Technische Universität München
Department of Mathematics

Bachelor's Thesis

Sequential Monte Carlo for time-dependent Bayesian Inverse Problems

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With my signature below, I assert that the work in this thesis has been composed by myself independently and no source materials or aids other than those mentioned in the thesis have been used.

München, April 14, 2018

Place, Date

Signature

Abstract

Titel auf Englisch wiederholen.

Es folgt die englische Version der Kurzfassung.

Contents

Chapter 1

Introduction

Understanding physical systems is often done through mathematical models, which parameters allow to predict and understand the behaviour of these systems. Estimating these parameters from a set of measurements, called an *inverse problem*, is a challenging task for which a large variety of methodologies and mathematical frameworks have been developed. This thesis is designed to be a rigorous but approachable guide for practitioners interested in solving inverse problems related to dynamical systems, often found in fields such as biology, robotics and many others. While being a natural fit for such time-dependent problems, the methods presented will also be shown to have desirable properties for static inverse problems.

Inspired by [ABL$^+$13], this thesis will be structured around a practical case study of solving an inverse problem, this time in the context of a dynamical system, in order to illustrate and validate theoretical concepts and numerical algorithms presented in the thesis. The dynamical system studied along this thesis is a simple pendulum, an idealized model for a pendulum in which the mass of the pendulum and its friction are ignored, described by the following second-order and non-linear differential equation;

$$\frac{\mathrm{d}^2\theta}{\mathrm{d}t^2} = -\frac{g}{l}\sin(\theta), \tag{1.1}$$

where θ is the angle of the pendulum to its resting point, l is the length of the pendulum and g is the acceleration due to gravity, called *gravitational acceleration*. The inverse problem will be to estimate the gravitational acceleration from an data of an experiment in which the pendulum is let go with no initial velocity from an angle θ_0, and where to measurements are the times $(t_1, \ldots, t_n)$ at which the pendulum reaches a zero angle.

This thesis will start with a presentation of *Bayesian data analysis*, a methodology with probabilistic modeling as its core, expressing the solution of the inverse problem as a probability distribution of the parameters, called the *posterior distribution*. We will show how to express inverse problems in this framework using the example of the estimation of the gravitational acceleration. Furthemore, this section will characterize the set of problems having a *well-posed* solution, using Hadamard's [Had02] definition, and will proove well-posedness of the pendulum inverse problem. For most practical applications the posterior distribution does not have a closed form solution, creating the need to consider numerical approximations of the real posterior. In Section 3, a construction of the *Sequential Monte Carlo* algorithm will be presented, showing it to provide a natural and efficient way of estimating the real posterior distribution. This section will also provide a proof of convergence of the approximation to the exact solution, and illustrate these results by approximating the solution to the pendulum's inverse problem.

Chapter 2

Bayesian Inverse Problem

(should this be 'bayesian filtering problem' ?

Chapter 3

Sequential Monte Carlo

Chapter 4

Case Study

Chapter 5

Conclusion

Appendix A

Appendix

Beispiel für eine Tabelle:

Table A.1: Beispiel für eine Beschriftung. Tabellenbeschriftungen sind üblicherweise über der Tabelle platziert.

left	center	right
entry	entry	entry
entry	entry	entry
entry	entry	entry

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Bibliography

[ABL+13] Moritz Allmaras, Wolfgang Bangerth, Jean Marie Linhart, Javier Polanco, Fang Wang, Kainan Wang, Jennifer Webster, and Sarah Zedler. Estimating parameters in physical models through bayesian inversion: A complete example. *SIAM Review*, 55(1):149–167, 2013.

[Had02] Jacques Hadamard. Sur les problèmes aux dérivés partielles et leur signification physique. *Princeton University Bulletin*, 13:49–52, 1902.