

Matthias Frühwirth

Institute for Retailing & Data Science

Recap and Intro

Last week

- Causal Inference
- DAGs & Diff-in-Diff
- Quarto Presentations

Today

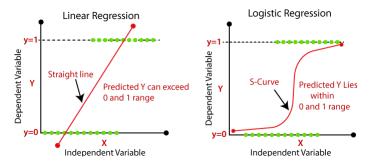
- Logistic Regression
- Quarto Documents

Intro to Logistic Regression (Logit)

Logistic Regression

- Can be used if outcome is a binary variable (e.g. 1 = Purchase Complete, 0 = Cart Abandoned)
- Generalized Linear Model (linear regression + non-linear link function, marginal effects are non-linear)
- Interested in modelling probabilities of outcomes:
 - Q: "How does making payment one-click affect the probability that the purchase is completed"? What about browsing time on the website?
- Sits right at the intersection of econometrics and machine learning
- Widely used baseline for inference, prediction/classification with category outcomes

Graphical



As we change X, we change the probability that Y becomes 1.

Basics

- ullet We assume that the outcome is Bernoulli distributed (outcome of a Bernoulli trial): this means with probability p the outcome is 1 and with probability 1-p it is 0.
- We are interested in p, so logistic regression models the probability of a binary outcome (Y = 0, 1). Probability Form:

$$P(Y = 1 \mid X) = \frac{1}{1 + e^{-\beta_0 + \beta_1 X_1 + \dots + \beta_k X_k}}$$

Equivalent log odds form: The model assumes a logit (log-odds)
 link between predictors and the probability:

$$\log\left(\frac{P(Y=1\mid X)}{1-P(Y=1\mid X)}\right) = \beta_0 + \beta_1 X_1 + \dots + \beta_k X_k$$

Odds and Interpretation

• We can interpret a coefficient β_k as changes to the log-odds; and $\exp(\beta_k)$ as changes to odds.

$$\log\left(\underbrace{\frac{P(Y=1\mid X)}{1-P(Y=1\mid X)}}_{\text{Odds}}\right) = \beta_0 + \beta_1 X_1 + \dots + \beta_k X_k.$$

- Odds reflect how likely an event is compared to it not happening ("ratio
 of 1 to 0"), rather than its share of all outcomes (which would be the
 probability).
- **Example**: The baseline odds for 100 customers (20 purchase complete, 80 cart abandoned). Then you estimate $\beta_k = 0.5$:

$${\rm Odds} = \frac{20}{80} = 0.25 \qquad e^{\beta_k = 0.5} \approx 1.65 \qquad {\rm New~Odds} = 0.25 \times 1.65 \approx 0.4125.$$

• For a one-unit increase in β_k , the odds increase by about $(\exp(\beta)-1)\cdot 100\%=65\%$. However, the effect on the actual probability depends on the starting value of the regressors.

Probabilites

- Coefficients from the regression output can be directly interpreted as changes to (log-)odds. If $\beta_1>0$, odds increase, as well as probabilities (and reverse). The change does not depend on the actual level of X.
- The reason: Odds are multiplicative, log-odds are additive (changes are "just added").
- However, the change in the outcome probability $P(Y=1\mid X)$ depends on the starting level of the covariates.
- Also note:

$$P(Y=1) = \frac{\mathsf{Odds}}{1 + \mathsf{Odds}}$$

• Previous Example (odds before: 0.25, odds after: 0.41)

$$p_1 = \frac{20}{20 + 80} = \frac{0.25}{1 + 1.25} = 0.2, \quad p_{\text{new}} \approx \frac{0.4125}{1.4125} \approx 0.291.$$

• Interpretation: For the given baseline probability/odds, increasing the explanatory variable by one unit increases the probability of Y by approx. 9.1%.

How to do in R:

Use the glm (generalized linear models) function from base R.

```
logit_model <- glm(Y ~ X1 + X2, data = df, family = binomial)
summary(logit_model)</pre>
```

• Model Fit (Pseudo- \mathbb{R}^2):

```
pscl::pR2(logit_model)
```

Calculate predicted probabilities:

```
predict(logit_model, type = "response")
```

 Get changes in probabilities (average marginal effects, keeps other covariates constant for each obs.)

```
#install.packages("margins")
library(margins)
marginal_effects <- margins(logit_model)
summary(marginal_effects)</pre>
```

Example Logit: Movie Box-Office-Bombs

- 1812 movies released in U.S. cinemas between 1995 and 2024 with a budget of at least 25 million dollars.
- We are interested in if a film is a box office flop ("bomb"). Defined as ROI < 66% and no international success \rightarrow Y = 1 (is a bomb).
- Explanatory variables (in the data) are audience and critical scores, as well as film properties like runtime and genre.
- Source: BoxOfficeMojo + IMDb + RottenTomatoes



John Carter (2012)

