

Report Assignment 2: Velocity Control of the Cart

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1 Introduction

In this report, two velocity controllers for a DC motor are designed, using frequency response methods. The main criterion states that the velocity controller yields a zero steady-state error on a constant velocity reference.

2 Design of the Controller

2.1 Type of Controller

To satisfy the criterion of zero steady-state error, multiple controllers can be used. A PI, PID and feedforward controller can all yield a zero steady-state error. For the PI and PID controllers, this is caused by the integration part. The feedforward controller can be especially useful for tracking and can eliminate errors induced by a changing reference value. However, as the controller must yield a zero steady-state error on a constant velocity reference and deal with errors caused by disturbances, the feedforward controller will not be used.

Since a large bandwidth yields a fast responding system, a high bandwidth seems advantageous. If the bandwidth is too high though, the high frequency noise has more influence. A trade-off between the two has to be made. Additionally, the overshoot M_p must be kept sufficiently low. High peaks in the transient response have to be avoided, as the Arduino can only deliver voltages in a range of $[-12V, 12V]$. If high peaks are present in the response, it is possible the controller wants to send out voltages outside this range. This is ofcourse impossible, which makes the cart unable to follow the correct response imposed by the controller. Because of the aforementioned reasons and because of simplicity the PI controller is chosen.

2.2 Design Parameters

To properly execute the design, some design parameters must be determined. The parameters that can be chosen free are the phase margin PM and the extra margin $d\phi$ to compensate for the phase lag of the PI controller. Using these, the new cross over frequency ω_c , the integration time T_i , the gain K and the gain margin GM of the controller can be calculated.

2.2.1 Phase Margin

The design value of the PM can vary from 40° to 50° . From chapter 7 of the course notes "Control Theory- Handouts" [2], one knows that by increasing the PM, the damping ratio ζ and T_i will increase. The peak value in the closed loop step response M_p and the new cross over frequency ω_c will decrease.

The increase in ζ causes the response of the signal to be slower, but have less oscillations. Since the value of ζ is still below the value of critical damping of $\zeta = 0.7$, the increase is in general advantageous for the transient response.

The decrease of ω_c reduces the influence of high frequency noise. However, $\omega_c \gg \frac{1}{T_i}$ is necessary to lower the influence of the phase lag introduced by the PI controller. Luckily by increasing the PM, $\frac{1}{T_i}$ decreases (see Equation 1) and thus the fact that ω_c reduces, is partly compensated by the decrease in $\frac{1}{T_i}$. The decrease in overshoot M_p improves the transient response. By processing the data in Matlab, it becomes clear that by increasing the PM, the GM increases and the transient errors decreases. For these reasons, the PM is chosen to be equal to 50° .

2.2.2 Extra Margin to compensate for the Phase Lag

As the PI controller has a negative phase, an extra margin $d\phi$ on the PM has to be included to calculate the new cross over frequency ω_c . The chosen margin can vary from 10° to 15° . By choosing a higher value, the GM increases and the gain decreases, which helps to reduce resonance. By using different values and

calculating the stepresponse on the closed loop system in Matlab, it is clear the transient errors decreases when the extra margin is increased. $d\phi = 15^\circ$ is chosen.

2.2.3 General Design Procedure

Now the PM and the extra margin on the PM have been chosen, the frequency where the uncompensated open loop system has a phase $\phi = -180^\circ + PM + d\phi$ can be determined. This is the new cross over frequency ω_c . Using this result, T_i can be calculated using Equation 1.

$$T_i = \frac{\tan(90^\circ - d\phi)}{\omega_c} \quad (1)$$

The transfer function of the PI controller can be written as:

$$D(s) = \frac{K}{s} \left(s + \frac{1}{T_i} \right) \quad (2)$$

The gain K can be determined, stating that the gain of the compensated system at ω_c indeed equals 1:

$$|G(j\omega_c)D(j\omega_c)| = 1 \quad (3)$$

The gain margin is a measure for stability and reduces resonance. The design value is $GM \approx 2 \approx 6dB$. The values obtained from these calculations can be found in Table 1.

Parameter	Value motor A	Value motor B
GM	1.6733	1.6241
PM	49.2211°	49.2477°
ω_c	60.9960 rad/s	62.9272 rad/s
T_i	0.0612 s	0.0592s
$\frac{1}{T_i}$	16.3473s ⁻¹	16.8808s ⁻¹
$d\phi$	15°	15°
gain K	1.0712	1.0225

Table 1: Design parameters and their calculated values using general design method

Next to the values in Table 1, the open loop (= serial connection of controller and model) and closed loop bodediagrams are shown in Figure 1 and Figure 2 respectively. The closed loop step responses are shown in Figure 3. Several factor show this controller is not optimal. First of all the GM does not meet the required value of 2. This means the system can become unstable or contain resonance. The latter is reflected in the resonance peak in the bodeplot of the closed loop system in Figure 2. In Figure 3, a high overshoot value M_p can be observed. This can cause troubles, as discussed earlier in Section 2.1.

