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Modeling of a Quadcopter Trajectory Tracking System Using PID Controller

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Abstract

A quadcopter is a type of Unmanned Aerial Vehicles (UAV) with four propellers and ability to take-off/landing in limited space. In the past few years, quadcopters have become an integral part of life. Therefore, in this paper, first, mathematical model of the quadcopter is created and it based on the equation of motion and forces of the moment using the Newton-Euler method. Secondly, a cascaded PID controller is designed to track the given trajectory. The chosen model is made to hover at an altitude where the nonlinear model is linearized. In addition to that, the response of the linear and nonlinear model is analyzed and the PID controller for the nonlinear model is designed and the results are analyzed.

Keywords: UAV; Quadcopter; Newton-Euler method; Cascaded PID controller.

1. Introduction

A quadcopter aircraft, also known as Unmanned Aerial Vehicles (UAVs) is a multirotor helicopter that is lifted and propelled with the aid of 4 rotors. It exists also in different sizes and shapes. [1], [2], [3], [4], [5]. In recent years, researchers have shown that it's possible to control the quadcopter using linear manipulate techniques via linearizing the dynamics around a running point [6]. However, a wider flight and better performances can be achieved by using nonlinear control techniques that are considered more general form of the dynamics of the vehicle in all flight zones.

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In the present work, mathematical model of the quadcopter is developed considering it dynamics. The fundamental equations of motion and forces of the quadcopter are derived and the design parameters for the given quadcopter are chosen. The model is created to be nonlinear for the quadcopter based on the equation of motion and forces of the moment using the Newton-Euler method. A PID controller was used for the control of a quadrotor in [7], [8], [9] and [10] for the altitude control. However, in this paper, the PID controller scheme has been applied to control not only the altitude, but also attitude, heading and position of the quadcopter in space. The chosen model is made to hover at an altitude where the nonlinear model is linearized. The aim is to find a controller method that permits the states of a quadrotor to converge to an arbitrary set of time-various reference states. Thus, a PID controller for the nonlinear model is designed and the results are analyzed, Also, a simple cascaded PID controller proposed for trajectory tracking to make the implementation easier.

The present paper is organized as follows. The first section is dedicated to the dynamic modeling of the drone and offer a description of a general structure of the mathematical model of the quadrotor. Indeed, a dynamic model of the drone was developed and the state model for the control of this system was given.

The second section is devoted to the general scheme of the drone made under Simulink software. Then, a review of the application of the PID controller technique to control the drone is provided. However, Simulink simulation results are shown in the third section.

Nomenclature

$(\varphi, \theta, \psi) \in \mathbb{R}^3$	Euler angles
$\Re_B$	Rigid body frame
$\Re_E$	Flat earth frame
$\xi \in \mathbb{R}^3$	Position of center of mass in flat earth coordinate
$\eta \in \mathbb{R}^3$	Attitude angles of body frame in the flat earth coordinate
$\omega_i \in \mathbb{R}$	Angular velocity of the i propeller
$V \in \mathbb{R}^3$	Velocity in the body frame
$R \in \mathbb{R}^{3\times3}$	Rotation transform matrix
$\Omega \in \mathbb{R}^3$	Angular velocity in the body frame
$F \in \mathbb{R}^3$	Total force acting on the quadcopter
$M \in \mathbb{R}^3$	Total torque and moment acting on the quadcopter
$I \in \mathbb{R}^{3\times3}$	Symmetrical inertia matrix
$m \in \mathbb{R}$	Mass of the quadcopter
$g \in \mathbb{R}$	Gravitational acceleration
$b \in \mathbb{R}$	Trust constant
$C_d \in \mathbb{R}^3$	Translational drag coefficients
$l \in \mathbb{R}$	Distance between the motor axis and the center of the mass of the quadcopter
$d \in \mathbb{R}$	Drag factor
$C_a \in \mathbb{R}^3$	Aerodynamic friction coefficients
$J_r \in \mathbb{R}$	Rotor inertia
$(K_p, K_I, K_D) \in \mathbb{R}^3$	Proportional, integral and derivative gain
$e(t)$	Error function
$y(t)$	Output of the dynamic model

2. Dynamic Model of the quadcopter

2.1. Coordinate Frames

Before developing a mathematical model of the drone, the two notation $\Re_E(O, \vec{I}, \vec{J}, \vec{K})$ and $\Re_B(O, \vec{i}, \vec{j}, \vec{k})$ need to be proceed as shown in Fig.1. Such that the reference $\Re_E$ is bound to the ground and the reference $\Re_B$ is a frame linked to the body of the drone and its centre coincides with the centre of mass of the drone.