• Budget: 250 million USD

Domestic Box Office: 73 million USD
Audience Score: 60 & Critic Score: 52

(RT), runtime: 132 minutes

 One of the biggest bombs of all time (Disney took a 200 million dollar write-off)

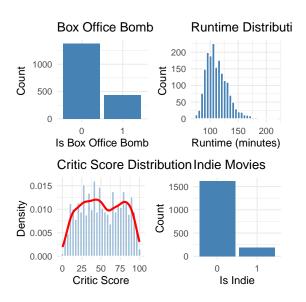
Data

```
df_movie <- read_csv("movie_select.csv")
df_movie %>%
  select(title,budget, domestic,runtime,audience_score,critic_score,is_bo_bomb) %>%
  arrange(-budget) %>% head(7)
```

A tibble: 7 x 7

title	budget	domestic	runtime	audience_score	critic_score	is_bo_bomb
<chr></chr>	<dbl></dbl>	<dbl></dbl>	<dbl></dbl>	<dbl></dbl>	<dbl></dbl>	<dbl></dbl>
1 Indiana Jones ~	387	174	154	88	70	1
2 Avengers: Endg~	356	858	181	90	94	0
3 Fast X(2023)	340	146	141	84	56	0
4 Avengers: Infi~	321	678	149	92	85	0
5 Pirates of the~	300	309	169	72	44	0
6 Mission: Impos~	291	172	163	94	96	0
7 Solo: A Star W~	275	213	135	63	69	0

Distributions



Estimate Model

11h

11hNu11

-888.6360852 -995.2245458 213.1769211

McFadden

0.1070999

r2ML

0.1109905

r2CU

0.1664965

G2

Model Summary

```
summary(logit model) # Print summary
Call:
glm(formula = is bo bomb ~ runtime + imdb avg rating + audience score +
   critic_score + is_indie + ge_action + ge_animation, family = binomial,
   data = df_movie)
Deviance Residuals:
   Min
            10 Median
                            30
                                   Max
-1.6731 -0.7543 -0.5323 -0.2811 2.8183
Coefficients:
               Estimate Std. Error z value Pr(>|z|)
(Intercept)
              1.470635 0.543181 2.707 0.006780 **
runtime
              0.010314 0.003738 2.759 0.005799 **
audience_score -0.018976  0.005279 -3.595  0.000325 ***
            -0.009036 0.003473 -2.602 0.009272 **
critic_score
is_indie
             0.820175 0.168235 4.875 0.00000109 ***
ge action
            -0.524517 0.124737 -4.205 0.00002611 ***
ge animation
            -0.402121 0.265862 -1.513 0.130401
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
(Dispersion parameter for binomial family taken to be 1)
   Null deviance: 1990.4 on 1811 degrees of freedom
Residual deviance: 1777.3 on 1804 degrees of freedom
ATC: 1793.3
Number of Fisher Scoring iterations: 4
```

Interpreting Coefficients (Odds)

• Runtime (Estimate = 0.0103):

For each additional minute of runtime, the log-odds of being a box office bomb increase by 0.0103. This corresponds to an odds ratio of $\exp(0.0103)\approx 1.01$, meaning about a 1% increase in the odds per minute.

• Audience Score (Estimate = -0.0189):

Each one-point increase in the RT audience score decreases the log-odds by 0.019. This also implies implies a $(\exp(-0.018976) - 1)*100\% = -1.879\%$ change (decrease) in the odds of being a box office bomb per point.

• Is Indie (Estimate = 0.8201):

Being an indie film (when (is_indie = 1)) increases the log-odds by 0.82 relative to non-indie films. This translates to an odds ratio of an odds ratio of 2.27, indicating that indie films have approximately 127% higher odds of being a box office bomb compared to non-indie films.

Marginal Effects (on P(Y = 1))

```
marginal_effects <- margins(logit_model)
summary(marginal_effects)</pre>
```

```
factor AME SE z p lower upper audience_score -0.0030 0.0008 -3.6310 0.0003 -0.0047 -0.0014 critic_score -0.0014 0.0006 -2.6134 0.0090 -0.0025 -0.0004 ge_action -0.0840 0.0197 -4.2652 0.0000 -0.1226 -0.0454 ge_animation -0.0644 0.0425 -1.5140 0.1300 -0.1477 0.0190 imdb_avg_rating -0.0555 0.0185 -2.9953 0.0027 -0.0918 -0.0192 is_indie 0.1313 0.0263 4.9858 0.0000 0.0797 0.1829 runtime 0.0017 0.0006 2.7765 0.0055 0.0005 0.0002
```

Interpretation. On average:

- a one-unit decrease in the audience score decreases the probability that the film is a box office bomb by 0.3%.
- a one-unit increase in the runtime increases the probability that the film is a box office bomb by 0.17%.
- switching from a non-indie to an indie film increases the predicted probability by roughly 13.1 [7.9, 18.2] percentage points.

