

W3S1 extra practice

About recursion

Recursion: definition

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- In other words, we can define the value of the **factorial function with value n** , by reusing the factorial function with value $(n - 1)$.

Recursion vs. **for** loop

- The **factorial function** with value n (denoted $n!$) is defined as
$$n! = 1 \times 2 \times \cdots \times n$$
- Following this definition, we could compute the value of the factorial using a simple **for** loop.
- And that would work just fine.

```
1 def factorial_fun_for(n):  
2     result = 1  
3     for i in range(1, n + 1):  
4         result *= i  
5     return result
```

```
1 print(factorial_fun_for(1))
```

1

```
1 print(factorial_fun_for(3))
```

6

```
1 print(factorial_fun_for(5))
```

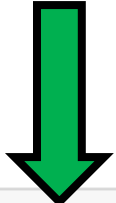
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Recursion vs. **for** loop

- But we could also use the recursive definition for the factorial function, defined as


$$n! = \begin{cases} 1 & \text{if } n = 1 \\ n \times ((n-1)!) & \text{if } n > 1 \end{cases}$$

- And it works as well, without any **for** loop!



```
1 def factorial_fun_rec(n):
2     if n == 1:
3         value = 1
4     else:
5         value = n*factorial_fun_rec(n - 1)
6     return value
```

```
1 print(factorial_fun_rec(1))
```



1

```
1 print(factorial_fun_rec(3))
```

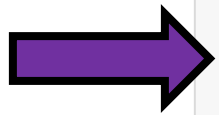
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```
1 print(factorial_fun_rec(5))
```

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Recursion vs. **for** loop

- What happens in practice? Multiple **concurrent runs** of a function.



```
1 def factorial_fun_rec(n):  
2     print("Function called, with n = {}".format(n))  
3     if n == 1:  
4         value = 1  
5     else:  
6         value = n*factorial_fun_rec(n - 1)  
7     print("Function completed, with n = {}, and return value = {}".format(n, value))  
8     return value
```



```
1 print(factorial_fun_rec(1))
```

```
Function called, with n = 1  
Function completed, with n = 1, and return value = 1  
1
```

