# Capacity of Multi-antenna Gaussian Channels (I. E. Telatar, 1999)

#### Mattia Lecci<sup>12</sup>

<sup>1</sup>Università degli Studi di Padova Department of Information Engineering (DEI-UniPD)

<sup>2</sup>Universitat Politècnica de Catalunya Escola Tècnica Superior d'Engenyeria de Telecomunicació de Barcelona (UPC-ETSETB)

January 2018





#### Overview

- Notation used in the course
  - Subsection one
  - Subsection two

Second section

#### Table of Contents

- Notation used in the course
  - Subsection one
  - Subsection two

2 Second section

#### Notation for sets, scalars and vectors

- Lowercase letters,  $x, y, \dots$  are used for constants and values of random variables.
- Sequences or column vectors are  $x_i^j=(x_i,x_{i+1},...x_j)$ . In case i=1 then  $x^j=(x_1,x_2,...x_j)$ .
- Let  $\alpha, \beta \in [0, 1]$ . Then  $\bar{\alpha} = (1 \alpha)$  and  $\alpha * \beta = \alpha \bar{\beta} + \beta \bar{\alpha}$ .
- Calligraphic letters  $\mathcal{X}, \mathcal{Y}, ...$  are used for finite sets and  $|\mathcal{X}|$  denotes the cardinality of the set  $\mathcal{X}$ .
- ullet  $[i:2^a]=\{i,i+1,...,2^{\lceil a \rceil}\}$ , where  $\lceil a \rceil$  is the smallest integer  $\geq a$ .

# Notation for probability and random variables (I)

- The probability of an event  $\mathcal{A}$  is  $\mathsf{P}(\mathcal{A})$  and the conditional probability of  $\mathcal{A}$  given  $\mathcal{B}$  is  $\mathsf{P}(\mathcal{A}|\mathcal{B})$ .
- ullet Uppercase letters,  $X,Y,\ldots$  are used for random variables.
- Random variables may take values from finite sets  $\mathcal{X}, \mathcal{Y}, ...$  or from the real line  $\mathbb{R}$ .
- $X = \emptyset$  means that X is a degenerate random variable (a constant).
- $\bullet$  The probability of the event  $X \in \mathcal{A}$  is  $\mathsf{P}\{X \in \mathcal{A}\}$
- Sequences or column vectors of random variables are  $X_i^j=(X_i,X_{i+1},...X_j).$  In case i=1 then  $X^j=(X_1,X_2,...X_j).$

# Notation for probability and random variables (II)

- $X^n \sim p(x^n)$  means that  $p(x^n)$  is the probability mass function (pmf) of the discrete random vector  $X^n$ .
- $X^n \sim f(x^n)$  means that  $f(x^n)$  is the probability density function (pdf) of the continuous random vector  $X^n$ .
- $(X^n,Y^n)\sim p(x^n,y^n)$  means that  $p(x^n,y^n)$  is the joint pmf of  $X^n$  and  $Y^n$ .
- ullet Given a random variable X, the expected value of a function g(X) is denoted by  $\mathsf{E}_X(g(X))$  or simply  $\mathsf{E}(g(X))$ .

# Notation for probability and random variables (and III)

- $X \sim \text{Bern}(p)$  means X is a Bernoulli random variable with parameter  $p \in [0,1]$ , i.e., X=1 with probability p and X=0 with probability 1-p.
- $X \sim \mathsf{Unif}(\mathcal{A})$  means X is a discrete uniform random variable over the set  $\mathcal{A}$ .
- $X \sim \mathsf{Unif}[i:j]$  for integers j > i means X is a discrete uniform random variable over [i:j].
- $X \sim \mathsf{Unif}[a,b]$  for b > a means X is a continuous uniform random variable over [a,b].
- $X \sim N(\mu, \sigma^2)$  means X is a Gaussian random variable with mean  $\mu$  and variance  $\sigma^2$ .

#### Common functions

- The function  $\log p$  is assumed to be the base 2 logarithm function of p.
- The binary entropy function:  $H(p) = -p \log p \bar{p} \log \bar{p}$  for  $p \in [0,1]$ .
- The Gaussian capacity:  $C(x) = \frac{1}{2} \log(1+x)$ , for  $x \ge 0$ .
- $[x]^+ = \max\{x, 0\}.$

This is a text in second frame. For the sake of showing an example.

• Text visible on slide 1

This is a text in second frame. For the sake of showing an example.

- Text visible on slide 1
- Text visible on slide 2

This is a text in second frame. For the sake of showing an example.

- Text visible on slide 1
- Text visible on slide 2
- Text visible on slides 3

This is a text in second frame. For the sake of showing an example.

- Text visible on slide 1
- Text visible on slide 2
- Text visible on slide 4

#### Table of Contents

- Notation used in the course
  - Subsection one
  - Subsection two

Second section

Second section January 2018 10 / 12

In this slide, some important text will be highlighted beause it's important. Please, don't abuse it.

#### Remark

Sample text

#### Important theorem

Sample text in red box

#### Examples

Sample text in green box. "Examples" is fixed as block title.

Second section January 2018 11 / 12

#### Two-column slide

This is a text in first column.

$$E = mc^2$$

- First item
- Second item

This text will be in the second column and on a second thought this is a nice looking layout in some cases.

Second section January 2018 12 / 12