Superconductivity in fractal geometries

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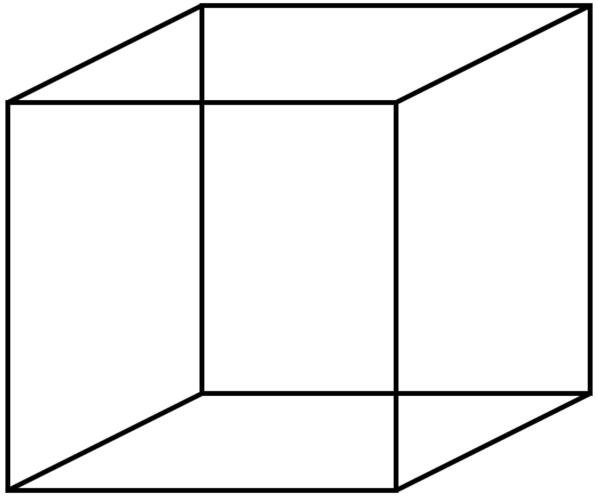
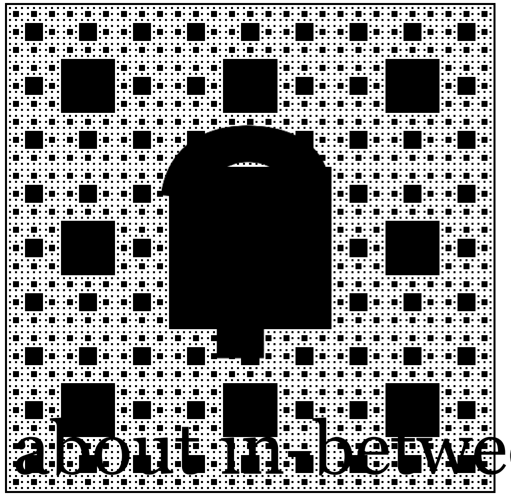
2nd corrector : dr. W. Löffler

Going between dimensions

2D



3D



How about in-between?

'ln(8) / ln(3)' (≃ 1.89)
dimensional?

Fractals:
e.g. Sierpiński carpet (left)

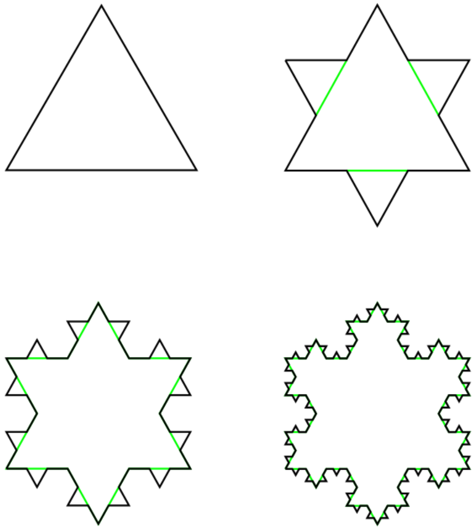
From: Wikipedia (CC)

Characterization of dimensionality

- Hausdorff dimension:

Generalisation of dimension from real vector spaces to arbitary metric spaces

Hausdorff dimension d reduces to usual dimension for non-fractal geometries (e.g. $d_{point} = 0$, $d_{line} = 1$ etc.)

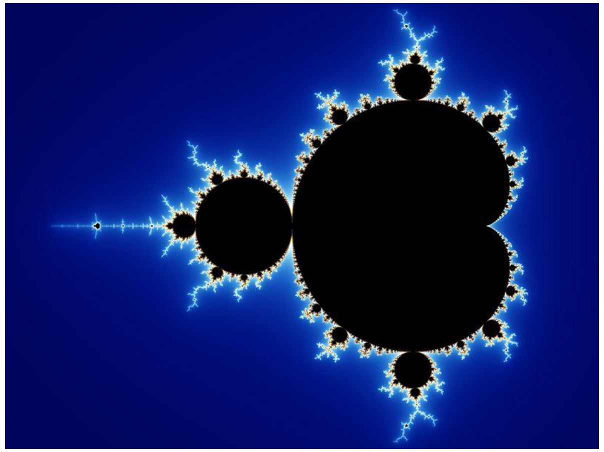


Koch snowflake, Hausdorff dim. $2\ln(2) / \ln(3) \simeq 1.26$

Wikipedia (CC)

- Fractals:

No consensus on definition ('never-ending pattern')



Mandelbrot set, Hausdorff dim. 2

Wikipedia (CC)

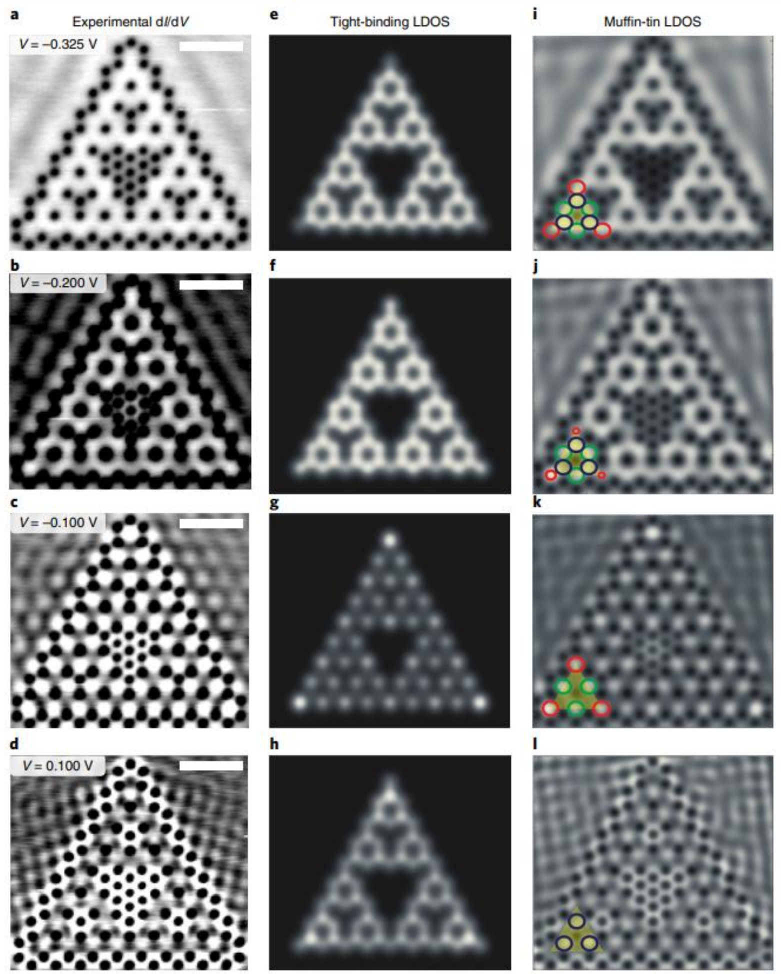
Motivation

- STM paper: advances in the exploration of fractal geometries in electronic quantum systems

Design and characterization of electrons in a fractal geometry

S. N. Kempkes[1,3], M. R. Slot[2,3], S. E. Freeney[2], S. J. M. Zevenhuizen[2], D. Vanmaekelbergh[2], I. Swart[2*] and C. Morais Smith[1*]

Utrecht University



Link with superconductivity

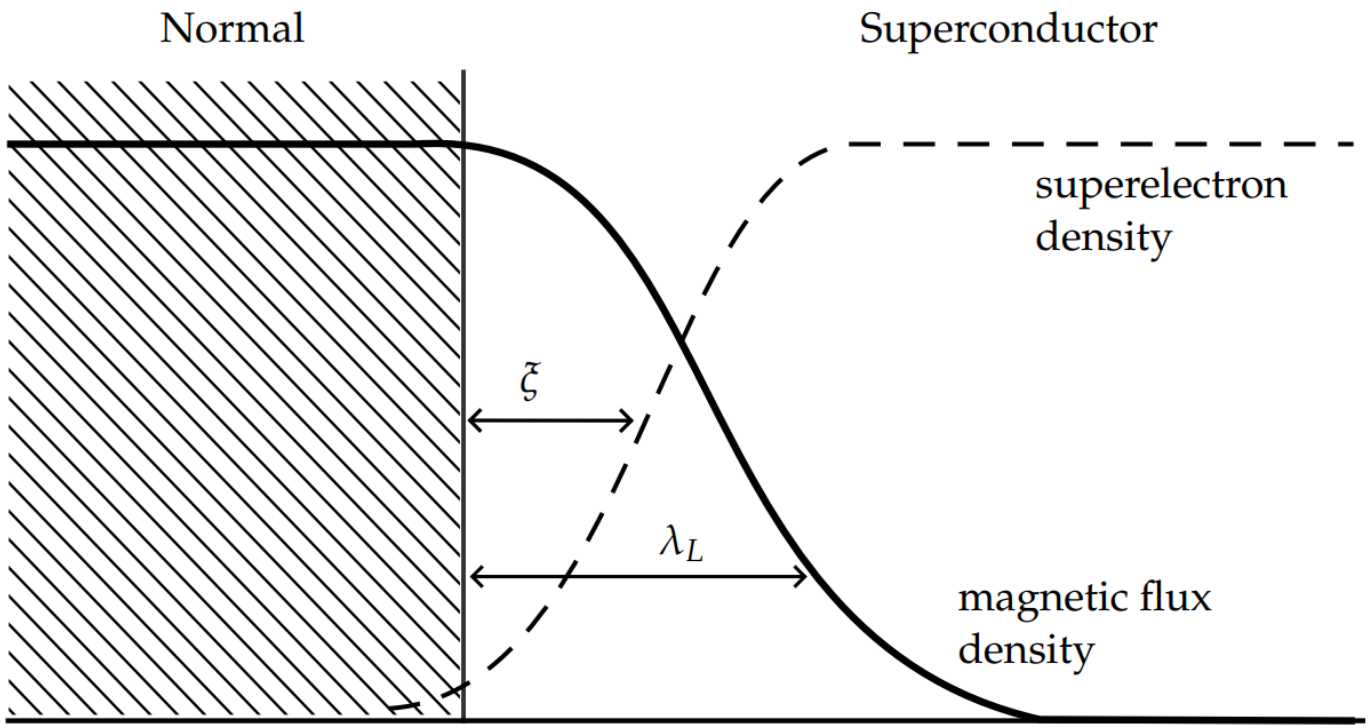
Brief intro superconductivity

- Condensate: (bosonic) cooper pairs
- Ginzburg-Landau equations

    2^{nd} G-L equation:

$$\boldsymbol{J} = -\frac{\Phi_0}{2\pi\mu_0\lambda^2}\left(\frac{2\pi}{\Phi_0}\boldsymbol{A} + \nabla\gamma\right)$$

- Characteristic length scales:
    ξ and λ
- Meissner effect



Josephson junctions, 3D

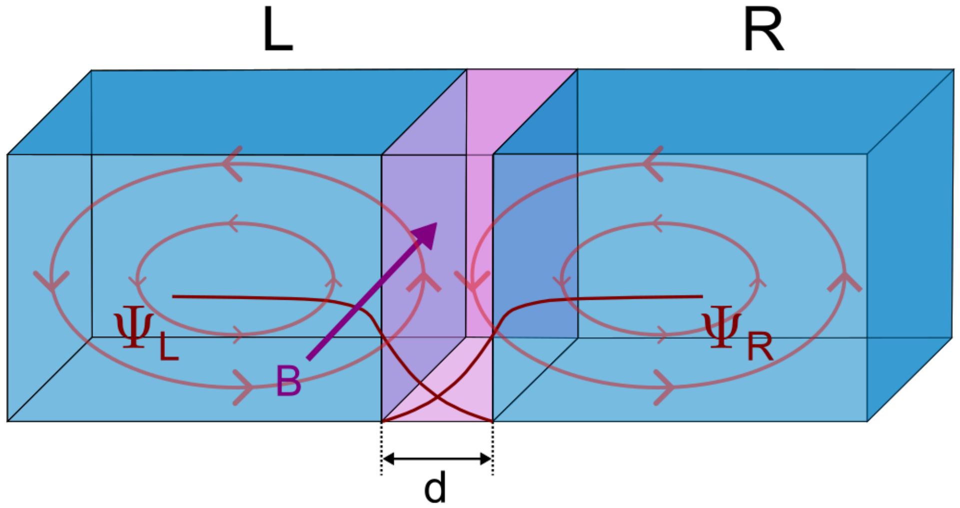
- Superconductors separated by weak-link

