Digital Epidemiology

Mobility impact on epidemics spread

Mattia Mazzoli - UniTo

Human mobility impact on epidemics

Basics of epidemic spreading

- A first look at the SIR model
- Epidemic threshold in homogeneous mixing: the reproductive number
- Epidemic threshold in homogeneous networks

Spatial invasion: Basic principles of spatial transmission

- Probability of invasion
- Country level arrival times: the hidden geometry of epidemic spread
- Epidemic threshold in heterogeneous networks
- Pathways of spatial invasion

Interventions

- Travel bans: slowing-down the spatial spread

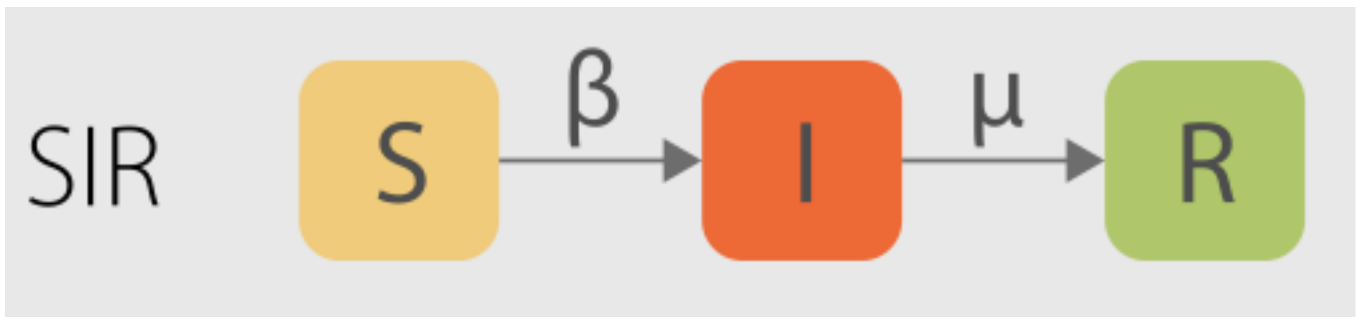
Playing with epidemics

Before going into this, let's first see what happens without mobility

Go to http://35.161.88.15/interactive/outbreak/
Let's play Outbreak!

Play with the simulator parameters and observe the interplay of transmissibility, contacts and finally add mobility!

- In the *Probabilistic infection* section:
  there is no mobility, agents interact with their grid neighbors
- In the *Travel* section:
  play with the travel radius parameter, check the patterns.
  See anything familiar?
- In the *Number of encounters* section, try the following combo:
  - low transmission rate + high n of encounters
  - medium transmission rate + low n of encounters



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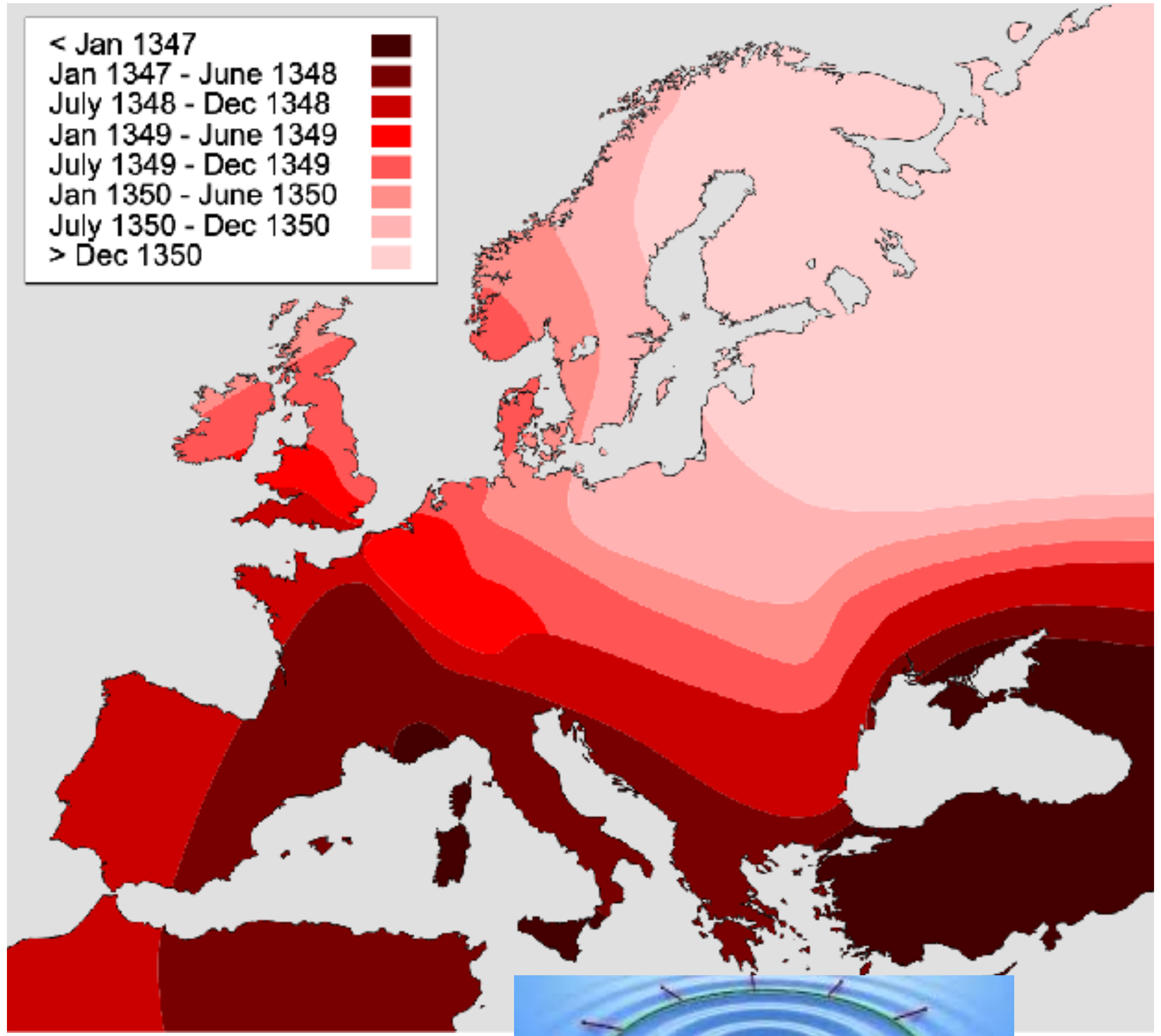
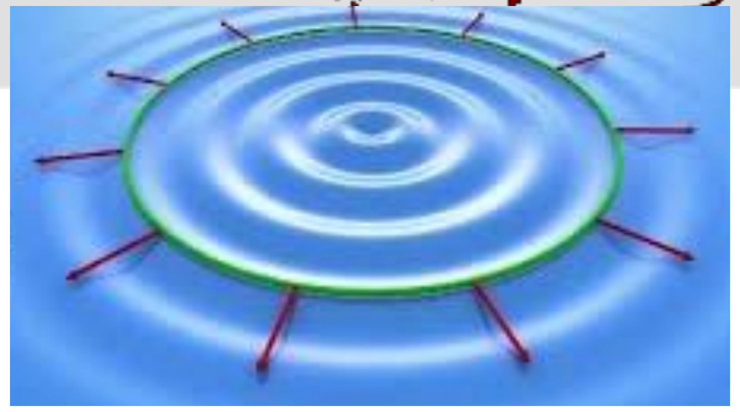
The spread of plague ("the Black Death") across Europe in the 14th century.





