Partinioning Orbitopes for Permutation of Rows and Columns – Term Paper

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1 Abstract

2 Introduction

Symmetry is a problem in integer programming. The introduction and analysis of orbitopes has been one recent step towards removing those symmetries. Orbitopes exploit the fact that one can introduce groups acting on the feasible solutions of a symmetric integer problem such that members of the same orbit share the same objective value. Obviously, any optimization algorithm that only considers at least one representative of each orbit will still find an optimal solution, if there is one. Orbitopes are the convex hulls of all representatives thus considered.

In this paper I will consider a certain kind of partitioning orbitopes. Partitioning orbitopes are polytopes that live in the set of 0/1-matrices with at most, resp. exactly, one 1-entry per row. Matrices that can be transformed into each other by permutation of columns are considered equal and belong to one orbit. The partitioning orbitopes are inclusion minimal polytopes that contain one and only one member from each orbit: the unique representative matrix with lexicographically sorted columns.

In [1] Faenza and Kaibel give compact extended formulations for the packing and partitioning orbitopes. In this paper I want to give extended formulations for symmetric partitioning orbitopes: In addition to column permutation, row permutations are allowed as well. More formally the group $S = S_R \times S_C$ acts on the solution, where S_R and S_C are the symmetric groups on rows and columns. The next section gives a description of the representatives contained in a symmetric partitioning orbitope.

3 Vertices of the Symmetric Partitioning Orbitope

Let p be the number of columns and q be the number of rows of matrices in the set $M \subseteq \{0,1\}^{p \times q}$ of all 0/1-matrices with exactly one 1-entry in each row. Let $S = S_R \times S_C$ be the Cartesian product of the symmetric groups on rows and columns. S acts on M.

Obviously we can require representatives to be lexicographically sorted along rows and columns. Thus we get representative matrices with 1 in the top-left. In each row the 1-entry stays in the column it dwelled in the row above – or wanders off one column right.

Using a very simple flow network one can easily find the facettes of the resulting *simple symmetric partition orbitope* – see **??**.

Unfortunately this way some orbits of S intersect the orbitope with more than one member. To guarantee exactly one representative per orbit we can require runs of 1-entries in the same column to descend in length.

Computer generated examples suggest that no polynomial in $p \cdot q$ limits the number of the resulting *exact symmetric partioning orbitope*'s facettes.

But - I will show an extended formulation for this exact symmetric partitioning orbitope that is polynomially bounded in number of both variables and facettes in $p \cdot q$.

4 Polynomial Optimization on the Exact Symmetric Partitioning Orbitope

Literatur

[1] Yuri Faenza and Volker Kaibel. Extended formulations for packing and partitioning orbitopes, 2008.