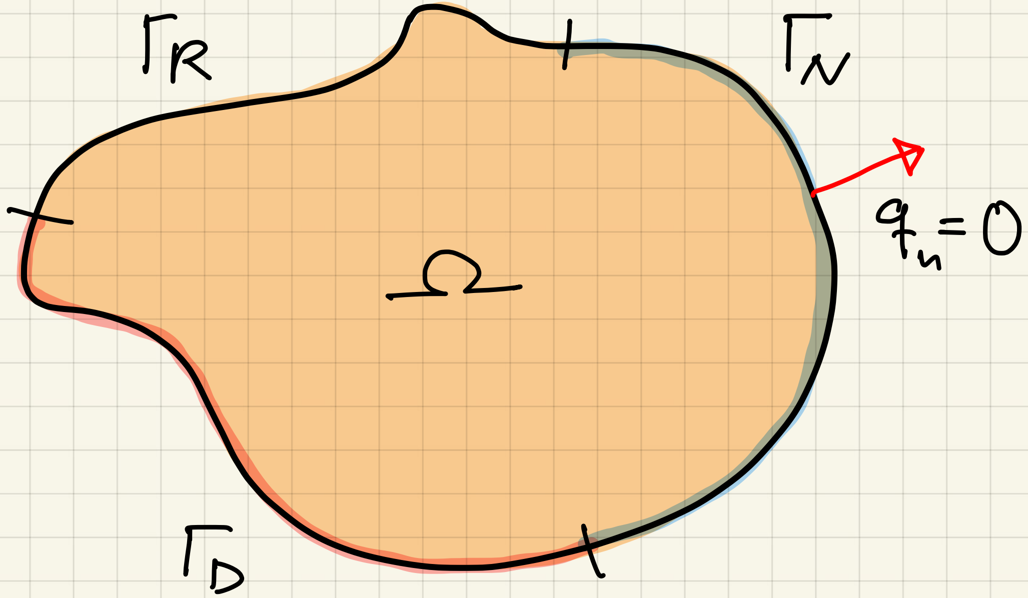
FEM for 2D problems

Heat conduction: Dirichlet boundary conditions

Background



(V): Find $\theta \in V_0$ such that

$$a(\theta, \delta\theta) = b(\delta\theta)$$

for all $\delta\theta \in V_0$. The space V_0 contains "nice" functions $u: \Omega \to \mathbb{R}$ with $u(x,y) = 0$ for $(x,y) \in \Gamma_D$.

Reminder

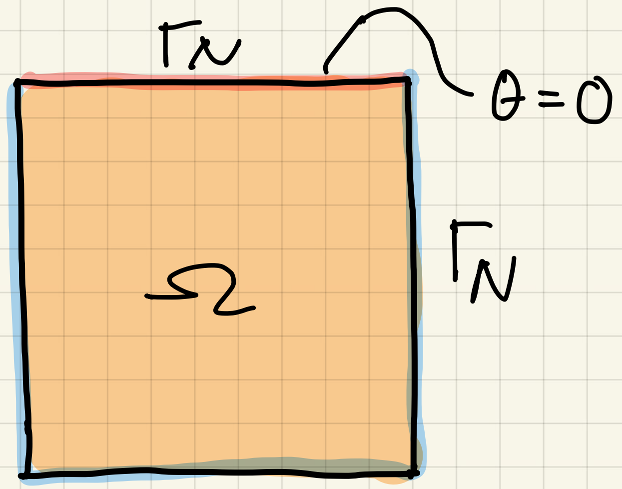
$\nabla\theta(x,y) \cdot \underline{n}(x,y) = 0$ for $(x,y) \in \Gamma_N$ is (automatically) satisfied by solution of (V). We consider Γ_R later.

Question

How to make sure that $\theta_h = \sum \varphi_i \hat{\theta}_i$ is in V_0?