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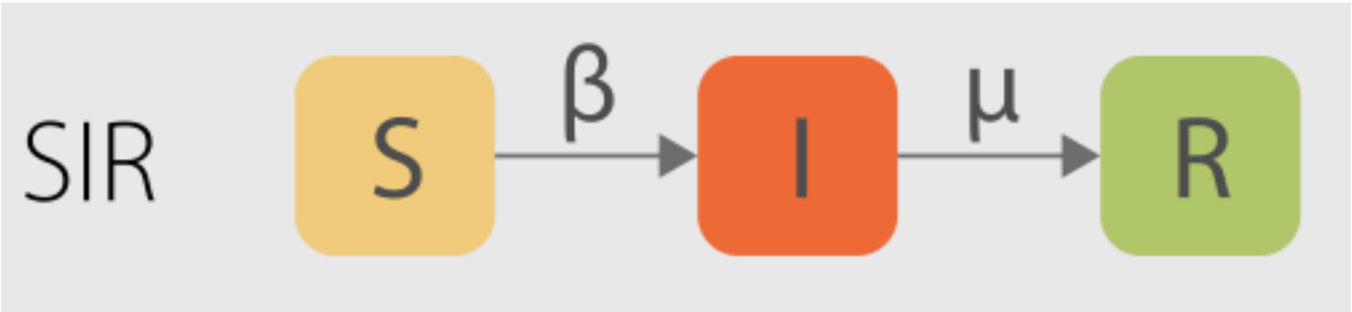


$$R_0, \beta, \mu, \langle k \rangle$$

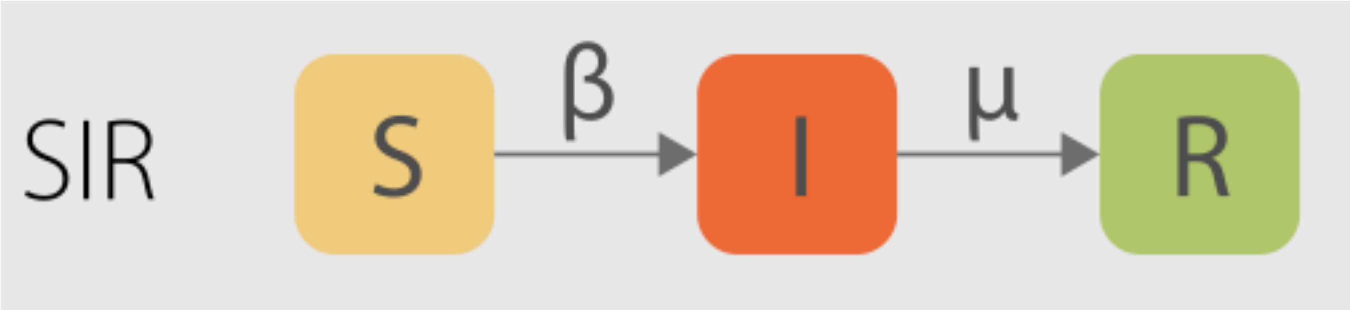
Pastor-Satorras, Romualdo, et al. "Epidemic processes in complex networks." *Reviews of modern physics* 87.3 (2015): 925.