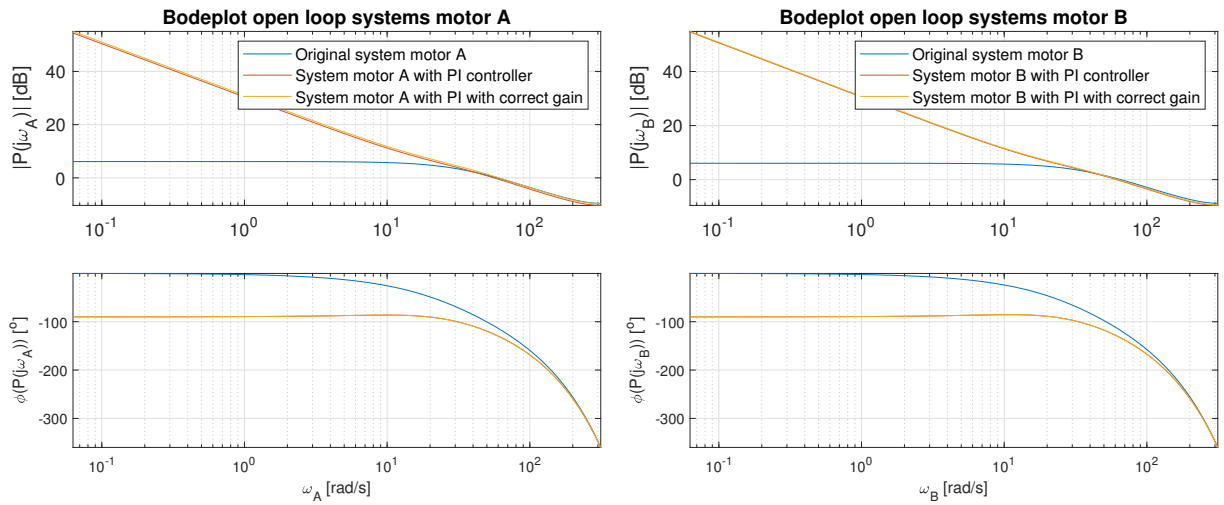


Figure 1: Bodeplot of the different open loop systems using the general design method

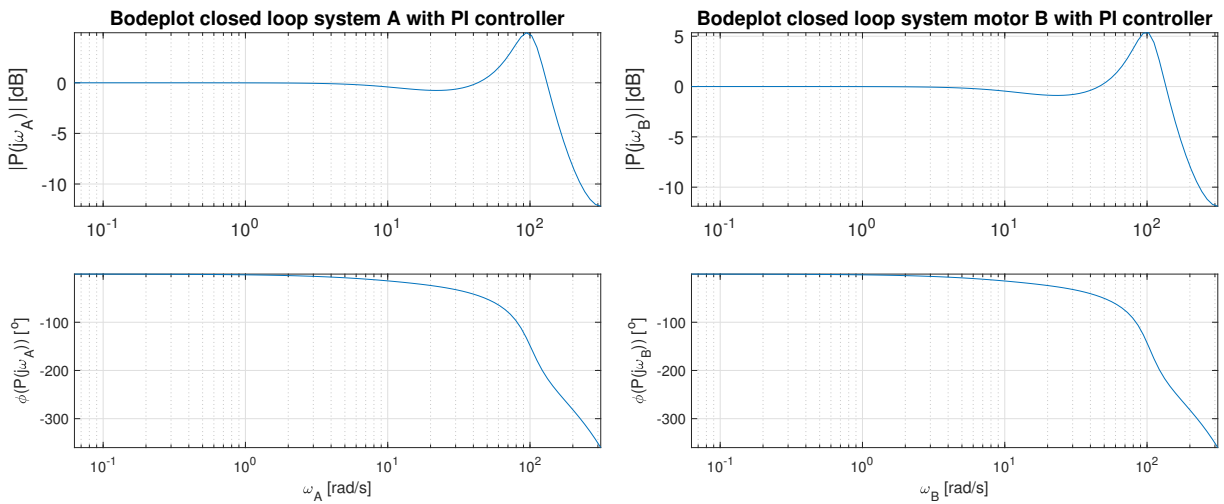


Figure 2: Bodeplot of the closed loop system with PI controller using the general design method

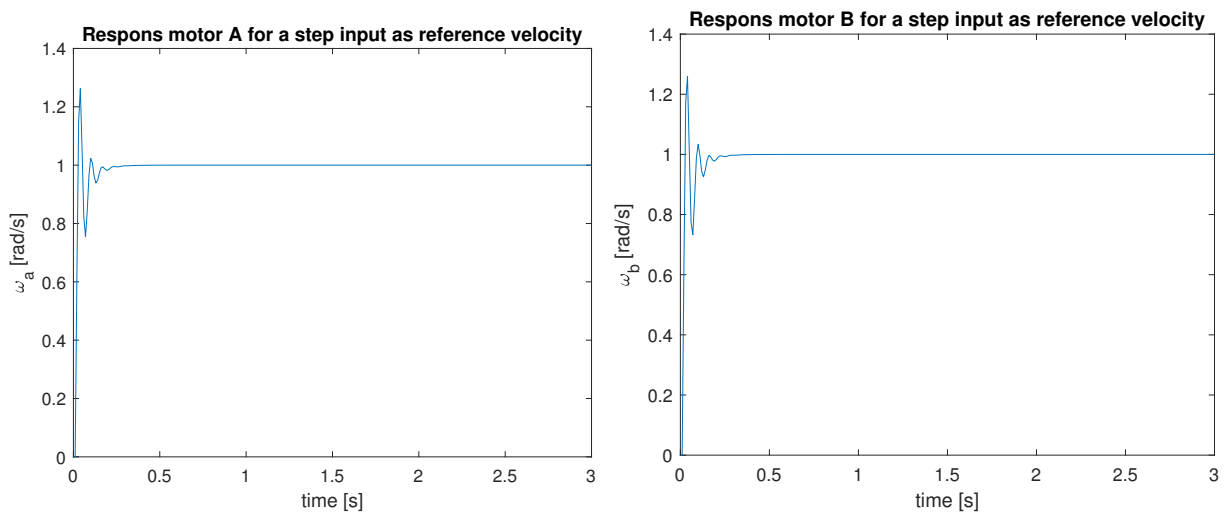


Figure 3: Step response of the closed loop system with PI controller using the general design method