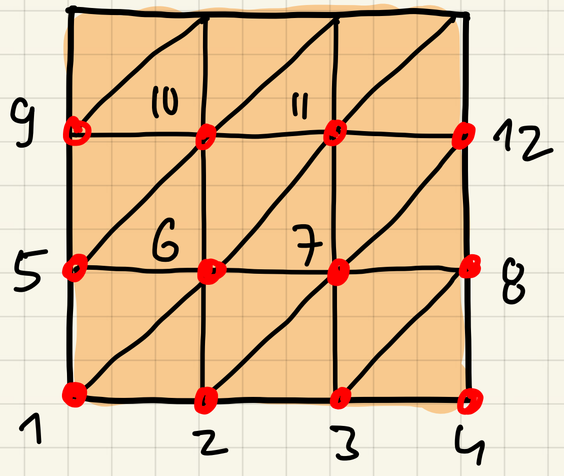
Possible approaches

Example



Some advantages but more programming

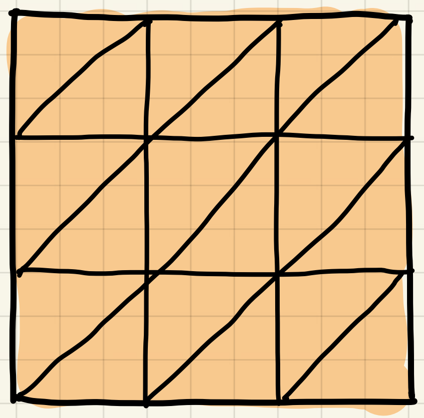
1. Only basis functions $\varphi_i \in V_0$, then $\theta_h \in V_0$

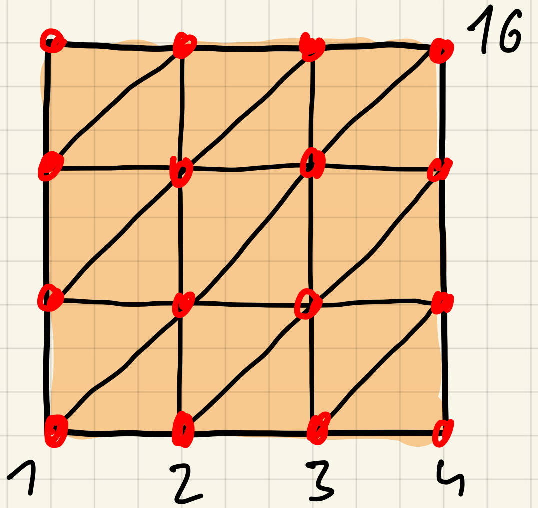


Exclude nodes on Γ_D when assembling the linear system $\underline{K}$ and $\underline{r}$. Requires explicit enumeration of DOFs.

→ Bit complicated for $\theta^* \neq 0$

2. Basis functions for all nodes





Assemble $\underline{K}$ and $\underline{r}$ for all nodes. Requires modification of linear system in order to ensure that $\hat{\theta}_i = 0$ for all nodes $(x_i, y_i) \in \Gamma_D$.

→ Easy to extend to $\theta^* \neq 0$

Modification of linear System

Original system

$$\begin{pmatrix} k_{11} & k_{12} & k_{13} & k_{14} & k_{15} \\ k_{21} & k_{22} & k_{23} & k_{24} & k_{25} \\ k_{31} & k_{32} & k_{33} & k_{34} & k_{35} \\ k_{41} & k_{42} & k_{43} & k_{44} & k_{45} \\ k_{51} & k_{52} & k_{53} & k_{54} & k_{55} \end{pmatrix} \begin{pmatrix} \hat{\theta}_1 \\ \hat{\theta}_2 \\ \hat{\theta}_3 \\ \hat{\theta}_4 \\ \hat{\theta}_5 \end{pmatrix} = \begin{pmatrix} r_1 \\ r_2 \\ r_3 \\ r_4 \\ r_5 \end{pmatrix}$$

$\hat{\theta}_2$ and $\hat{\theta}_5$ should be zero.

Modified system

$$\begin{pmatrix} k_{11} & k_{12} & k_{13} & k_{14} & k_{15} \\ 0 & 1 & 0 & 0 & 0 \\ k_{31} & k_{32} & k_{33} & k_{34} & k_{35} \\ k_{41} & k_{42} & k_{43} & k_{44} & k_{45} \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \hat{\theta}_1 \\ \hat{\theta}_2 \\ \hat{\theta}_3 \\ \hat{\theta}_4 \\ \hat{\theta}_5 \end{pmatrix} = \begin{pmatrix} r_1 \\ 0 \\ r_3 \\ r_4 \\ 0 \end{pmatrix}$$

Alternative

- Lagrange-Multiplikatoren