Modeling Continuous Bounded Interval Responses With a Multivariate Logit-Function

Matthias Kloft & Daniel W. Heck

DGPS 2024, Vienna

Motivation

When **ONE** Response Value is **NOT** Enough

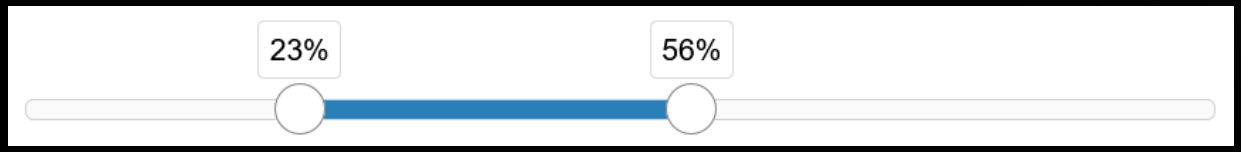
"What percentage of your daily work time did you spend on preparing for DGPs 2024 in the last week?"

Interval Responses

Dual-range slider (**DRS**)

Interval Responses

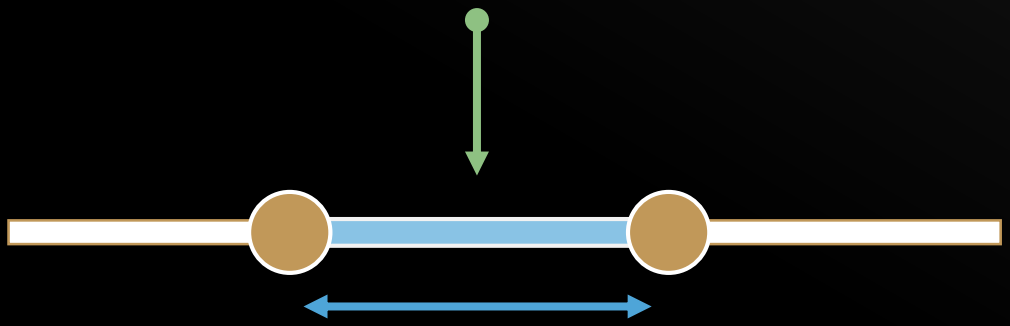
Dual-range slider (**DRS**)



noUISlider JavaScript range slider (Gersen, 2024)

Interval Responses

DRS **Location**: $\frac{y^{(L)}+y^{(U)}}{2}$



DRS **Width**: $y^{(U)} - y^{(L)}$

Interval Responses

Variability / plausible range:

- Self-ratings, stimuli

Uncertainty / expertise:

- Estimation (e.g, forecasting)

Ambiguity:

- Item content unclear
- No clear-cut true answer (e.g., verbal quantifiers like “seldom” or “likely”)

Topics of the Talk

1. A link function for interval responses (Smithson & Broomell, 2024)

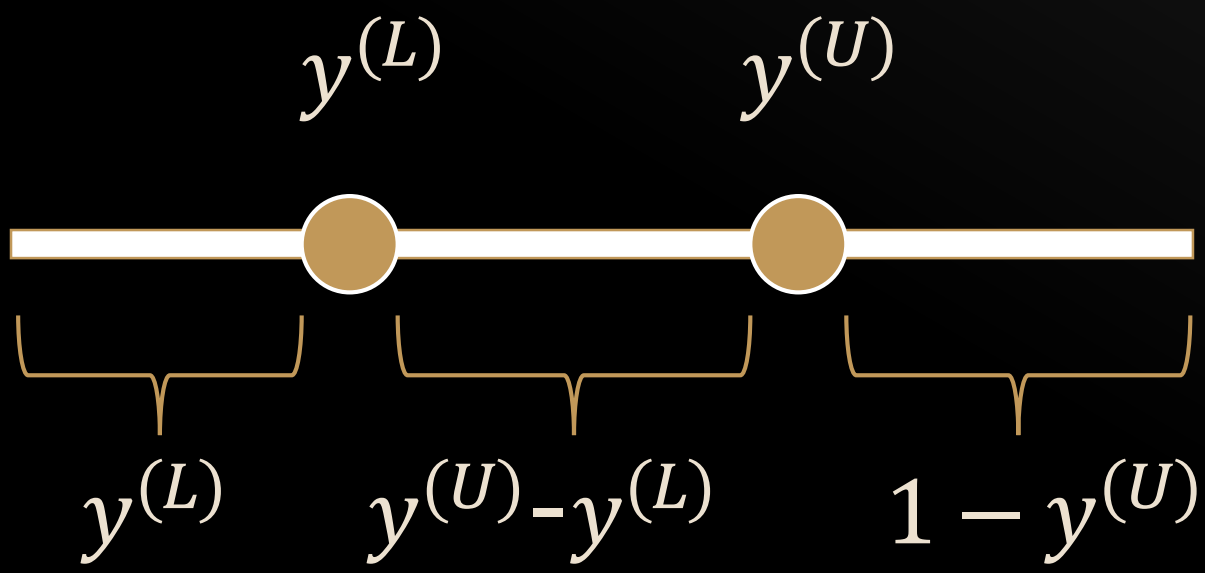
2. Application: factor analysis (Kloft & Heck, in press)

