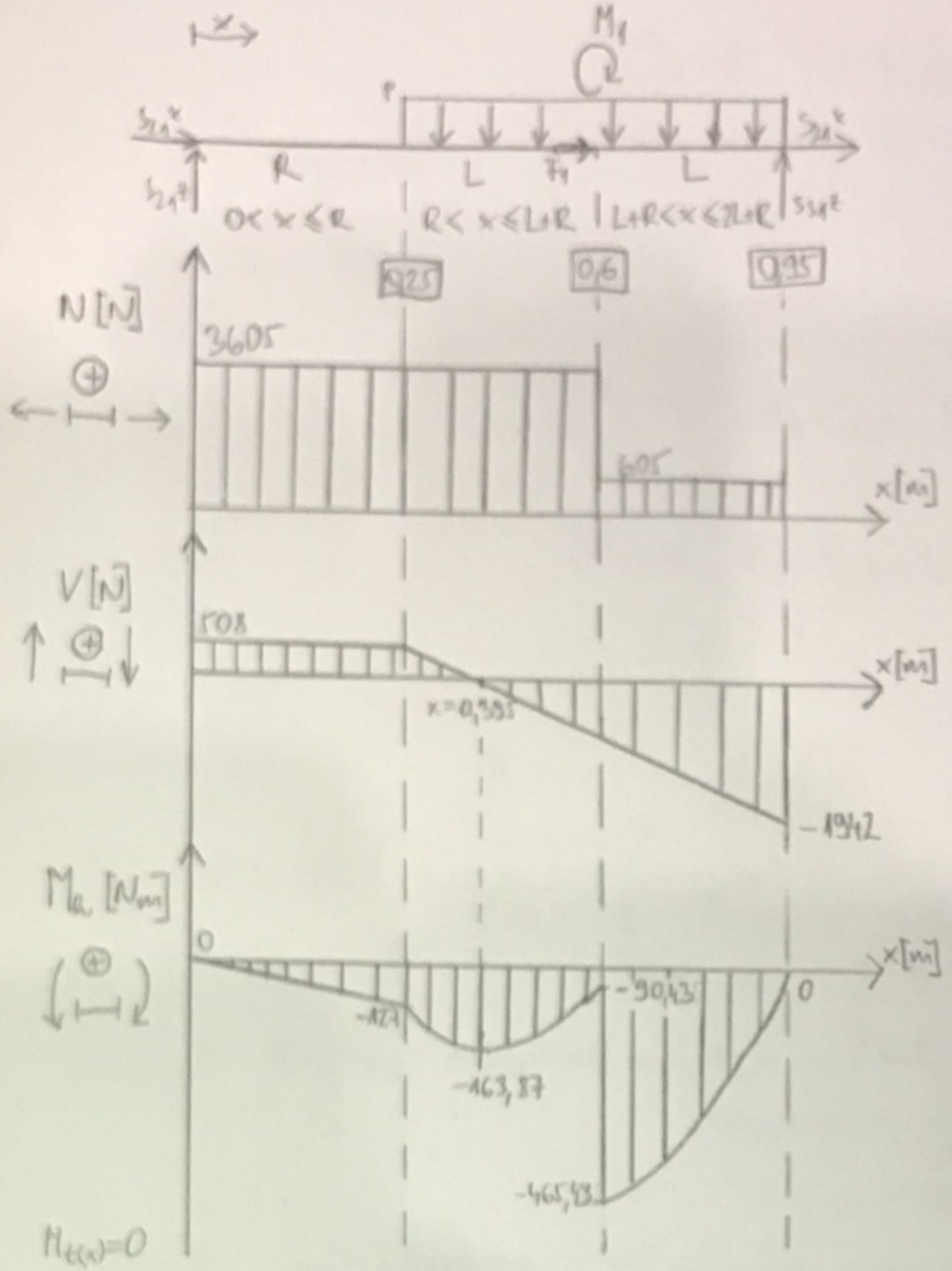
QU950

3.,

SzTÁ:



$0 \leq x \leq 2L+R$

$N_1 = N_2 = -S_{21}x = 3605$ [N]

$N_3 = -S_{21}x - F_1 = 605$ [N]

$V_1 = S_{21}z = 508$ [N]

$V_2 = V_3 = S_{21}z - p(x-R) =$

$= 508 - 3500x + 875 = 1383 - 3500x$ [N]

$1383 - 3500x = 0$

$\frac{1383}{3500} = 0{,}395 \Rightarrow x = 0{,}395$-nél 0 a fv.