- Overlap between wave functions

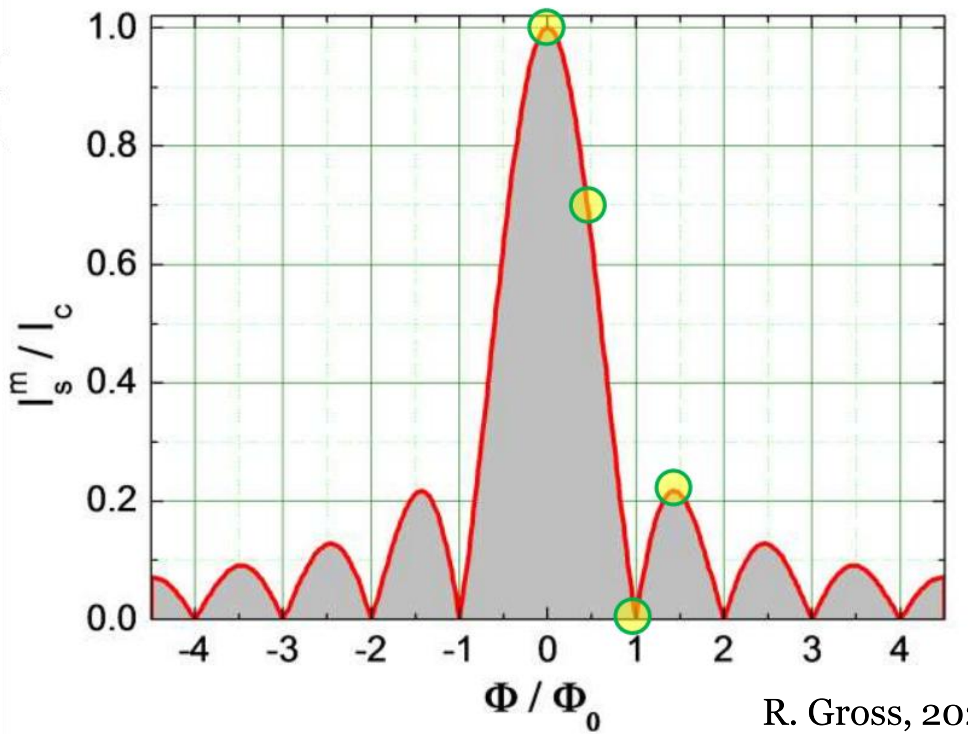
- Interference:

- Fraunhofer pattern when plotting applied magnetic field vs critical current.

- Periodicity determined by effective junction length $2\lambda + d$.



Fermin, phd thesis, 2022



R. Gross, 2020

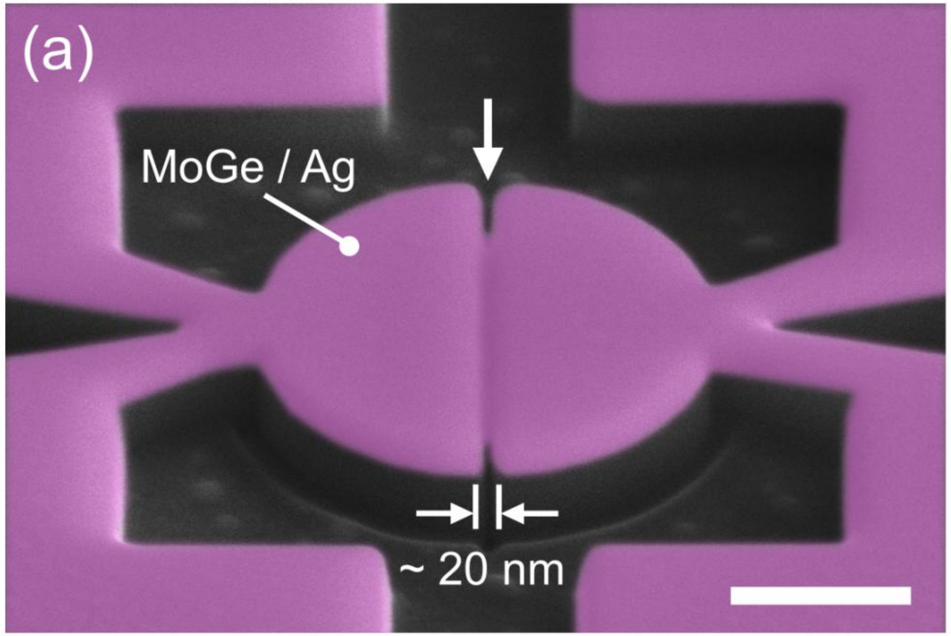
2D: thin film planar Josephson junctions

- Critical current solely determined by geometry (not λ)

- Thin film planar junctions + restricted in lateral size:

in the asymptotic limits critical current solely determined by area (not even geometry anymore!)

- Why 2D: we can simulate



TOP: Schematic of thin film eliptical laterally constrained Josephson junction
BOTTOM: Side view

Theory: background

- Finding γ also from 2nd Ginzburg-Landau equation, assuming:

supercurrent conservation (1) Coulomb gauge (2)

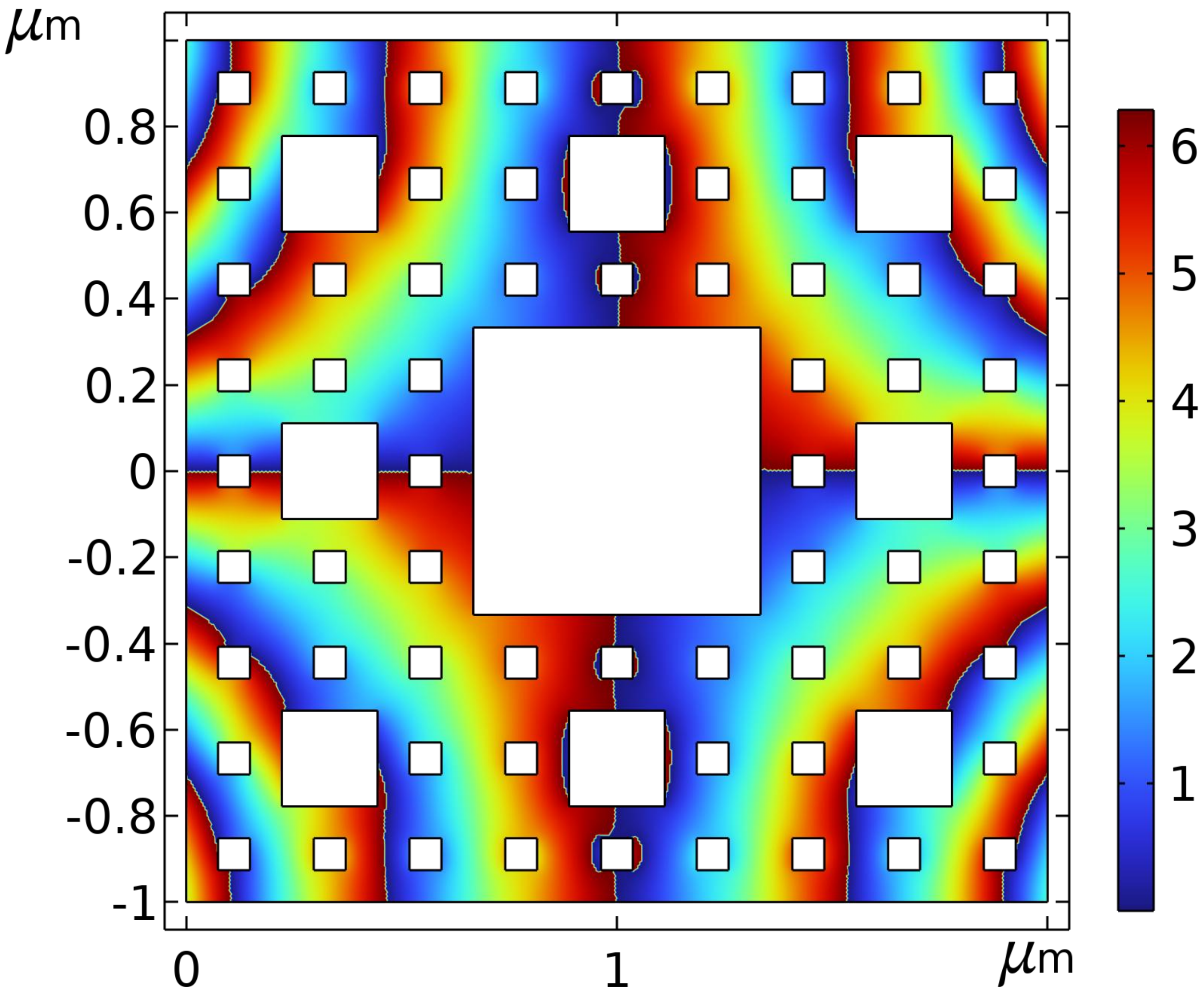
$$\nabla \cdot \boldsymbol{J} = 0 \qquad \nabla \cdot \boldsymbol{A} = 0$$

- G-L mapped onto Laplace equation:

$$\boldsymbol{J} = -\frac{\Phi_0}{2\pi\mu_0\lambda^2}\left(\frac{2\pi}{\Phi_0}\boldsymbol{A} + \nabla\gamma\right) \Rightarrow \boxed{\nabla^2\gamma = 0}$$

we can solve this (with appropriate B.C.'s)