A Link Function for Interval Responses

Smithson & Broomell (2024)

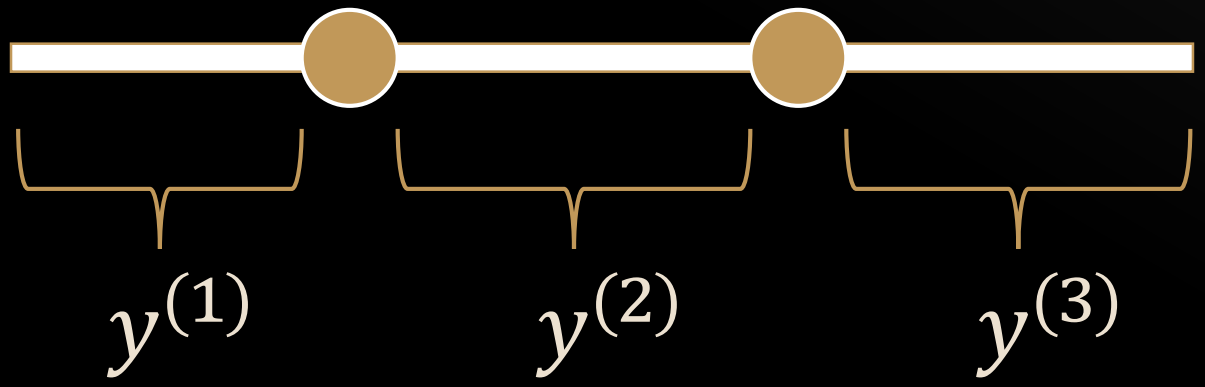
Compositional Data

- Components must sum to one: simplex



Compositional Data

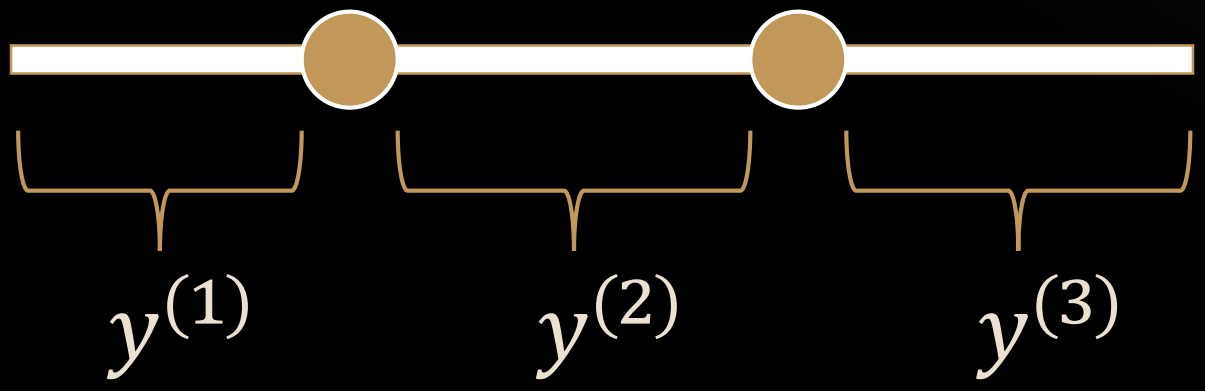
- Components must sum to one: simplex



Log-Ratios

Unbounded **Location**: $\log\left(\frac{y^{(1)}}{y^{(3)}}\right)$

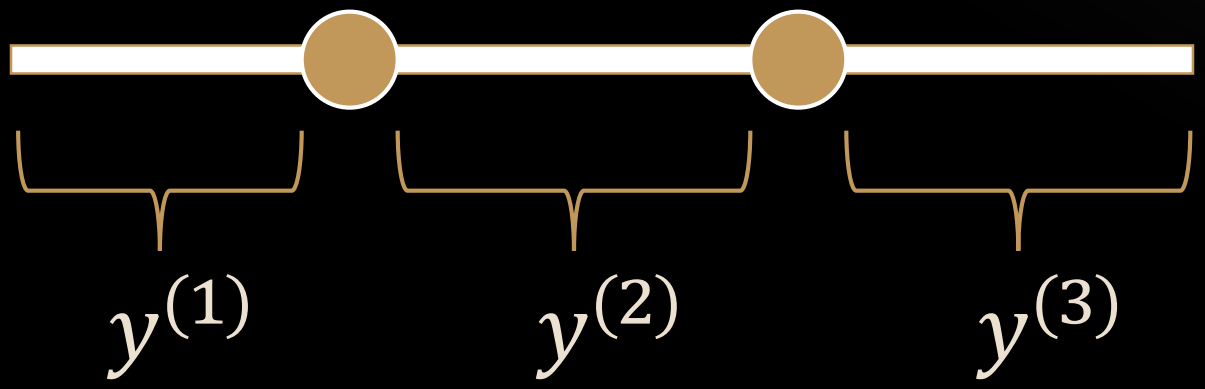
- Compares outer components



Log-Ratios