```
1 print(factorial_fun_rec(3))
```



```
Function called, with n = 3  
Function called, with n = 2  
Function called, with n = 1  
Function completed, with n = 1, and return value = 1  
Function completed, with n = 2, and return value = 2  
Function completed, with n = 3, and return value = 6  
6
```


Recursion vs. **for** loop

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- In some cases, the value of a function **can only be computed** using a recursive approach.

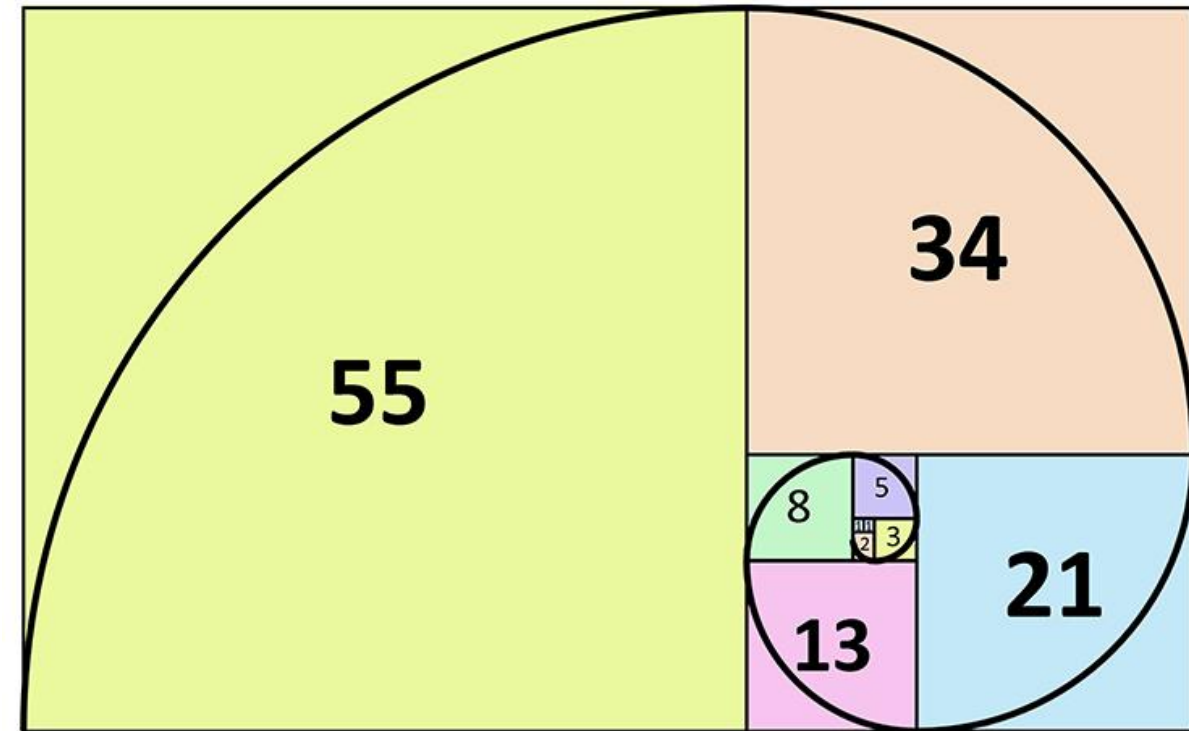
Recursion vs. **for** loop

- **Recursion** is an interesting trick for computing the value of a function, which would normally require an **iterative for loop**.
- In some cases, the value of a function **can only be computed** using a recursive approach.
- In some cases, recursion might be **faster** than an iterative for loop. Sometimes it is not.
 - **Something we will discuss once we investigate computational complexity (a.k.a. the science of designing the best code for a task)**

Practice activities: recursion vs. **for** loops

Let us practice a bit

Activity 5 – Fibonacci sequence.ipynb

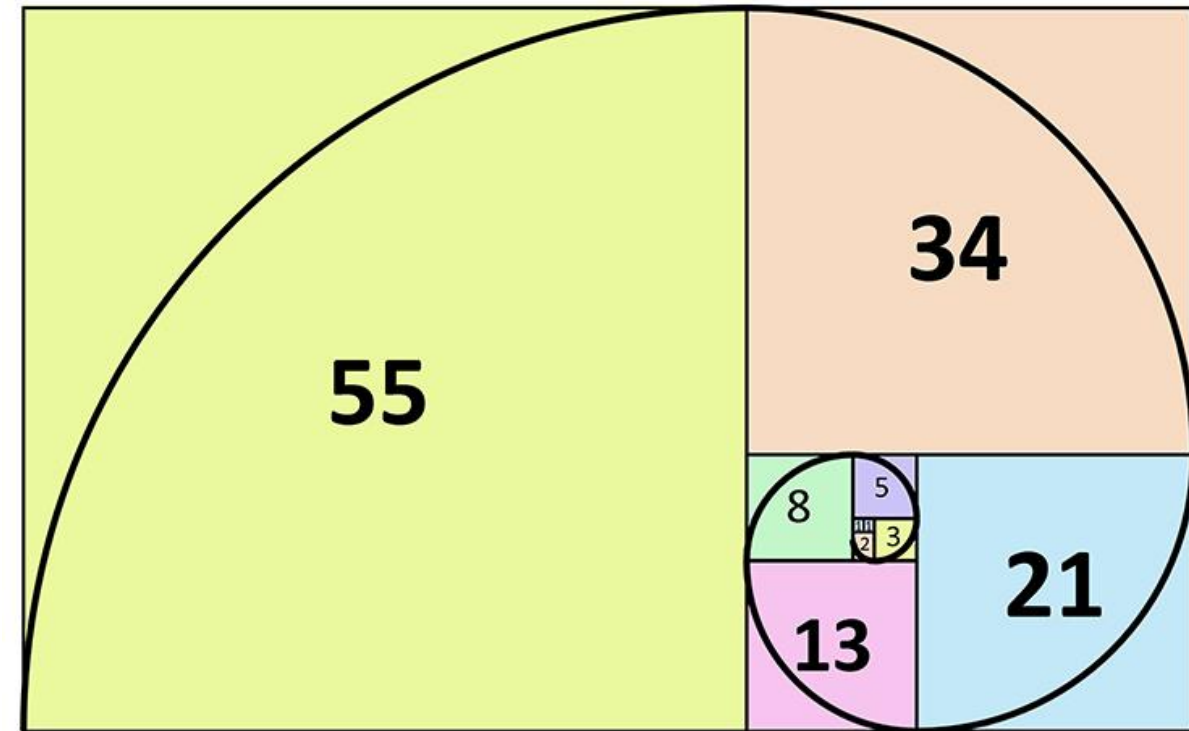


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The Fibonacci sequence F is a mathematical curiosity.

- The first and second elements of the sequence are equal to 1, i.e.
 $F(1) = 1$ and $F(2) = 1$.

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Activity 5 – Fibonacci sequence.ipynb

- The n -th Fibonacci element, with $n \geq 3$, is defined as the **sum of its previous two elements**, i.e.
$$F(n) = F(n - 1) + F(n - 2).$$

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- The first and second elements of the sequence are equal to 1, i.e. $F(1) = 1$ and $F(2) = 1$.
- The n -th Fibonacci element, with $n \geq 3$, is defined as the **sum of its previous two elements**, i.e. $F(n) = F(n - 1) + F(n - 2)$.
- **Task:** Write **two functions** that compute and return the n -th Fibonacci element, for any value of $n \geq 1$.

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- The n -th Fibonacci element, with $n \geq 3$, is defined as the **sum of its previous two elements**, i.e. $F(n) = F(n - 1) + F(n - 2)$.
- **Task:** Write **two functions** that compute and return the n -th Fibonacci element, for any value of $n \geq 1$.
 - The first function uses a **for** loop, the second uses **recursion**.