Online Participatory Budgeting

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Outline

Online LP

Borodin and El-Yaniv [2005]

$$
\begin{aligned}
& \max c^T x \\
s.t. \quad & Ax \leq b \\
& x \geq 0
\end{aligned}
$$

Object x come 1 by 1

Online Knapsack/ Secretary problem

Marchetti-Spaccamela and Vercellis [1995], Bateni et al. [2013]

- Utility function f
- Constraint (Cardinal or Knapsack)
- Look for approx guarantees

Results

- no constant approx Marchetti-Spaccamela and Vercellis [1995], Chakrabarty et al. [2008]
- $^1/_{10e}$ approx by Babaioff et al. [2007] for random order
- $^1/_{2e}$ approx by Vaze [2017] for small individual contributions

The setting

Budget $b = 50$

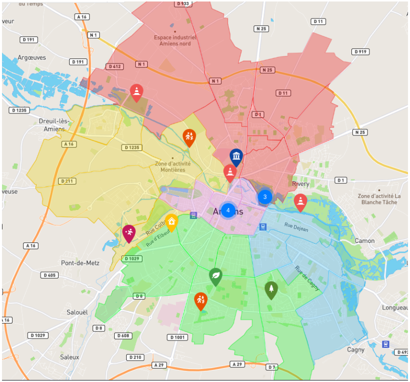
project	cost	votes
chocolate cake	50	
carrot cake	20	
banana bread	30	

Most common type of vote $\rightarrow$ approval ($A = A_1, \ldots, A_n$)

How to take votes into account ?

Welfarist rules
(score maximization)

- Approval Voting: number of approved projects
- Proportional Approval Voting: $1 + \frac{1}{2} + \ldots + \frac{1}{\mathrm{AV}}$



Method of Equal Shares

Peters et al. [2021]

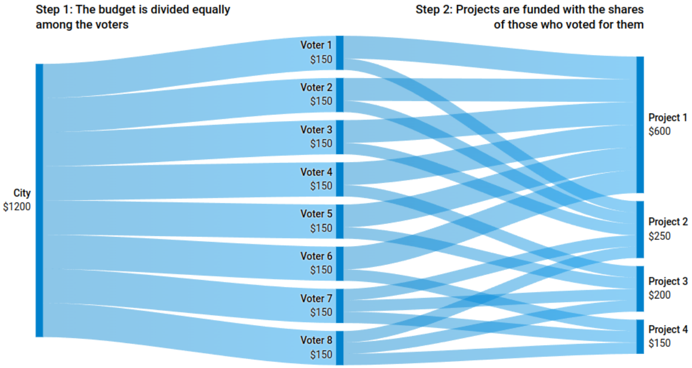


Figure: https://equalshares.net

How to evaluate an outcome ?

Rey and Maly [2023]

Definition

Priceability: Possible to explain outcome by a contribution graph

Definition

Local Fair Share: equal contribution function $\Rightarrow$ no more project affordable

Definition

Cohesiveness: For $S \subseteq N, Q \subseteq P$ if $\frac{|S|}{n} \geq \frac{c(Q)}{b}$ and $\forall i \in S, Q \subseteq A_i$
Proportionality: S is Q-cohesive, $\text{sat}_S(\pi) \geq \text{sat}_S(Q)$.

About proportionality

Definition

Satisfaction Function: mapping from $\mathcal{P}(P)$ to $\mathbb{R}$

Different variations

- Perfect Justified Representation: $\text{sat}_S(\pi) = \min_{i \in S} \text{sat}(A_i \cap \pi)$
  Not always possible
- Extended Justified Representation: $\text{sat}_S(\pi) = \max_{i \in S} \text{sat}(A_i \cap \pi)$
- Proportional Justified Representation:
  $\text{sat}_S(\pi) = \text{sat}(\bigcup_{i \in S} A_i \cap \pi)$

Results for Offline PB

Axiom	Priceability	PJR-1	EJR-1	LFS
MES	✓	✓	✓	✗
MES(share)	✓	✓	✓	✓
PAV	✗	✗	✗	✗

The framework

From Online ABC elections Do et al. [2022]
Example: vote for funding missions

project	cost	voter 1	voter 2	voter 3
IJCAI (South Corea)	3k	✓	✓	✓
ECAI (Spain)	1k	✗	✓	✓
SAGT (Amsterdam)	1k	✓	✗	✓