2.2.4 Improvement of the Controller

In section 2.2.3, it became clear the general design procedure does not provide an acceptable controller. To improve the controller, two methods were considered. First of all, the gain could be lowered to increase the GM and PM, while keeping the same T_i . Secondly the desired GM given in the design could be increased. Comparison of the two methods for several values in Matlab showed that the first method (decreasing gain K) produced the best system parameters, for example higher GM for the same PM, lower overshoot and a far better transient response to a step input in general. By a method of trial and error, following values for the gain were selected:

$$\begin{aligned} \text{Motor A : } K_a &= 0.8 \\ \text{Motor B : } K_b &= 0.775 \end{aligned} \quad (4)$$

These gains yield the values given in Table 2.

Parameter	Values motor A	Values motor B
GM	2.2406	2.1427
PM	68.5401°	69.1969°
ω_c	42.1655 rad/s	43.65972 rad/s
T_i	0.0612 s	0.0592 s
$\frac{1}{T_i}$	16.3473 s ⁻¹	16.8808 s ⁻¹
$d\phi$	15°	15°
gain K	0.8000	0.7750
Closed loop bandwidth	121.5664 rad/s	128.2576 rad/s

Table 2: Design parameters and their calculated values using the improved design method

Figure 4 depicts the bodeplot of the open loop system (= serial connection of the controller and the model). The improved values for the GM and PM from Table 2 are shown in this figure. Apart from that, from Figure 4 it becomes clear the DC-value of the open loop system goes to infinity. This proves the steady-state error of the closed loop goes to zero. In Figure 5, the different open loop systems are shown and in Figure 6, the bodeplots of the closed loop systems are displayed. Comparing Figures 2 and 6, the resonance peak is clearly suppressed using the improved controller. In addition, the overshoot is lowered, as can be seen in Figure 7. The only disadvantage of this method is the decrease in open loop bandwidth.

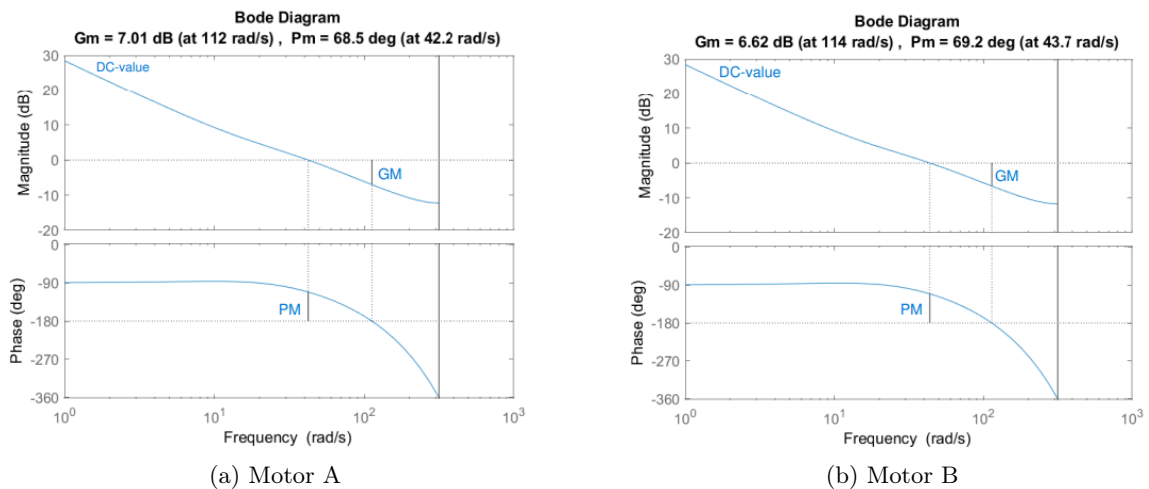


Figure 4: Bodeplot of the open loop systems using the improved controller

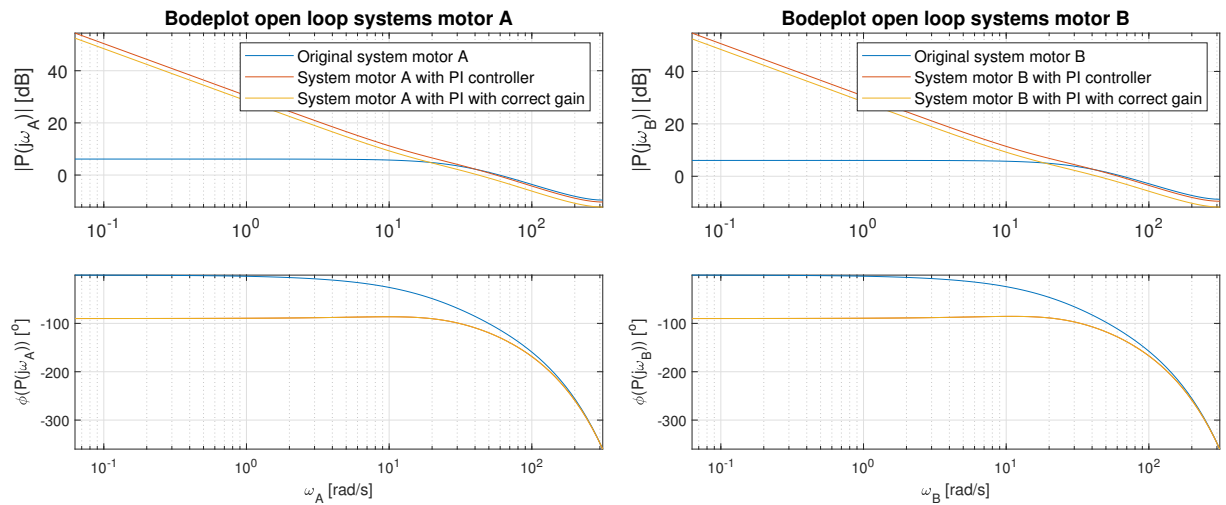


Figure 5: Bodeplot of the different open loop systems using the improved controller