$M_{h1} = -S_{21}z \cdot x = -508x$ [Nm]

$M_{h2} = -S_{21}z \cdot x + p \cdot \frac{(x-R)^2}{2} =$

$= -508x + 1750x^2 - 875x + 109{,}375 =$

$= 1750x^2 - 1383x + 109{,}375$ [Nm]

$M_{h3} = -S_{21}z \cdot x + p \cdot \frac{(x-R)^2}{2} - M_1 =$

$= 1750x^2 - 1383x - 265{,}625$ [Nm]

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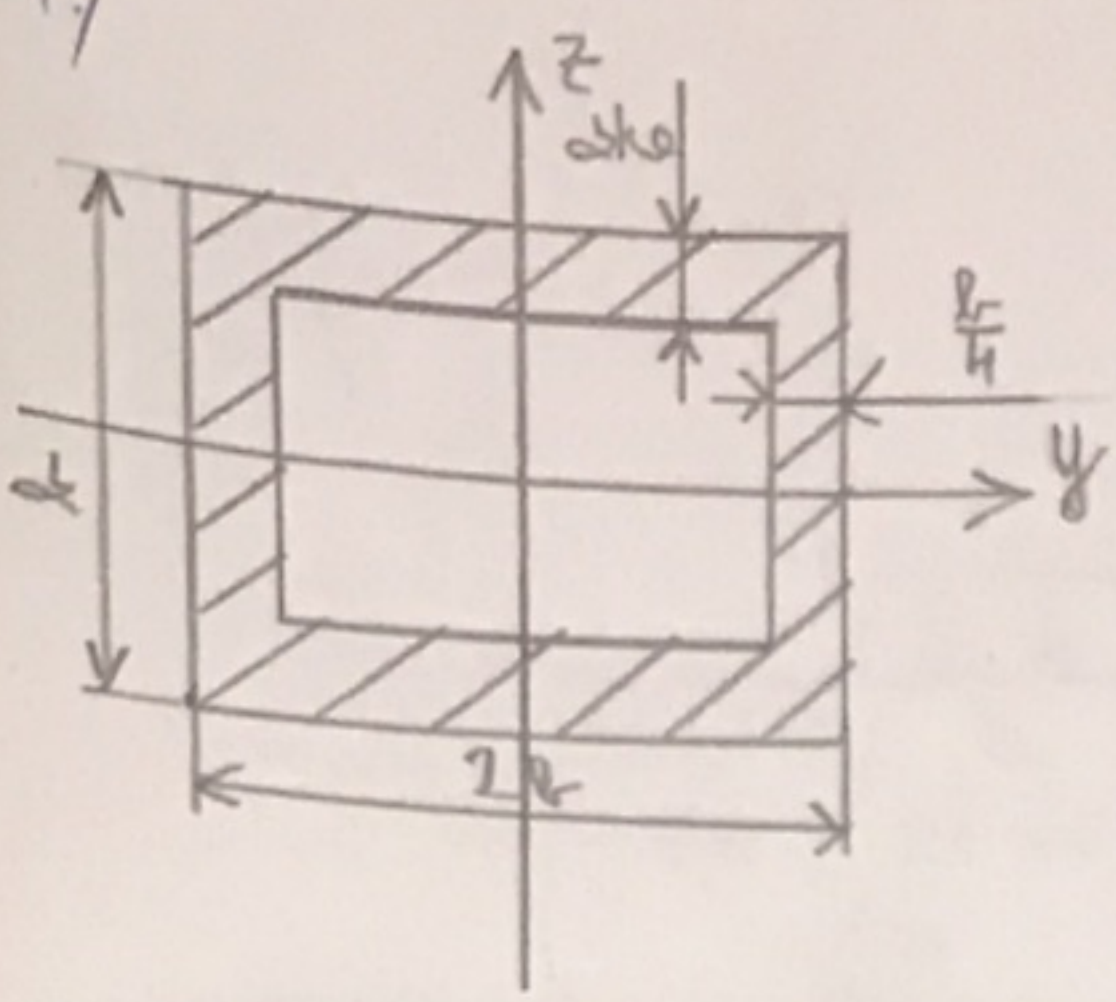
QIA9S0

4,



$|M_h^{(1)}\ max| = 465{,}4\ [Nm] = 4{,}654 \cdot 10^5\ [Nmm]$

$x = 0{,}6\ [m]$

$$K_{min} = \frac{M_{h\ max}}{\sigma_{meg}} = 4230{,}91\ [mm^3] = 4{,}231\ [cm^3]$$

$$K_{min} = \frac{I_x}{e} \qquad e = \frac{b}{2} \qquad I_y = \frac{b^3 \cdot 2b}{12} - \frac{\left(\frac{2}{3}b\right)^3 \cdot \frac{3b}{2}}{12}$$

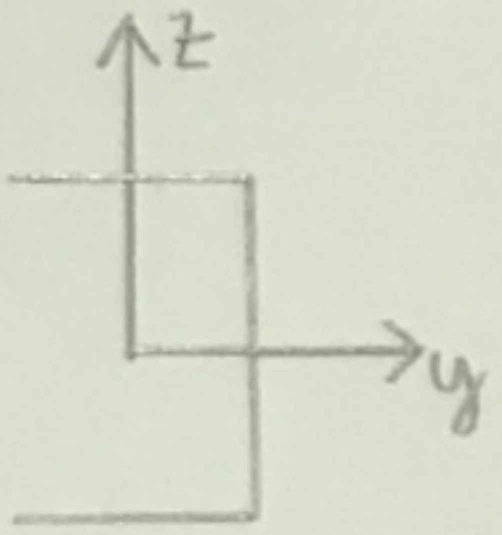
$$K_{min} = \frac{2I_y}{b}$$

$$b \cdot K_{min} = \frac{2b^4}{6} - \frac{4b^4}{54}$$

$$54\,K_{min} = 14\,b^3$$

$$b = \sqrt[3]{\frac{27}{7}K_{min}} = 25{,}36\,[mm] \Rightarrow b \simeq 26\,[mm]$$

5,



sorszám: 70

Az előző feladatban kiszámolt „K min" értéknél keresem a legkisebb eltéréssel nagyobb keresztmetszeti tényezőjű U-szelvényt a megadott táblázatból. Tiszta hajlítás esetén a zárt szelvény a 70-es számú U-szelvénnyel helyettesíthető az ábrán jelölt módon.

