Solution of Homework #1

Mattia Lecci, Federico Mason

October 29, 2017

1 Problem 3.2

Given a Z channel with probabilities $p_{Y|X}(0|0) = 1$, $p_{Y|X}(1|0) = 0$, $p_{Y|X}(1|1) = p_{Y|X}(0|1) = \frac{1}{2}$, we want to find the channel capacity C. We know that the channel capacity for a discrete memory channel is defined as follows.

$$C = \max_{p_X} I(X;Y) = H(Y) - H(Y|X) \tag{1}$$

First we compute the value of $H(Y)$ and $H(Y|X)$ in function of p_X.

$$\begin{aligned}
H(Y) &= -\,Prob\{y=1\} \cdot \log(Prob\{y=1\}) - Prob\{y=0\} \cdot \log(Prob\{y=0\}) \\
&= -\,p_{Y|X}(1|1) \cdot p_X(1) \cdot \log(p_{Y|X}(1|1) \cdot p_X(1)) \\
&\quad - (p_{Y|X}(0|0) \cdot p_X(0) + p_{Y|X}(0|1) \cdot p_X(1)) \cdot \log(p_{Y|X}(0|0) \cdot p_X(0) + p_{Y|X}(0|1) \cdot p_X(1)) \\
&= -\,\frac{1}{2} \cdot p_X(1) \log\left(\frac{1}{2} \cdot p_X(1)\right) - \left(p_X(0) + \frac{1}{2} \cdot p_X(1)\right) \cdot \log\left(p_X(0) + \frac{1}{2} \cdot p_X(1)\right) \\
&= -\,\frac{1}{2} \cdot p_X(1) \log\left(\frac{1}{2} \cdot p_X(1)\right) - \left(1 - \frac{1}{2} \cdot p_X(1)\right) \cdot \log\left(1 - \frac{1}{2} \cdot p_X(1)\right)
\end{aligned}$$

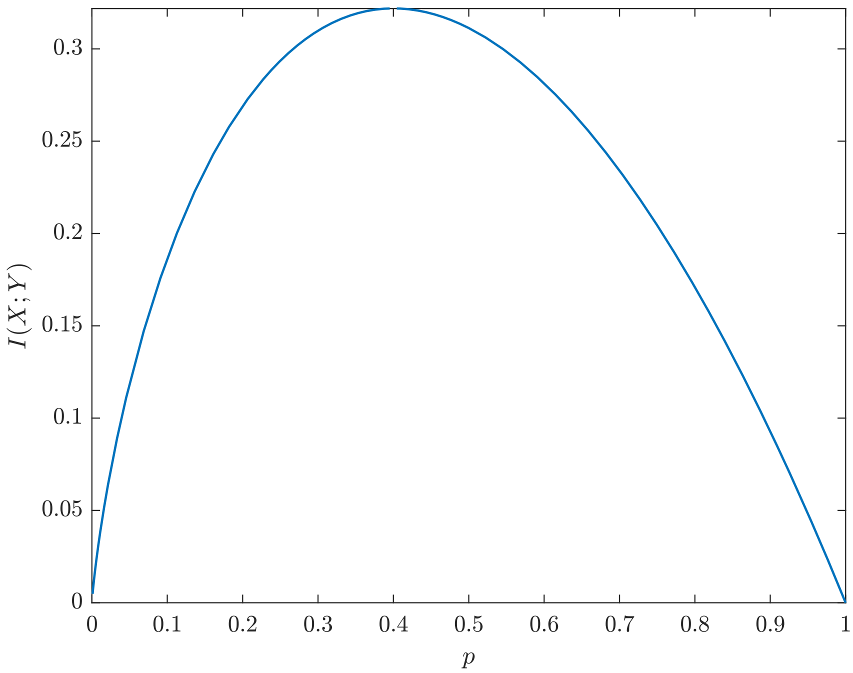

Figure 1: Mutual information as a function of $p \triangleq p_X(1)$