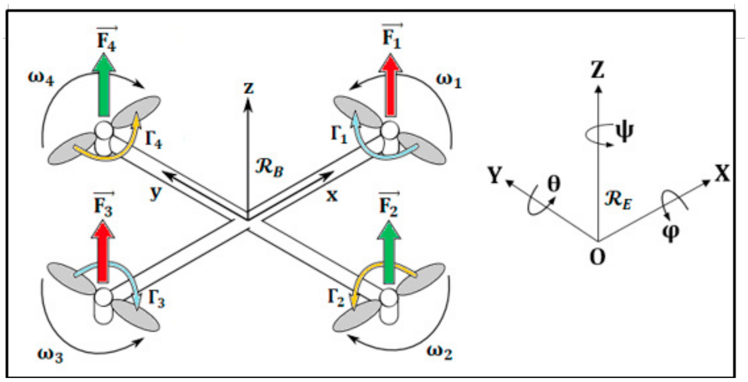

Fig. 1. Quadcopter structure and coordinate systems.

2.2. Applied forces and torques

To work with a realistic quadcopter model, aerodynamic forces must be included in addition to gravity and thrust. The movements of the quadcopter are governed by mechanical or aerodynamic effects. The main effects on the drone are shown in Table 1.

Table 1: Mechanical actions and sources.

Mechanical Action	Source
Aerodynamic effect	Rotation of the propellers
Inertial counter torque	Speed change of propellers
Gravity	Universal Law of Gravitation
Gyroscopic effects	Direction change of the quadcopter
Friction	Air resistance

To obtain the equations of motion of the studied system, the following assumptions were made:

- The quadcopter is a rigid body and it has a symmetrical structure.
- Center of gravity and center of mass coincide with a quadcopter geometrical center.
- Moment of inertia of the propellers is neglected.

For recapitulating the mathematical model of the quadcopter, the Newton-Euler formalism [11] is used. The equations can be expressed as:

$$\begin{bmatrix} F \\ M \end{bmatrix} = \begin{bmatrix} mI_{3\times3} & 0_{3\times3} \\ 0_{3\times3} & I \end{bmatrix} \begin{bmatrix} \dot{V} \\ \dot{\Omega} \end{bmatrix} + \begin{bmatrix} \Omega \wedge mV \\ \Omega \wedge I\,\Omega \end{bmatrix} \tag{1}$$

The equation (1) is divided into two parts, the first one is the translation dynamic (Newton’s second law) and the second one is the rotational dynamic (Euler's rotation equations).

2.3. Translation dynamic

The forces applied on the studied system are:

- The weight of the drone:

$$W = [0 \quad 0 \quad -mg]^T \tag{2}$$

- The trust of the rotors:

$$F_t = R\sum_{i=1}^{4} F_i = b\sum_{i=1}^{4} \omega_i^2 \begin{bmatrix} \cos\psi \sin\theta \cos\varphi + \sin\psi \sin\varphi \\ \cos\varphi \sin\psi \sin\theta \ - \ \cos\psi \sin\varphi \\ \cos\theta \cos\varphi \end{bmatrix} \tag{3}$$

- The drag force and the air friction:

$$F_d = C_d\dot{\xi} = \begin{bmatrix} -C_{dx} & 0 & 0 \\ 0 & -C_{dy} & 0 \\ 0 & 0 & -C_{dz} \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix} = -\begin{bmatrix} C_{dx}\,\dot{x} \\ C_{dy}\,\dot{y} \\ C_{dz}\,\dot{z} \end{bmatrix} \tag{4}$$