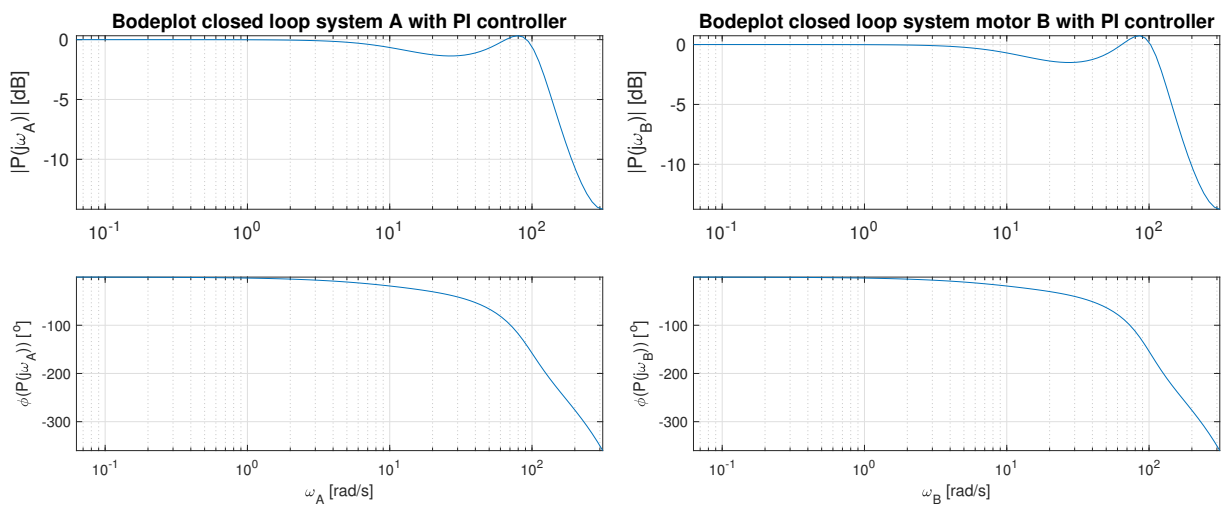


Figure 6: Bodeplot of the closed loop system with PI controller using the improved controller

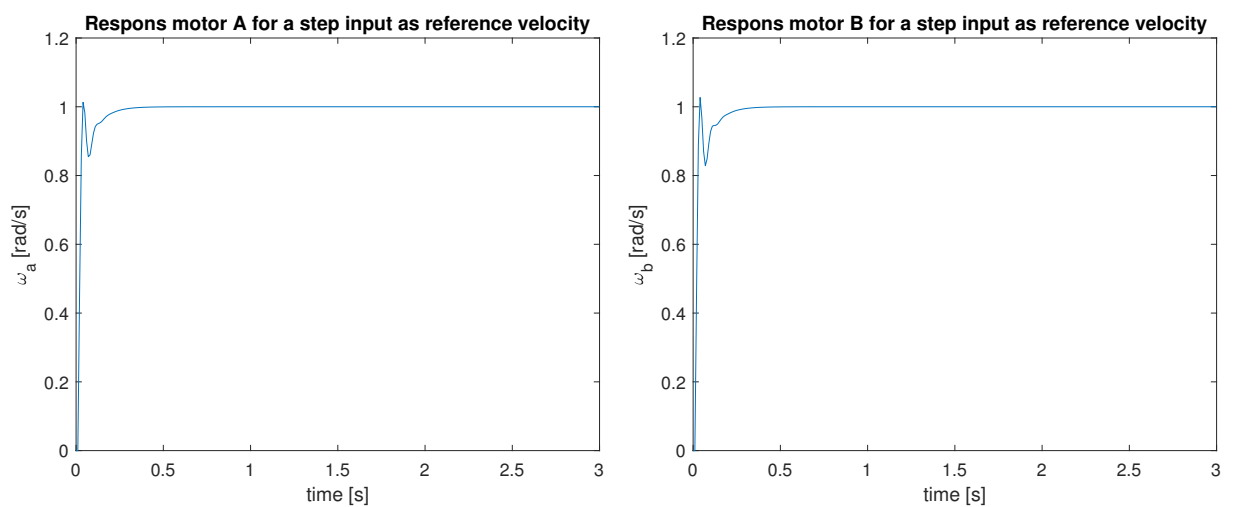


Figure 7: Step response of the closed loop system with PI controller using the improved controller

2.3 Limitations on the Bandwidth

The Nyquist theorem states that a signal can be constructed without aliasing if the sampling frequency f_s is at least two times higher than the highest frequency in the system. This means the theoretical limitation on the closed-loop bandwidth is $\frac{f_s}{2}$, which equals 50 Hz in our case as the sampling frequency equals 100 Hz. In practice, increasing the bandwidth to the theoretical limit can pose problems, since the PM decreases with increasing bandwidth. However, the PM has a great influence on stability of the system and other parameters such as the overshoot and damping ratio. This means a minimal PM is often required, since the system has to be stable or the overshoot can't be too large because of the finite voltage range of the controller. For this reason the practical limit for the closed loop bandwidth can be lower than the theoretical limit.