NB: We can have predictions

The Greedy Budgeting rule

Split budget and pay for affordable projects

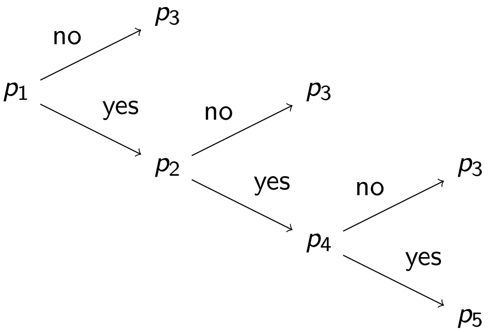
- Satisfies Priceability (then PJR[cost]-1)
- 2 versions
- Introducing Efficient Priceability

project	1	2	3	price
p_1	1	0	1	$5k$
p_2	1	0	1	$2k$
p_3	0	1	1	$4k$
p_4	1	0	0	$1k$

Priceability and Local FS

budget $b = 6001$

project	1	2	3	price
p_1	0	0	1	$2k$
p_2	0	1	1	$2k$
p_3	0	0	0	$3k$
p_4	1	1	0	$2k$
p_5	1	0	0	$1k$



Approximating axioms

Definition

α-Cohesiveness: For $S \subseteq N, Q \subseteq P$ if $\frac{|S|}{n} \geq \alpha \frac{c(Q)}{b}$ and $\forall i \in S, Q \subseteq A_i$
Proportionality: S is α-Q-cohesive, it deserves $\text{sat}_S(\pi) \geq \text{sat}_S(Q)$ of the budget.

	JR	PJR	EJR
sat^{card}	**JR[**sat^{card}**]**	γ_b-**PJR[**sat^{card}**]**	γ_b-**EJR[**sat^{card}**]**
sat^{cost}	**2-JR[**sat^{cost}**]**	**2-PJR[**sat^{cost}**]**	τ_b-**EJR[**sat^{cost}**]**

$^{b}/_{\tau_b}(\log_2(^{b}/_{\tau_b}) + 1) = b = \lfloor ^{b}/_{\gamma_b} H(^{b}/_{\gamma_b}) \rfloor$
$b = 1000 \implies \gamma_b \simeq \tau_b = 8$

Conclusion

Pros	Some optimal fairness guarantees Online better than repeated one-shot on PJR[cost]
Cons/ Further Questions	optimal $\not\Rightarrow$ good How to integrate predictions ? Perpetual PB JR+ properties

References

M. Babaioff, N. Immorlica, D. Kempe, and R. Kleinberg. A knapsack secretary problem with applications. In *International Workshop on Approximation Algorithms for Combinatorial Optimization*, pages 16–28. Springer, 2007.

M. Bateni, M. Hajiaghayi, and M. Zadimoghaddam. Submodular secretary problem and extensions. *ACM Transactions on Algorithms (TALG)*, 9(4):1–23, 2013.

A. Borodin and R. El-Yaniv. *Online computation and competitive analysis*. cambridge university press, 2005.

D. Chakrabarty, Y. Zhou, and R. Lukose. Online knapsack problems. In *Workshop on internet and network economics (WINE)*, 2008.

References (cont.)

V. Do, M. Hervouin, J. Lang, and P. Skowron. Online approval committee elections. *arXiv preprint arXiv:2202.06830*, 2022.

A. Marchetti-Spaccamela and C. Vercellis. Stochastic on-line knapsack problems. *Mathematical Programming*, 68(1-3): 73–104, 1995.

D. Peters, G. Pierczyński, and P. Skowron. Proportional participatory budgeting with additive utilities. *Advances in Neural Information Processing Systems*, 34:12726–12737, 2021.

S. Rey and J. Maly. The (computational) social choice take on indivisible participatory budgeting. *arXiv preprint arXiv:2303.00621*, 2023.

R. Vaze. Online knapsack problem and budgeted truthful bipartite matching. In *IEEE INFOCOM 2017-IEEE Conference on Computer Communications*, pages 1–9. IEEE, 2017.