Fractal Josephson junctions



- **Fractals in COMSOL**

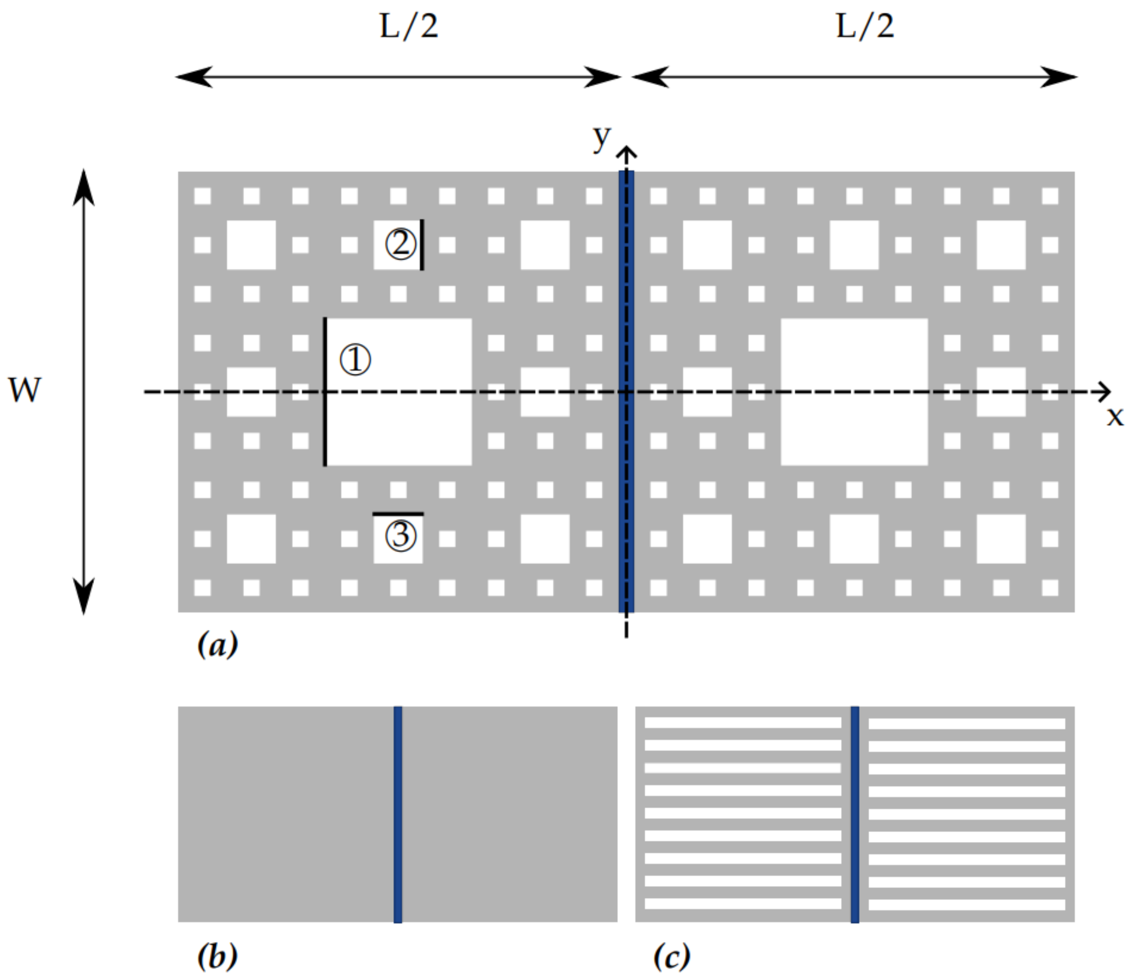
Sierpinski carpet: Hausdorff dim. 1.89

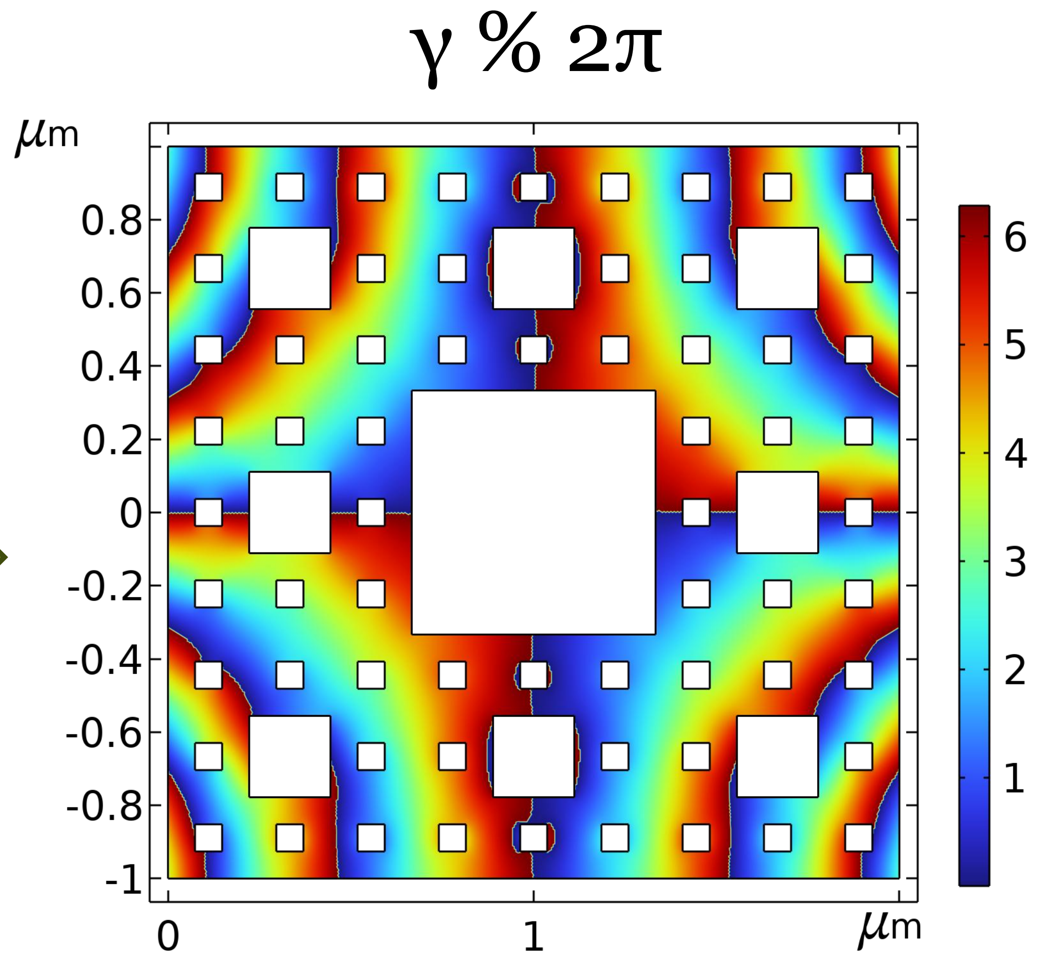
- **Probing the limits**

Looking for asymptotic behaviour

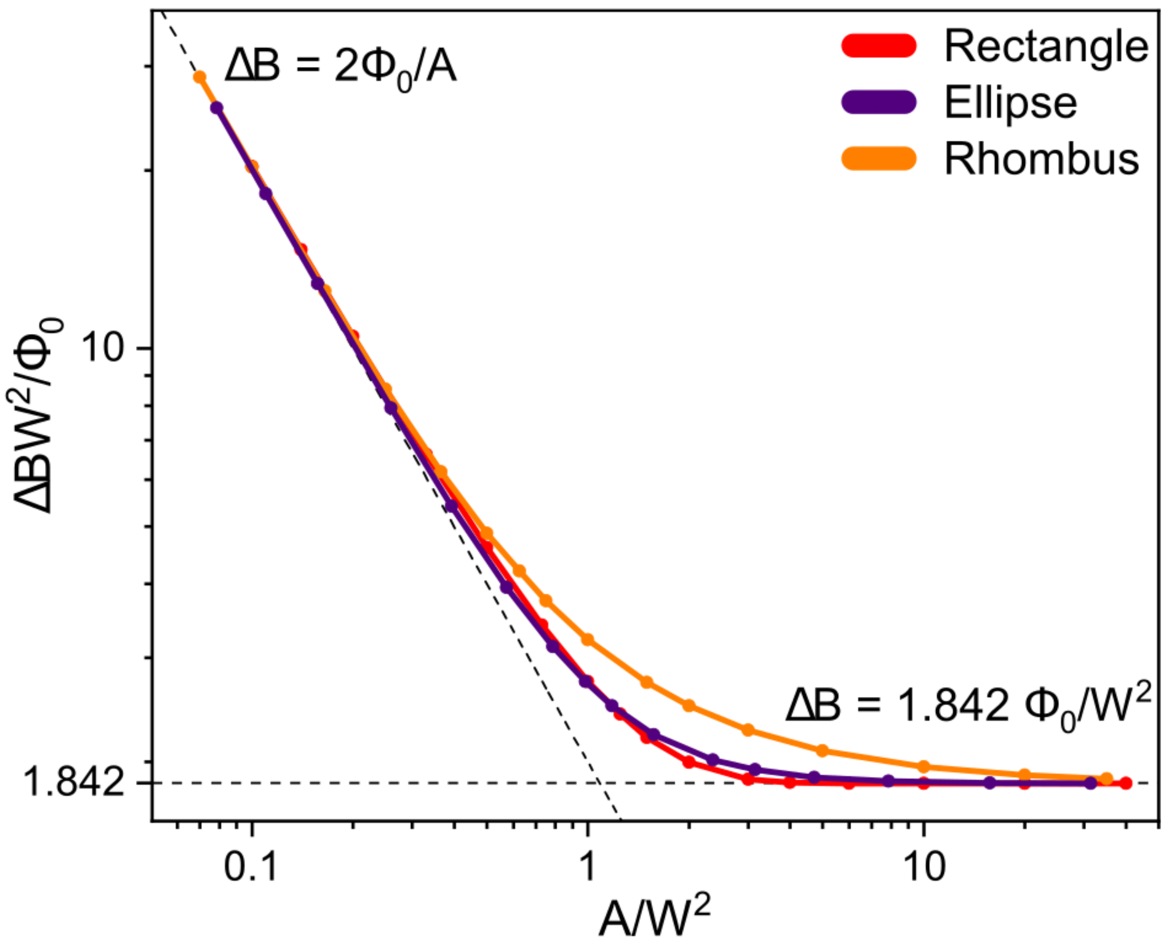
Distribution of γ % 2π on the surface of a 3rd iteration Sierpiński carpet in COMSOL