3 Validation of the Controller

In this section the controller designed in the previous sections is validated experimentally. To do this, the controller is implemented in Arduino and a step input of 6 rad/s is applied for the reference velocity. In Figure 8 the different responses are shown, with a more zoomed in view on the transient response in Figure 9. The tracking error of the step reference is shown in Figure 10 with a close-up in Figure 11. From the aforementioned figures, it is clear the steady-state error of the simulated response goes to zero. For steady-state value of the measured response, there are still some deviations from the step reference. These are caused by measurement noise. Since the deviations occur around the step reference value, the conclusion can be made the measured steady-state error also equals zero, excluding measurement noise. Figure 11 shows that the tracking error is maximal in the beginning for two time delays and equal to the value of the step reference. This is caused by the 2 timedelays between the input voltages and output wheel velocity of the system, which is discussed in Assignment 1.

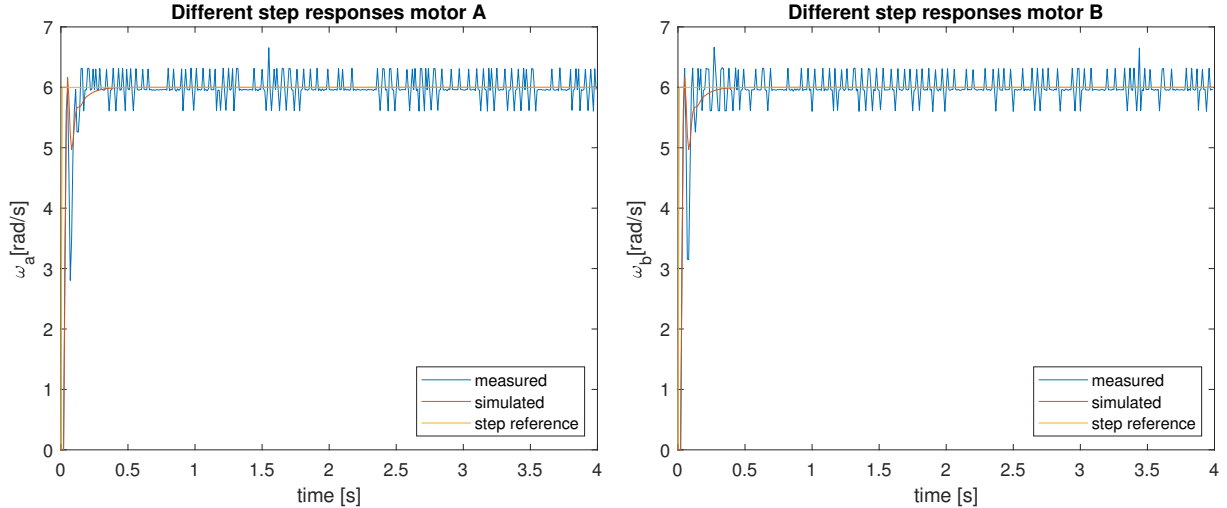


Figure 8: Measured and simulated responses of the closed loop system to a step input of 6 rad/s as reference velocity

Figure 12 shows the measured and simulated voltages (control signals) for the step input reference. One knows that as the time increases, the error of the simulated model response goes to zero. This can be seen in Figure 12, as the simulated voltage goes to a constant value. There are still variations in the measured voltages, these do not converge to a constant value due to the fact that the error on the measured response does not go to zero exactly.

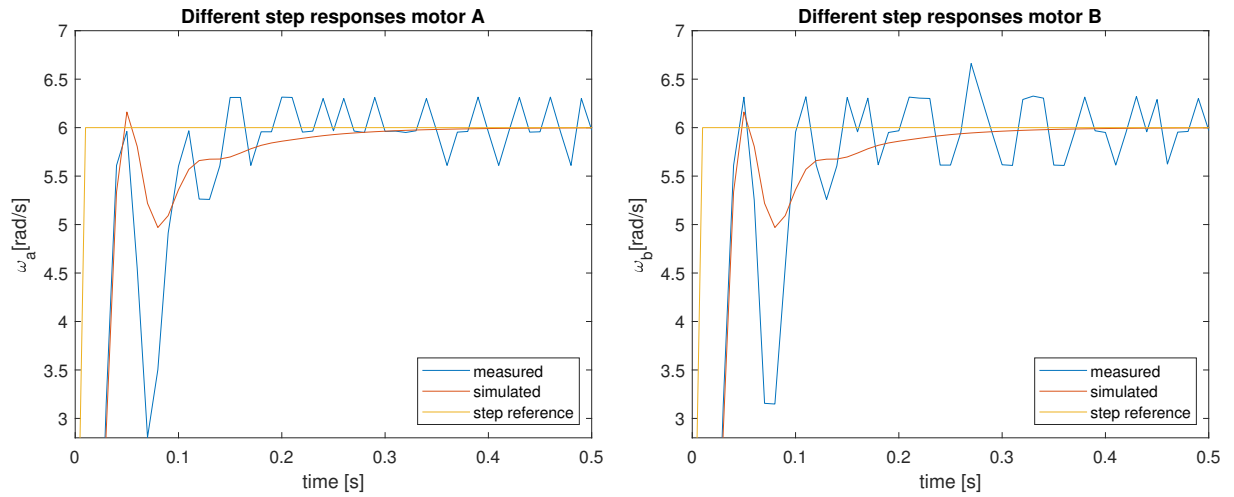


Figure 9: Close-up of the measured and simulated responses of the closed loop system to a step input of 6 rad/s as reference velocity

