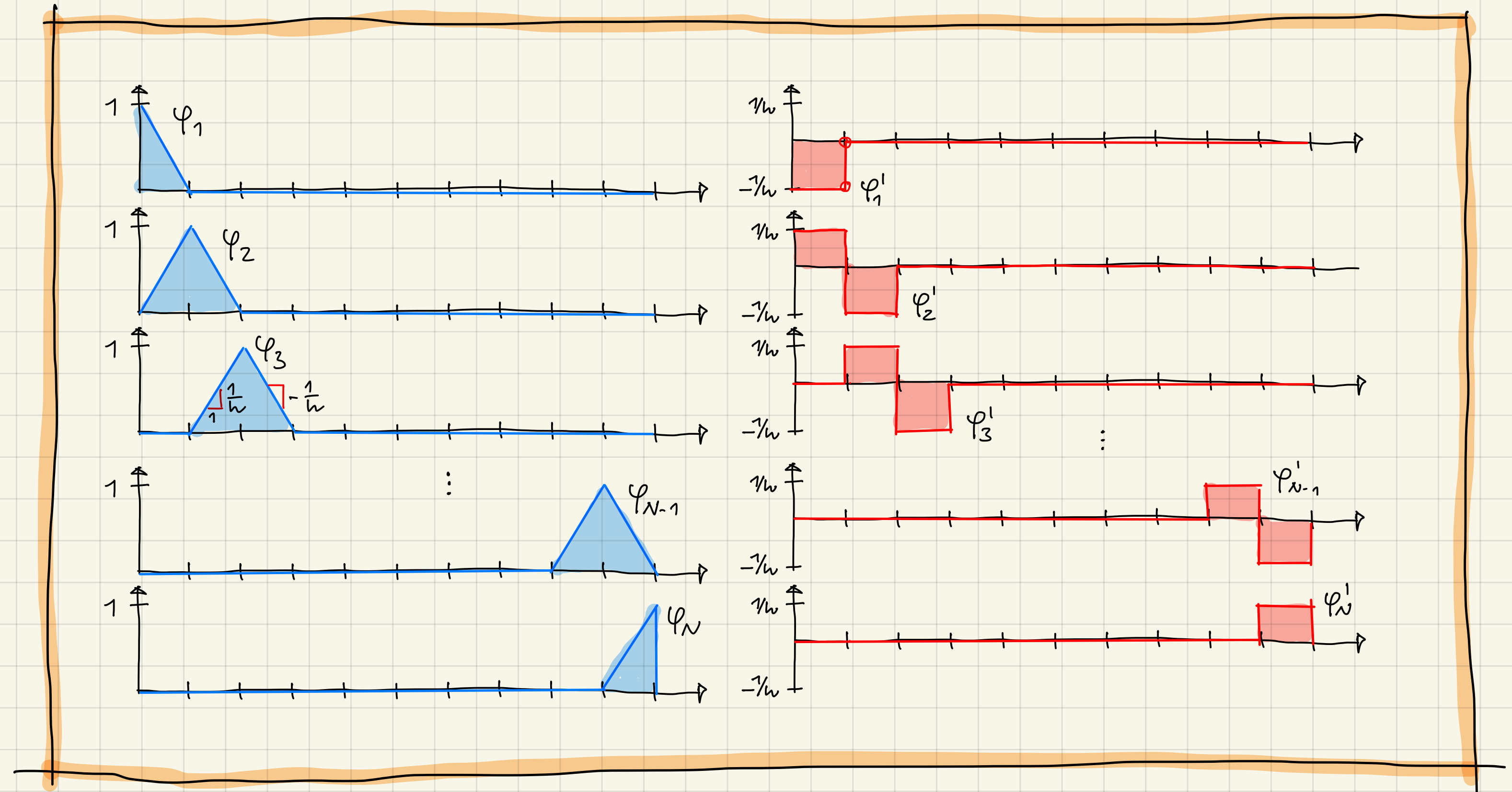
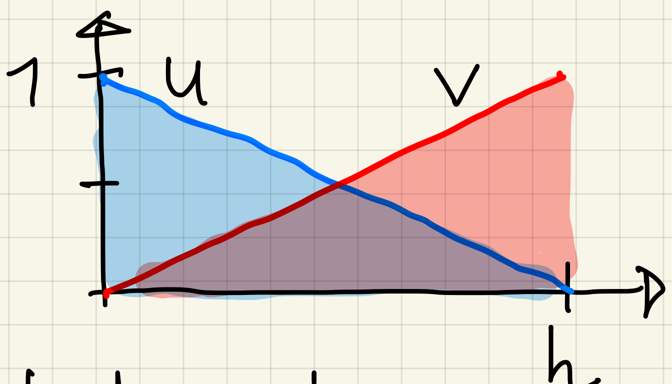
Contribution of distributed spring C