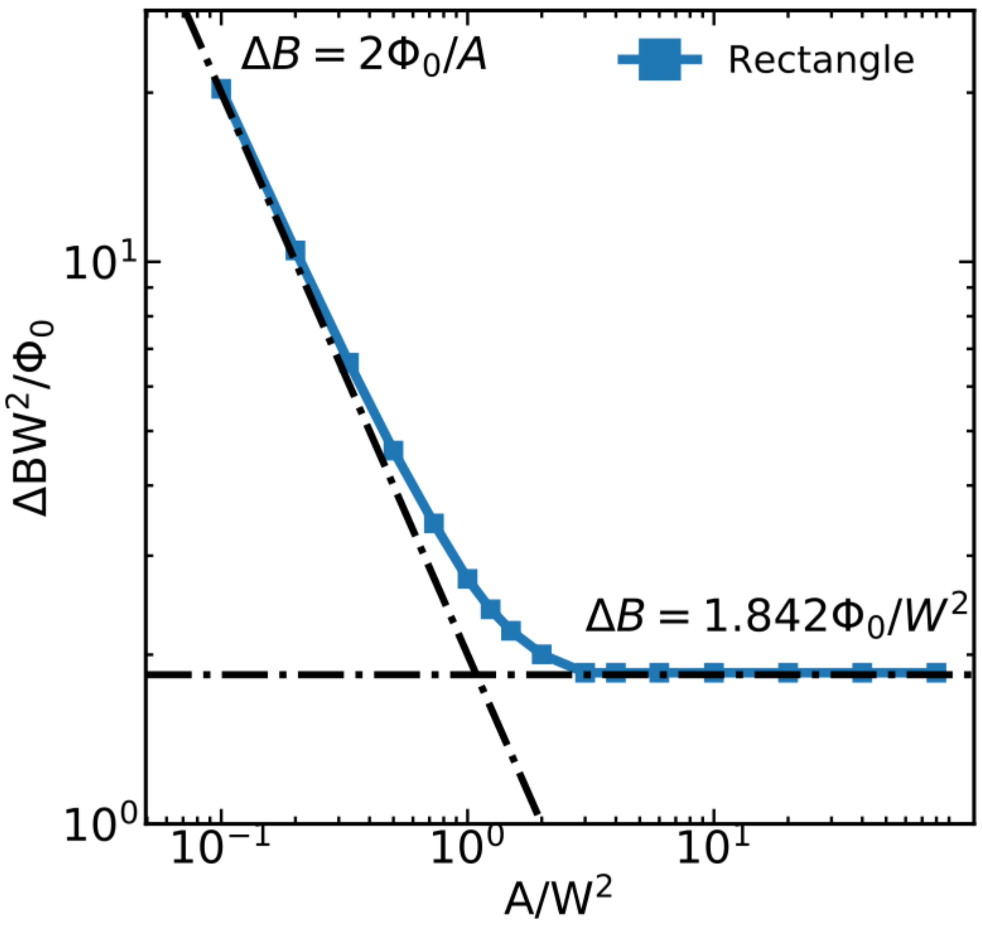
Simulations: phase distribution





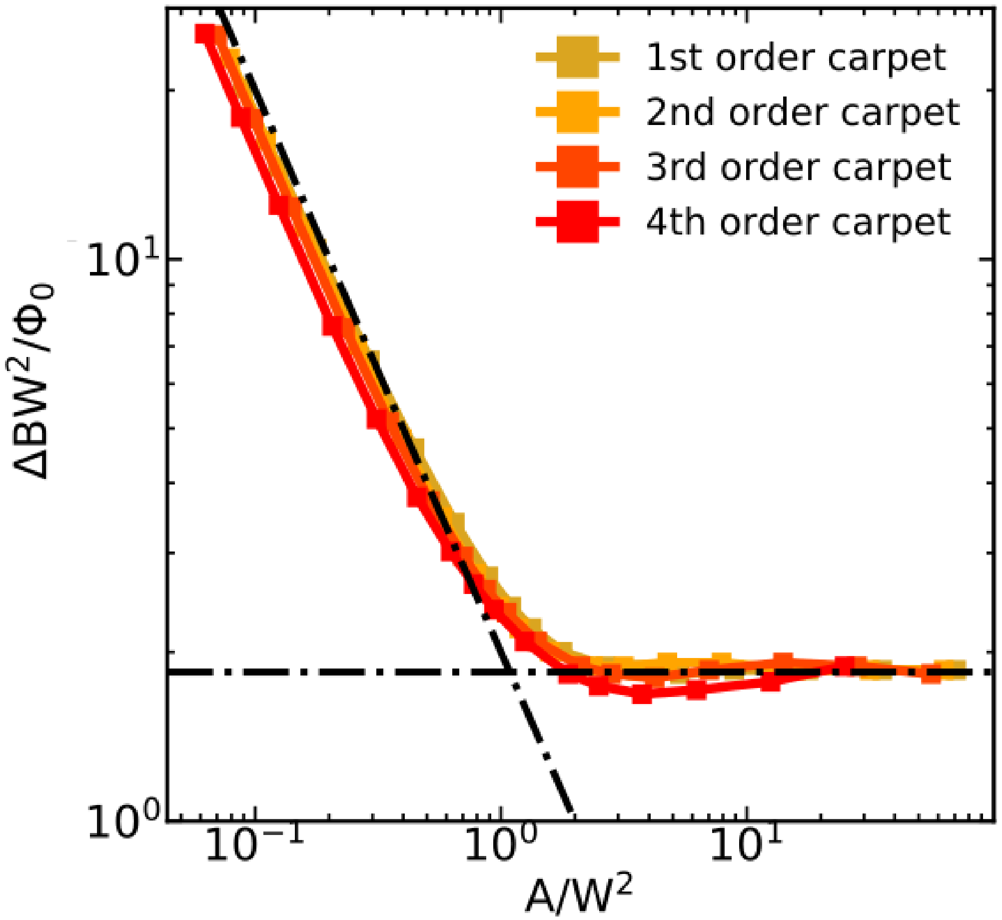
Simulations: periodicity (I)





LEFT: dependence of periodicity on sample area for various geometries of the electrodes (Fermin, 2023)
RIGHT: independent confirmation of same behaviour for rectangular electrodes

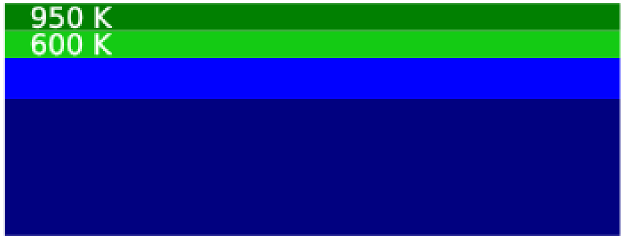
Simulations: periodicity (II)



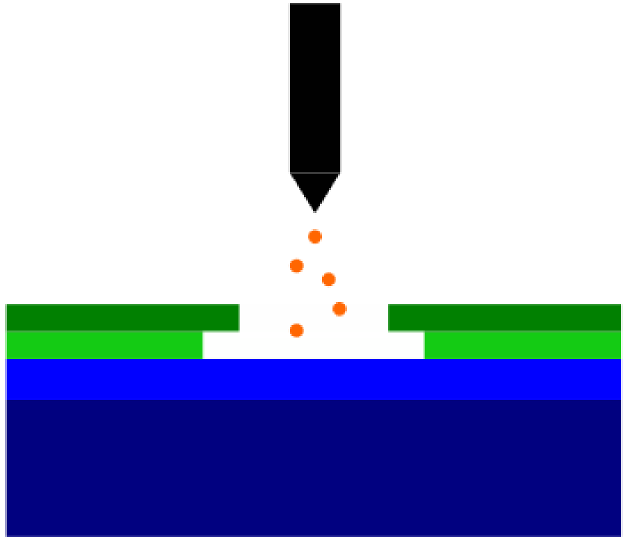
LEFT: dependence of periodicity on sample area for various iterations of the Sierpinski carpet
RIGHT: dependence of periodicity on sample with non-fractal geometry, but identical surface-to-hole area

Nanofabrication

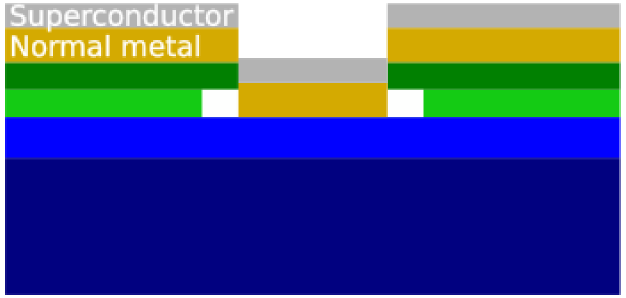
Empty substrate



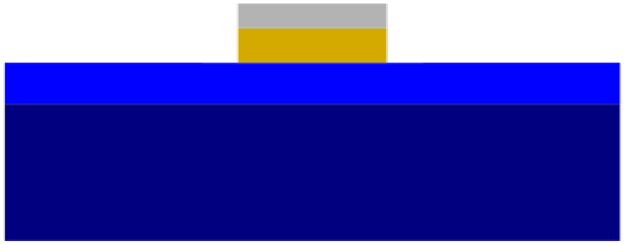
Spin-coating



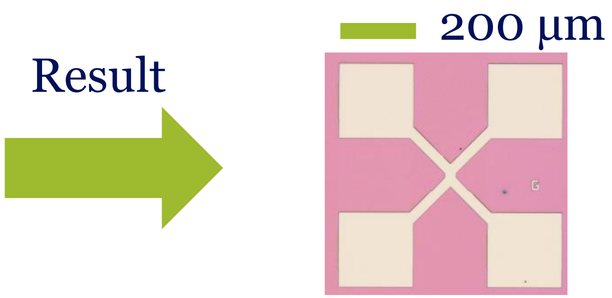
E-beam lithography



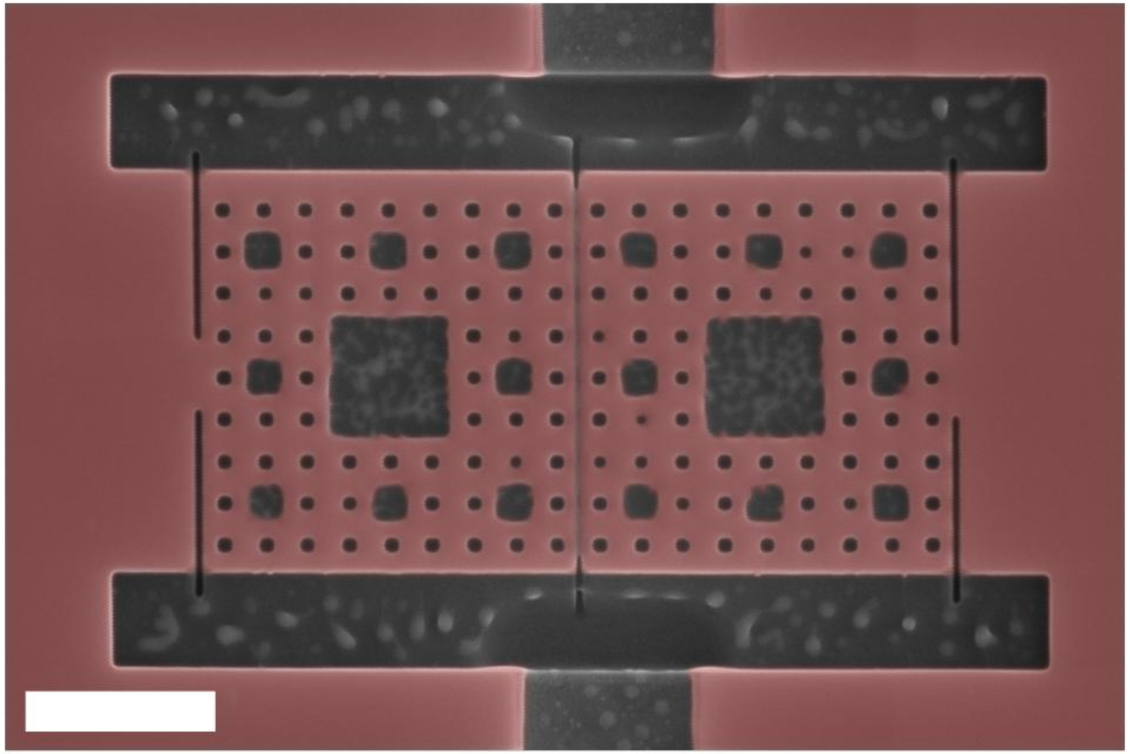
Sputtering

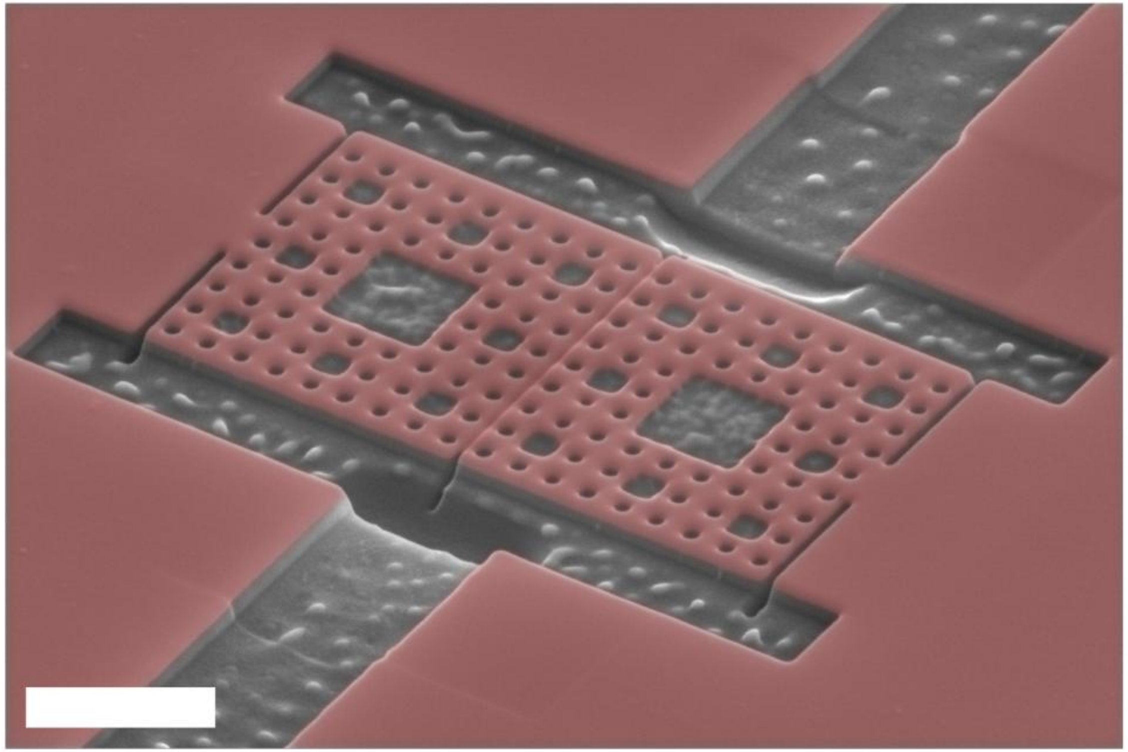


Lift-off



FIB results (I)