Unbounded **Width**: $\log\left(\frac{y^{(2)}}{\sqrt{y^{(1)} \times y^{(3)}}}\right)$

- Compares interval width to geometric mean of outer components



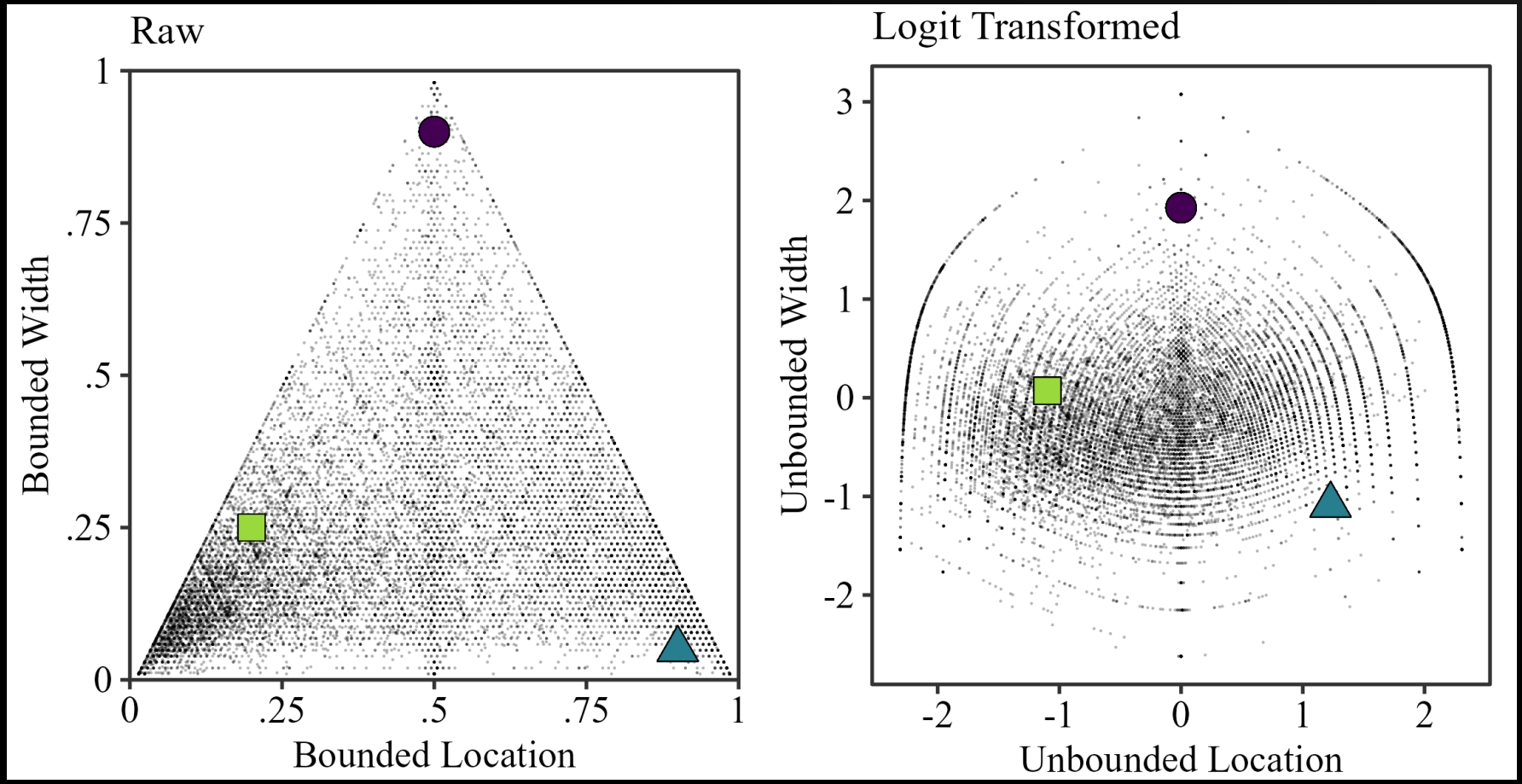
Isometric Log-Ratio Transformation

- Smithson & Broomel (2024)

$$\mathbf{z} = \begin{pmatrix} z^{loc} \\ z^{wid} \end{pmatrix} = \begin{pmatrix} \sqrt{\frac{1}{2}} \log\left(\frac{y^{(1)}}{y^{(3)}}\right) \\ \sqrt{\frac{2}{3}} \log\left(\frac{y^{(2)}}{\sqrt{y^{(1)} \times y^{(3)}}}\right) \end{pmatrix}$$