6.)

$b = 26\ [mm]$

$\hookrightarrow I_y = \frac{2b^4}{12} - \frac{4b^4}{108} = 76162{,}67 - 16925{,}04 = 59237{,}63\ [mm^4]$

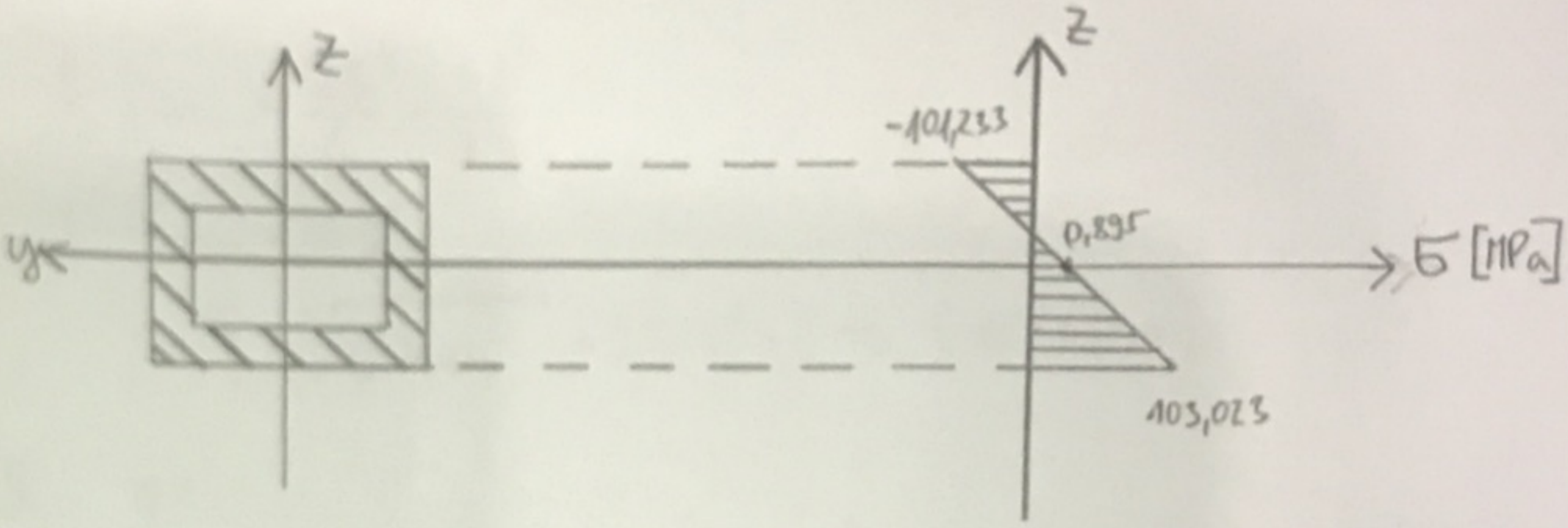
$A = b \cdot 2b - \frac{2b}{3} \cdot \frac{3b}{2} = 1352 - 676 = 676\ [mm^2]$

$\sigma(z) = \frac{N}{A} + \frac{M_{h\,max}^{(1)}}{I_y} \cdot z = \frac{605}{676} + \frac{-4{,}654 \cdot 10^5}{59237{,}63} \cdot z = 0{,}895 - 7{,}856\,z\ [MPa]$

$z_1 = \frac{b}{2} \rightarrow \sigma(z_1) = -101{,}233\ [MPa]$

$z_2 = -\frac{b}{2} \rightarrow \sigma(z_2) = 103{,}023\ [MPa] \Rightarrow$ ez a σ_{max}

$\sigma_{max} < \sigma_{meg}$ ✓



7.)