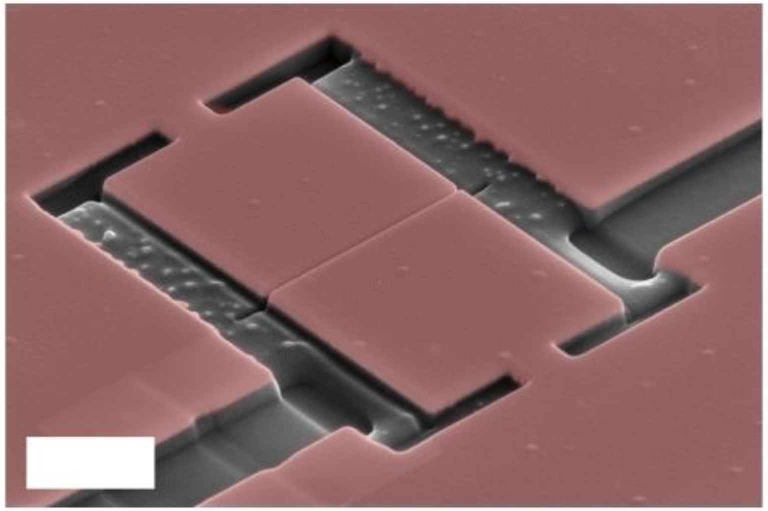


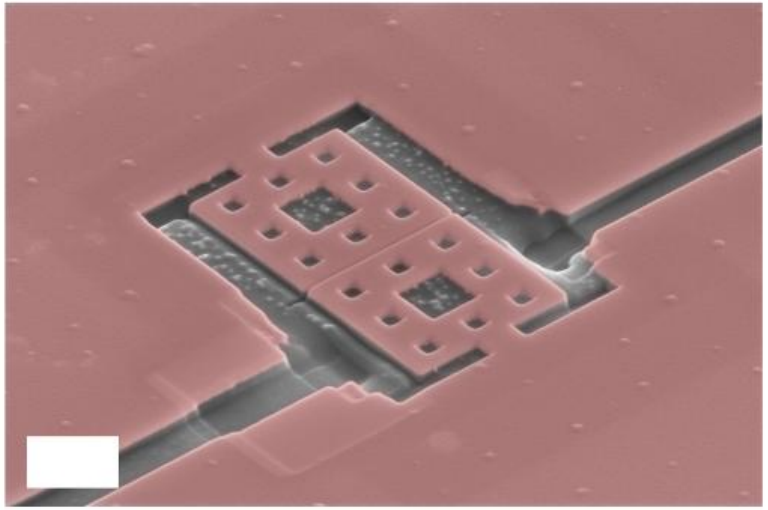


Third order Sierpiński carpet electrodes:

- Non-unifom holes…
- Damage to weak link…

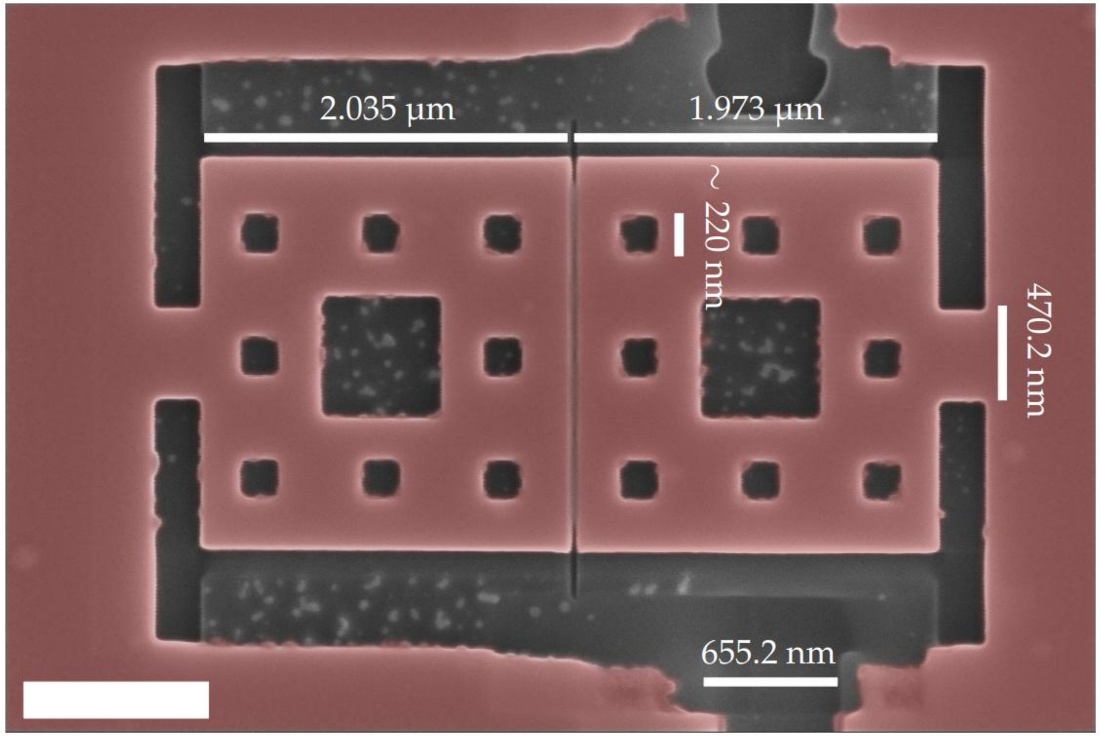
FIB results (II)



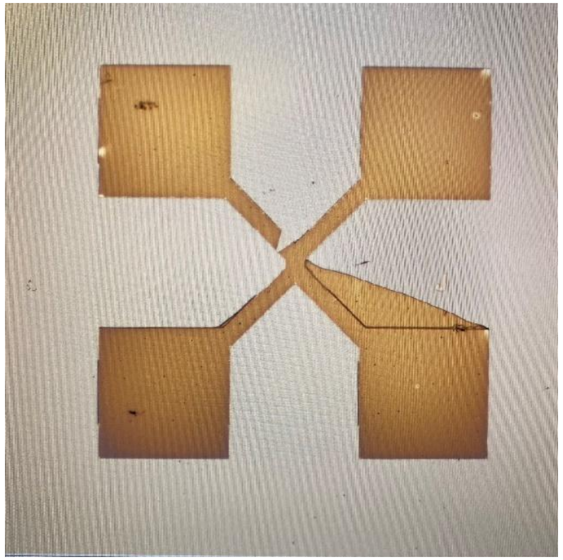


Second order Sierpiński carpet electrodes:
- Uniform holes
- No damage to weak link

Rectangular electrodes:
- Compare results with literature



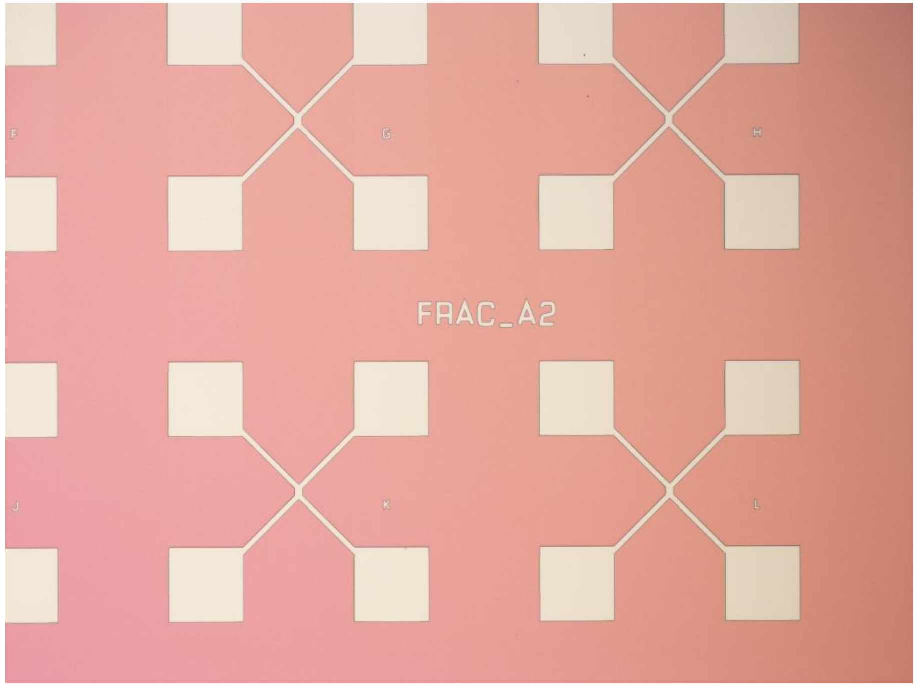
Challenges (I)

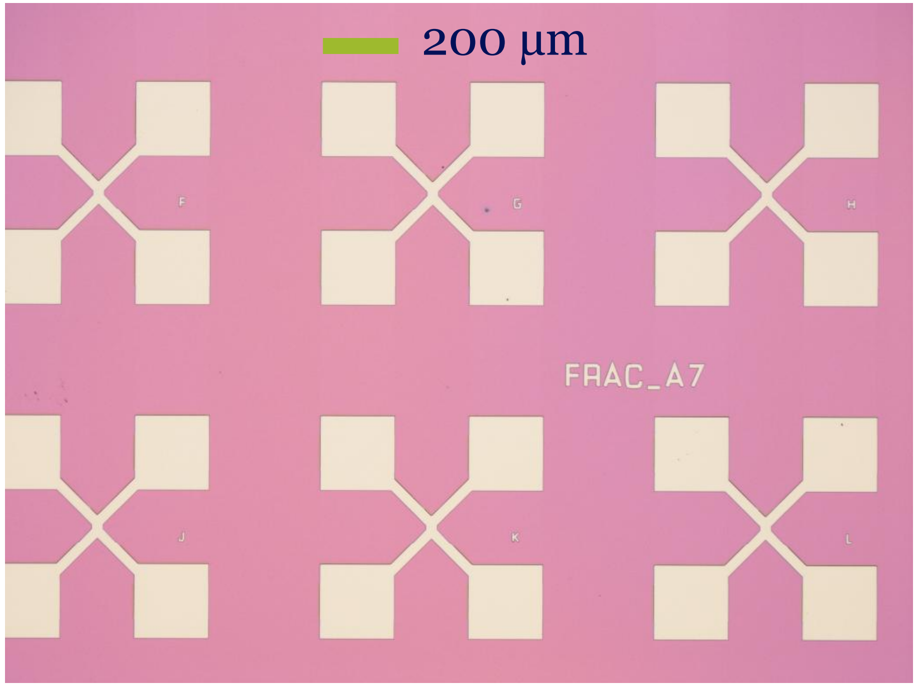


LEFT: unsuccessful lift-off

Weak attachment:

- Thicker elements





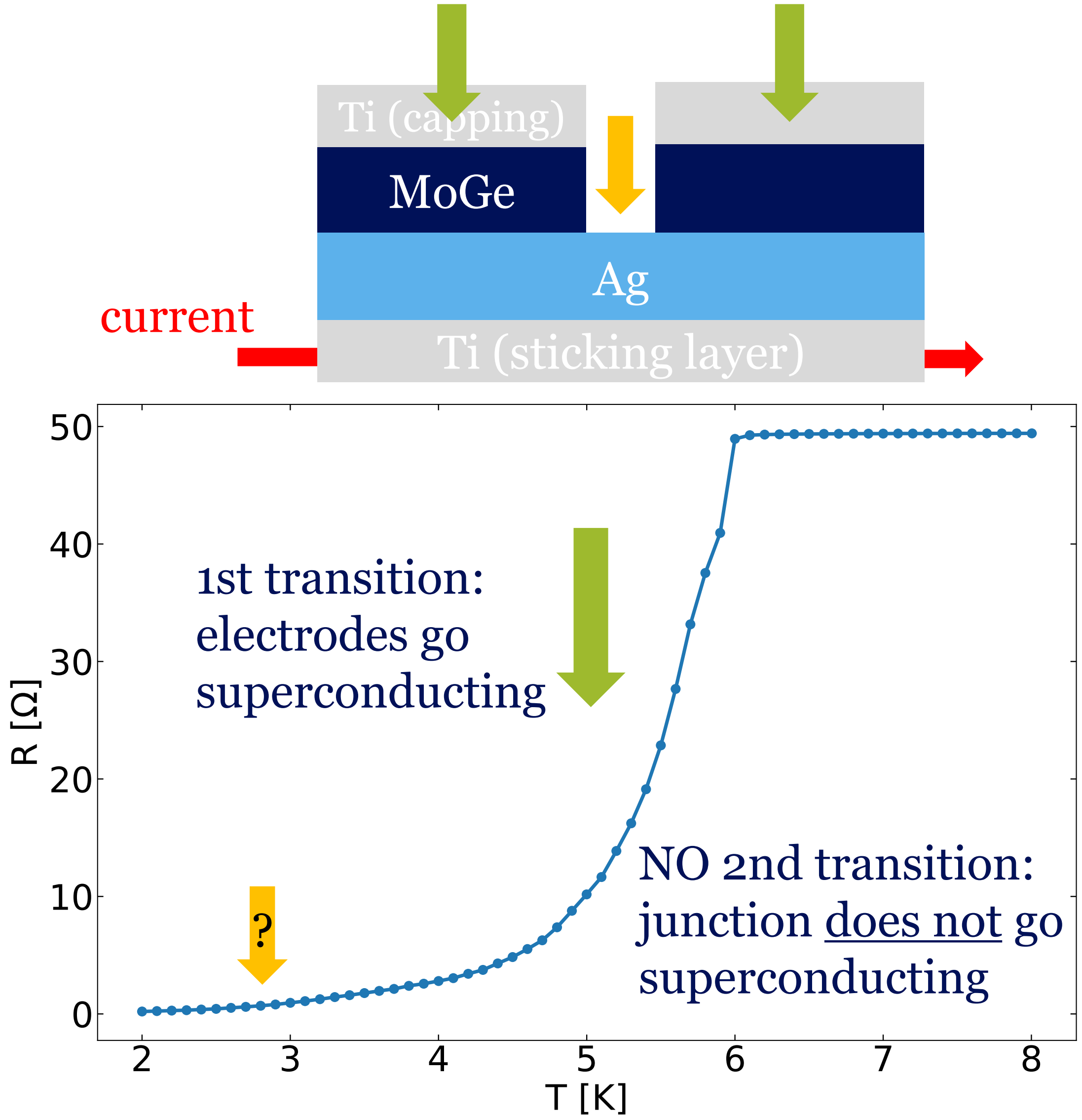
Challenges (II)

Solve weak attachment:

- Sticking layer: 4 nm MoGe
- MoGe superconducting > shorts the junction
- Switch to 5 nm Ti

Prevent FIB damage:

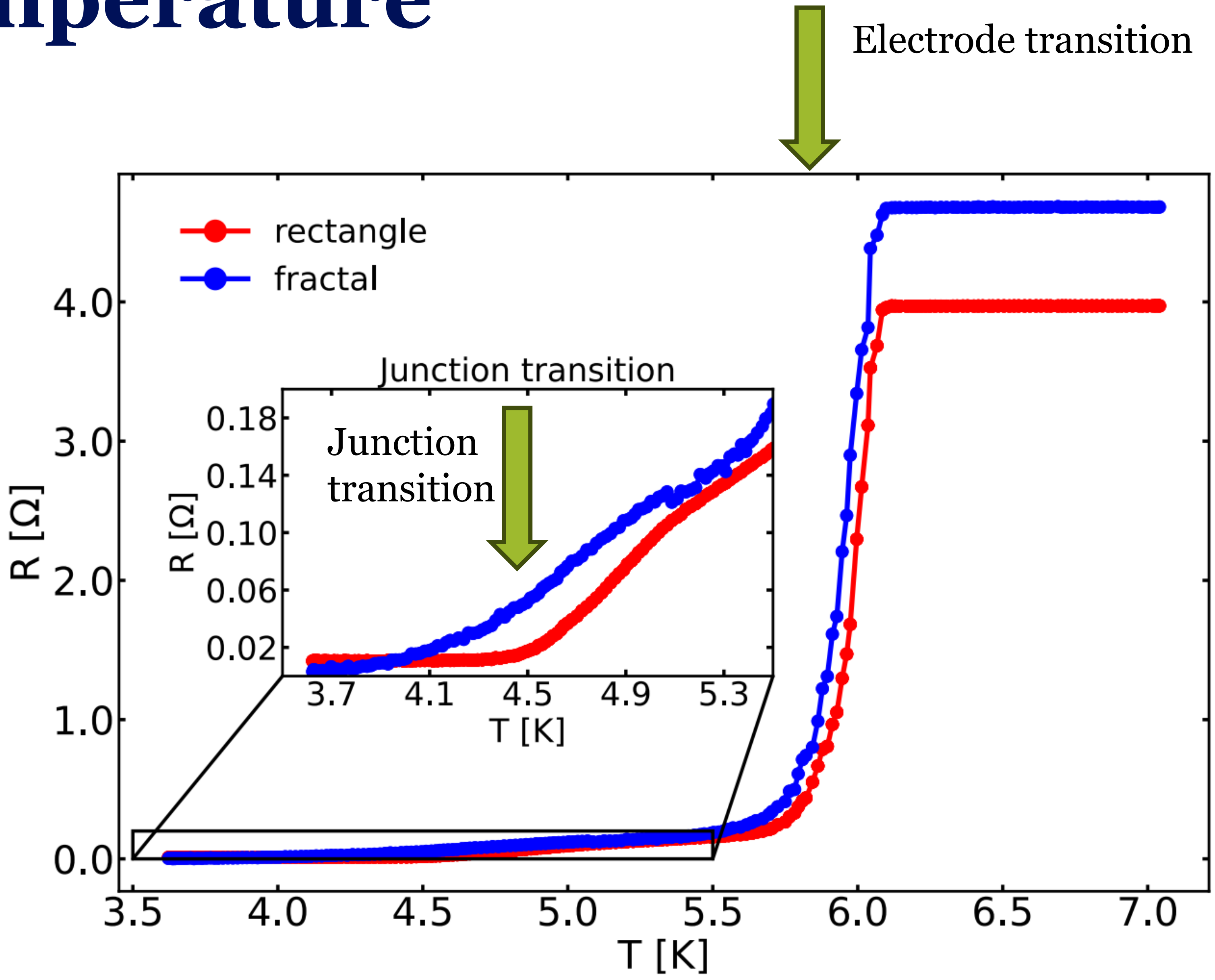
- 5 nm Ti capping layer against FIB-damage



MoGe sticking layer shorts the junction: no second transition

Resistance vs temperature

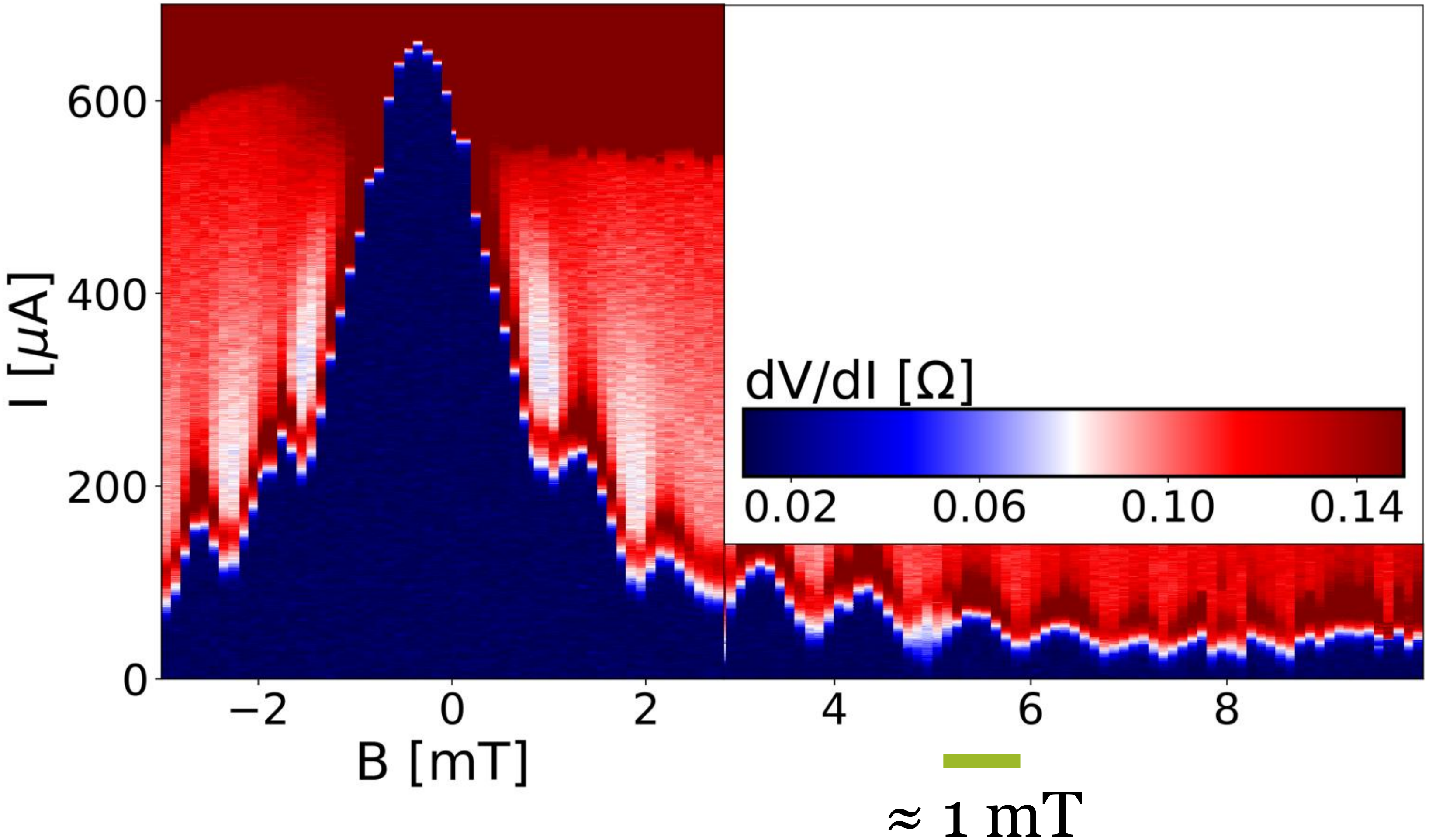
- Two transitions:
  - ❑ electrodes
  - ❑ junction
- Fractal: higher normal state resistance
- Fractal: slightly lower T_c