Reproductive number

$R_0 =$ Average number of individual that an infector will infect during their infectious period

Naive population assumption: all individuals are susceptible
Valid only at the early stage of the epidemic

$$\begin{cases} \dfrac{dS}{dt} = -\beta \dfrac{SI}{N} & \text{Susceptible} \\ \dfrac{dI}{dt} = \beta \dfrac{SI}{N} - \mu I & \text{Infected} \\ \dfrac{dR}{dt} = \mu I & \text{Recovered} \end{cases}$$



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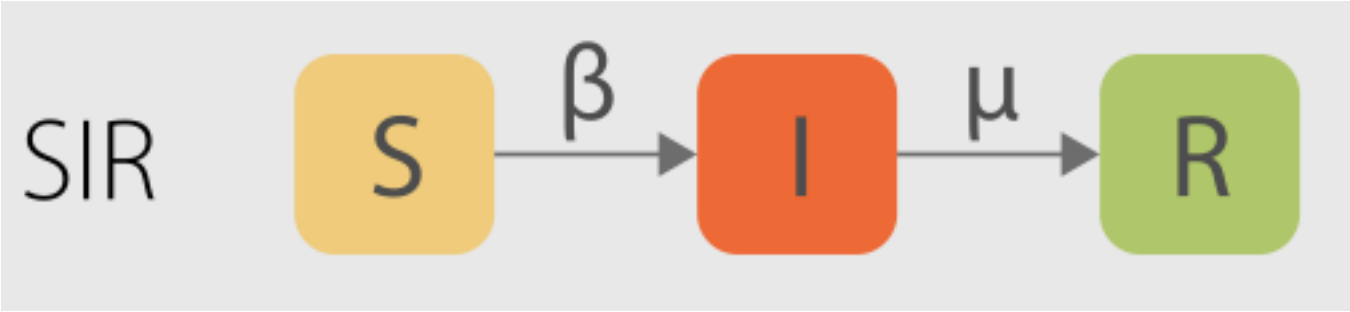
Early stage approximation

$$\begin{cases} t \to 0 \\ I(0) \ll N \\ S(0) \simeq N \end{cases}$$

$$\frac{dI}{dt} \simeq (\beta - \mu) I$$

$$I(t) \simeq I_0 e^{(\beta - \mu)t}$$



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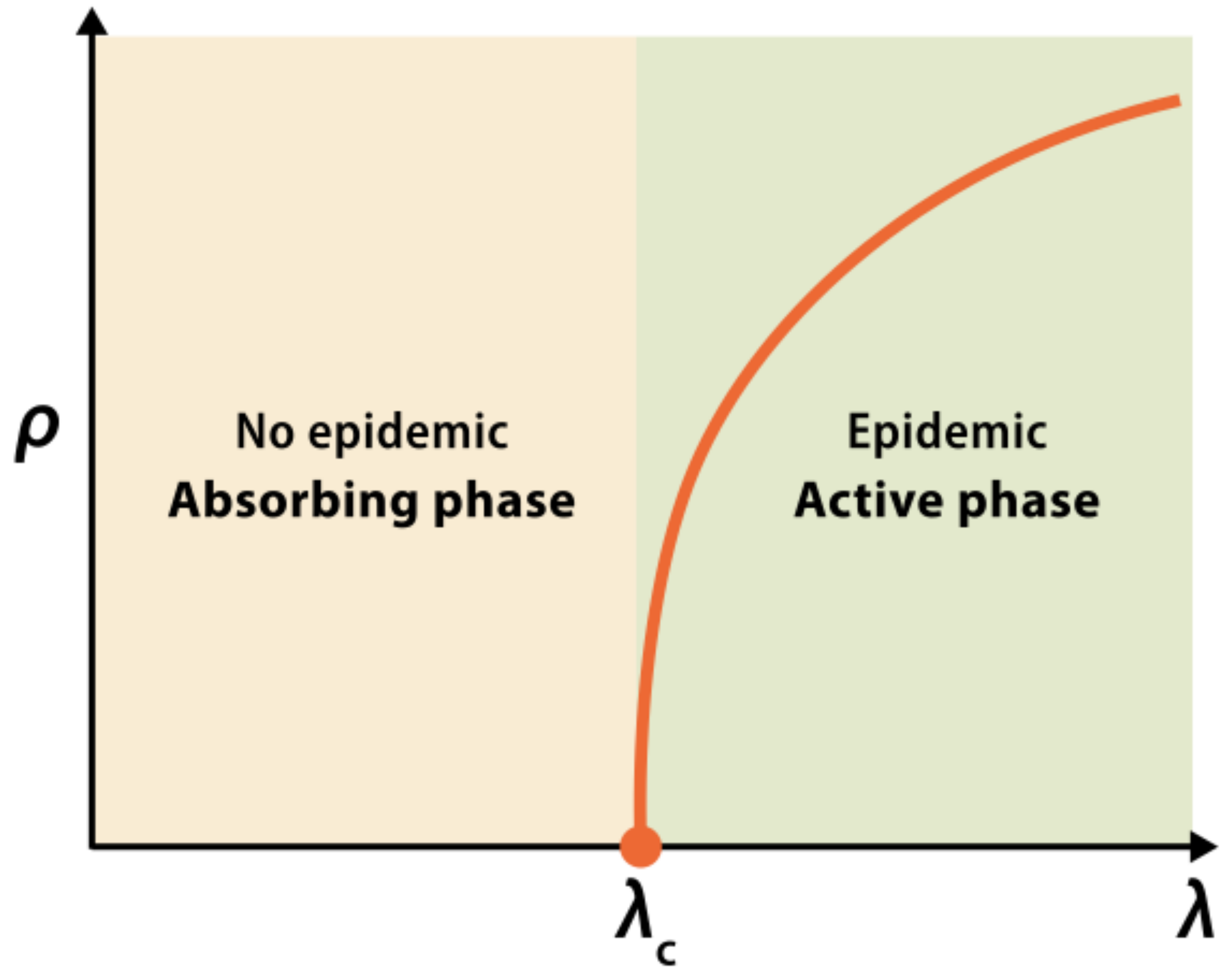
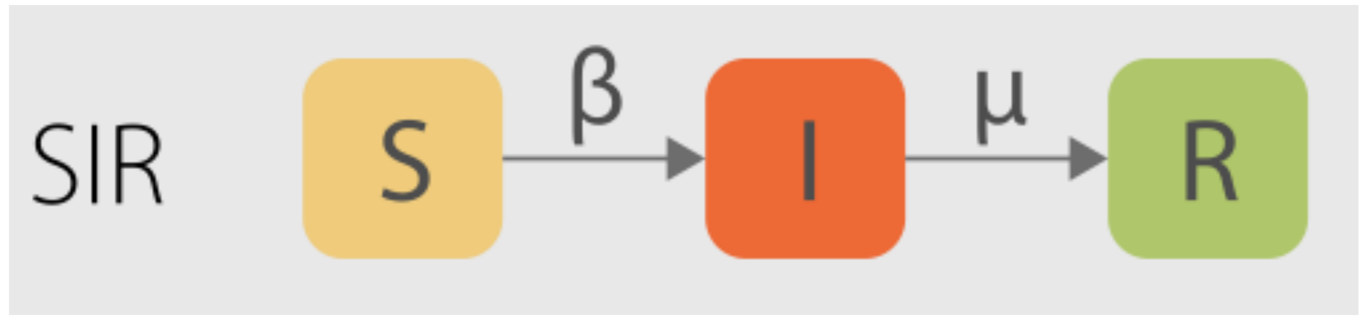
$$\beta - \mu > 0$$

