Sequential Monte Carlo for time-dependent Bayesian Inverse Problems

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This document is a detailed plan of the Bachelor thesis I intend to write under the supervision of Prof. Ullmann and J. Latz covering Sequential Monte Carlo methods for static Bayesian inverse problems.

The goal of this Bachelor thesis will be to give the mathematical reader a self-contained guide for infering static parameters of time-dependent models from a set of observations. To do this, I will present the Bayesian inverse problem associated to the time-dependent inference task, as well as theoretical properties of this formulation. I will follow with a presentation and analysis of Sequential Monte Carlo methods as a numerical approximation of the solution of the Bayesian inverse problem. I will then conclude with a practical application on a real-world inference problem in order to illustrate theoretical results and assess performances of the numerical method.

The plan of the Bachelor thesis will be the following:

- Introduction
  - Inverse problem and ill-posedness, refering to Hadamard's definition [4]
  - Pendulum inverse problem
  - Litterature review
- Bayesian Inverse Problem
  - Filtering problem for static problems, similar to [5] and [2]
  - Time-dependent Bayesian updating
  - Characterization of the solution and its well-posedness
- Sequential Monte Carlo
  - Monte Carlo to approximate a solution
  - Importance Sampling and Sequential Importance Sampling
  - Sequential Monte Carlo, presenting the general method defined in [3]
  - Kernel choice (focus on MCMC kernels)
  - Proof of convergence, probably an adaptation of [1]

- Application
  - Presentation and bayesian formulation of the Pendulum inverse problem
  - Proof that the problem is well-posed
  - Presentation and analysis of the SMC numerical solution

References

[1] Alexandros Beskos, Ajay Jasra, Ege A Muzaffer, and Andrew M. Stuart. Sequential monte carlo methods for bayesian elliptic inverse problems. *Statistics and Computing*, 25(4):727–737, 2015.

[2] Nicolas Chopin. A sequential particle filter method for static models. *Biometrika*, 89(3):539–552, 2002.

[3] Pierre Del Moral, Arnaud Doucet, and Ajay Jasra. Sequential monte carlo samplers. *Journal of the Royal Statistical Society: Series B (Statistical Methodology)*, 68(3):411–436, 2006.

[4] Jacques Hadamard. Sur les problèmes aux dérivés partielles et leur signification physique. *Princeton University Bulletin*, 13:49–52, 1902.

[5] Andrew M. Stuart. Inverse problems: A bayesian perspective. *Acta Numerica*, 19:451–559, 2010.