*W*ireless Communications

Simulation of Multipath Fading Channels

Author:

Mattia Lecci *(1153428)*

Abstract

1 Introduction

The report is structured as follow: in Section 2 I will present the technical aspects of the project, starting from Subsection 2.1 which delineates the main objectives, the in Subsection 2.2 the mathematical models of all the implemented simulators will be carefully described and in Subsection 2.3 an idea of the code structure will be presented. Finally, in Subsection 2.4 I will briefly talk about a few complications encountered while completing this project. In Section 3, then, the results will be presented and lastly in Section 4 the conclusions will be drawn.

2 Technical Approach

In this Section all the technical aspects of the project will be presented. A quick explanation of the various simulators implemented will follow in Subsection 2.2, following the history of the subject. That is the reason why the order of the citations will be different from the one proposed. Note that all of the equations have been slightly modified in order to obtain unit power fading channels and to unify the notation.

2.1 Objectives

The main objective of the project is to evaluate the performance of different types of wireless channel simulators. In particular, I'm interested, here, in simulating a Rayleigh fading channel, namely a channel where no Line-Of-Sight (LOS) component is present. I will show both statistical and performance results for all of the 8 simulators implemented, comparing them to one another.

2.2 Mathematical models used

Almost all of the references start by introducing the ideal statistical properties that a Rayleigh channel should have, obtained for the classical model of such channel. This model is presented in [Cla68] and it is often referred to as **Clarke**'s 2D isotropic (both scattering and antenna gain) Rayleigh fading model, given by

$$X(t) = \frac{1}{\sqrt{N}} \sum_{n=1}^{N} e^{j(2\pi f_d \cos\alpha_n t + \phi_n)} \tag{1}$$

where N is the number of propagation paths, f_d is the maximum Doppler frequency, α_n is the angle of arrival of the n-th ray and ϕ_n its initial phase. Both α_n and ϕ_n are uniformly distributed in $(-\pi,\pi]$ for all n and they are mutually independent, fo a total of $2\times N$ random variables. Since in general many rays reach the receiver at the same time, the *Central Limit Theorem* (*CLT*) justifies the approximation of the channel to a Complex Normal ($\mathscr{CN}$) distribution. Actually the independence of real and imaginary part is not trivial, but it will not be further clarified here. From this, we know that the magnitude of a Complex Normal random variable yields a Rayleigh distributed one (since it's equivalent to the euclidean norm of a 2D Gaussian random vector) and, by symmetry of the distribution (given by the independence of real and imaginary part of the Complex Gaussian), the phase is uniformly distributed in $(-\pi,\pi]$. In formulas,

$$f_{|X|}(x) = 2x\, e^{-x^2}, \quad x \geq 0 \tag{2a}$$

$$f_{\theta}(x) = \frac{1}{2\pi}, \quad x \in (-\pi,\pi] \tag{2b}$$

As N tends to infinity, defining $X(t) = X_c(t) + jX_s(t)$ it is possible to prove the following correlations:

$$R_{X_cX_c}(\tau) = E[X_c(t)X_c(t-\tau)] = \frac{1}{2}J_0(2\pi f_d \tau) \tag{3a}$$

$$R_{X_sX_s}(\tau) = \frac{1}{2}J_0(2\pi f_d \tau) \tag{3b}$$

$$R_{X_cX_s}(\tau) = R_{X_sX_c}(\tau) = 0 \tag{3c}$$

$$R_X(\tau) = E[X(t)X^*(t-\tau)] = J_0(2\pi f_d \tau) + j0 \tag{3d}$$

$$R_{|X|^2}(\tau) = 1 + J_0^2(2\pi f_d \tau) \tag{3e}$$

Where $J_0(x)$ is the zero-order Bessel function of the first kind, defined as

$$J_0(x) = \frac{1}{\pi}\int_0^{\pi} \cos(x\ \cos\theta)\,\mathrm{d}\theta \tag{4}$$

As you can see, all of these correlations are obtained from a *Wide Sense Stationary* (*WSS*) process, since they only depend on the variable τ.

Another two very interesting properties which can be extracted from Clarke's model are called *Level Crossing Rate* (*LCR*) and *Average Fade Duration* (*AFD*). They both characterize what happen at certain thresholds for the wireless channel and it's generally a higher order behavior: while LCR determines the rate at which the envelope crosses a thresholds with positive slope, AFD indicates for how long the channel will stay below the given threshold in average. They are clearly important parameters to consider when designing a wireless system and the appropriate channel coding. Ideally, their formulas are respectively:

$$L_{|X|}(\lambda) = \sqrt{2\pi} f_d \lambda e^{-\lambda^2} \tag{5}$$

$$T_{|X|}(\lambda) = \frac{e^{\lambda^2} - 1}{\sqrt{2\pi} f_d \lambda} \tag{6}$$

where λ is the normalized fading envelope threshold defined as $\lambda = |X_{thr}|/|X_{rms}|$. Since we are dealing with unit power simulations, then, λ is simply equal to the threshold itself.