$$R_0 = \frac{\beta}{\mu} > 1$$





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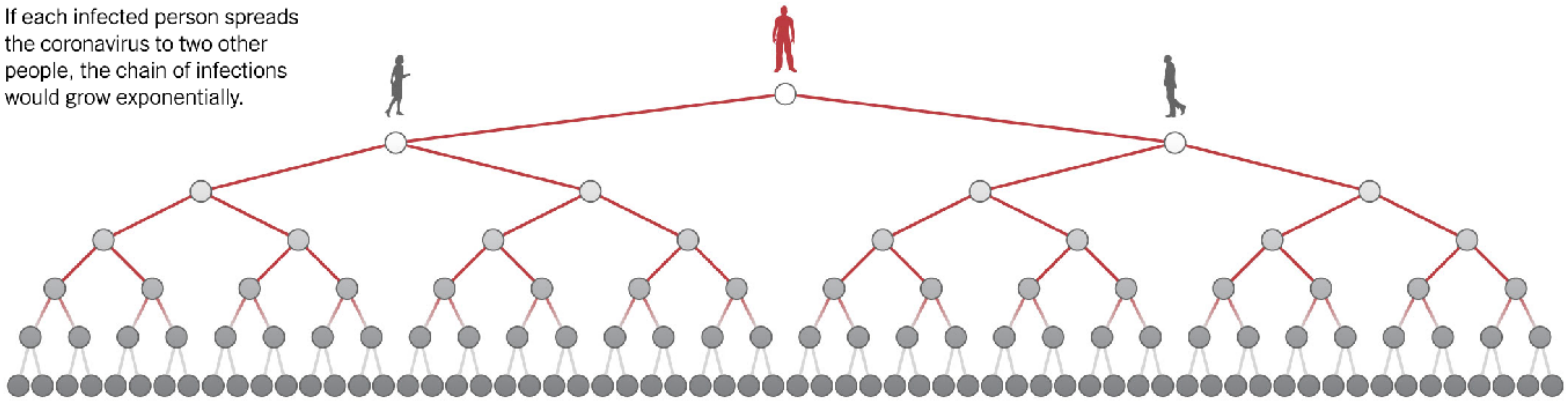
Reproductive number

$R_0 =$ Average number of individuals that an infector will infect during their infectious period in a fully naive population