Data Example

- More suitable for models using a normal distribution



Application: Factor Analysis

Kloft et al. (in press)

Application of Logit Transformation

1. Transform interval responses with the isometric log-ratio function

2. Factor Analyses: EFA, CFA
   - We use **only** the unbounded **widths**
   - Transformation acts as a **link function**
     - Accounts for boundedness
     - Adjusts the width for the location

Research Questions

- Focus on DRS **Widths**:
  - **Sensitivity** to different tasks?
  - Just a preference response style?
  - **Dimensionality**?

- Different tasks of varying similarity

Study Design: Applications / Tasks

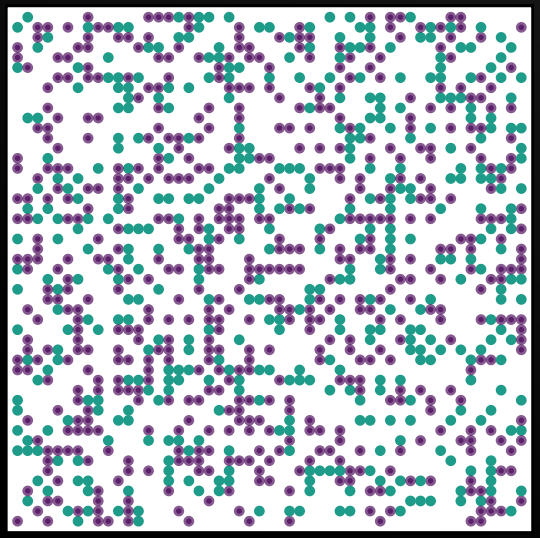
Extraversion & Conscientiousness:

Talkative = [75% – 93%] applicable

Color Dot Estimation:

% purple dots = [46% – 73%]

(true = 60%)



Election Forecasting: outcomes for 6 parties

- *Green party* = [10% – 19%]

Study Design: Applications / Tasks

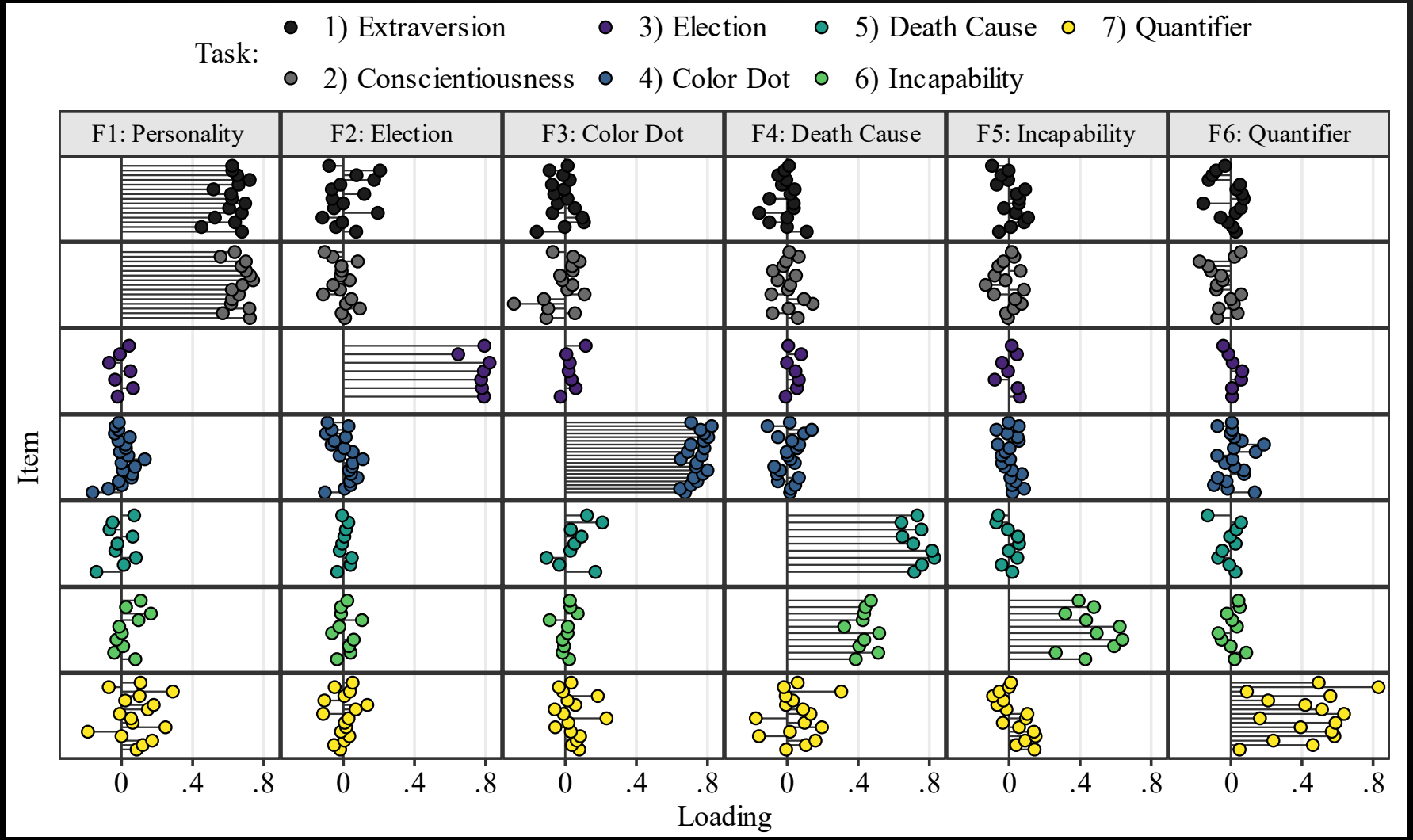
Estimation of percentages:

- Death Causes: *heart diseases*
- Reasons for Incapability for work: *mental health*

Rating of verbal quantifiers as probabilites that a so described event would occur:

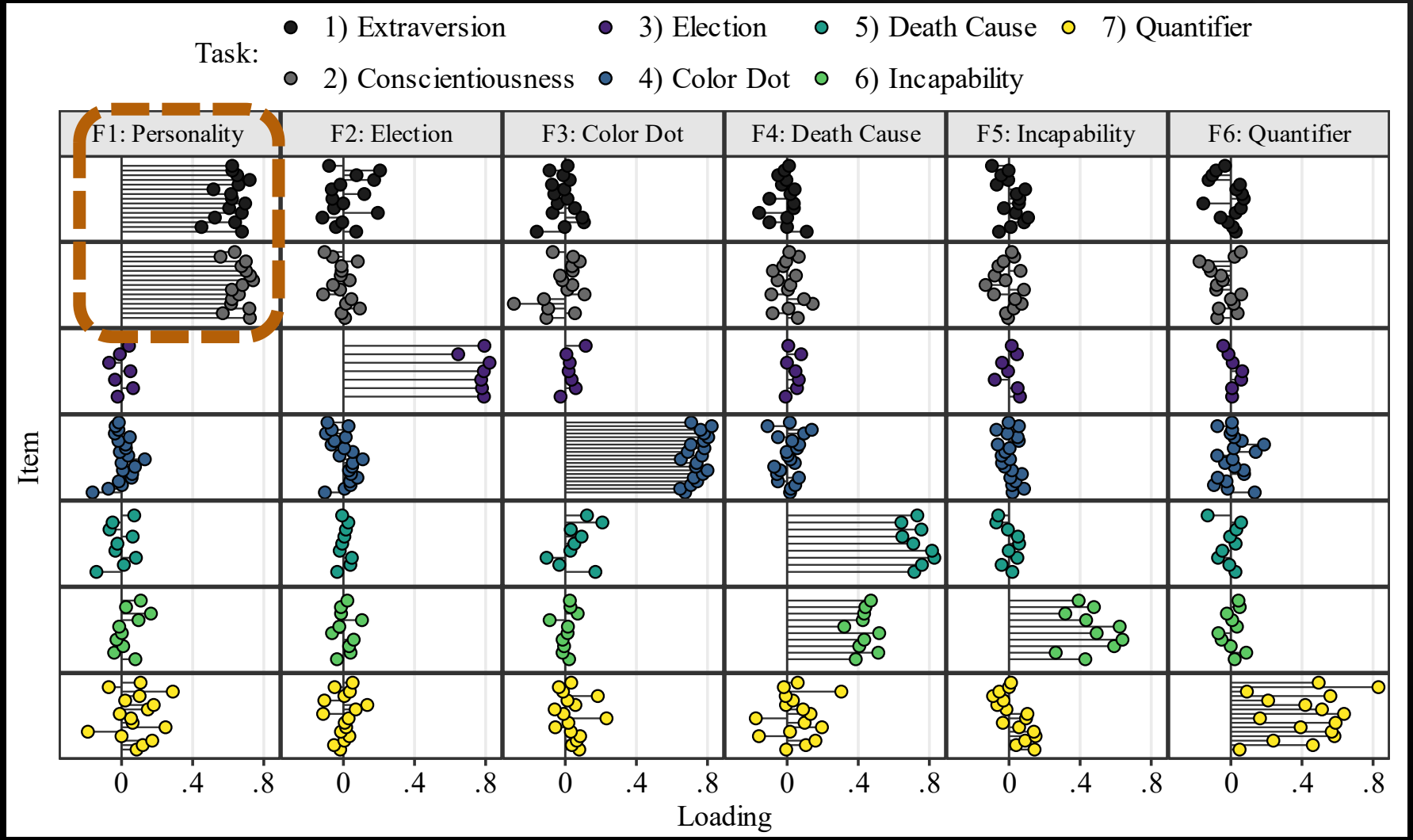
- *Seldom* = $[5\% - 30\%]$

Interval Width: EFA Loadings



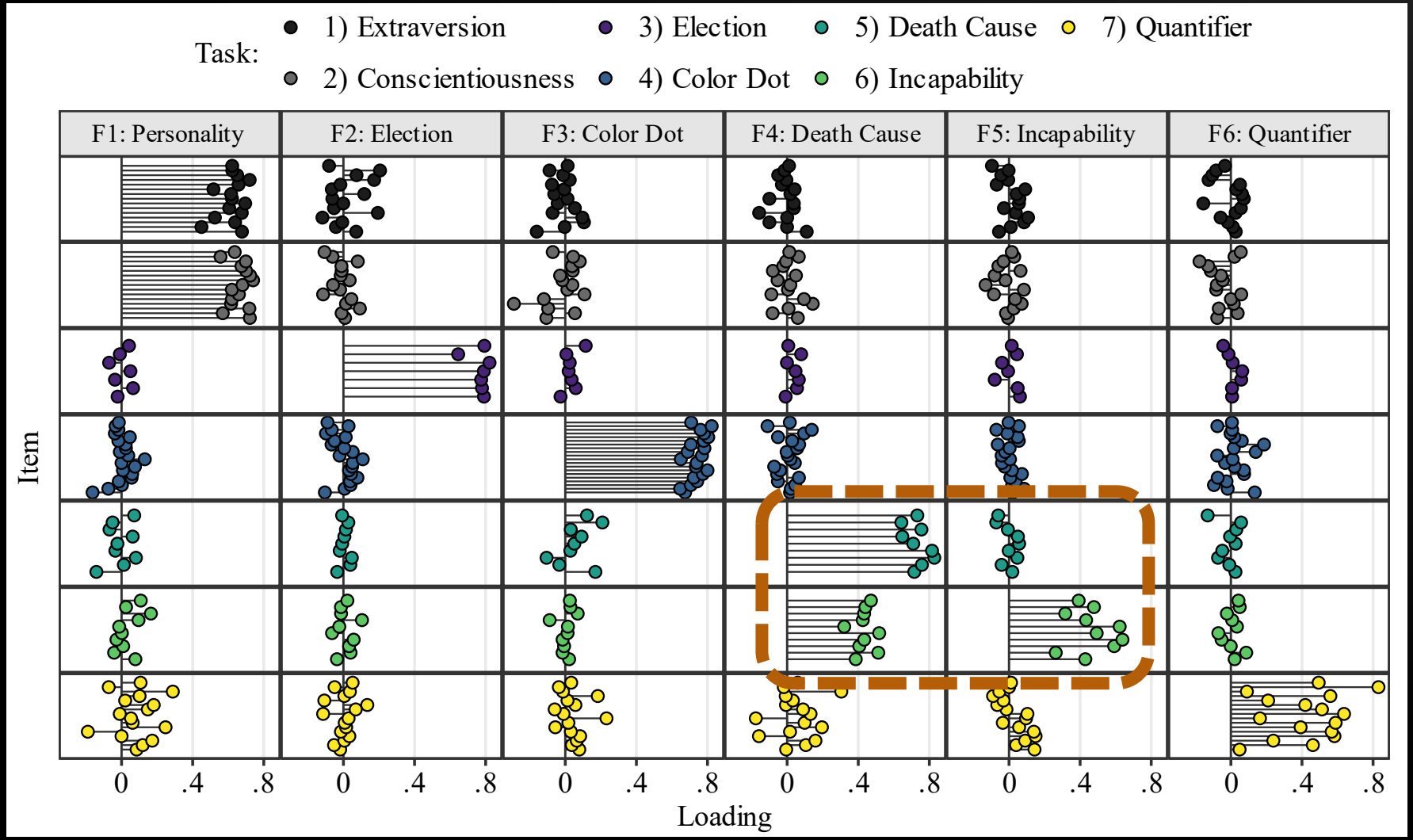
DRS **Width** only!