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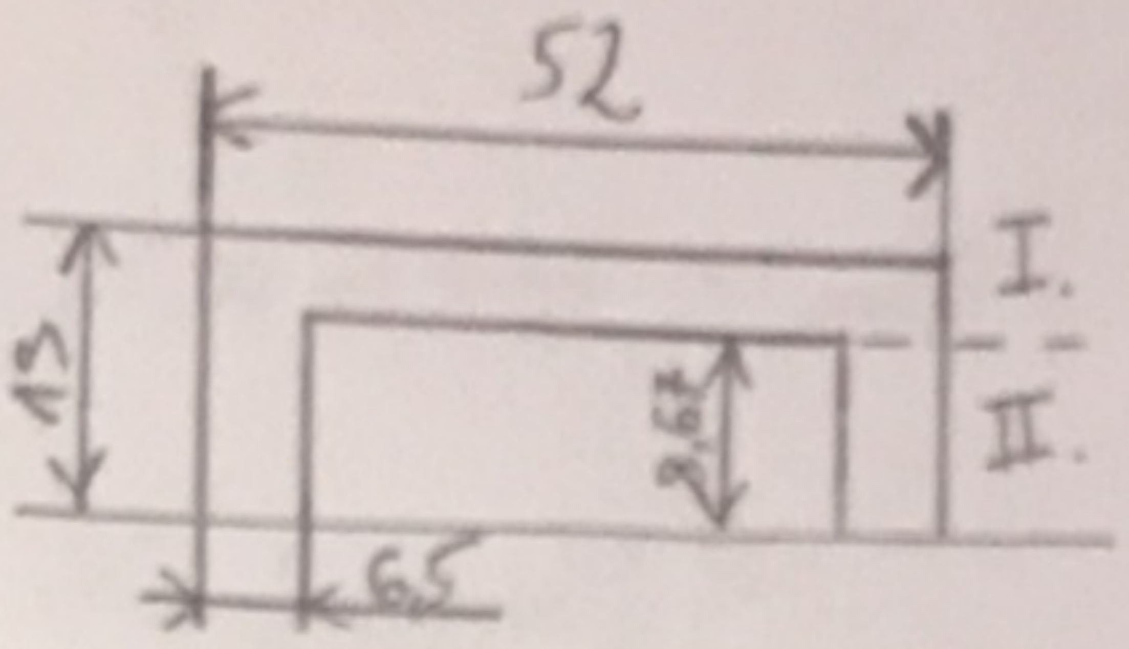
$V^{(1)}_{max} = -1942 \, [N]$

$x = 0{,}95 \, [m]$

$\tau_1(z) = \frac{V^{(1)}_{max} \cdot S(z)}{I_y \cdot a(z)}$



→ ott veszem kettő, ahol változik a falvastagság

I. rész: $a_1 = 52 \, [mm]$

$S_1 = A_1 \cdot z_1 = 26(13-z) \cdot \left(\frac{13+z}{2}\right) = 13(13-z)(13+z) = 26(169-z^2) = 4394 - 26z^2 \, [mm^3]$

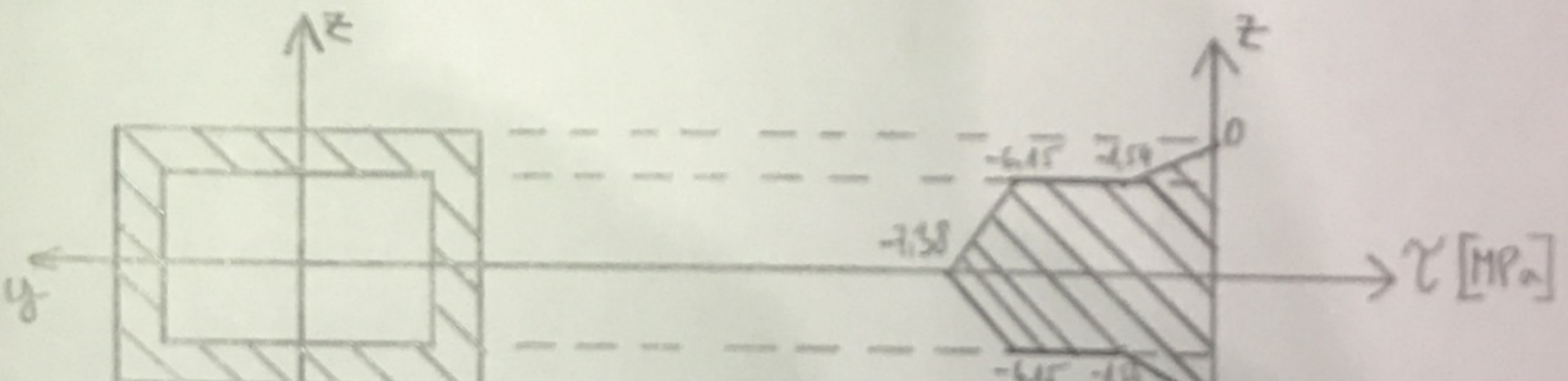
II. rész: $a_2 = 13 \, [mm]$

$S_2 = \frac{13}{2} \cdot (8{,}67 - z) \cdot \frac{8{,}67 + z}{2} + S_1(8{,}67) = \frac{13}{4}(8{,}67-z)(8{,}67+z) + S_1(8{,}67) =$

$= 6{,}5(75{,}17 - z^2) + 2439{,}61 = 488{,}61 - 6{,}5z^2 + 2439{,}61 = 2928{,}22 - 6{,}5z^2 \, [mm^3]$

$\tau_1 = \frac{-1942 \cdot (4394 - 26z^2)}{59237{,}63 \cdot 52} = \frac{-8533148 + 50492z^2}{3080356{,}76} = -2{,}77 + 1{,}639 \cdot 10^{-2} z^2 \, [MPa]$