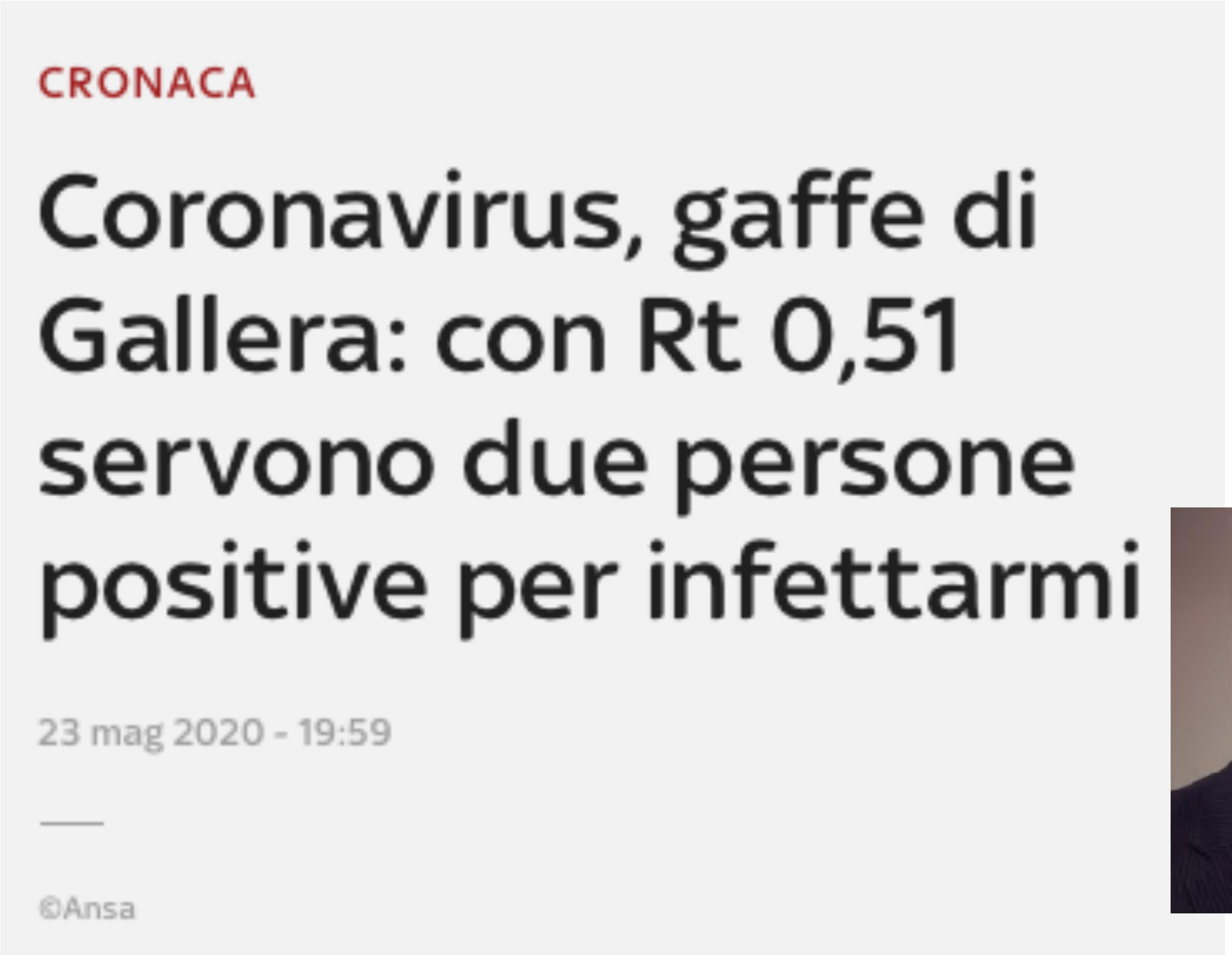
$$R_0 = \frac{\beta}{\mu} > 1$$



$R_t =$ Effective reproductive number: average number of individuals that an infector will infect during their infectious period when the population is no longer naive

Effective reproductive number

Common pitfalls…

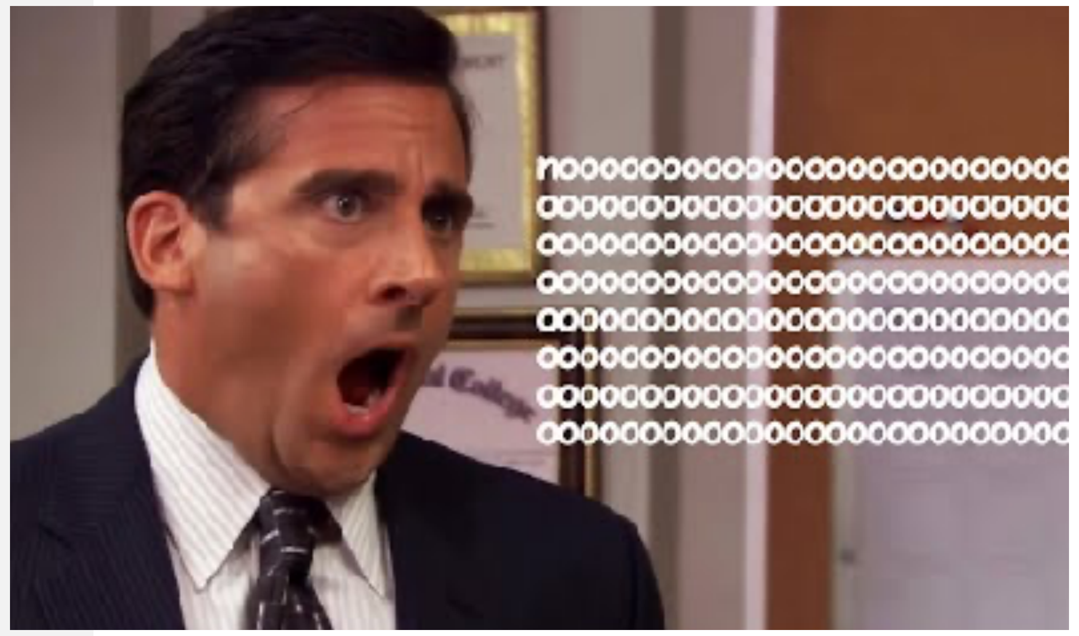


CRONACA

Coronavirus, gaffe di Gallera: con Rt 0,51 servono due persone positive per infettarmi