By applying the second law of Newton $\left(F = m\ddot{\xi} = \mathrm{W} + F_t + \mathrm{F_d}\right)$, the equation of motion that governs the translational motion of the quadcopter is given by:

$$\begin{cases} \ddot{x} = \frac{b}{m}({\omega_1}^2 + {\omega_2}^2 + {\omega_3}^2 + {\omega_4}^2)(\cos\psi \sin\theta \cos\varphi + \sin\psi \sin\varphi) - \frac{C_{dx}}{m}\dot{x} \\ \ddot{y} = \frac{b}{m}({\omega_1}^2 + {\omega_2}^2 + {\omega_3}^2 + {\omega_4}^2)(\cos\varphi \sin\psi \sin\theta \ - \ \cos\psi \sin\varphi) - \frac{C_{dy}}{m}\dot{y} \\ \ddot{z} = \frac{b}{m}({\omega_1}^2 + {\omega_2}^2 + {\omega_3}^2 + {\omega_4}^2) \cos\theta \cos\varphi - \frac{C_{dz}}{m}\dot{z} - g \end{cases} \tag{5}$$

2.4. Rotational dynamic

The torques acting on the drone are:

- Roll torque:

$$\tau_x = \begin{bmatrix} 0 \\ -l \\ 0 \end{bmatrix} \wedge \begin{bmatrix} 0 \\ 0 \\ \mathrm{F_2} \end{bmatrix} + \begin{bmatrix} 0 \\ l \\ 0 \end{bmatrix} \wedge \begin{bmatrix} 0 \\ 0 \\ \mathrm{F_4} \end{bmatrix} = \begin{bmatrix} lb\,({\omega_4}^2 - {\omega_2}^2) \\ 0 \\ 0 \end{bmatrix} \tag{6}$$

- Pitch torque:

$$\tau_y = \begin{bmatrix} l \\ 0 \\ 0 \end{bmatrix} \wedge \begin{bmatrix} 0 \\ 0 \\ \mathrm{F_1} \end{bmatrix} + \begin{bmatrix} -l \\ 0 \\ 0 \end{bmatrix} \wedge \begin{bmatrix} 0 \\ 0 \\ \mathrm{F_3} \end{bmatrix} = \begin{bmatrix} 0 \\ lb\,({\omega_3}^2 - {\omega_1}^2) \\ 0 \end{bmatrix} \tag{7}$$

- Yaw torque:

$$\tau_z = \begin{bmatrix} 0 \\ 0 \\ d({\omega_1}^2 - {\omega_2}^2 + {\omega_3}^2 - {\omega_4}^2) \end{bmatrix} \tag{8}$$

- Torque resulting from aerodynamic friction:

$$\tau_a = C_a \begin{bmatrix} \dot{\varphi}^2 \\ \dot{\theta}^2 \\ \dot{\psi}^2 \end{bmatrix} = \begin{bmatrix} C_{ax} & 0 & 0 \\ 0 & C_{ay} & 0 \\ 0 & 0 & C_{az} \end{bmatrix} \begin{bmatrix} \dot{\varphi}^2 \\ \dot{\theta}^2 \\ \dot{\psi}^2 \end{bmatrix} = \begin{bmatrix} C_{ax}\,\dot{\varphi}^2 \\ C_{ay}\,\dot{\theta}^2 \\ C_{az}\,\dot{\psi}^2 \end{bmatrix} \tag{9}$$

- Gyroscopic effect from propeller:

$$\tau_{gp} = J_r\dot{\eta} \wedge \begin{bmatrix} 0 \\ 0 \\ \sum_{i=1}^{4}(-1)^{i+1}\omega_i \end{bmatrix} = J_r\Omega_r \begin{bmatrix} \dot{\theta} \\ -\dot{\varphi} \\ 0 \end{bmatrix} \text{ with } \Omega_r = \sum_{i=1}^{4}(-1)^{i+1}\omega_i \quad (10)$$