SQI-patterns: Rectangular junction

- Periodicity agrees with simulations
- Long junction limit

$$W > \frac{\Phi_0}{4\pi\mu_0\lambda^2 J_c(0)}$$

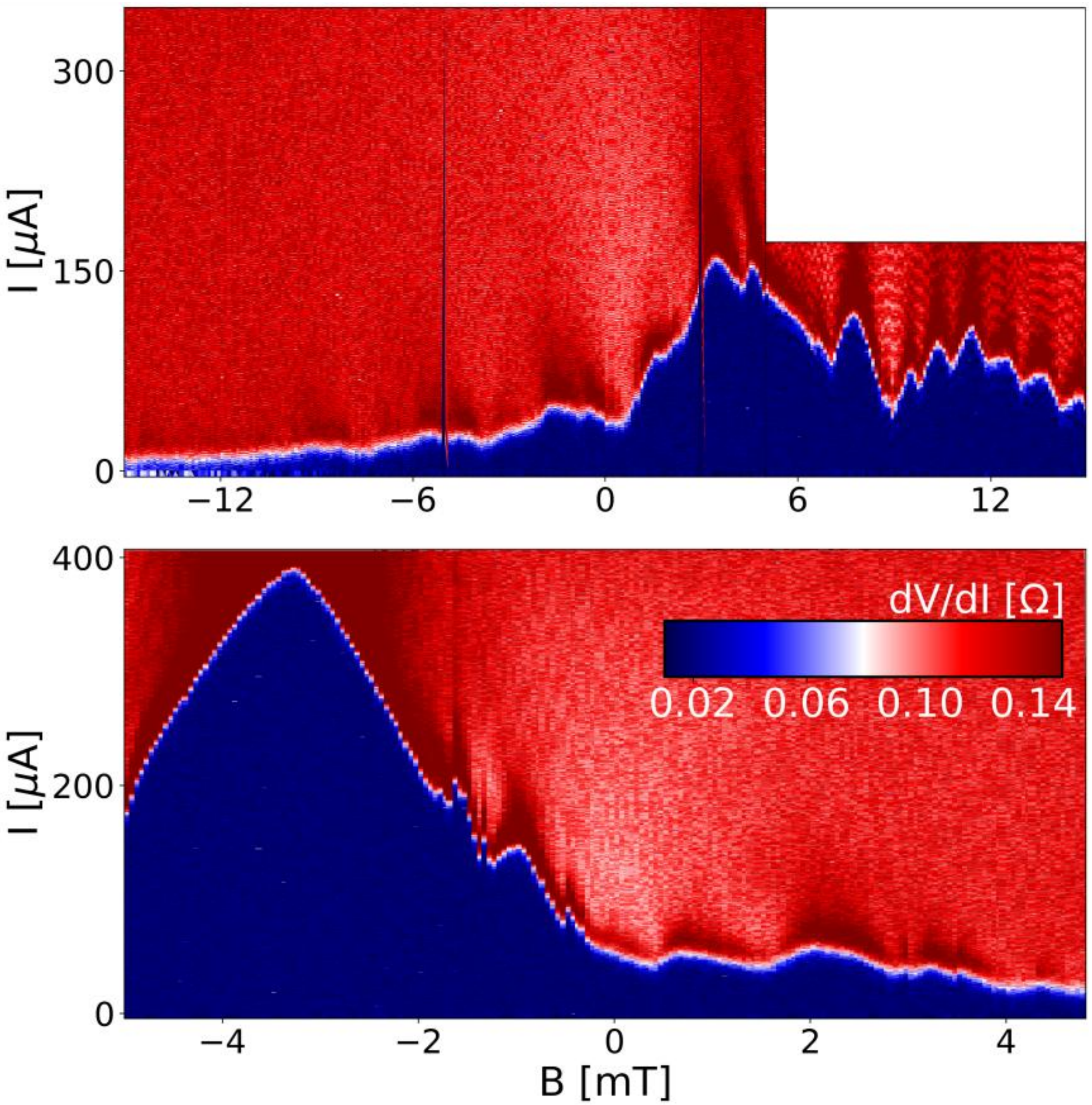


SQI-patterns: Fractal junction

- Maximum not @ zero T
- Visible oscillations have irregular periodicity
- Pattern irreproducible

Conclusion:

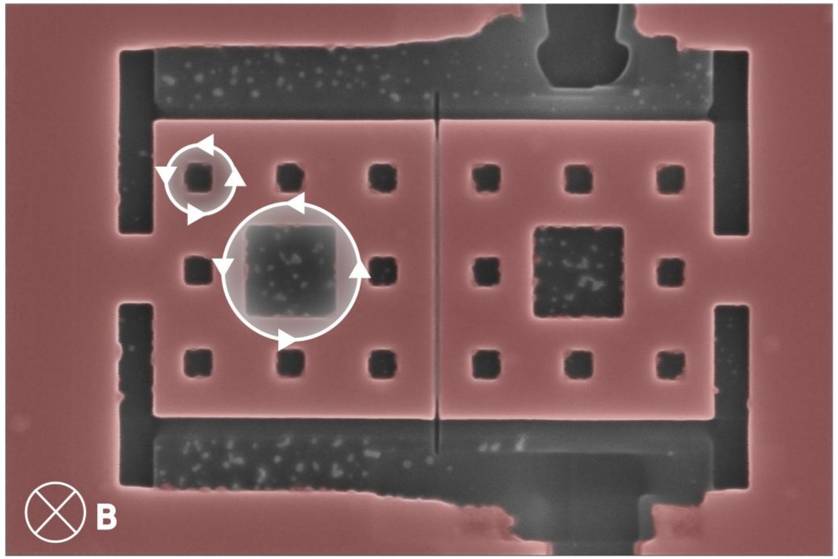
- <u>Vortex trapping</u>

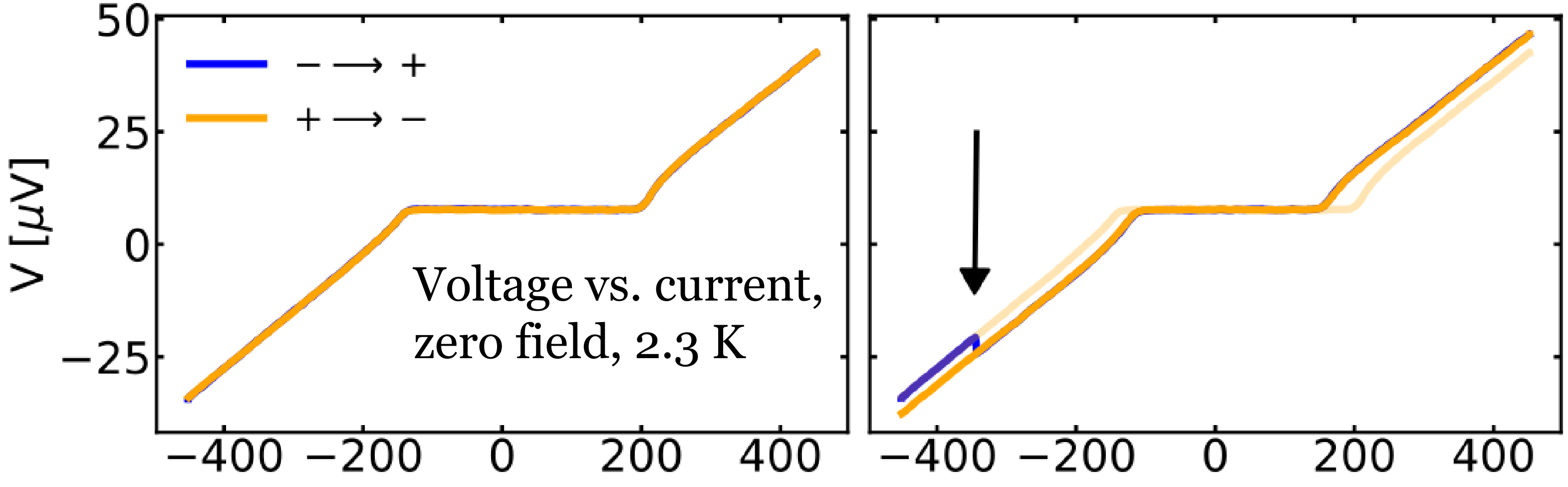


Vortex trapping

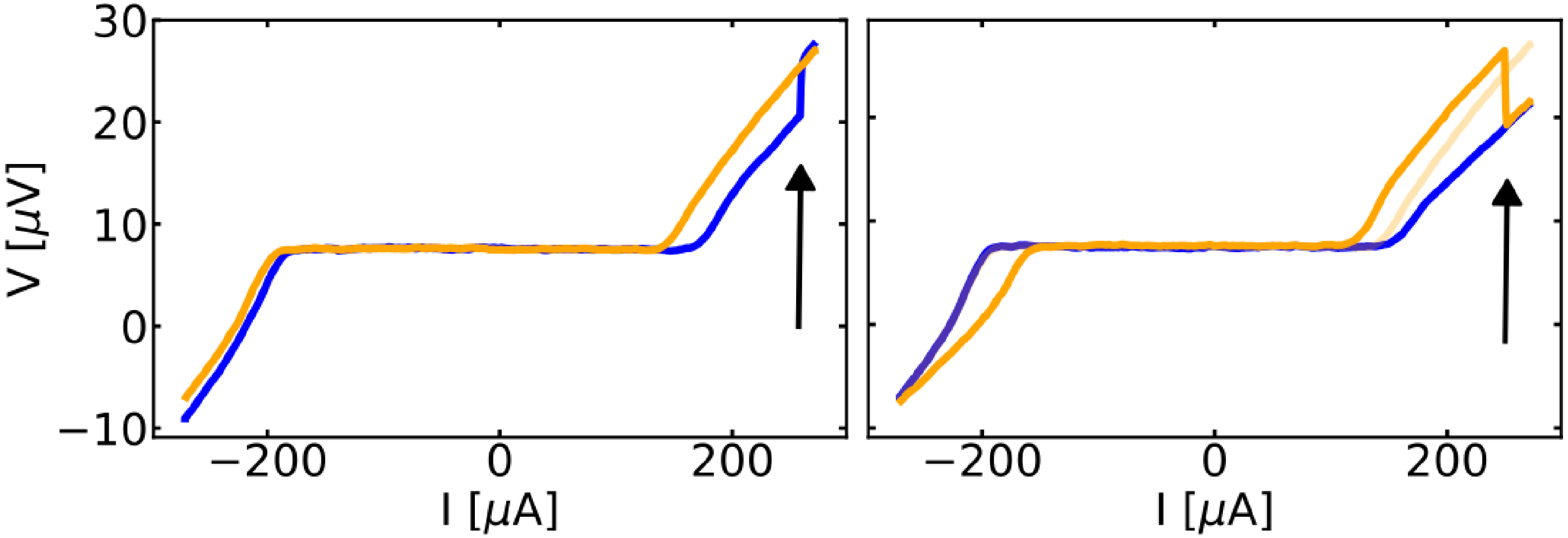
Vortex trapping

- Trapped vortex locally changes magnetic field
- Trapped vortex redistributes phase in vicinity > change in shielding currents





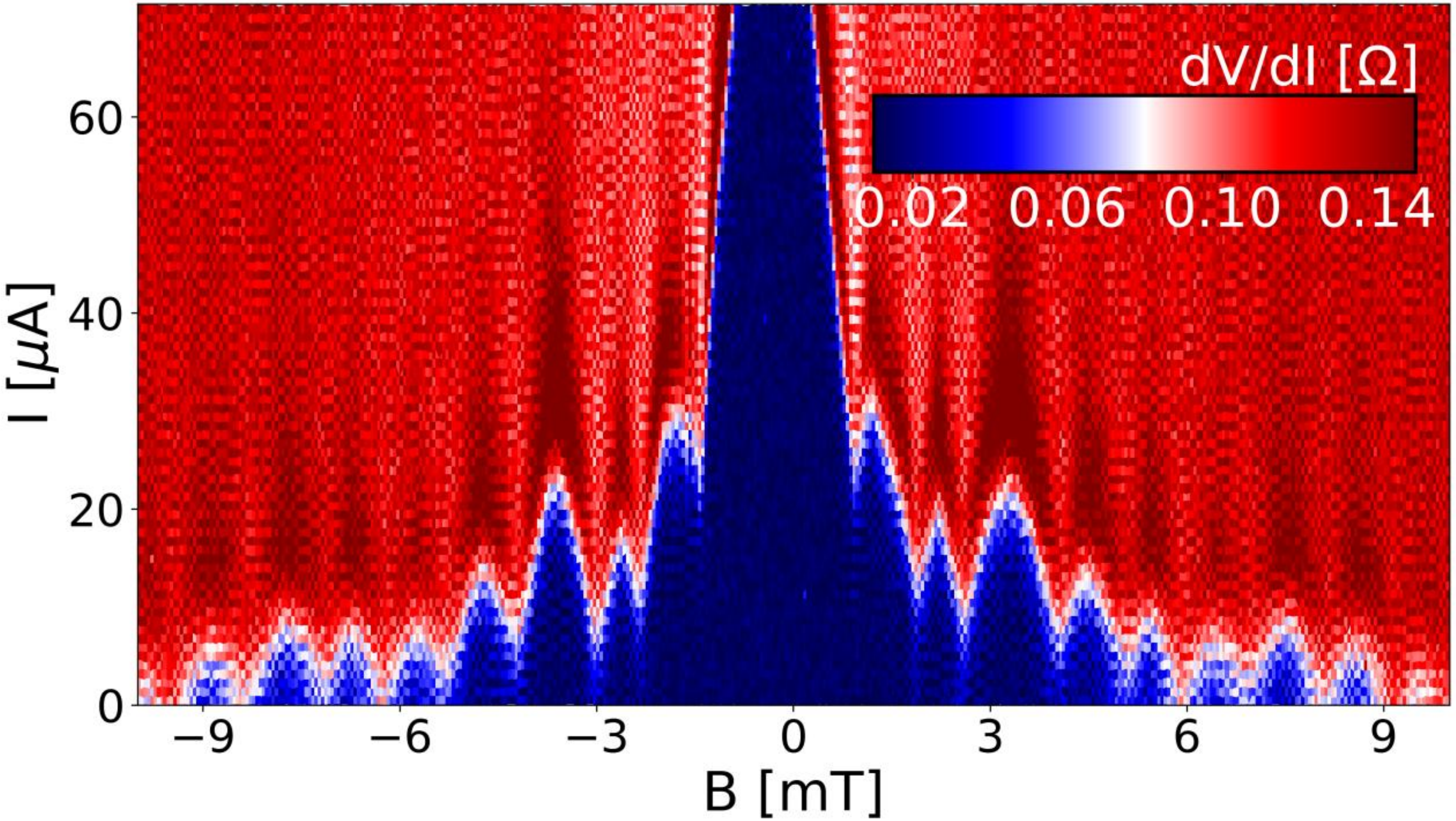
Heat sample to $T > T_c$
And slow cooldown