Prediction: Predict/classify new data

Create a stinker (long and bad reviews)

```
new_bad_film <- data.frame(
  runtime = 240,
  imdb_avg_rating = 6.2, audience_score = 22, critic_score = 34,  # BAD REVIEWS!
  is_indie = 1, ge_action = 0, ge_animation = 0
)

predicted_prob <- predict(logit_model, newdata = new_bad_film, type = "response")
cat("The probability of bombing is: ", predicted_prob)</pre>
```

The probability of bombing is: 0.868991

Create a masterpiece (great reviews and not too long)

```
new_great_film <- data.frame(
   runtime = 120,
   imdb_avg_rating = 9.2, audience_score = 89, critic_score = 79,
   is_indie = 0, ge_action = 0, ge_animation = 1
)

predicted_prob <- predict(logit_model, newdata = new_great_film, type = "response")
cat("The probability of bombing is: ", predicted_prob)</pre>
```

The probability of bombing is: 0.03605312

Prediction and Classification (on existing data) and Model Eval

- Since logit is a common classifier, we can also evaluate our model on the quality of the models predicition (not just probabilities)
- We can compare actual classes to predicted classes and look at how often the model is wrong ("cross entropy loss").

Add the predicted probability as a new column and construct the predicted class (threshold 0.5)

```
set.seed(100)
df_movie <- df_movie %>%
  mutate(
    predicted_prob = predict(logit_model, type = "response"),
    predicted_class = if_else(predicted_prob >= 0.5, 1, 0)
)

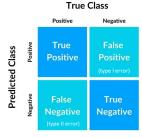
df_movie %>% select(title,is_bo_bomb,predicted_prob,predicted_class) %>%
    slice_sample(n = 6)
```

```
# A tibble: 6 x 4
 title
                            is bo bomb predicted prob predicted class
  <chr>>
                                 <dbl>
                                                <dbl>
                                                                 <dh1>
1 Mumford(1999)
                                               0.184
                                                                     0
2 The Social Network(2010)
                                               0.0749
                                                                     0
3 Sanctum(2011)
                                               0.277
                                                                     0
4 Lucky Numbers (2000)
                                               0.554
                                                                     1
5 Nanny McPhee (2005)
                                               0.154
6 Barney's Version(2010)
                                               0.135
                                                                     0
```

Confusion Matrix and Metrics (from package yardstick)

```
df_movie <- df_movie %>%
  mutate(
    truth = factor(is_bo_bomb, levels = c(1, 0), labels = c("Yes", "No")),
    predicted = factor(predicted_class, levels = c(1, 0), labels = c("Yes", "No"))
)
cm <- yardstick::conf_mat(df_movie, truth, predicted)</pre>
```

 $\begin{array}{ccc} & & \text{Truth} \\ \text{Prediction} & \text{Yes} & \text{No} \\ & \text{Yes} & 63 & 50 \\ & \text{No} & 369 & 1330 \end{array}$



Metrics

There are several metrics than can be used to evaluate a binary classification model (can also use yardstick functions), which depend on these 4 values.

Truth Prediction Yes No Yes 63 50 No 369 1330

```
TP <- 63 # True positives
TN <- 1330 # True negatives
FP <- 50 # False positives
FN <- 369 # False negatives
```

Accuracy (overall correctness):

```
(TP + TN) / (TP + TN + FP + FN)
```

[1] 0.7687638

Sensitivity/Recall: indicates how well the model identifies actual positives.

```
TP / (TP + FN)
```

[1] 0.1458333

• Specificity/True Negative Rate: measures how well the model identifies actual negatives.

```
TN / (TN + FP)
```

[1] 0.9637681

• Precision: reflects the accuracy of the positive predictions.

```
TP / (TP + FP)
```

[1] 0.5575221

Final thoughts

- Our model misses a lot of positives. We could experiment with lowering the threshold, if our goal is to find more positives (trade-off between recall and precision).
- Use XGBoost or similarly flexible model (always use out-of-sample testing).
- Probably lots of missing confounders :(
- We could try to add or engineer more features (regressors) like interaction terms, "text-based stuff" and crew info.

Intro to Quarto Documents

- Quarto documents are very similar to Quarto Presentations
- perfect for writing assignments, reports, your empirical thesis
- yaml (beginning of document) for PDF with Table of Contents, numbered sections and external bibliography

```
title: "Predicting Box Office Bombs: A Logistic Regression Analysis"
author: "Matthias"
date: "2025-03-25"
format:
   pdf:
    toc: true
    number-sections: true
bibliography: references.bib
```

 Sections and Subsections are created by using #Section and ##Subsection (similar to Quarto Pres)

Referencing

- Quarto automatically generates the "References" section at the end of your document, if you do the following:
- Create a file called references.bib and add it in your YAML settings (see Slide before)
- Add a reference in BibTex format to you reference file @article{smith2020, author = {Smith, John}, title = {Predicting Movie Success with Machine Learning}, journal = {Journal of Data Science}, year = {2020}, volume = {18}, pages = {101-115}}
- · Cite with:

As shown in recent studies [@smith2020], and following @smith2020