Since Clarke's model deals with multiple complex sinusoids and random variables, which are both computationally expansive to calculate, **Jakes** proposed in [Jak74] its well known simplification of such model, which basically became a standard for wireless channel simulation for over 20 years. In order to cut down on computational complexity he makes some assumptions: instead of being random variables, he forces $\alpha_n = \frac{2\pi n}{N}$ and correlates ϕ_n in quadruplets in order to obtain the following simplified model:

$$X_c(t) = \sqrt{\frac{2}{N}}\left[\cos(2\pi f_d t) + \sum_{n=1}^{M} 2\cos\left(\frac{\pi n}{M}\right)\cos(2\pi f_d \cos\alpha_n\, t)\right] \tag{7a}$$

$$X_s(t) = \sqrt{\frac{2}{N}}\left[\cos(2\pi f_d t) + \sum_{n=1}^{M} 2\sin\left(\frac{\pi n}{M}\right)\cos(2\pi f_d \cos\alpha_n\, t)\right] \tag{7b}$$

You can see that the model is now fully deterministic and there are about a quarter of the oscillators of the corresponding Clarke's model. In fact, by defining $N = 4M + 2$, there are only $M + 1$ low frequency oscillators needed. Note that the directions with maximum Doppler spread are forcefully kept.

This, though, comes at a price: **Pop** and **Beaulieu** state in [A1] that Jakes' simulator is not even stationary and yields poor higher order statistics. To overcome this phenomena a simple modification is proposed: the addition of an initial random phase to the low frequency oscillators. From Eqs. 7a,7b the oscillator terms become: $\cos(2\pi f_d t + \phi_0)$ and $\cos(2\pi f_d \cos\alpha_n\, t + \phi_n)$, where ϕ_n are mutually independent uniform random variables in $(-\pi, \pi]$.
Now, a small addition that I did with respect to the original paper is the addition of the multichannel support. This may be useful is different interesting scenarios: multiple independent channels are usually used to model a frequency selective fading and MIMO systems. I, instead, used this feature to estimate all the statistics of the simulators without relying on any ergodicity. For the case of the Pop-Beaulieu simulator, this addition is very simple: calling $X_k(t)$ the k-th channel ($k = 1, 2, ..., K$), we just need to generate $K \times (M+1)$ random variables $\phi_{k,n}$.

In 2002, **Zheng** and **Xiao** [C2]. In this paper a model for multichannel simulation is directly given, with the following real and imaginary components:

$$X_{k,c}(t) = \frac{1}{\sqrt{M}} \sum_{n=1}^{M} \cos\left(2\pi f_d t \cos\left(\frac{2\pi n - \pi + \theta_k}{4M}\right) + \phi_{n,k}^{(c)}\right) \tag{8a}$$

$$X_{k,s}(t) = \frac{1}{\sqrt{M}} \sum_{n=1}^{M} \cos\left(2\pi f_d t \sin\left(\frac{2\pi n - \pi + \theta_k}{4M}\right) + \phi_{n,k}^{(s)}\right) \tag{8b}$$

where θ_k, $\phi_{n,k}^{(c)}$ and $\phi_{n,k}^{(s)}$ are mutually independent random variables uniformly distributed in $(-\pi, \pi]$, for a total of $K \times (2M+1)$ random variables. This ensures that all the channels have the same statistical properties while being uncorrelated. Note that with respect to the Pop-Beaulieu model it adds randomness on the angle of arrival α_n and uncorrelates the initial phases of real and imaginary components, while not retaining the angles with maximum Doppler spread, having then $N = 4M$. This means more than double the quantity of random variables required to perform the simulation.

Later in 2002, **Li** and **Huang** [C1] proposed another approach to a multichannel simulator. Instead of randomizing the directions of arrival, they follow a deterministic approach, similar to Jakes'. Defining $\alpha_{n,k} = \frac{2\pi n}{N} + \frac{2\pi k}{NK} + \alpha_{0,0}$ for $n = 0, ..., N-1$ and $k = 0, ..., K-1$, the formulas for the k-th ray are:

$$X_{k,c}(t) = \frac{1}{\sqrt{M}} \sum_{n=0}^{M-1} \cos\left(2\pi f_d t \cos\alpha_{n,k} + \phi_{n,k}^{(c)}\right) \tag{9a}$$

$$X_{k,s}(t) = \frac{1}{\sqrt{M}} \sum_{n=0}^{M-1} \sin\left(2\pi f_d t \sin\alpha_{n,k} + \phi_{n,k}^{(s)}\right) \tag{9b}$$

