Homogeneous vehicle schedules

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Abstract

Although most railroad timetables remain in use for half a year, they tend to repeat themself each week. Thus one can also repeat their implementations in rolling stock, the vehicle schedulings, each week.

But timetables show even more structure: All the working days of the week are usually identical. This work sets out to investigate how to preserve this similarity in vehicle schedulings. We define homogeneous vehicle schedulings, establish a measure of partial homogenity and search for ways to efficiently find the most homogeneous vehicle scheduling for a given timetable.

1 Some definitions

Let $G = (V, A)$ be a digraph of connected stations, *e.g.* ordinary railway stations, maintenance shops. Now look at *trains* $(v, t_v, w, t_w) \in (V, \mathbb{R})^2$ where v, t_v name the station and time of departure, and w, t_v name the station and time of arrival. We call a set of trains $T \subseteq (V, \mathbb{R})^2$ a timetable. (In practise T can also be a multiset.)

We want to focus our attention to cyclic timetables. A timetable T is *cyclic*, iff there exists a period $w \in \mathbb{R}^+$, so that for each train $(v, t_v, w, t_w) \in T$ the trains $\{(v, t_v + nw, w, t_w + nw) \mid n \in \mathbb{Z}\}$ are also in T. We will identify cyclic timetables with their image under the canonic morphism $\varphi\colon (V, \mathbb{R})^2 \to (V, \mathbb{R}/(w\mathbb{Z}))^2$.

A timetable only fixes the services offered by the railroad company. But it does not specify where the locomotives and waggons in use come from or where they go afterwards. Naturally one wants to re-use rolling stock from one train for another one. So one seeks to establish a digraph $G_S = (T, S \subseteq T^2)$, where each train has exactly one predecessor and exactly one successor. Thus G_s consists of cycles.

In practise timetables repeat every week *i.e.* $w = 7$ days. This work will deal with timetables that show more structure: Fix an interval $d := \frac{w}{n}$ with $n \in \mathbb{N}^+$ (*e.g.*for days in a week, let $n = 7$). We partition the trains into equivalence classes that share the same arrival and departure stations, and whose arrival and departure times only differ by an integral multiply of w (*i.e.* whole days):

$$[(v, t_v, w, t_w)]_d := \{(v, t_v + nd, w, t_w + nd) \mid n \in \mathbb{Z}\} \tag{1}$$

Let C be the set of all those classes.

Ideally each class has exactly one member for each day of the week. That would allow $w = 1$day instead of $w = 7$days and enable a schedule that repeats perfectly every day. But more typical me observe a lot of classes in a timetable with one member each day from Monday to Friday and none on the weekends. Or even less regular arrangements.

To guarantee homogenity one might naïvely propose that all members of one class c_1 only be succeeded by members of the same class c_2. But that would not be practial, because *e.g.*a class that is offered all week would be banned from being connected to a class of trains that are not offered on the weekend.

So we introduce a weaker condition. A schedule S is called homogenous iff:

- Given classes $z, \overline{z} \in C$
- and trains $r_1, r_2 \in z$ and $\overline{r}_1, \overline{r}_2 \in \overline{z}$,
- then either there is no path between neither r_1 and $\overline{r}_2$ nor r_2 and $\overline{r}_2$ in S; or they are connected by paths of the same length:

$$\Delta_S(r_1, \overline{r_1}) = \Delta_S(r_2, \overline{r_2}) \tag{2}$$

with some distance function $\Delta_S \colon (T, T) \to \mathbb{R}_0^+ \dot{\cup} \infty$ on paths in S.

We will define $\Delta_S(s, t)$ as the sum of arc lengths δ in the path between the trains s and t or ∞ if no such path exists. We can talk about *the* path $P \subseteq S$ between s and t, because G_S consists only of cycles.

$$\Delta_S(s, t) = \sum_{(v,w) \in P} \delta(v, w)$$

Of course we have only moved the problem. We still have to define the length of a single arc: $\delta(t, \overline{t})$ shall be the minimal time that a vehicle needs to provide t and be ready for the departure of $\overline{t}$:

$$\delta((\cdot, t_{\text{dep}}, \cdot, t_{\text{arr}}), (\cdot, \overline{t_{\text{dep}}}, \cdot, \cdot) := \min(\{\overline{t_{\text{dep}}} - t_{\text{dep}} + nw \mid \overline{t_{\text{dep}}} - t_{\text{arr}} + nw \geq 0, n \in \mathbb{Z}\} \cap \mathbb{R}_0^+)$$

In most cases this will be just the difference $\overline{t_{\text{dep}}} - t_{\text{dep}}$, but we may need to add a whole number of weeks if $\overline{t}$ departs before {t} arrives.