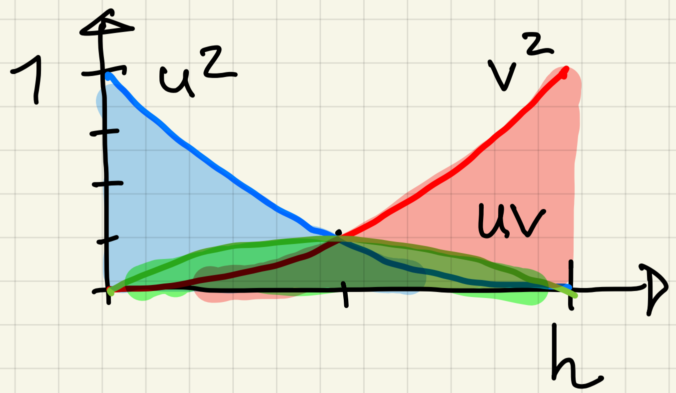
$$K^c_{ij} = a^c(\varphi_i, \varphi_j) = C \cdot \int_0^l \varphi_i \cdot \varphi_j \, dx$$



Preliminary considerations

- $u(x) = 1 - \frac{x}{h}$ and $v(x) = \frac{x}{h}$





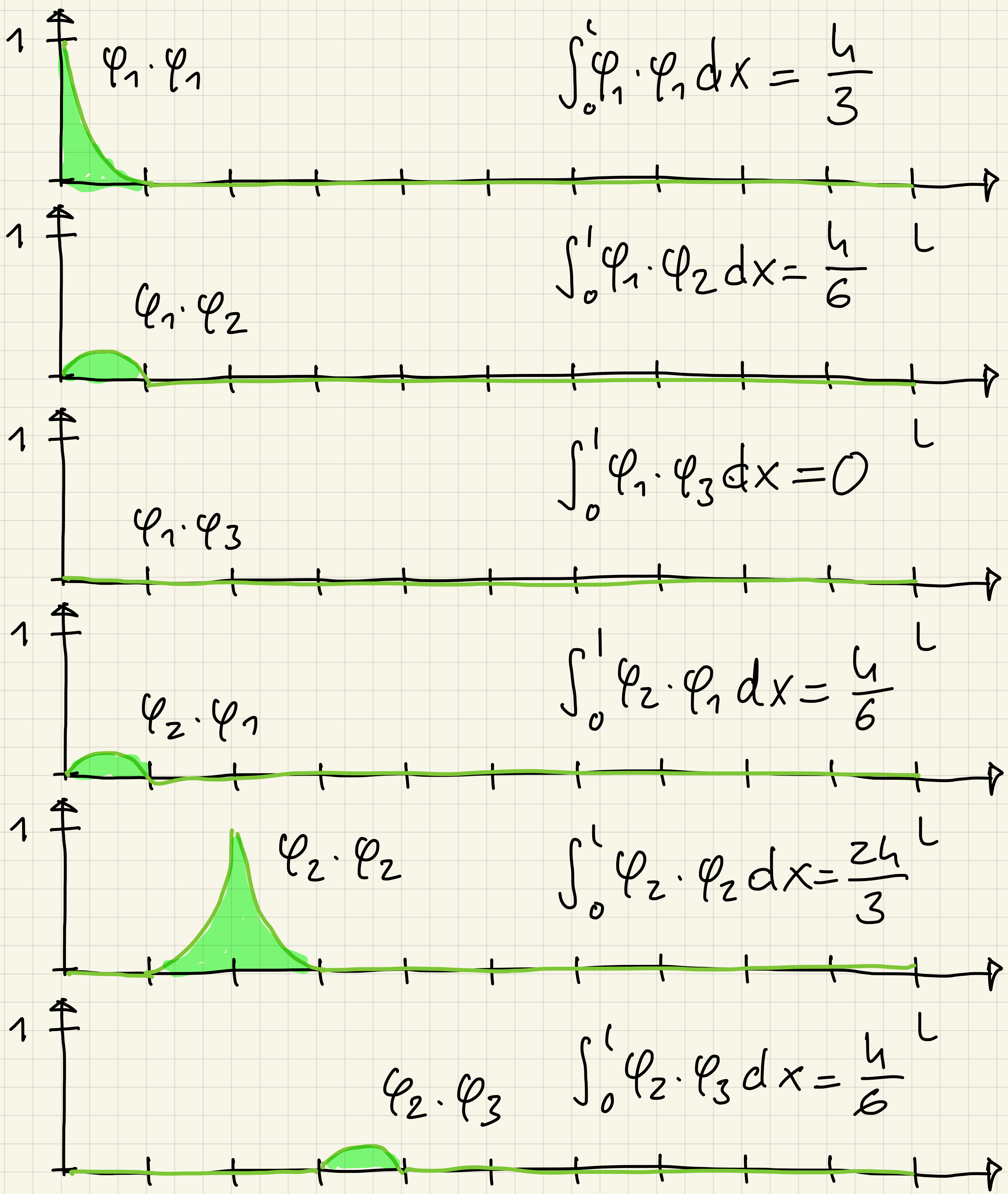
- Integrals

$$\int_0^h v^2(x)\,dx = \int_0^h \frac{x^2}{h^2}\,dx = \frac{1}{h^2}\left[\frac{1}{3}x^3\right]_0^h = \frac{h}{3}$$

$$\int_0^h u^2(x)\,dx = \frac{h}{3} \quad \text{(symmetry)}$$

$$\int_0^h u(x)\cdot v(x)\,dx = \int_0^h \left(1-\frac{x}{h}\right)\cdot\frac{x}{h}\,dx = \int_0^l \left(\frac{x}{h} - \frac{x^2}{h^2}\right)dx = \left[\frac{x^2}{2h} - \frac{x^3}{3h^2}\right]_0^h = \frac{h}{2} - \frac{h}{3} = \frac{h}{6}$$

