Classification Aggregation without unanimity

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Outline

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History of Classification Aggregation

- Question "Who is a J?" $\rightarrow$ Group Identification
- Studied after the Law of return in Israel
- Generalization, objects instead of individuals

Objects and Notations

- N individuals, $|N| = n$
- X objects, $|X| = m$
- P categories, $|P| = \rho$
- $\mathcal{C}$ surjective classifications $c : X \to P$
- $\alpha : \mathcal{C}^N \to \mathcal{C}$ a Classification Aggregation Function (CAF)

Example (MAJ is not a CAF)

$N = \{1, 2, 3\}, P = \{p, q\}, X = \{x, y, z\}$.

object	1	2	3	MAJ result
x	p	q	p	p
y	q	p	p	p
z	p	p	q	p

Why?

Properties

Definition

- *Unanimity* (U): How would you define it ?

Properties

Definition

- *Unanimity* (U): $\forall c \in C, \alpha(c, \dots c) = c$
- *Independence* (I):
  $\forall x \in X, \boldsymbol{c}, \boldsymbol{c}' \in \mathcal{C}^N, \boldsymbol{c}_x = \boldsymbol{c}'_x \implies \alpha_x(\boldsymbol{c}) = \alpha_x(\boldsymbol{c}')$

Example (Independence)

	Preference $\boldsymbol{c}$				Preference $\boldsymbol{c}'$			
object	1	2	3	$\alpha(\boldsymbol{c})$	1	2	3	$\alpha(\boldsymbol{c}')$
x	p	p	p	p by U	p	p	q	
y	p	q	p	$\alpha_y(\boldsymbol{c})$	p	q	p	$\alpha_y(\boldsymbol{c})$ by I
z	q	q	q	q by U	q	p	p	

Outline

A simple supposition with a big impact

Example

Reminder: MAJ is not a CAF

object	1	2	3	$\alpha(\boldsymbol{c})$
x	p	q	p	q (supposed wlog)
$\dots$				

Build a new similar pref $\boldsymbol{c}'$

object	1	2	3	$\alpha(\boldsymbol{c}')$
x	p	q	p	q (by I)
y	q	q	q	q (by U)
z	q	p	q	p (by surjectivity)

Big impact

Example

object	1	2	3	
x	p	p	p	p (by U)
y	p	q	p	q (by S)
z	q	p	q	p (by I)

By playing arround, one can show 2 is a Global dictator for α.

Previous Results

Theorem (from Rubinstein and Fishburn [1986] and Kasher and Rubinstein [1997])

For $|X| > |P| = 2$, any CAF that satisfies independence and unanimity is a dictatorship.

Theorem (Maniquet and Mongin [2016])

For $|X| \geq |P| \geq 3$, any CAF that satisfies independence and unanimity is a dictatorship.

Proof with ultrafilter technique, doable with pivotal voter.

Outline

Generalizing U and D

Definition

- *Generalized Unanimity* (GU):
  $\exists \pi \in P \leftrightarrow P, \forall c \in C, \alpha(c, \dots c) = \pi \circ c$
- *Generalized Dictatorship* (GD):
  $\exists \pi \in P \leftrightarrow P, \exists d \in N \mid \forall \boldsymbol{c} \in \mathcal{C}^N, \alpha(\boldsymbol{c}) = \pi \circ \boldsymbol{c}_d$

Lemma (Generalized Impossibility)

For $|X| > |P| \geq 2$ any CAF that satisfies independence and Generalized unanimity is a generalized dictatorship.

Proof by pivotal voter.
Still, GU is close to U...

CS under Independence

Definition

An independent CAF α is CS iff
$\forall x \in X, \forall p \in P, \exists \boldsymbol{k}_{x,p} \in P^N \mid \alpha_x(\boldsymbol{k}_{x,p}) = p$

Example

	Preference $\boldsymbol{c}$		Preference $\boldsymbol{c}'$	
object	preferences	result	preferences	result
x	$(p, \ldots, p)$		$(p, \ldots, p)$	
y	$(q, \ldots, q)$		$(q, \ldots, q)$	
z	$\boldsymbol{k}_{z,p}$	p (by CS)	$\boldsymbol{k}_{z,q}$	q (by CS)

By I, $\exists \pi \in P \leftrightarrow P \mid \alpha_x(p, \ldots, p) = \pi(p), \alpha_y(q, \ldots, q) = \pi(q)$.

High cost of Independence

Example

	Preference $\boldsymbol{c}$		Preference $\boldsymbol{c}'$	
object	preferences	result	preferences	result
x	$(p,\dots,p)$	$\pi(p)$ by I	$\boldsymbol{k}_{x,\pi(q)}$	$\pi(q)$ by CS
y	$\boldsymbol{k}_{y,\pi(p)}$	$\pi(p)$ by CS	$(q,\dots,q)$	$\pi(q)$ by I
z	$(q,\dots,q)$	$\pi(q)$ by S	$(p,\dots,p)$	$\pi(p)$ by S

One can show that $\forall t \in X, r \in P\alpha_t(r,\dots,r) = \pi(r)$ (GU under I)

New result

Lemma

For $|X| > |P| \geq 2$, every citizen sovereign and independent CAF satisfies generalized unanimity.

Proof similar to the tricks just used

Theorem

For $|X| > |P| \geq 2$, every citizen sovereign and independent CAF is a generalized dictatorship.

Opening

- Study anonymity, neutrality [Ozkes and Sanver, 2021]
- Solution of fuzzy CAFs [Alcantud and de Andrés Calle, 2017]
- Committee elections with diversity [Bredereck et al., 2018]

Thank you for your attention!

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