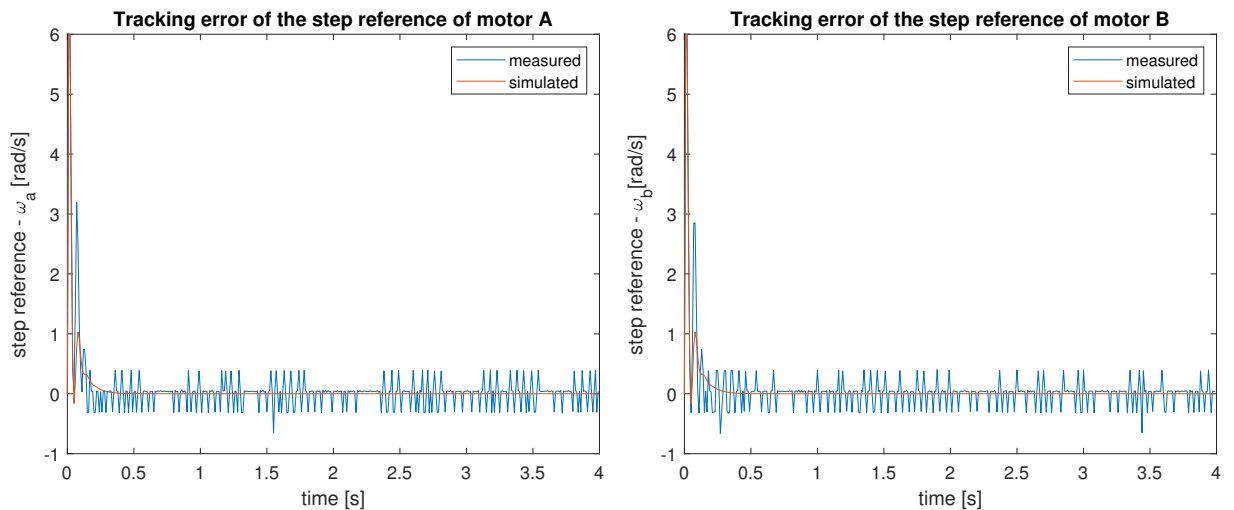


Figure 10: Measured and simulated tracking error of the closed loop system to a step input of 6 rad/s as reference velocity

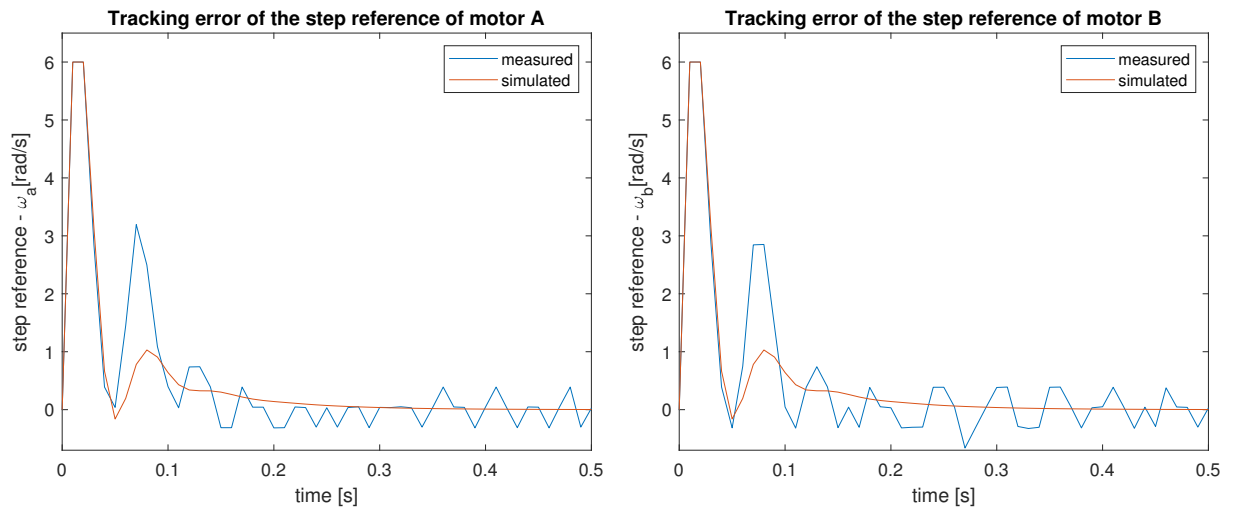


Figure 11: Close-up of the measured and simulated tracking error of the closed loop system to a step input of 6 rad/s as reference velocity