By applying Euler's rotation equations, the equation of motion that governs the rotational motion of the quadcopter is given by:

$$\begin{cases} \ddot{\varphi} = \frac{lb\,({\omega_4}^2-{\omega_2}^2)}{I_x} - \frac{C_{ax}}{I_x}\dot{\varphi}^2 - \frac{J_r\,\Omega_r}{I_x}\dot{\theta} - \frac{(I_z-I_y)}{I_x}\dot{\theta}\dot{\psi} \\ \ddot{\theta} = \frac{lb\,({\omega_3}^2-{\omega_1}^2)}{I_y} - \frac{C_{ay}}{I_y}\dot{\theta}^2 + \frac{J_r\,\Omega_r}{I_y}\dot{\varphi} - \frac{(I_x-I_z)}{I_y}\dot{\varphi}\dot{\psi} \\ \ddot{\psi} = \frac{d}{I_z}({\omega_1}^2 - {\omega_2}^2 + {\omega_3}^2 - {\omega_4}^2) - \frac{C_{az}}{I_z}\dot{\psi}^2 - \frac{(I_y-I_x)}{I_z}\dot{\varphi}\,\dot{\theta} \end{cases} \quad (11)$$

2.5. Total system model

As a result, the complete dynamic model governing the quadrotor is as follows:

$$\begin{cases} \ddot{x} = \frac{u_1 u_x}{m} - \frac{C_{dx}}{m}\dot{x} \\ \ddot{y} = \frac{u_1 u_y}{m} - \frac{C_{dy}}{m}\dot{y} \\ \ddot{z} = \frac{u_1}{m}\cos\theta\cos\varphi - \frac{C_{dz}}{m}\dot{z} - g \\ \ddot{\varphi} = \frac{u_2}{I_x} - \frac{C_{ax}}{I_x}\dot{\varphi}^2 - \frac{J_r\Omega_r}{I_x}\dot{\theta} - \frac{(I_z-I_y)}{I_x}\dot{\theta}\dot{\psi} \\ \ddot{\theta} = \frac{u_3}{I_y} - \frac{C_{ay}}{I_y}\dot{\theta}^2 + \frac{J_r\Omega_r}{I_y}\dot{\varphi} - \frac{(I_x-I_z)}{I_y}\dot{\varphi}\,\dot{\psi} \\ \ddot{\psi} = \frac{u_4}{I_z} - \frac{C_{az}}{I_z}\dot{\psi}^2 - \frac{(I_y-I_x)}{I_z}\dot{\varphi}\,\dot{\theta} \end{cases} \text{ with } \begin{cases} u_x = \cos\psi\sin\theta\cos\varphi + \sin\psi\sin\varphi \\ u_y = \cos\varphi\sin\psi\sin\theta - \cos\psi\sin\varphi \\ u_1 = b{\omega_1}^2 + b{\omega_2}^2 + b{\omega_3}^2 + b{\omega_4}^2 \\ u_2 = -lb{\omega_2}^2 + lb{\omega_4}^2 \\ u_3 = -lb{\omega_1}^2 + lb{\omega_3}^2 \\ u_4 = d({\omega_1}^2 - {\omega_2}^2 + {\omega_3}^2 - {\omega_4}^2) \end{cases} \quad (12)$$

3. PID controller of the quadcopter using Simulink

The quadrotor is an under-actuated system [12], which means, the six degrees of freedom in space are controlled only with four motors. Therefore, the control of these drones must be established for a subset of four of its degrees of freedom. In addition, the control of the X and Y coordinates depends on the pitch and roll orientation respectively. In considering this coupling, the control of a quadrotor is usually done for two different subsets of the coordinates.

The command is made for the three position coordinates plus the yaw orientation. Nevertheless, the control mode uses both roll and pitch orientation controllers. In short, the control signals of three position controllers define a force vector (thrust) in the inertial coordinate system. The orientation of this vector defines the setpoint sent to the roll and pitch controllers. The diagram in Fig.2 summarizes what was previously described.