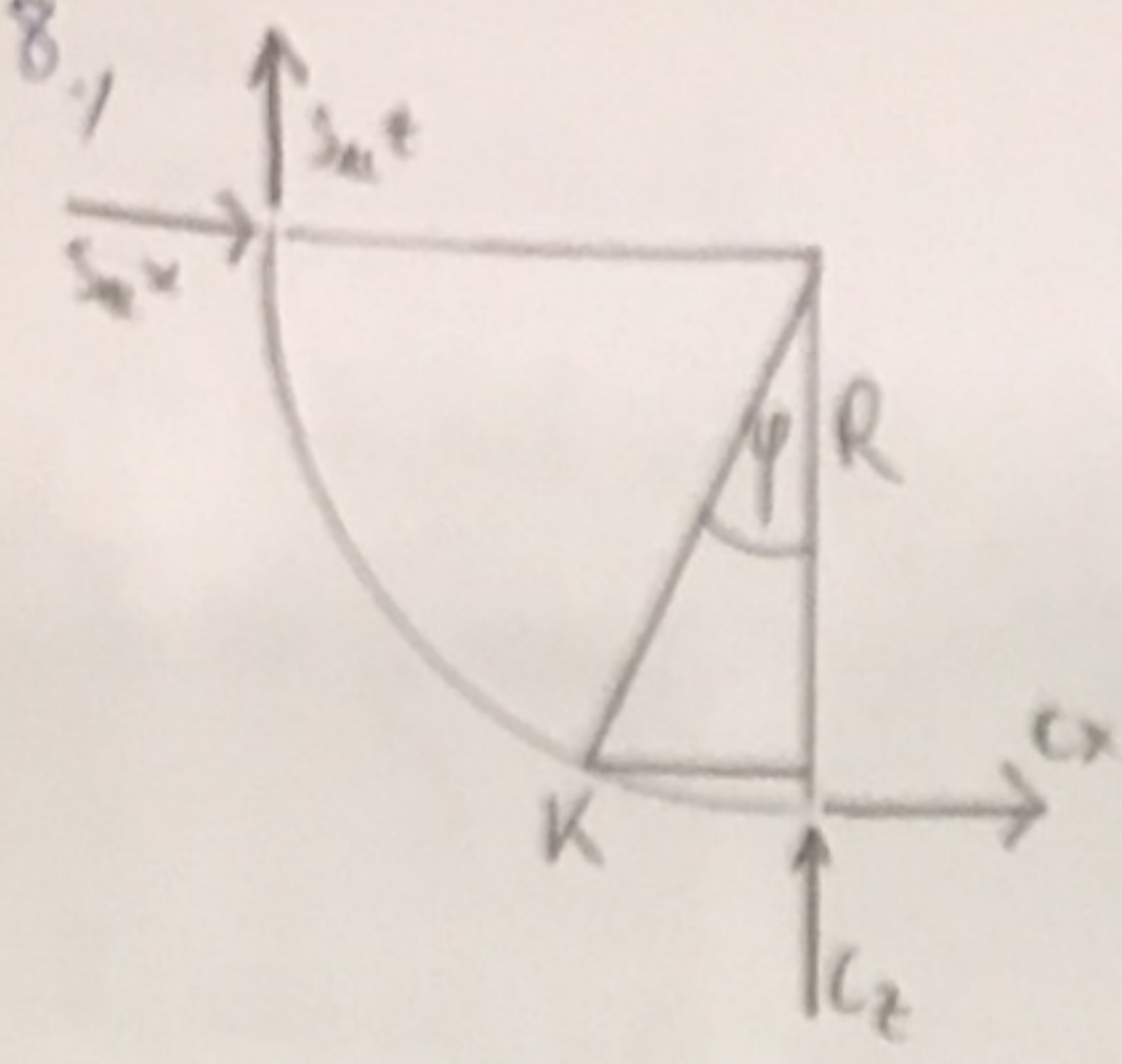
$\tau_2 = \frac{-1942 \cdot (2928{,}22 - 6{,}5z^2)}{59237{,}63 \cdot 13} = \frac{-5686603{,}24 + 12623z^2}{770089{,}19} = -7{,}384 + 1{,}639 \cdot 10^{-2} z^2 \, [MPa]$