23 mag 2020 - 19:59

©Ansa



Epidemic threshold with homogeneous mixing

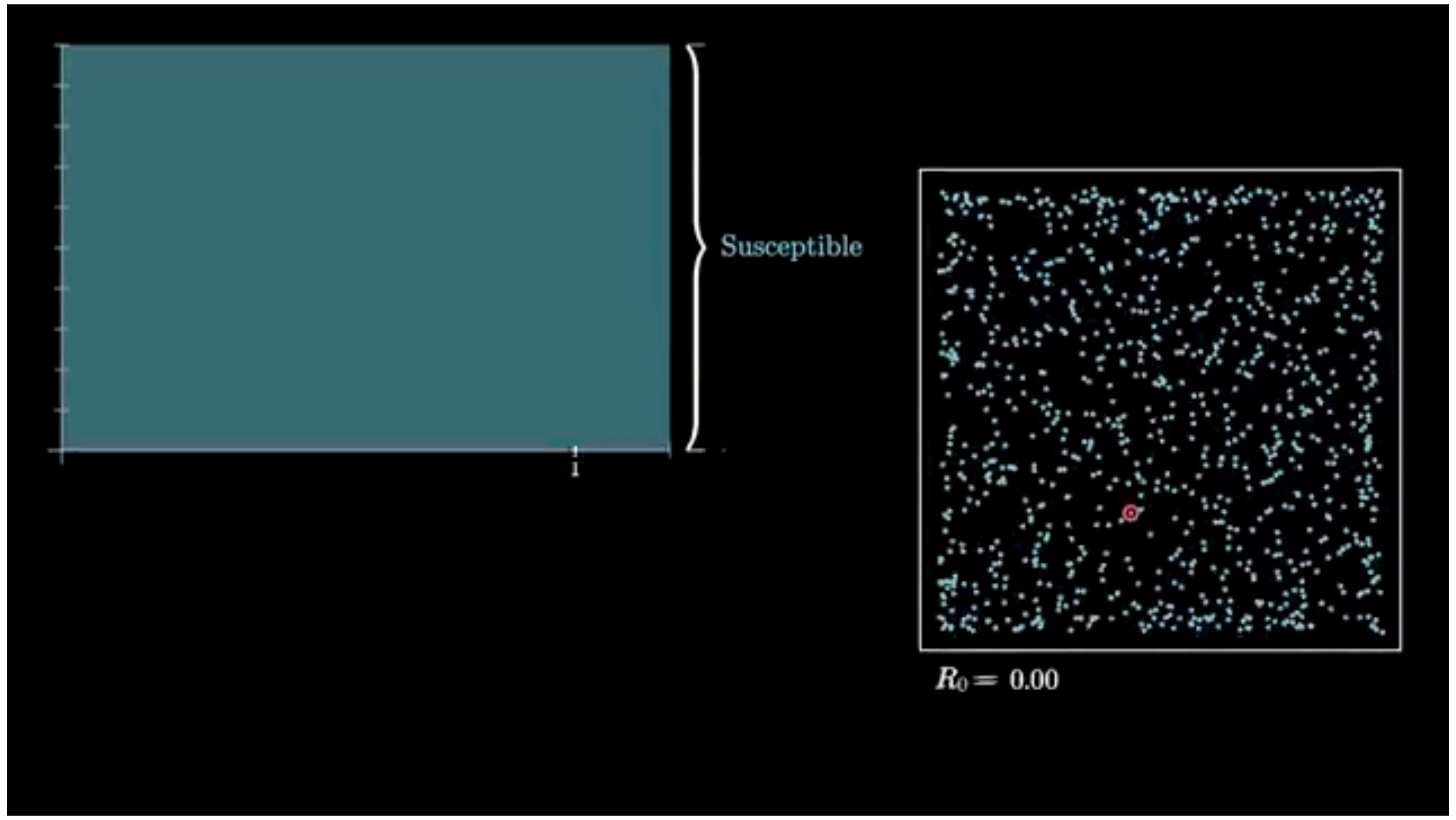
Homogeneous mixing assumption:
all individuals behave the same and have random contacts

~ phase transition as in statistical physics

control parameter (spreading rate) $\lambda = \frac{\beta}{\mu}$

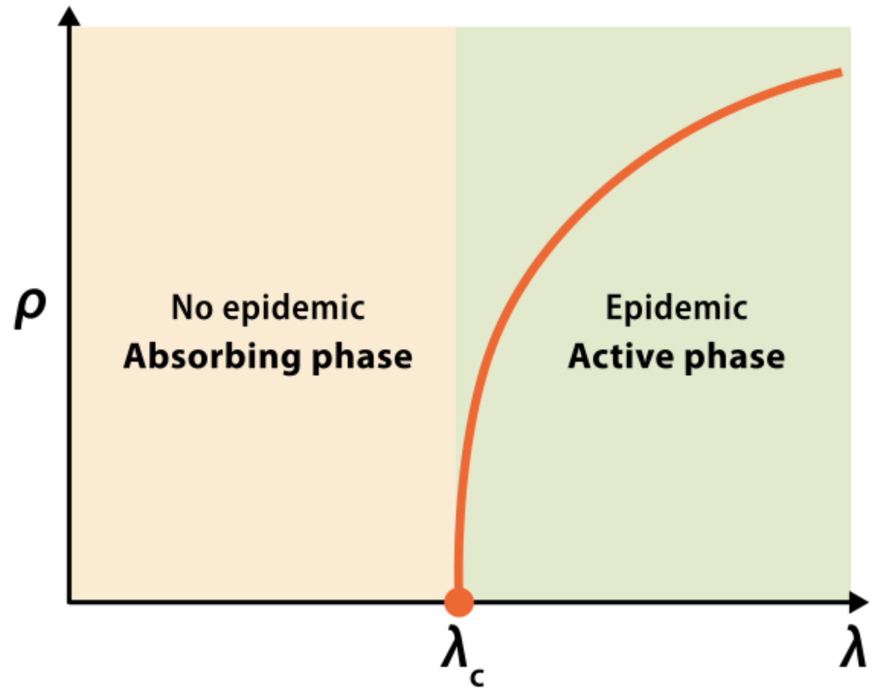
order parameter (fraction of infected) ρ

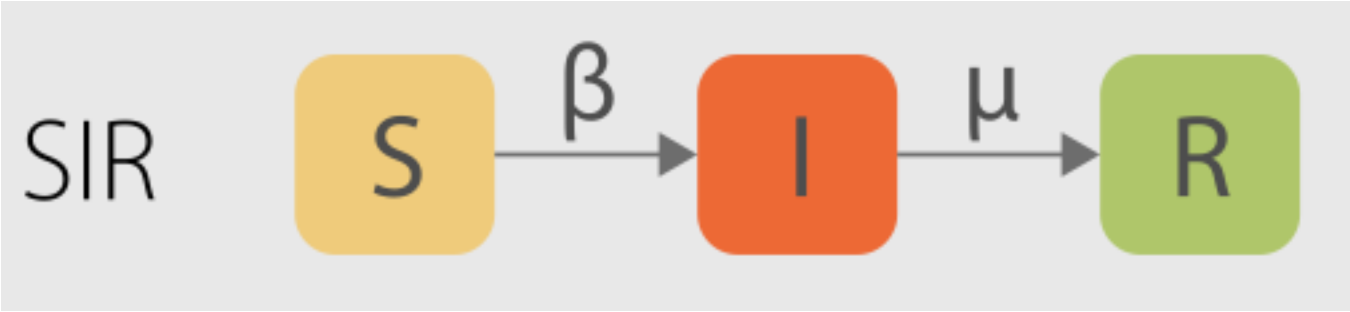
There exists a critical threshold (Epidemic threshold) λ_c
for the control parameter above which the epidemic spreads