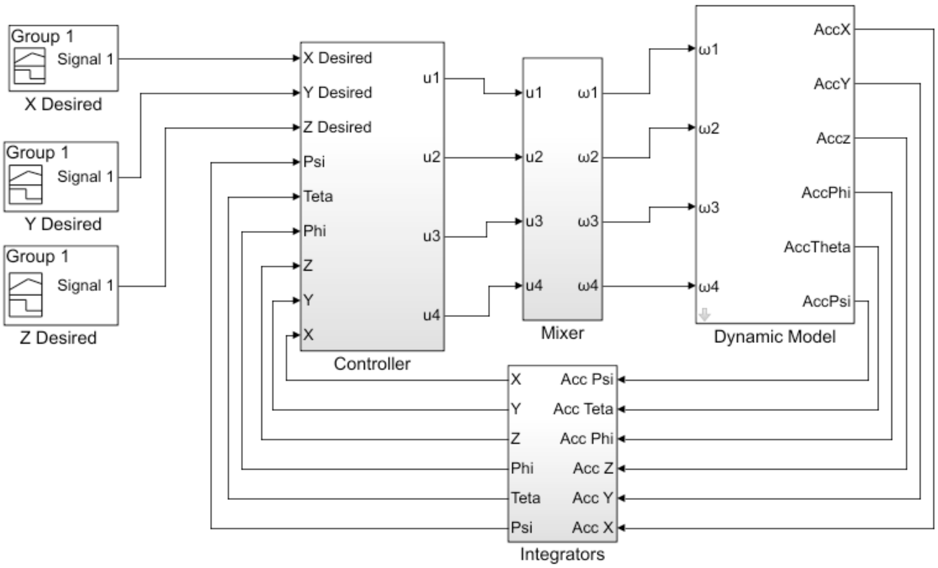

Fig. 2. Full architecture of the quadcopter using Simulink.

The chosen controller is the PID type [13] which is very simple to implement and which is still widely used in the industry. The PID controller involves three distinct parameters: the Proportional term, the Integral term and the Derivative term. The proportional term determines the direct action with respect to the calculated error, the integral term considers the sum of recent errors to react, and the derivative term determines the reaction with respect to the rate of change of the error. The equation of the regulator is given by the following formula:

$$U(t) = K_p e(t) + K_I \int e(t)dt + K_D \frac{de(t)}{dt} \text{ such us } e(t) = Setpoint - y(t) \tag{13}$$

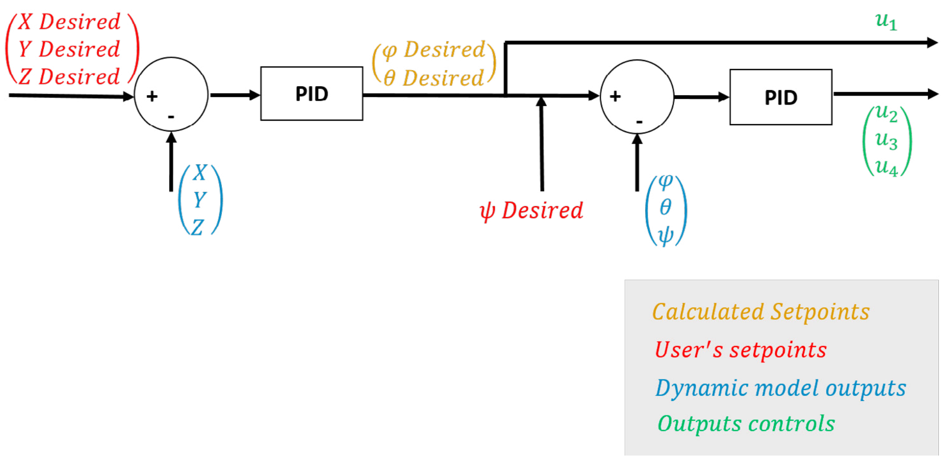

Fig. 3. Architecture of the controller block.

The PID control architecture (shown in Fig.3) for the specific case of drones consists of two regulators installed in cascade. Depending on the position references (X Desired, Y Desired and Z Desired), the first PID generates the orientation instructions (φ Desired, and θ Desired) for stabilization and horizontal displacements and also generate the thrust controller u_1. The follow-up of these instructions in roll, pitch and yaw is then the mission of a second

PID which makes it possible to generate the commands u_2, u_3 and u_4.

Finally, a block mixer provide the converting of the control outputs into the rotational speed of each motor.

This approach has been successfully tested on several experimental platforms such as helicopters [14], [15] and coaxial rotor drones [16].