as usual $\phi_{n,k}^{(c)}$ and $\phi_{n,k}^{(s)}$ are mutually independent random variables uniformly distributed in $(-\pi, \pi]$, for a total of $2 \times M \times K$ random variables. It is highlighted the fact that any combination of sine and cosine functions will not affect the actual statistics of the channels. The choice $\alpha_{0,0}$ is suggested by the authors to be in $\left(0, \frac{2\pi}{NK}\right) \setminus \left\{\frac{\pi}{NK}\right\}$. In the end I decided to use their same initial angle, meaning $\alpha_{0,0} = \frac{\pi}{2NK}$. The paper then also tries to reduce the high cost of calculating trigonometric functions by proposing different approximations and comparing then the results. I decided, though, to not implement this further in-depth analysis.

Going on to 2003, **Zheng** and **Xiao** [A2] further enhance their simulator by adding a random gain to each oscillator:

$$X_{k,c}(t) = \frac{1}{\sqrt{M}} \sum_{n=1}^{M} \cos(\psi_{n,k}) \cos\left(2\pi f_d t \cos\left(\frac{2\pi n - \pi + \theta_k}{4M}\right) + \phi_k\right) \tag{10a}$$

$$X_{k,s}(t) = \frac{1}{\sqrt{M}} \sum_{n=1}^{M} \sin(\psi_{n,k}) \cos\left(2\pi f_d t \cos\left(\frac{2\pi n - \pi + \theta_k}{4M}\right) + \phi_k\right) \tag{10b}$$

Note that the randomization of the angle of arrival is just the same as their 2002 paper but there are a couple of differences: the initial phase of the oscillators is now constant for all of the oscillators of the same channel (and equal for real and imaginary parts, similarly to the Pop-Beaulieu model), whereas the difference between oscillators of the same channel is given by the random amplitude, which, even though it's different, it is correlated between real and imaginary components ($\psi_{n,k}$ is the same for both of them). A total of $K \times (M+2)$ is needed.

In 2006, the most recent paper about *Sum of Sinusoids* simulator that I considered was written by **Xiao**, **Zheng** and **Beaulieu** [B1]. Once again, the simulator is a *SoS* and introduces slight differences with respect to the others

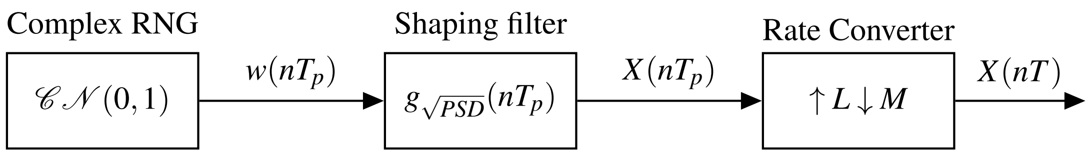
$$X_{k,c}(t) = \frac{1}{\sqrt{M}} \sum_{n=1}^{M} \cos\left(2\pi f_d t \cos\left(\frac{2\pi n + \theta_{n,k}}{M}\right) + \phi_{n,k}\right) \tag{11a}$$

$$X_{k,s}(t) = \frac{1}{\sqrt{M}} \sum_{n=1}^{M} \sin\left(2\pi f_d t \cos\left(\frac{2\pi n + \theta_{n,k}}{M}\right) + \phi_{n,k}\right) \tag{11b}$$

The main differences with previous models are: no random amplitude for the oscillators is present, both θ and ϕ are independent between channels and oscillators (with a total number of $2 \times K \times M$ random variables required) although they are the same for real and imaginary components, the angle of arrival $\alpha_{n,k}$ has changed using only M instead of $4M$ as the denominator. Similarly to the 2 previous papers from Zheng and Xiao, the angle of arrival is randomized but within sectors, in order to force a "more uniform" distribution, particularly for small values of M, with respect to the pure uniform distribution of the original Clarke's model.

As the last one, I decided to keep the **Komninakis** simulator [A3] since it's completely different from all the others, even though it's actually from 2003. In fact, while all the previous ones are based on the *Sum of Sinusoids* method, this one works based on a *filtering* method.