Homogeneous mixing $R_0 = \frac{\beta}{\mu}$

If $R_0 > 1$
The epidemic spreads, otherwise it fades





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Epidemic threshold in homogeneous networks

Homogeneous networks:
Each individual has its own *k* contacts

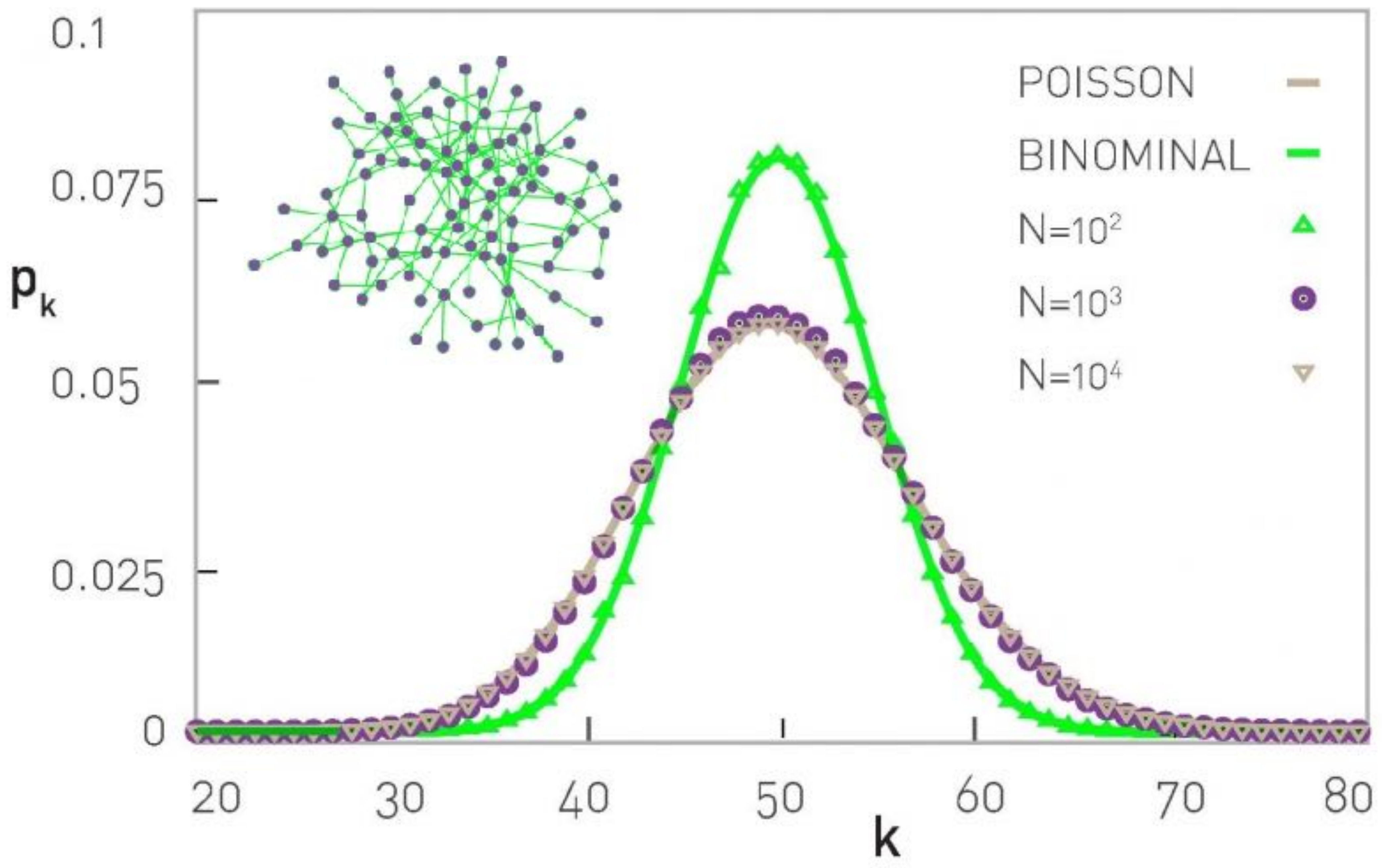
The control parameter is $\lambda = \frac{\beta}{\mu}$

$$R_0 = \langle k \rangle \frac{\beta}{\mu}$$

$\langle k \rangle$ average contacts per individual

If $R_0 > 1$
The epidemic spreads, otherwise it fades

so $\lambda_c = \frac{1}{\langle k \rangle}$



… and this is why when playing Outbreak, balancing contacts and transmissibility has similar outcomes

Barabási, Albert-László. "Network science book." *Network Science* 625 (2014)
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The impact of nowadays mobility