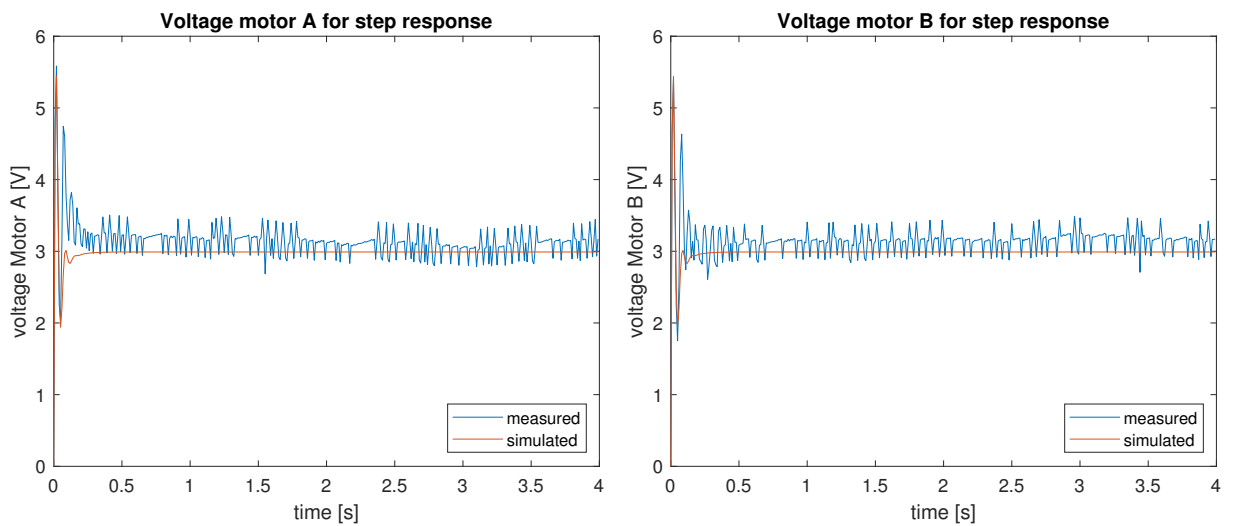


Figure 12: Measured and simulated voltages for a step reference input

3.1 Force Disturbance

To apply a constant force disturbance, the cart is placed on a slope instead of a horizontal surface. When the cart drives uphill, gravity creates a constant force opposite to the driving direction. The magnitude of the force is given by Equation 5, where m is the mass of the cart and α is the slope angle ($= 3.518^\circ$ in our case).

$$F = m * g * \sin(\alpha) \quad (5)$$

The disturbance enters the loop after the system, but is included within the feedback loop. This is shown in the block diagram in Figure 13.

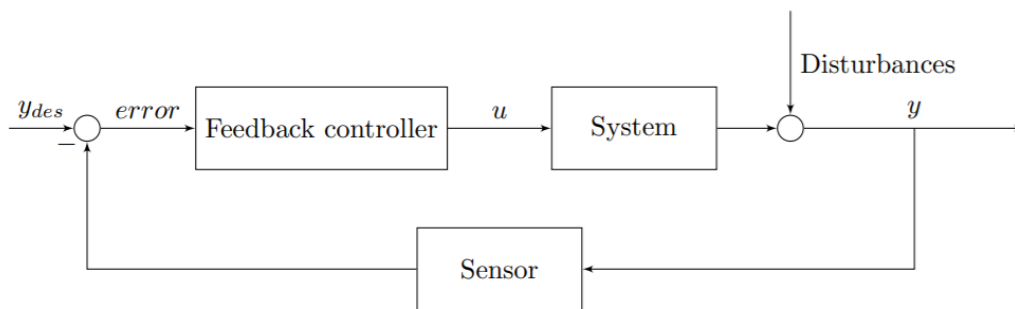


Figure 13: Block diagram of control configuration with force disturbance [1]

In Figure 14 the measured and simulated responses to the step input as reference velocity are shown, while Figure 15 shows the tracking errors. These figures clearly indicate that the output velocity still follows the reference velocity with a steady-state error equal to zero. This can be explained looking at the block diagram in Figure 13. As the force disturbance is included within the feedback loop, the controller can take into account the error caused by the constant force disturbance and thus ensure the steady-state error to go to zero. Figure 16 shows the measured control signal is higher than the simulated one. This makes sense, as a higher voltage is needed to overcome the force disturbance while maintaining the same speed.

In this section, the cart was placed on a slope to create a constant force disturbance. Other possibilities to achieve the same result are for example adding a weight to the cart itself, or letting the cart drag an additional weight. Both cases increase the drag force, between the wheels and the ground or between the extra weight and the ground respectively. Using the coulomb friction model, this increase in the drag force causes a constant friction force disturbance. Following the same reasoning as for the cart on a slope, the steady-state error will be zero as well.

There are however also disturbances possible that do affect the steady-state tracking error. For example, if the cart is placed on a hill with an increasing slope, the force disturbance won't be constant and the steady-state error will not equal zero. At last, undulations of the road may cause a non-constant disturbance, which will affect the steady-state tracking error.

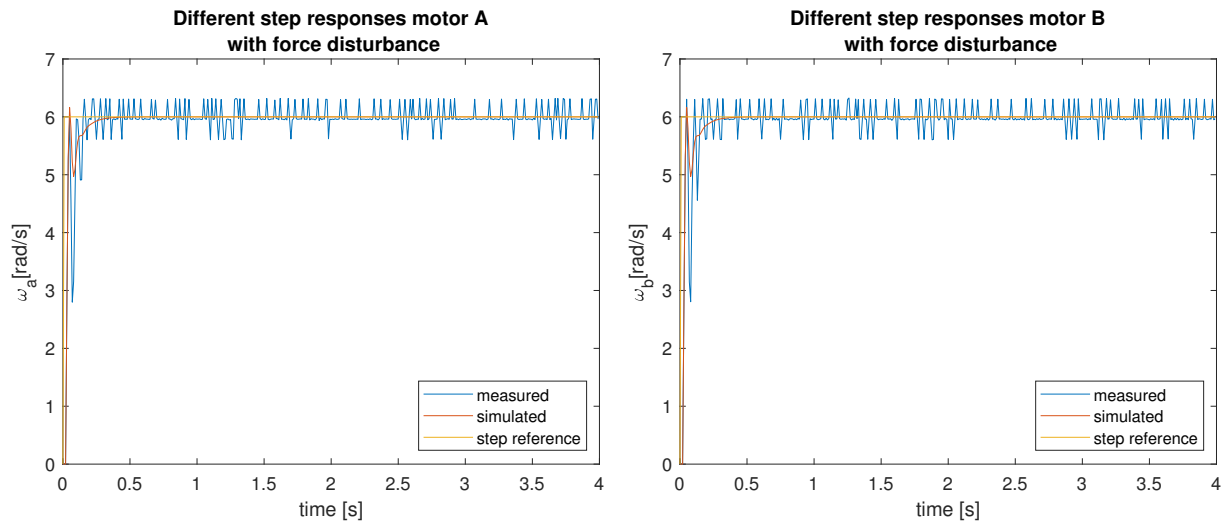


Figure 14: Measured and simulated responses of the closed loop system to a step input of 6 rad/s as reference velocity with a constant force disturbance