4. Result and discussions

In order to allow the drone to follow a path entered by the user, the generation of three setpoints was established, and after adjusting the PID controller gains and using the parameters mentioned in Table 2, the simulation of Fig.4.(a) has been obtained.

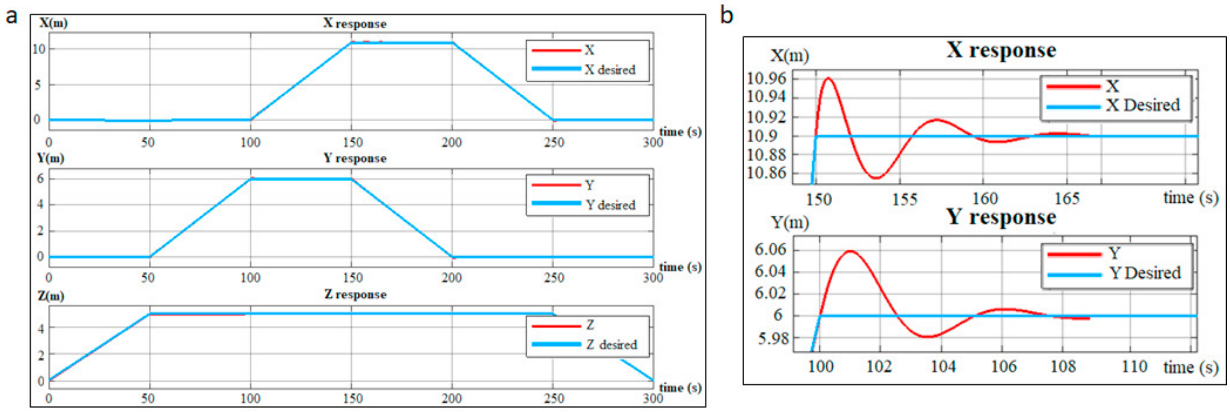

Fig. 4. (a) The response of the drone to user's setpoints; (b) Response of X and Y.

We can see from Fig.4.(b) that the response of the system is very satisfactory. It is characterized by a very small overshoot (0.6% for X and 1% for Y) and a very low response time (15 seconds for X and 10 seconds for Y).

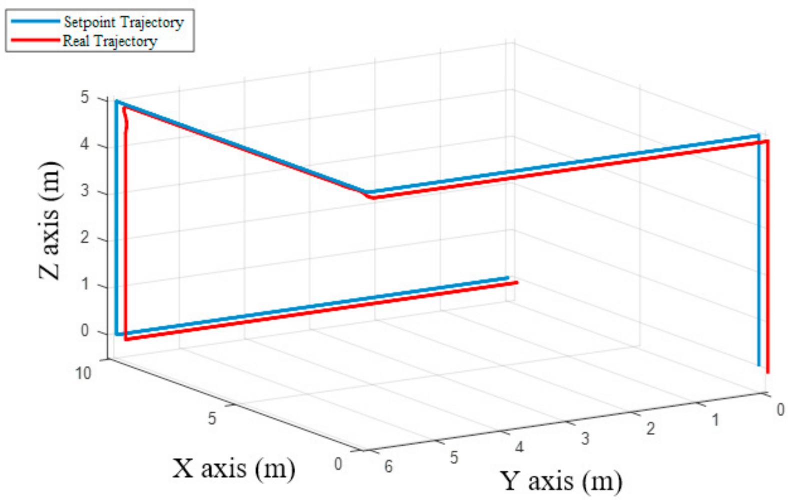

Fig. 5. Setpoint and real trajectory.

The result shown in the Fig.5 illustrates the trajectory tracking performance of PID controller.

Table 2. Parameters of the quadcopter used in the simulation.