The basic idea is the following: starting from a white process $w(t)$ (which has constant *Power Spectral Density*), in principle it is possible to obtain a stochastic process $X(t)$ with arbitrary PSD filtering it through an *LTI* system with frequency response $G_{\sqrt{PSD}}(f)$ since $P_X(f) = P_w |G_{\sqrt{PSD}}(f)|^2$, where the magnitude of



the frequency response of the filter equals the square root of the desired PSD. In practice, though, it is not trivial to create a filter with arbitrary shape. In its paper, Komninakis proposes a way of approximating the magnitude of an IIR filter to a desired shape (in particular the classical PSD given by Clarke's model).
There is one more problem to solve: particularly when modeling bit-level simulations, the sampling period T may be much smaller than the *coherence period* usually defined as $T_{coh} = \frac{1}{f_d}$. Equivalently, it means that the sampling frequency $F = \frac{1}{T} \gg f_d$, thus a very narrowband filter $g_{\sqrt{PSD}}$ would be required in order to correctly produce the desired output. It is widely known, though, that narrowband filters tend to require very high order which mean lots of calculations per sample to be done, very long transients and possible numerical instabilities given by the fact that poles have to be very close to the unit circle. This is definitely not the way to proceed. It is much wiser to compute only one or two filters with fixed values for the product $f_d T_p$ in order to balance order, stability and complexity, considering that an interpolation (or in general a rate conversion) will have to be done later. Note that the most correct way to proceed with the interpolation would be an up-sampling with zero padding followed by a low pass filter (either using low order IIR filter or a polyphase FIR filter). If the product $f_d T$ is small, though, it means that the channel will be slowly changing with respect to the chosen sampling period. This would allow us to use a simpler *spline* interpolation.
The big disadvantage of this simulator, then, becomes the high number of random variables needed. This depends, though, on the value of the product $f_d T_p$ for the reference shaping filter as well as the the actual product $f_d T$ needed.

2.3 Scenario and Implementation

The project has been implemented using MATLAB® R2017a using the student license provided by the department.

A separate function for each simulator has been created as well as the interface `createChannel` to create a realization of a channel with all different kinds of parameters, such as f_d, T, duration (either in number of samples, seconds or multiples of T_{coh}), type of simulator, number of oscillators (i.e. the parameter $M \approx N/4$ for most simulators), number of independent channels and the interpolation method for the Komninakis simulator (either by `filter`, `spline`, `pchip` or `linear` interpolation). For what concerns the Komninakis simulator, shaping filter's coefficients were taken from [CB02, p. 317] which assumes $f_d T_p = 0.1$.

Separate functions for the computation of all the statistics required have been done and again all of them have multiple optional parameters to be set. Note that when talking about correlation I mean the statistical one. Hence, I also created a function that computes an estimate of the statistical correlations (all of those described in Eqs. 3) given a number of independent channels (which can be easily created exploiting the Name-Value parameter `NChannels` from the function `createChannel`). The only exception to this rule is Jakes' simulator since it's fully deterministic. The standard temporal correlation has been used in this specific case.

More functions were created in order to properly and simply plot all of the numerous statistics and to ensure a homogeneous look. Finally, different `mains` were created to obtain different types of results in an orderly manner and the possibility to store and load statistics has been added in order to promptly plot previously computed statistics.

2.4 Complications found

No major complications were found while doing this project. The only thing to be careful about was the different notation and normalization used in the various papers. Towards the end it has also been non trivial how to choose all of the numerous parameters for the simulation in order to properly highlight strengths and weaknesses of all the different models and how certain parameters affect the results.

3 Results

4 Conclusions

References

[A1] M. F. Pop and N. C. Beaulieu. "Limitations of Sum-of-Sinusoids Fading Channel Simulators". In: *IEEE Transactions on Communications* 49.4 (Apr. 2001).

[A2] Y. R. Zheng and C. Xiao. "Simulation Models With Correct Statistical Properties for Rayleigh Fading Channels". In: *IEEE Transactions on Communications* 51.6 (June 2003).

[A3] C. Komninakis. "A Fast and Accurate Rayleigh Fading Simulator". In: *IEEE GLOBECOM* (Dec. 2003).

[B1] C. Xiao, Y. R. Zheng, and N. C. Beaulieu. "Novel Sum-of-Sinusoids Simulation Models for Rayleigh and Rician Fading Channels". In: *IEEE Transactions on Wireless Communications* 5.12 (Dec. 2006).

[C1] Y. Li and X. Huang. “The Simulation of Independent Rayleigh Faders”. In: *IEEE Transactions on Communications* 50.7 (Sept. 2002).

[C2] Y. R. Zheng and C. Xiao. “Improved Models for the Generation of Multiple Uncorrelated Rayleigh Fading Waveforms”. In: *IEEE Communications Letters* 6.6 (June 2002).

[CB02] G. Cherubini and N. Benvenuto. *Algorithms for Communications Systems and their Applications*. John Wiley & Sons Inc., 2002. ISBN: 0-470-84389-6.

[Cla68] R. H. Clarke. “A statistical theory of mobile-radio reception”. In: *Bell Syst. Tech J.* (July 1968), pp. 957–1000.

[Jak74] W. C. Jakes. *Microwave Mobile Communications*. Reissued in 1994. Piscataway, NJ: IEEE Press, 1974.