***W*ireless Communications**

# Simulation of Multipath Fading Channels

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## Abstract

# 1 Introduction

The report is structured as follow: in Section 2 I will present the techninical aspects of the project, starting from Subsection 2.1 which delineates the main objectives, the in Subsection 2.2 the mathematical models of all the implemented simulators will be carefully described and in Subsection 2.3 an idea of the code structure will be presented. Finally, in Subsection 2.4 I will briefly talk about a few complications encountered while completing this project. In Section 3, then, the results will be presented and lastly in Section 4 the conclusions will be drawn.

# 2 Technical Approach

In this Section all the technical aspects of the project will be present.

## 2.1 Objectives

The main objective of the project is to evaluate the performance of different types of wireless channel simulators. In particular, I'm interestd, here, in simulating a Rayleigh fading channel, namely a channel where no Line-Of-Sight (LOS) component is present. I will show you both statistical and performance results for all of the 8 simulators implemented, comparing them to one another.

## 2.2 Mathematical models used

Almost all of the references start by intrducing the ideal statistical properties that a Rayleigh channel should have, obtained for the classical model of such channel. This model is presented in [Cla68] and it is often referred to as Clarke's 2D isotropic (both scattering and antenna gain) Rayleigh fading model, given by

$$X(t) = \frac{1}{\sqrt{N}} \sum_{n=1}^{N} e^{j(2\pi f_d \cos\alpha_n t + \phi_n)} \tag{1}$$

where N is the number of propagation paths, $f_d$ is the maximum Doppler frequency, $\alpha_n$ is the angle of arrival of the $n$-th ray and $\phi_n$ its initial phase. Both $\alpha_n$ and $\phi_n$ are uniformly distributed in $(-\pi, \pi]$ for all $n$ and they are mutually independent. Since in general many rays reach the receiver at the same time, the *Central Limit Theorem* (*CLT*) justifies the approximation of the channel to a Complex Normal distribution. Actually the independence of real and imaginary part is not trivial, but it will not be further clarified here. From this, we know that the magnitude of a Complex Normal random variable yields a Rayleigh distributed one (since it's equivalent to the euclidean norm of a 2D Gaussian random vector) and, by symmetry of the distribution (given by the independence of real and imaginary part of the Complex Gaussian), the phase is uniformly distributed in $(-\pi, \pi]$. In formulas,

$$f_{|X|}(x) = \frac{x}{2}\, e^{-x^2}, \quad x \geq 0 \tag{2}$$

$$f_{\theta}(x) = \frac{1}{2\pi}, \quad x \in (-\pi, \pi] \tag{3}$$

As $N$ tends to infinity, defining $X(t) = X_c(t) + jX_s(t)$ it is possible to prove the following equations:

$$R_{X_cX_c}(\tau) = E[X_c(t)X_c(t-\tau)] = \frac{1}{2} J_0(2\pi f_d \tau) \tag{4a}$$

$$R_{X_sX_s}(\tau) = \frac{1}{2} J_0(2\pi f_d \tau) \tag{4b}$$

$$R_{X_cX_s}(\tau) = R_{X_sX_c}(\tau) = 0 \tag{4c}$$

$$R_X(\tau) = E[X(t)X^*(t-\tau)] = J_0(2\pi f_d \tau) + j0 \tag{4d}$$

$$R_{|X|^2}(\tau) = 1 + J_0^2(2\pi f_d \tau) \tag{4e}$$

Where $J_0(x)$ is the zero-order Bessel function of the first kind, defined as

$$J_0(x) = \frac{1}{\pi} \int_0^{\pi} \cos(x\ \cos(\theta))\ d\theta \tag{5}$$

As you can see, all of these correlations are obtained by a *Wide Sense Stationary* (*WSS*) process, since they only depend on the variable $\tau$.

### 2.3 Scenario

### 2.4 Complications found

## 3 Results

## 4 Conclusions

## References


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