$\tau^{(1)}_{max} = -7{,}38 \, [MPa]$

$|\tau^{(1)}_{max}| = 7{,}38 \, [MPa]$

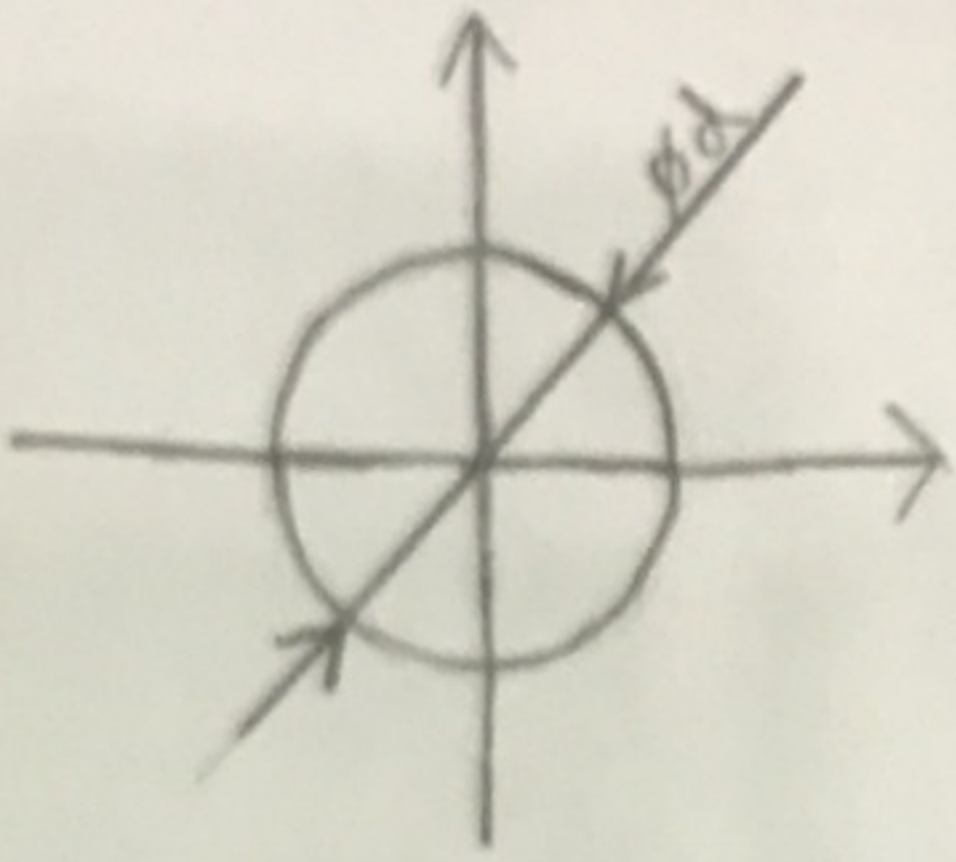
8.1



$N = C_x \cdot \cos\varphi - C_z \cdot \sin\varphi = 395 \cdot \cos 30° - (-395) \cdot \sin 30° =$
$= 342{,}08 + 197{,}5 = 539{,}58\,[N]$

$V = -C_x \cdot \sin\varphi - C_z \cdot \cos\varphi = -395 \cdot \sin 30° - (-395) \cdot \cos 30° =$
$= -197{,}5 + 342{,}08 = 144{,}58\,[N]$

$M_h = C_z \cdot R \cdot \sin\varphi + C_x \cdot (R - R \cdot \cos\varphi) = -395 \cdot 0{,}25 \cdot \sin 30° +$
$395 \cdot (0{,}25 - 0{,}25 \cdot \cos 30°) = -49{,}38 + 13{,}23 = -36{,}15\,[Nm] = -36150\,[Nmm]$



