Chapter 0. Course Notation

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Overview

Notation for sets, scalars and vectors

- Lowercase letters, $x, y, ...$ are used for constants and values of random variables.
- Sequences or column vectors are $x_i^j = (x_i, x_{i+1}, ...x_j)$. In case $i = 1$ then $x^j = (x_1, x_2, ...x_j)$.
- Let $\alpha, \beta \in [0, 1]$. Then $\bar{\alpha} = (1 - \alpha)$ and $\alpha * \beta = \alpha\bar{\beta} + \beta\bar{\alpha}$.
- Calligraphic letters $\mathcal{X}, \mathcal{Y}, ...$ are used for finite sets and $|\mathcal{X}|$ denotes the cardinality of the set $\mathcal{X}$.
- $[i : 2^a] = \{i, i + 1, ..., 2^{\lceil a \rceil}\}$, where $\lceil a \rceil$ is the smallest integer $\geq a$.

Notation for probability and random variables (I)

- The probability of an event $\mathcal{A}$ is $\mathsf{P}(\mathcal{A})$ and the conditional probability of $\mathcal{A}$ given $\mathcal{B}$ is $\mathsf{P}(\mathcal{A}|\mathcal{B})$.
- Uppercase letters, $X, Y, ...$ are used for random variables.
- Random variables may take values from finite sets $\mathcal{X}, \mathcal{Y}, ...$ or from the real line $\mathbb{R}$.
- $X = \emptyset$ means that X is a degenerate random variable (a constant).
- The probability of the event $X \in \mathcal{A}$ is $\mathsf{P}\{X \in \mathcal{A}\}$
- Sequences or column vectors of random variables are $X_i^j = (X_i, X_{i+1}, ...X_j)$. In case $i = 1$ then $X^j = (X_1, X_2, ...X_j)$.

Notation for probability and random variables (II)

- $X^n \sim p(x^n)$ means that $p(x^n)$ is the probability mass function (pmf) of the discrete random vector X^n.
- $X^n \sim f(x^n)$ means that $f(x^n)$ is the probability density function (pdf) of the continuous random vector X^n.
- $(X^n, Y^n) \sim p(x^n, y^n)$ means that $p(x^n, y^n)$ is the joint pmf of X^n and Y^n.
- Given a random variable X, the expected value of a function $g(X)$ is denoted by $\mathsf{E}_X(g(X))$ or simply $\mathsf{E}(g(X))$.

Notation for probability and random variables (and III)

- $X \sim \text{Bern}(p)$ means X is a Bernoulli random variable with parameter $p \in [0, 1]$, i.e., $X = 1$ with probability p and $X = 0$ with probability $1 - p$.
- $X \sim \text{Unif}(\mathcal{A})$ means X is a discrete uniform random variable over the set $\mathcal{A}$.
- $X \sim \text{Unif}[i : j]$ for integers $j > i$ means X is a discrete uniform random variable over $[i : j]$.
- $X \sim \text{Unif}[a, b]$ for $b > a$ means X is a continuous uniform random variable over $[a, b]$.
- $X \sim \text{N}(\mu, \sigma^2)$ means X is a Gaussian random variable with mean μ and variance σ^2.

Common functions

- The function $\log p$ is assumed to be the base 2 logarithm funcion of p.
- The binary entropy function: $H(p) = -p \log p - \bar{p} \log \bar{p}$ for $p \in [0, 1]$.
- The Gaussian capacity: $\mathcal{C}(x) = \frac{1}{2} \log(1 + x)$, for $x \geq 0$.
- $[x]^+ = \max\{x, 0\}$.