Homogeneous train schedules

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Abstract

Although most railroad schedules remain in use for half a year, they tend to repeat themself each week. Thus one can also repeat their implementations, the vehicle schedulings, each week.

But railroad schedules show even more structure: All the working days of the week are usually identical. This work sets out to investigate how to transport this similarity in relations to their vehicle scheduligs. We define homogeneous vehicle schedulings, establish a measure of partial homogenity and search for ways to efficiently find the most homogeneous vehicle scheduling for a given schedule.

1 Some definitions

Let $G = (V, A)$ be a digraph of connected stations, i.e. railway stations, maintenance shops. Now look at *relations* $(v, t_v, w, t_w) \in (V, \mathbb{R})^2$ where v, t_v name the station and time of departure, and w, t_v name the station and time of arrival. We call a set of relations $S \subseteq (V, \mathbb{R})^2$ a schedule.

We want to focus our attention to cyclic schedules. A schedule S is *cyclic*, iff there exists a period $w \in \mathbb{R}^+$, so that for each relation $(v, t_v, w, t_w) \in S$ the relations $\{(v, t_v + nw, w, t_w + nw) \mid n \in \mathbb{Z}\}$ are also in S. For a cyclic schedule S we will identify S with its image under the canonic morphism $\varphi\colon (V, \mathbb{R})^2 \to (V, \mathbb{R}/w\mathbb{Z})^2$.

In practise schedules repeat every week i.e. $w = 7$days.

This work will deal with schedules that show more structure: Fix an interval $d := \frac{w}{n}$ with $n \in \mathbb{N}^+$; in addition to the schedule S introduce the set of trains $T \subseteq \mathcal{P}(S)$ partitioning the relations of the schedule. Relations in each *train* $z \in T$ all share the same arrival and departure stations, also their arrival and departure times only differ by an integral multiply of w. That is

$$z \subseteq [(v, t_v, w, t_w)]_d := \{(v, t_v + nd, w, t_w + nd) \mid n \in \mathbb{Z}\} \qquad (1)$$

We want to achieve homogenity in our schedules. A schedule S is called homogenous iff:

- Given trains $z, \overline{z} \in T$
- and relations $r_1, r_2 \in z$ and $\overline{r}_1, \overline{r}_2 \in \overline{z}$,

- then either there is no path between neither r_1 and $\overline{r}_2$ nor r_2 and $\overline{r}_2$ in S; or they have the paths have the same length:

$$\Delta(r_1, \overline{r_1}) = \Delta(r_2, \overline{r_2}) \tag{2}$$