- Products of basis functions



$$\int_0^L \varphi_1 \cdot \varphi_1 \, dx = \frac{h}{3}$$

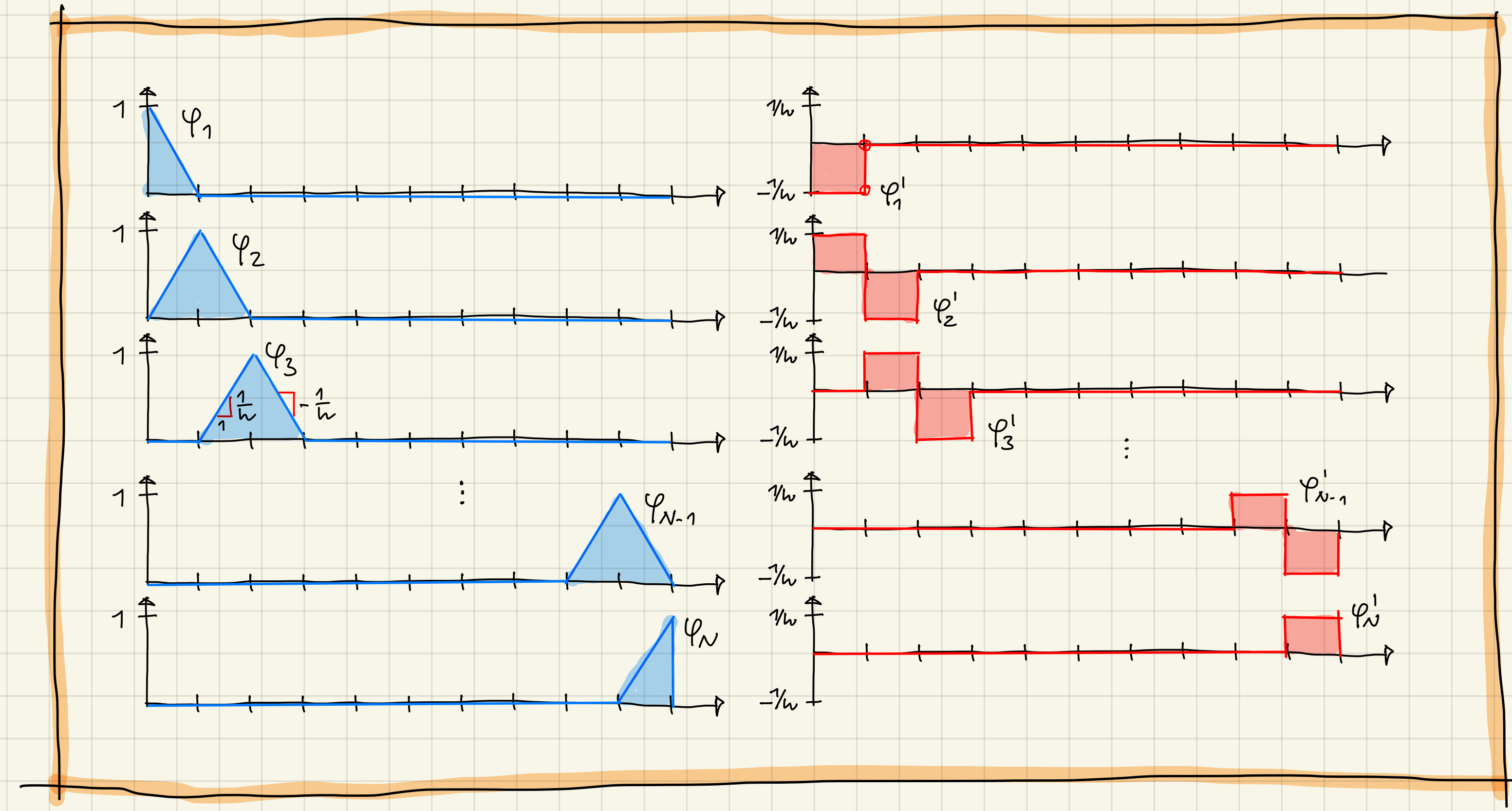
$$\int_0^L \varphi_1 \cdot \varphi_2 \, dx = \frac{h}{6}$$

$$\int_0^L \varphi_1 \cdot \varphi_3 \, dx = 0$$

$$\int_0^L \varphi_2 \cdot \varphi_1 \, dx = \frac{h}{6}$$

$$\int_0^L \varphi_2 \cdot \varphi_2 \, dx = \frac{2h}{3}$$

$$\int_0^L \varphi_2 \cdot \varphi_3 \, dx = \frac{h}{6}$$



Stiffness matrix $\underline{K}^c$

$$K^c_{ij} = c \cdot \int_0^L \varphi_i \cdot \varphi_j \, dx$$

$$\underline{K}^c = \frac{c \cdot h}{6} \begin{bmatrix} 2 & 1 & 0 & 0 & \cdots & 0 & 0 \\ 1 & 4 & 1 & 0 & \cdots & 0 & 0 \\ 0 & 1 & 4 & 1 & \cdots & 0 & 0 \\ 0 & 0 & 1 & 4 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & 4 & 1 \\ 0 & 0 & 0 & 0 & \cdots & 1 & 2 \end{bmatrix}$$