$d = 50\,[mm]$

$A = \frac{d^2 \pi}{4} = 1963{,}5$

$I_y = \frac{d^4 \pi}{4} = 306796{,}16$

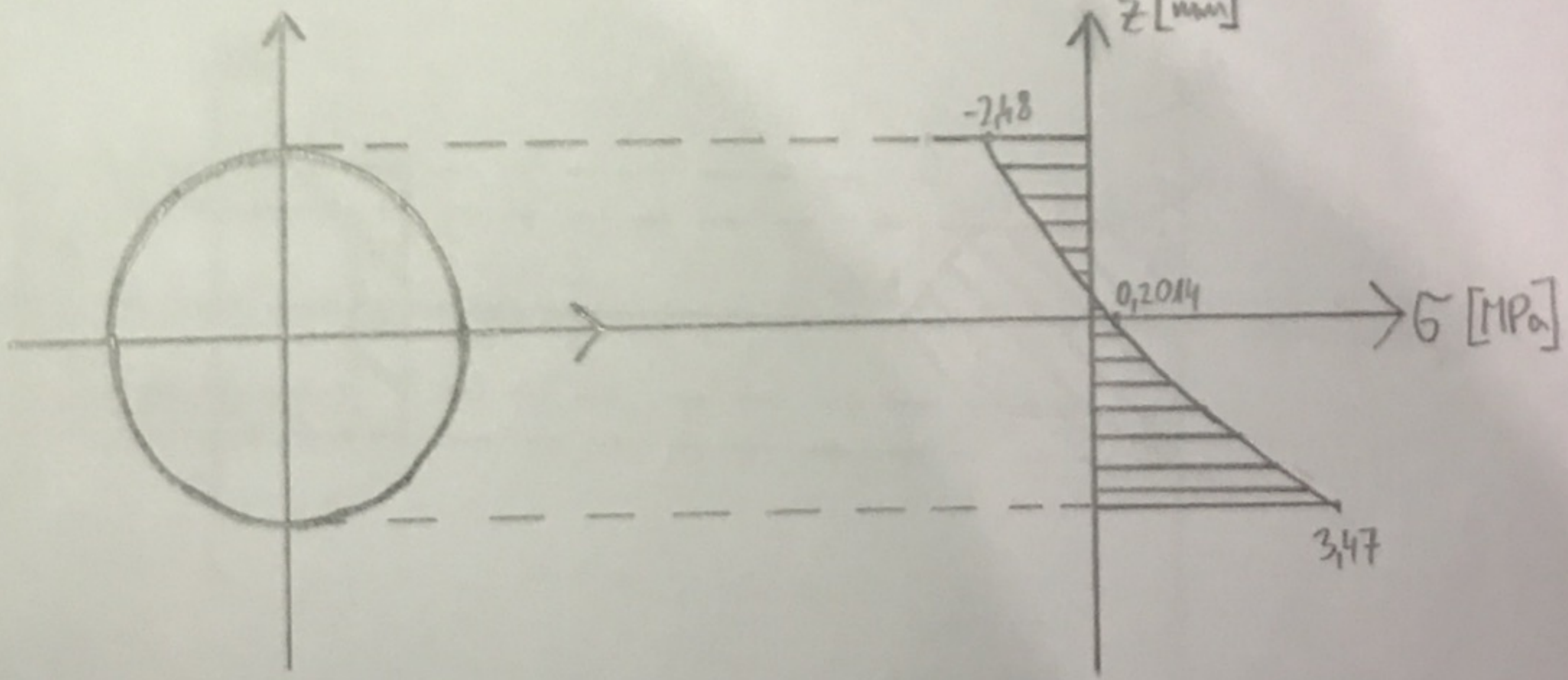
$\frac{R}{h} = \frac{250}{50} = 5$

$2 < \frac{R}{h} < 8 \Rightarrow$ Grashof-képlet

$\sigma_{M_h}(z) = \frac{M_h}{R \cdot A} + \frac{M_h}{I_y} \cdot \frac{R \cdot z}{R + z} = \frac{-36150}{250 \cdot 1963{,}5} + \frac{-36150}{306796{,}16} \cdot \frac{250z}{250+z} = -0{,}0736 - \frac{29{,}46z}{250+z}\,[MPa]$

$\sigma_N = \frac{N}{A} = 0{,}275$

$\sigma = \sigma_N + \sigma_{M_h} = 0{,}2014 - \frac{29{,}46z}{250+z}$



$\sigma^{(2)}_{K,max} = 3{,}47\,[MPa]$

9.

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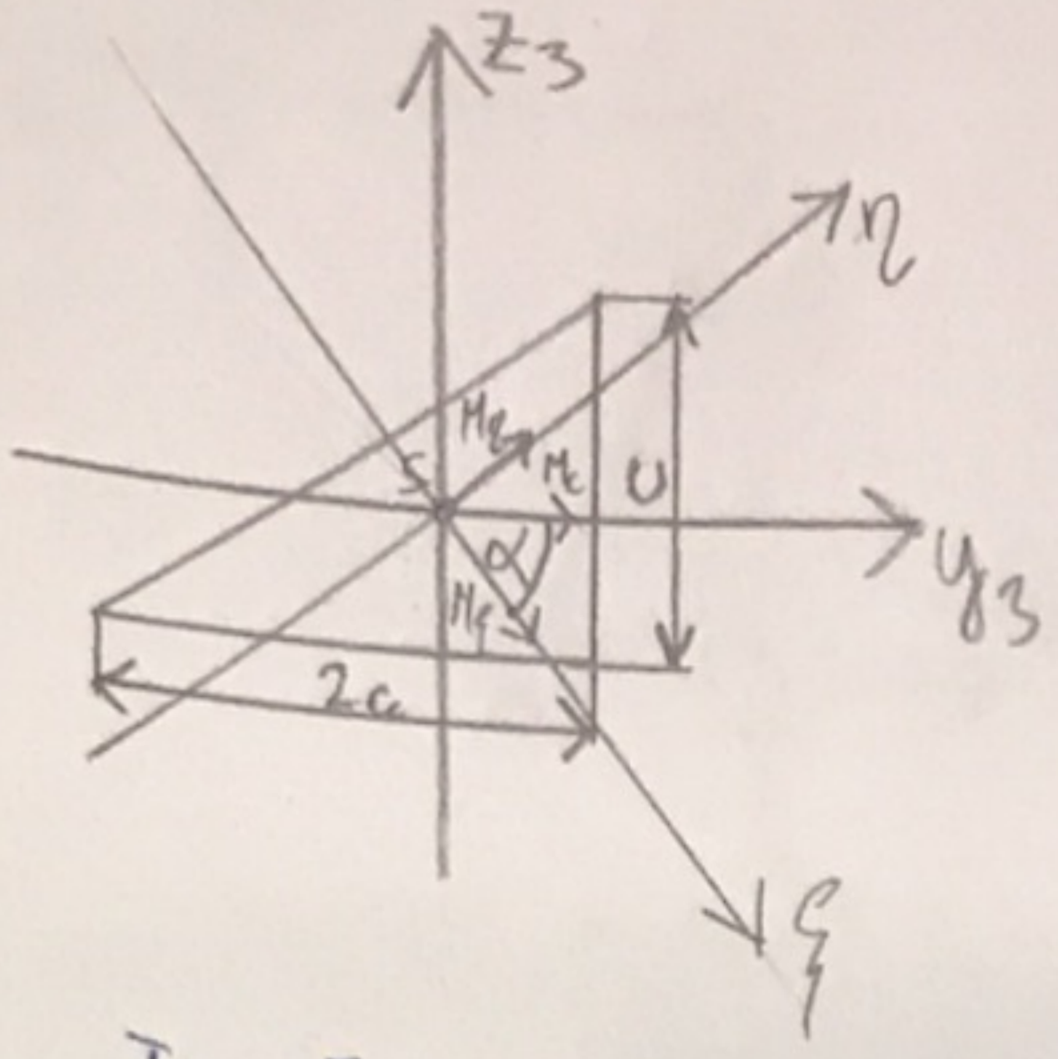

