Independent study NNs + Symmetry: summary initial Literature review

1. *“Learning SO(3) Equivariant Representations with Spherical CNN’s”*

- Addresses the 3D rotation equivariance in CNN’s. This means that recognition still works when objects are rotated.
- SO(3) is the group of all rotations about the origin of 3D Euclidean space.
- Currently, CNN’s are great with translations! Other nuisances are normally addressed with data augmentation. Equivariant CNN’s allow rotations as well, the paper proposes the first NN with spherical convolutions.
- Pooling = a layer in CNN that reduces the spatial size, summarizing multiple features. Here a spectral pooling is used instead of spatial pooling, as this retains the equivariance. A special smoothing approach is used to select only a few ‘anchor frequencies’, which makes the number of weights independent of the input resolution.
- Uses ‘Spherical harmonics’, which comes down to the use of Spherical Fourier Transform (SFT). The convolution of signal f with a filter h is computed by first using SFT to f and h, computing their point-wise multiplication, and then using the inverse SFT. For this, they use equiangular samples.
- They parametrize the filters in the spectral domain and obtain localized filters by parametrizing the spectrum with anchor frequencies.
- They use spectral pooling, as this preserves equivariance.
- 64 citations in a year, seems like a very big paper.

2. *“Rotation equivariant vector field networks”*

- Tackles the same problem as previous paper, being developing a CNN that has rotational equivariance by design. Here, the built Rotation Equivariant Vector Field Networks (RotEqNet). Each convolutional filter is applied at multiple orientations and returns a vector field representing magnitude and angle of the highest scoring orientation at every spatial location.
- They make a distinction between rotation equivariance (rotating input, same rotation in output), invariance (same classification score rotated or not) and covariance (some relationship is present between rotated input and output).
- Traditional CNN’s are translation equivariant thanks to the nature of their convolution operator.
- Trade-off. Doing R rotations of each filter leads to too large model size. Could also only retain max value, but then no idea about orientation. So, combo: keeping max value as 2D vector! This is called orientation pooling or OP.
- Two options for rotational equivariance: transform the representation or rotate the filters. Challenge of the first is loss of relative orientation of objects with respect to surroundings (not always relevant). Of the second, is dimensionality of the model. Traditional trade-off of the latter comes down to depth of network vs number of orientations included. RotEqNet bypasses this compromise.
- Note that data augmentation comes down to rotate the images.
- Segmentation problem in general comes down to identifying what part of the image belongs to what segment.

- In all examples provided by the paper, RotEqNet achieves better performance than previous work with significantly less parameters.

3. *“Tensor field networks: Rotation- and translation-equivariant neural networks for 3D point clouds”*

- A point cloud is a set of data points in space. Typically, this is produced by 3D scanners.
- Impressive that this works for high-dimensional tensors! Their filters are built from spherical harmonics. Each layer accepts as input scalars, vectors and higher-order tensors.
- Contains mathematically rigorous understanding and proof of equivariance.
- They use point convolution, which means that the convolution is executed on each point taking all other points and their relative location as input.
- Spherical harmonics = functions that are equivariant in SO(3).
- A cool application they show is the calculation of acceleration vectors of point masses (under Newtonian gravity and the moment of inertia tensor for every mass). Their network leads to perfect agreement with the exact solution.
- Another cool thing is their prediction of the location of a randomly removed point from a point cloud.
- Code is directly useable from github.
- 45 citations in a year, seems like a very big paper.

4. *“Harmonic Networks: Deep Translation and Rotation Equivariance”*

- From 2017, a year earlier than the other papers I have the feeling.
- They use Harmonic networks or H-nets to embed equivariance in CNN’s, using circular harmonics.
- Filters are steerable if they can be constructed at any rotation as a finite, linear combination of base filters.
- Again, ‘learning generalized transformations’ from the input data is considered as alternative of playing with filters. But this leads to less interpretable and reliable results.
- H-nets hard-bake 360°-rotation invariance into their feature representation, by constraining the convolutional filters of a CNN to be from the family of circular harmonics. Proof is in appendix.
- They use filters expressed in polar form, having a rotation order, radial profile and phase offset term, the two latter of which are learned during training. Note that this means that the filters are complex-valued.
- They use cross-correlation, which is an alternative for convolution.
- They claim a better interpretability of the feature maps.
- They have a Tensorflow implementation ready on their website.

5. *“On the generalization of Equivariance and Convolution in Neural Networks to the Action of Compact Groups”*

- Gives a rigorous, theoretical treatment of convolution and equivariance in NNs with respect to the action of any compact group.
- Topology is the branch of mathematics concerned with the properties of a geometric object that are preserved under continuous deformations. A compact group is a topological group whose topology is compact being closed and bounded.
- Defines a general formulation of a convolution for a compact group and claims that a FFNN is equivariant to the action of any compact group if and only if each of its layers implements a general form of their convolution. If this condition is met, the NN is called a G-CNN, with G representing the compact group.
- Interestingly, the claim that if a network is fully equivariant, the network must be convolutional with respect to the specific compact group.
- Mentions an example of NN learning from graphs, and specifically so-called Message-Passaging NNs, or MPNNs, claiming that is also a generalized CNN.

6. *“Hamiltonian Neural Networks for solving differential equations”*

- Paper by Sondak, Protopapas, Mattheakis and Dhogra
- They want to solve ODE’s governing dynamical systems. The model is data-free and unsupervised.
- Hamiltonian mechanics is a theory developed as a reformulation of classical mechanics and predicts the same outcomes as non-Hamiltonian classical mechanics
- They embed the Hamiltonian equations into the loss function of the NN, therefore conserving energy. Also, they use the sine as activation function, resulting in less iterations to reach the same performance than the sigmoid. This has ‘global support similar to the Fourier Series’ and introduces periodic, and empirically considered as equal, minima, which makes it easier to converge to a local minimum.