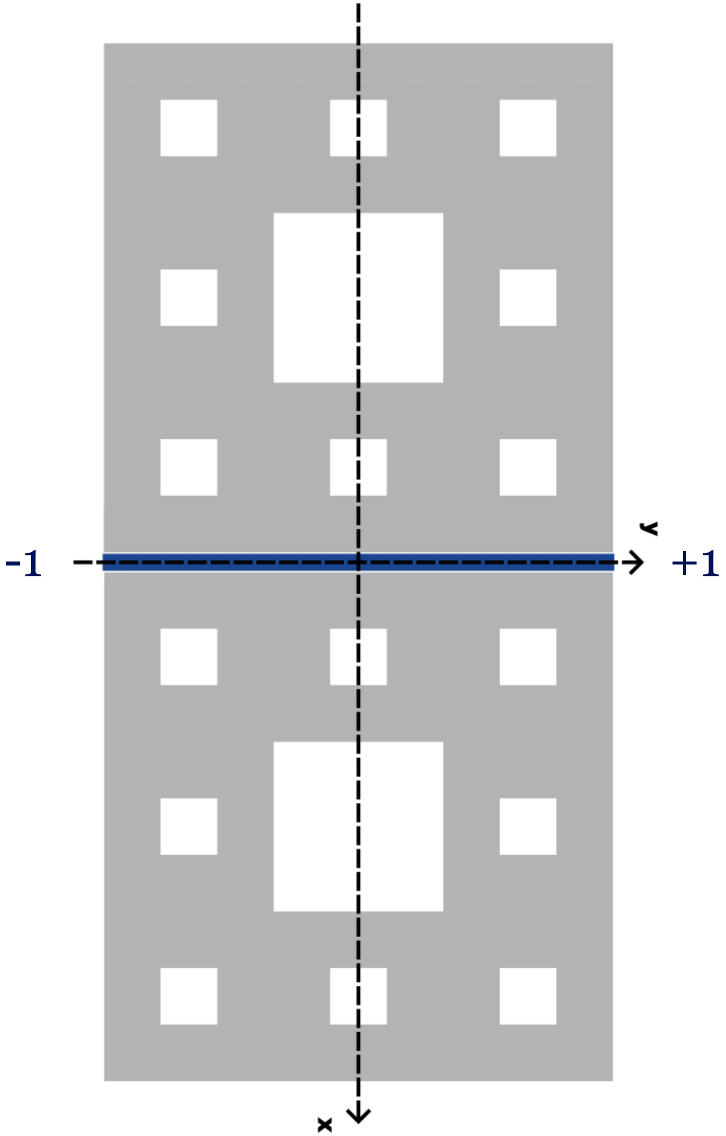
Isolating the contribution of geometry

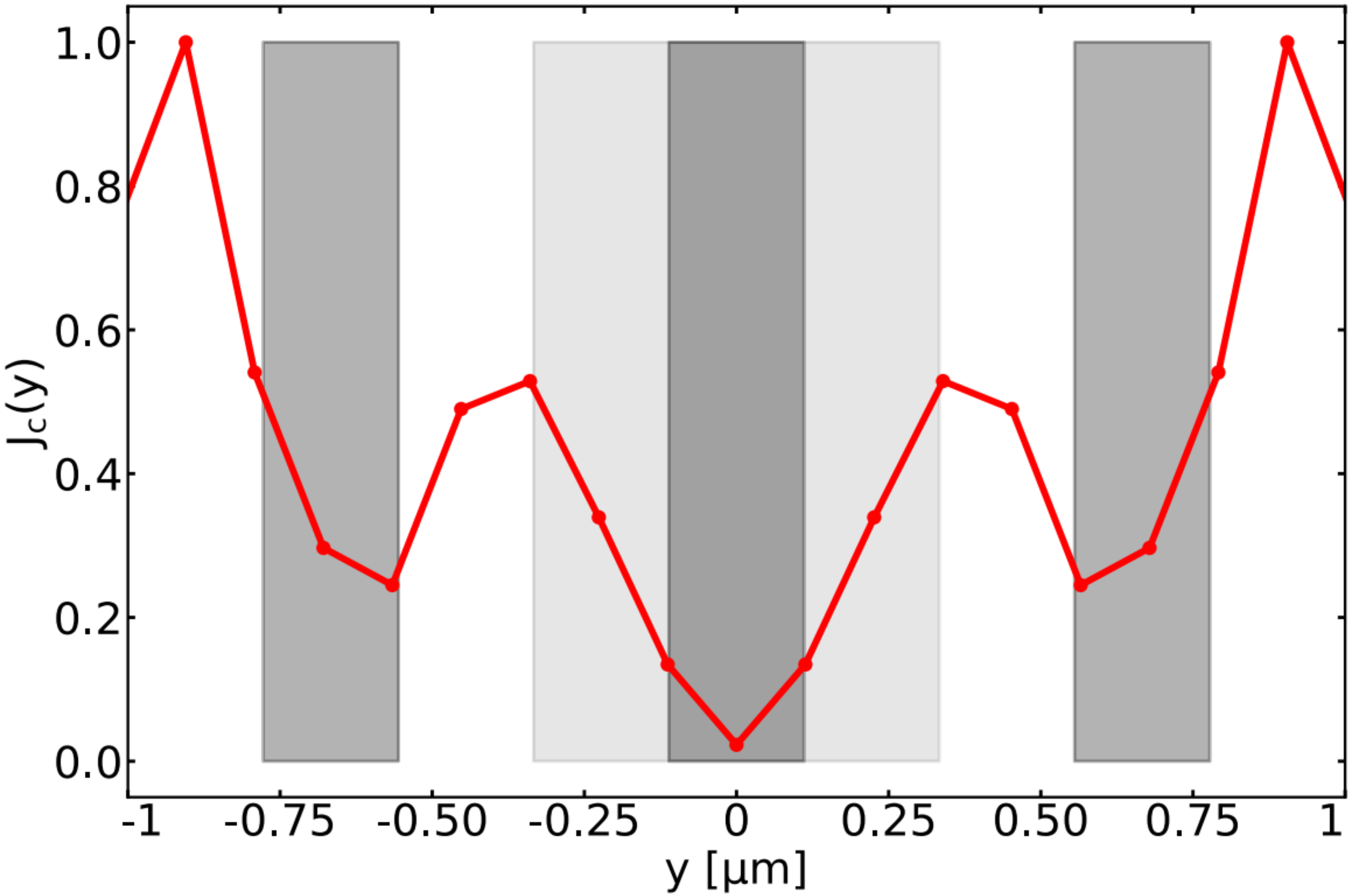
- Higher temperatures:

  2.3 K → 3.5 K
- Limit probe currents below 100 µA
- Pattern reproduced multiple times with intermittent reheating to $T > T_c$
- Multiple periodicities



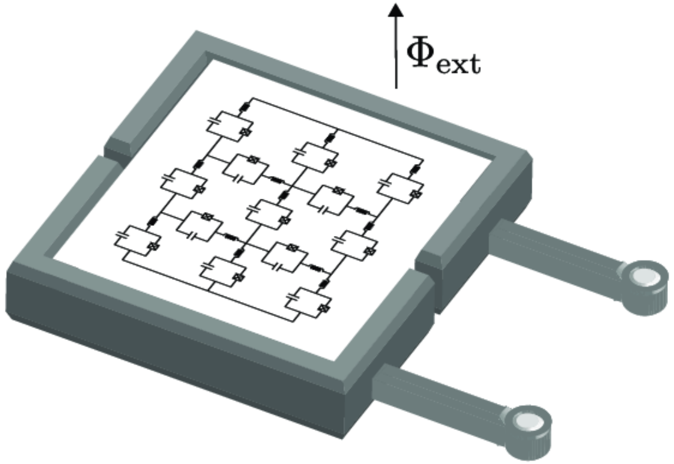
Critical current density distribution





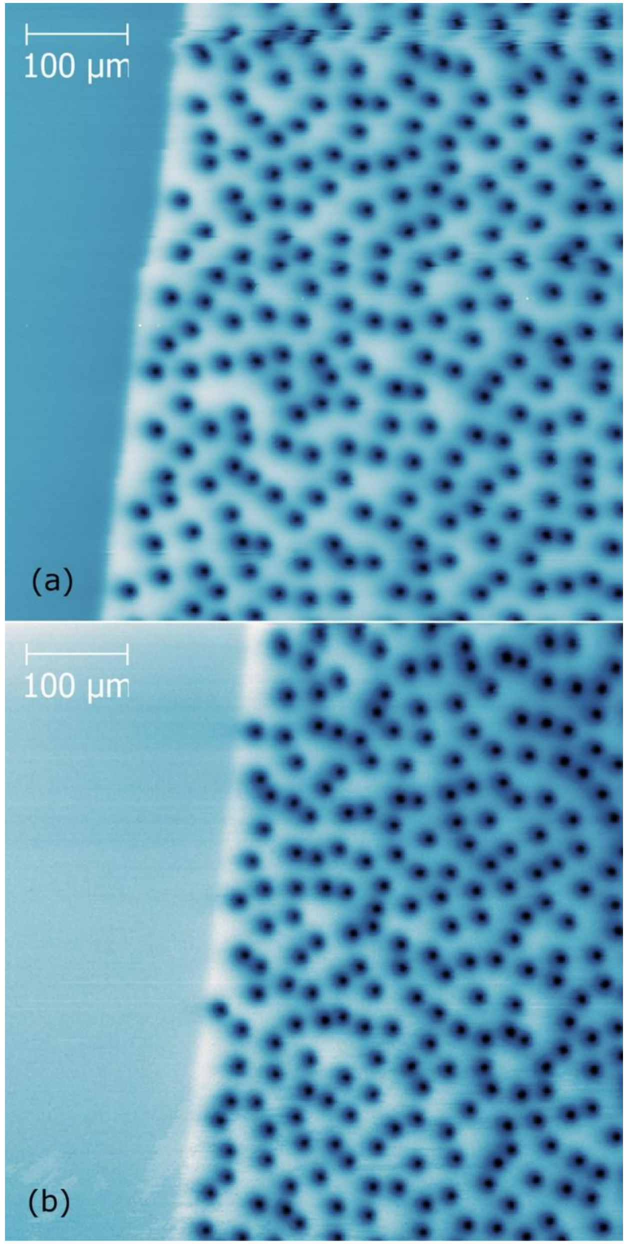
Outlook

- Josephson junction networks: junction arrays
  - ❑ large: easier to structure
  - ❑ exciting physics: Giant (fractional) Shapiro steps, commensurability effects, collective rf-response



S. A. Wilkinson, 2018

- Direct imaging of vortex trapping with SQUID on tip?



F. S. Wells, 2015

Thank you

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Final layer composition

Layer	Thickness (nm)	Sputtertime (s)	Presputtertime (s)
Ti	5	100	120
Ag	60	150	120
MoGe	55	589	120
Ti	5	100	120