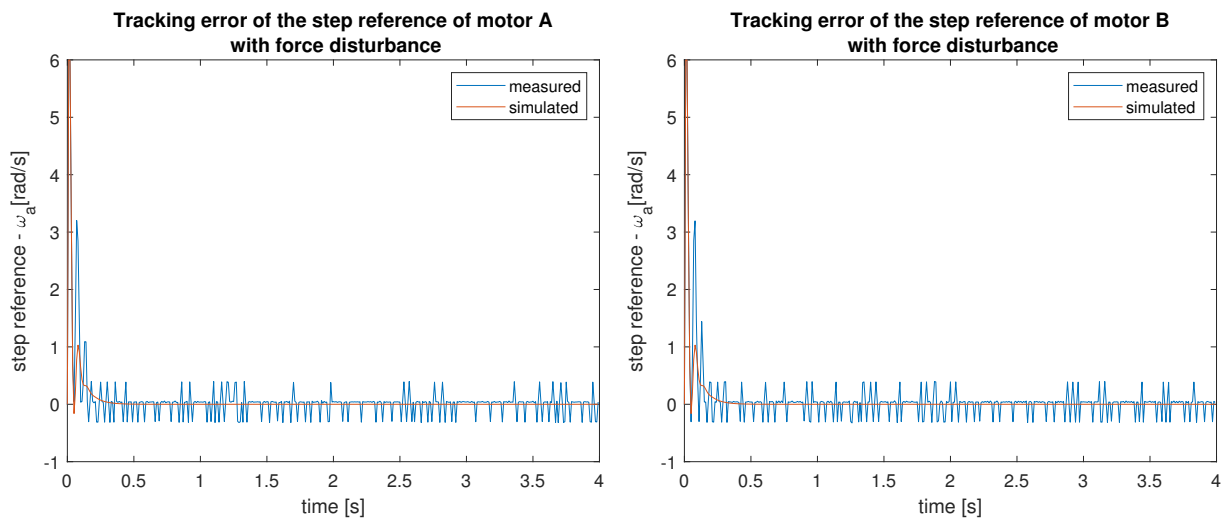


Figure 15: Measured and simulated tracking error of the closed loop system to a step input of 6 rad/s as reference velocity with a constant force disturbance

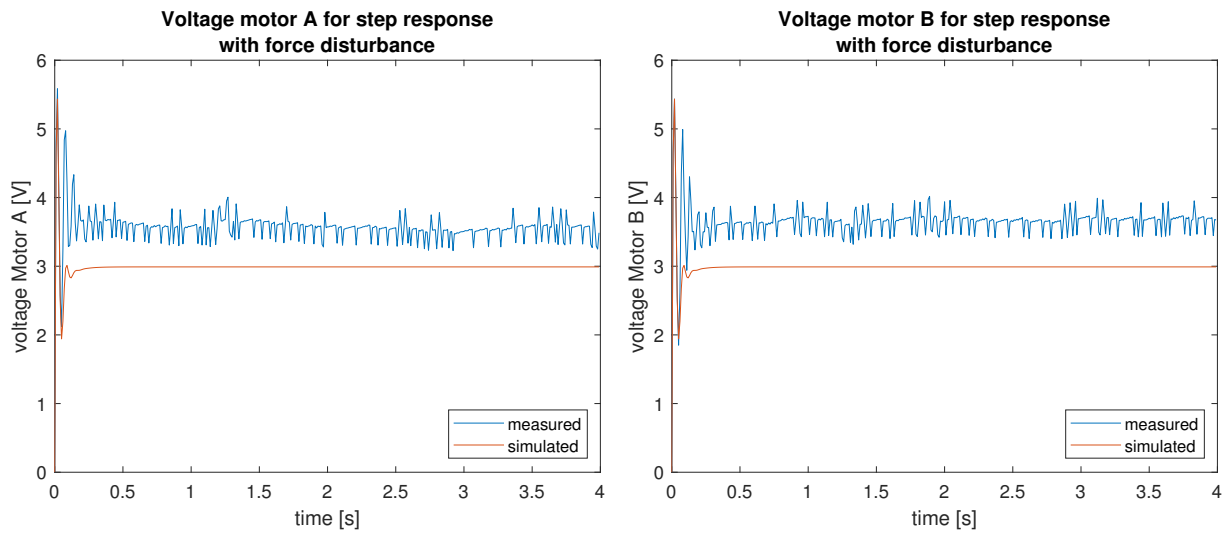


Figure 16: Measured and simulated voltages for a step reference input with a constant force disturbance

References

- [1] Siang Lim. Control block diagram with tikz. <https://www.overleaf.com/latex/examples/control-block-diagram-with-tikz/jfdcfxhgjmtz>, 2017. Last accessed December 8th 2021.
- [2] Swevers J. Pipeleers G. *Control Theory - Handouts*. Cursusdienst VTK, 2021.