Labs
Optimization for Machine Learning
Spring 2021

EPFL
School of Computer and Communication Sciences
Martin Jaggi & Nicolas Flammarion
github.com/epfml/OptML_course

Problem Set 10, due May 14, 2021 (Duality)

Prove the following property from the lecture slides:
If f is closed and convex, then for any $\mathbf{x}, \mathbf{y}$,

$$\begin{aligned} \mathbf{y} \in \partial f(\mathbf{x}) &\Leftrightarrow \mathbf{x} \in \partial f^*(\mathbf{y}) \\ &\Leftrightarrow f(\mathbf{x}) + f^*(\mathbf{y}) = \mathbf{x}^\top \mathbf{y} \end{aligned}$$

Hint: if function $f(\mathbf{x})$ is of the following form: $f(\mathbf{x}) = \max_{\alpha \in \mathcal{A}} f_\alpha(\mathbf{x})$, then its subgradient is given by

$$\partial f(\mathbf{x}) = \textbf{Co}\left[\cup \{\partial f_\alpha(\mathbf{x}) | f_\alpha(\mathbf{x}) = f(\mathbf{x})\}\right],$$

where **Co** is taking a convex hull of the set.