$$I_y = \frac{c^3 2c}{36} = \frac{36^3 \cdot 72}{36} = 93312\,[mm^4]$$

$$I_z = \frac{c \cdot (2c)^3}{36} = \frac{36 \cdot 72^3}{36} = 373248\,[mm^4]$$

$$I_{yz} = -\frac{c^2 (2c)^2}{72} = -\frac{36^2 \cdot 72^2}{7} = -93312\,[mm^4]$$

$$I_{1,2} = \frac{I_y + I_z}{2} \pm \sqrt{\left(\frac{I_y - I_z}{2}\right)^2 + I_{yz}^2} = \frac{466560}{2} \pm \sqrt{19591041024 + 8707129344} =$$

$$= 233280 \pm 168220{,}6 = \nearrow I_1 = 401500{,}6\,[mm^4]$$

$$\searrow I_2 = 65059{,}4\,[mm^4]$$

$$\alpha = \mathrm{arctg}\left(\frac{I_y - I_1}{I_{yz}}\right) = 73{,}155^\circ$$

$$M_c = \vec{CD} \times \vec{F_2} = \begin{bmatrix} -L \\ 0 \\ -\frac{R}{2} \end{bmatrix} \times \begin{bmatrix} F_2 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -350 \\ 0 \\ -125 \end{bmatrix} \times \begin{bmatrix} 1000 \\ 0 \\ 0 \end{bmatrix} = 125000\,[Nmm]$$

$$M_\xi = M_c \cdot \cos\alpha = 36222{,}95\,[Nmm]$$

$$M_\eta = M_c \cdot \sin\alpha = 119636{,}52\,[Nmm]$$

$$\left.\begin{aligned} \sigma_1(\eta) &= \frac{M_\xi}{I_1}\,\eta = 0{,}0902\,\eta \\ \sigma_2(\xi) &= \frac{M_\eta}{I_2} \cdot \xi = 1{,}8389\,\xi \end{aligned}\right\} \sigma = 0{,}0902\,\eta + 1{,}8389\,\xi\,[MPa]$$

SZIGECSÁN MÁTÉ

QIA950

$\overset{y_1\ z_1}{S(0;0)}$

$A(24;-12)$

$B(-48;-12)$

$C(24;24)$

B:

$\xi_1 = 24 \cdot \cos\alpha = 6{,}95$

$\eta_1 = 24 \cdot \sin\alpha = 22{,}97$

$\xi_2 = -12 \cdot \sin\alpha = -11{,}49$

$\eta_2 = 12 \cdot \cos\alpha = 3{,}48$

$B(\xi_1 + \xi_2 ; \eta_1 + \eta_2) = (-4{,}54; 26{,}45)$

A:

$\xi_1 = -48 \cdot \cos\alpha = -13{,}91$

$\eta_1 = -48 \cdot \sin\alpha = -45{,}94$

$\xi_2 = -12 \cdot \sin\alpha = -11{,}49$

$\eta_2 = 12 \cdot \cos\alpha = 3{,}48$

$A(-25{,}4; -42{,}46)$

C:

$\xi_1 = 24 \cdot \cos\alpha = 6{,}95$

$\eta_1 = 24 \cdot \sin\alpha = 22{,}97$

$\xi_2 = 24 \cdot \sin\alpha = 22{,}97$

$\eta_2 = -24 \cdot \cos\alpha = -6{,}95$

$C(29{,}92; 16{,}02)$

$\quad\ \ \xi \qquad \eta$

$A(-25{,}4; -42{,}46)$

$B(-4{,}54; 26{,}45)$

$C(29{,}92; 16{,}02)$

$|\sigma^{(3)}_{max,C}| = 56{,}47$ [MPa]

$\sigma_A = -46{,}71 - 3{,}83 = -50{,}54$ [MPa]

$\sigma_B = 2{,}39 - 8{,}35 = -5{,}96$ [MPa]

$\sigma_C = 1{,}45 + 55{,}02 = 56{,}47$ [MPa]

$0{,}0902\,\eta + 1{,}8389\,\xi = 0$

$\eta = -\frac{1{,}8389}{0{,}0902}\,\xi$

$\beta_{\text{zérus}} = \operatorname{arctg}\left(-\frac{1{,}8389}{0{,}0902}\right) = -87{,}19^\circ$

$\beta = \alpha + \beta_{\text{zérus}} = -14{,}036^\circ$