Interval Width: EFA Loadings



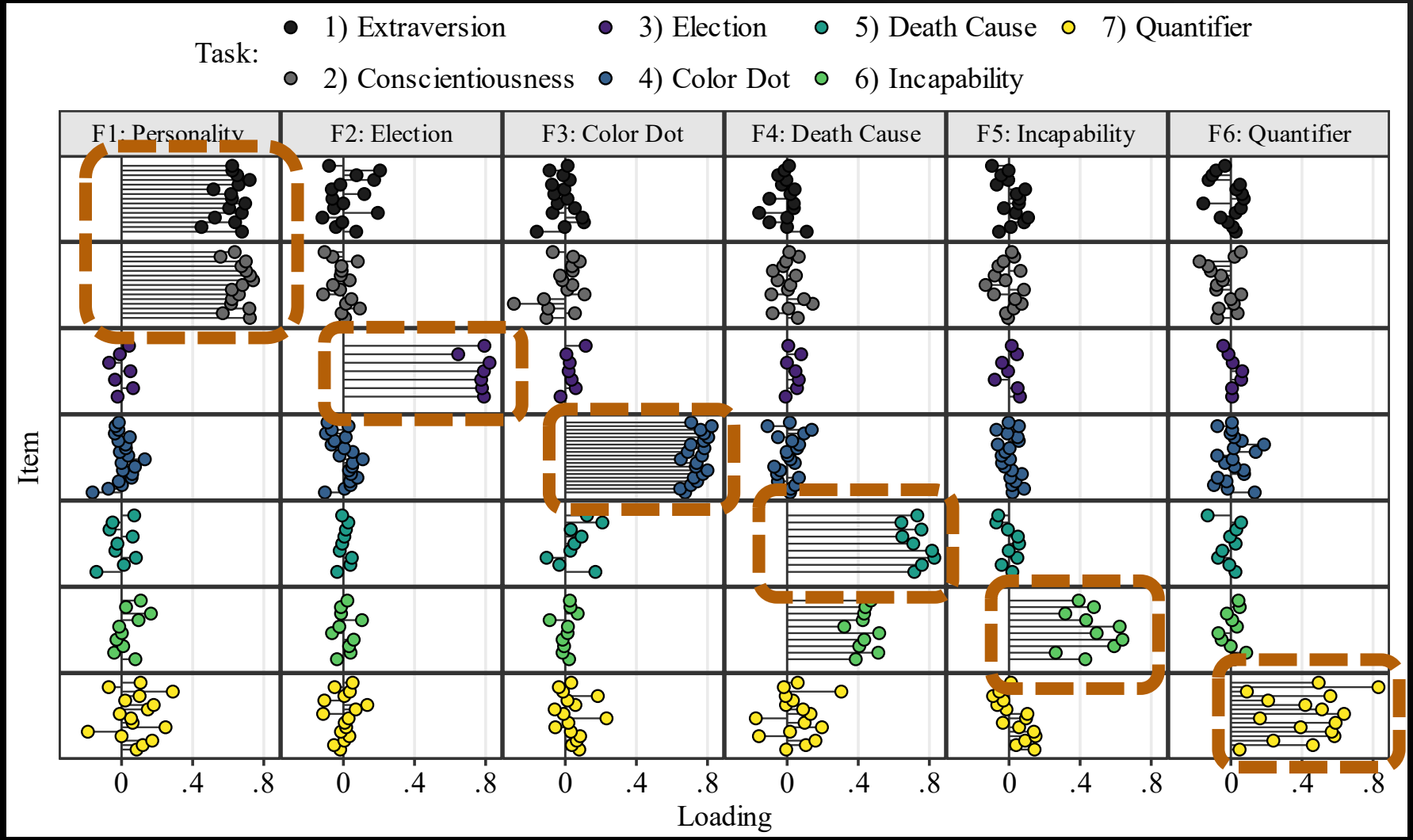
Common factor for Extraversion & Conscientiousness

Interval Width: EFA Loadings



High cross loadings: similar tasks

Interval Width: EFA Loadings



Factor structure **follows** task structure

Further Confirmatory Models

- Again only transformed DRS **Width**

6-Factor Model

- Assignment by theoretical task
- Simple structure

Bifactor Model

- 6 **Specific** Factors
  - Assignment by theoretical task
- **General** Factor: preferred width / response style

Model Comparison

Model	RMSEA [95%CI]	
	Transformed	Untransformed
EFA	.054 [.051,.056]	.062 [.059,.064]
CFA	.053 [.050, .055]	.062 [.059, .064]
Bifactor	.052 [.049, .054]	.059 [.057, .061]

Modelfit is better using the transformation

Take-Home Points

- Isometric log-ratio transformation makes interval responses **more suitable** for modeling frameworks using **normal** distributions

- Isometric log-ratio transformation as a **link function:** adapt existing models to interval reponses

Thanks to:

- Prof. Dr. Daniel W. Heck

- Björn Siepe

Contact:

kloft@uni-marburg.de

Slides:

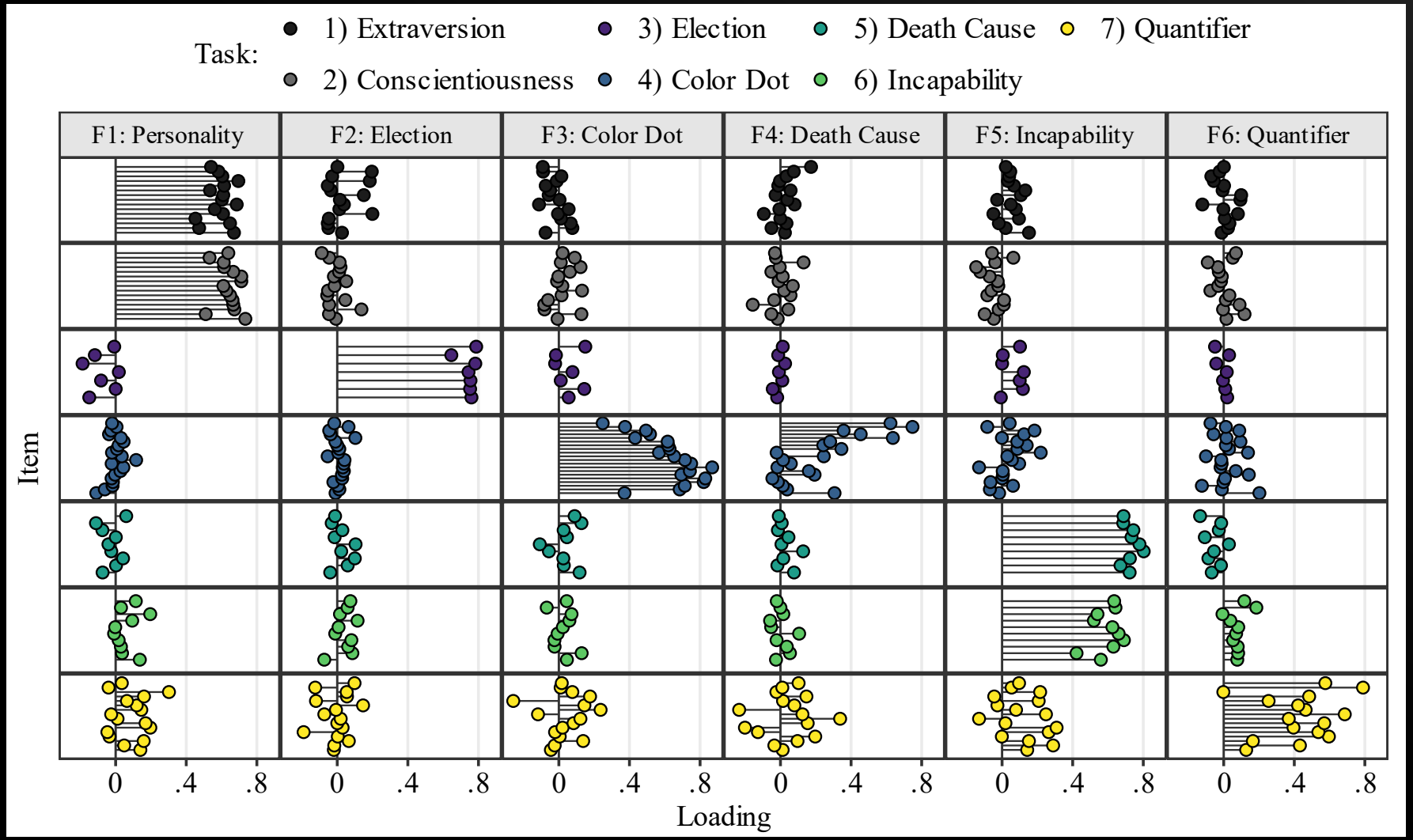
https://github.com/matthiaskloft/

References

- **Kloft, M.** & Heck, D.W. (in press). Discriminant validity of interval response formats: Investigating the dimensional structure of interval widths. Educational and Psychological Measurement. https://doi.org/10.31234/osf.io/esvxk
- **Kloft, M.**, Siepe, B.S., & Heck, D.W. (2024). The interval truth model: A consensus model for continuous bounded interval responses [Manuscript in preparation]. Department of Psychology, University of Marburg
- Gersen, L. (2024). Leongersen/noUiSlider [Software]. https://github.com/leongersen/noUiSlider
- Smithson, M., & Broomell, S. B. (2024). Compositional data analysis tutorial: Psychological Methods. *Psychological Methods, 29* (2), 362–378. https://doi.org/10.1037/met0000464

Additional Slides

Interval Width: EFA Loadings



DRS **Width** untransformed