Parameter	Value	Unit
I	$diag(3.8 \times 10^{-3}\ 3.8 \times 10^{-3}\ 7.1 \times 10^{-3})$	$kg.m^2$
m	5.2	kg
g	9.81	$m.s^{-2}$
b	3.13×10^{-5}	$kg.m$
C_d	$diag(0.1\ 0.1\ 0.15)$	$kg.s^{-1}$
l	0.32	m
d	7.5×10^{-7}	$kg.m$
C_a	$diag(0.1\ 0.1\ 0.15)$	$kg.m$
J_r	6×10^{-5}	$kg.m^2$

5. Conclusion

The objective of this work is to give a mathematical model of the drone by taking into account different mechanical actions and disturbances that act on the system, thus, to simulate the behaviour of the drone under Simulink, which requires the use of a cascaded PID controller to govern the manipulation of the drone for trajectory tracking to make a smooth implementation. The results of the simulations made were very satisfactory and the drone manages to follow perfectly the instructions of the user.

References

[1] R. Lozano, Unmanned Aerial Vehicles : Embedded Control. ISTE Ltd and John Wiley and Sons Inc, (2010).
[2] R. K. Barnhart, S. B. Hottman, D. M. Marshall et E. Shappee, Introduction to Unmanned Aircraft Systems. Taylor and Francis, (2012).
[3] K. Nonami, F. Kendoul, S. Suzuki, W. Wang et D. Nakazawa, Autonomous Flying Robots - Unmanned Aerial Vehicles and Micro Aerial Vehicles, Springer, (2010).
[4] R. Austin, Unmanned Aircraft Systems - UAVS Design, Development and Deployment, Wiley, (2010).
[5] K. P. Valavanis, Advances in Unmanned Aerial Vehicles - State of the Art and the Road to Autonomy, Springer, (2007).
[6] J. Kim, M. Kang, and S. Park, Accurate modeling and robust hovering control for a quadrotor vtol aircraft, Journal of Intelligent and Robotic Systems, 57(1-4): (2010) 9-26.
[7] H. Bolandi, M. Rezaei, R. Mohsenipour, H. Nemati, and S. M. Smailzadeh, Attitude Control of a Quadrotor with Optimized PID Controller, Intelligent Control and Automation (4): (2013) 335-342.
[8] A. L. Salih, M. Moghavvemi, H. A. Mohamed, and K. S. Gaeid, Modelling and PID controller design for a quadrotor unmanned air vehicle, 2010 IEEE International Conference on Automation, Quality and Testing, Robotics (AQTR), Cluj-Napoca: (2010) 1-5.
[9] R. A. Garcia, F. R. Rubio, M. G. Ortega, Robust PID control of the quadrotor helicopter, IFAC Proceedings Volumes (45): 229234 (2012).
[10] G. Bo, L. Xin, Z. Hui, W. Ling, Quadrotor helicopter Attitude Control using cascade PID, Chinese Control and Decision Conference (CCDC), Yinchuan: (2016) 5158-5163.
[11] S. János, G. Martinovic, Navigation of Mobile Robots Using WSN's RSSI Parameter and Potential Field Method, Acta Polytechnica Hungarica, Journal of Applied Sciences Vol.10, No.4. (2013) 107-118.
[12] H. Hou, J. Zhuang, H. Xia, G. Wang, and D. Yu, A simple controller of minisize quad-rotor vehicle. In Mechatronics and Automation (ICMA), International Conference on, (2010) 1701doi:10.1109/ICMA.2010.5588802.
[13] M. M. Atheer, L. Salih, A. F. Mohamed and K. S. Gaeid, Modelling and PID Controller Design for a Quadrotor Unmanned Air Vehicle, in IEEE International Conference on Automation Quality and Testing Robotics (AQTR) Cluj-Napoca (2010).
[14] D. H. Shim, T. J. Koo, F. Hoffmann et S. S. Sastry, A Comprehensive Study of Control Design for an Autonomous Helicopter, Proceedings of the 37th IEEE Conference on Decision and Control, Tampa, Florida, USA, vol. 4, (1998) 3653-3658.
[15] H. J. Kim et D. H. Shim, A Flight Control System for Aerial Robots : Algorithms and Experiments, Control Engineering Practice, vol. 11, no. 12, (2003) 1389-1400.
[16] L. Lipera, J. Colbourne, M. Tischler, M. H. Mansur, M. Rotkowitz et P. Patangui, The Micro Craft iSTAR Micro Air Vehicle : Control System Design and Testing, Proceedings of the 57th Annual Forum of the American Helicopter Society, Washington, District of Columbia, USA, (2001) 1-11.