14th century

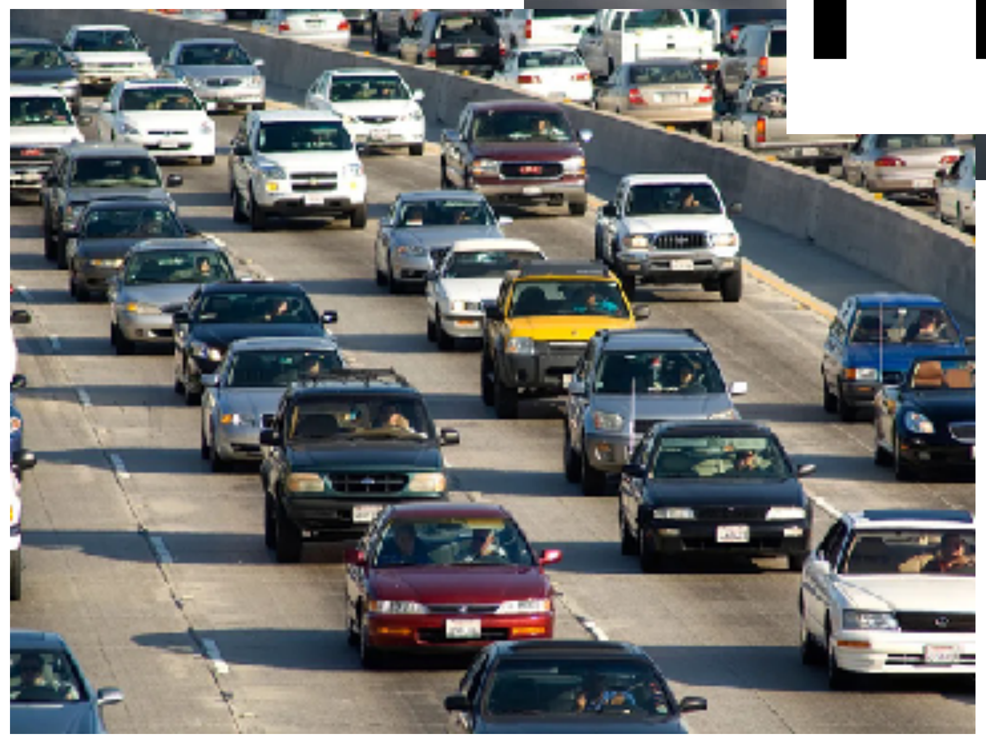
Meanwhile in 2024…

The impact of nowadays mobility

14th century

Meanwhile in 2024…

HUBS

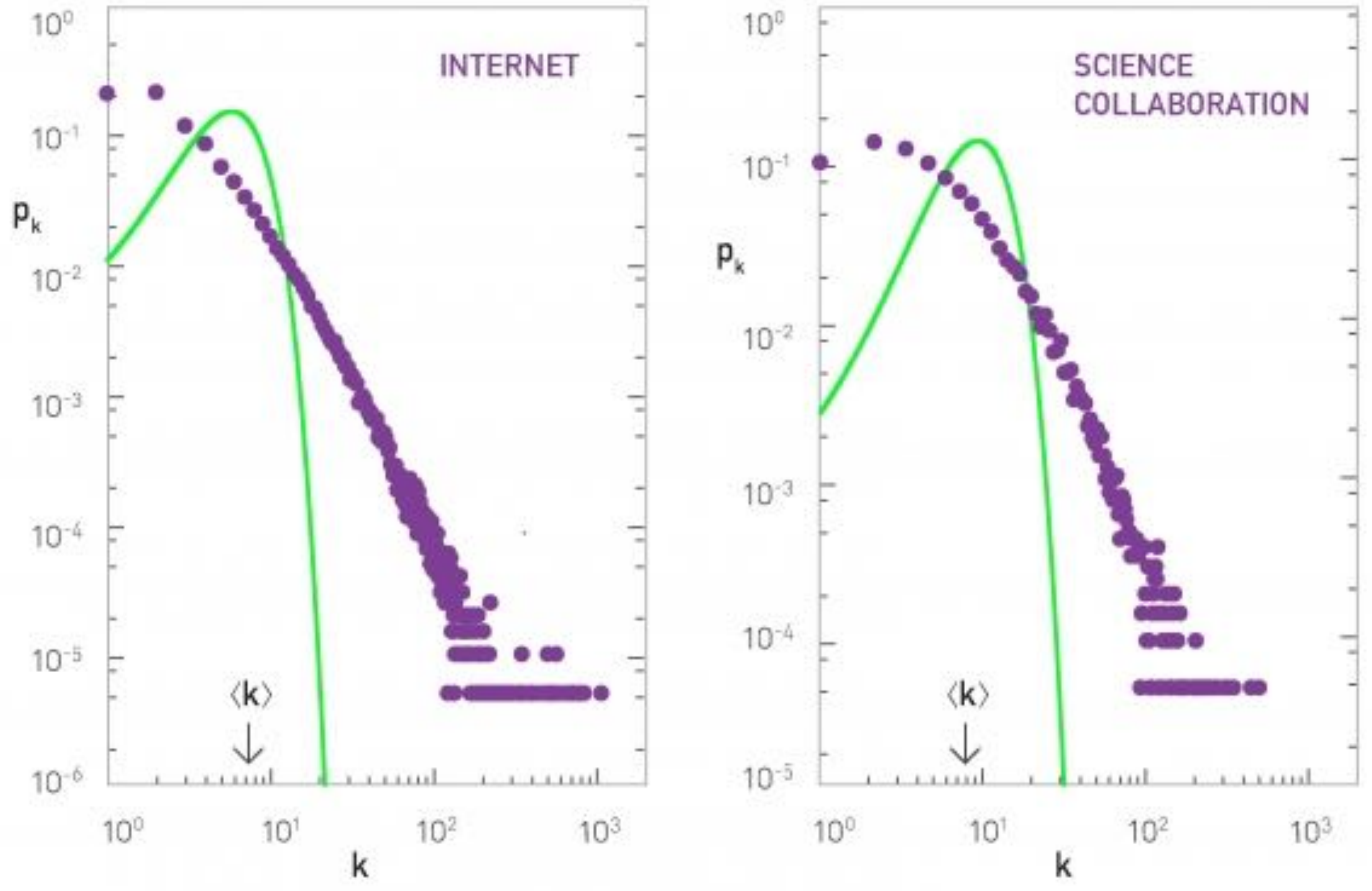


Epidemic threshold in heterogeneous networks

One population

each individual has its own *k* contacts -> heterogeneous network

“heavy tailed network”

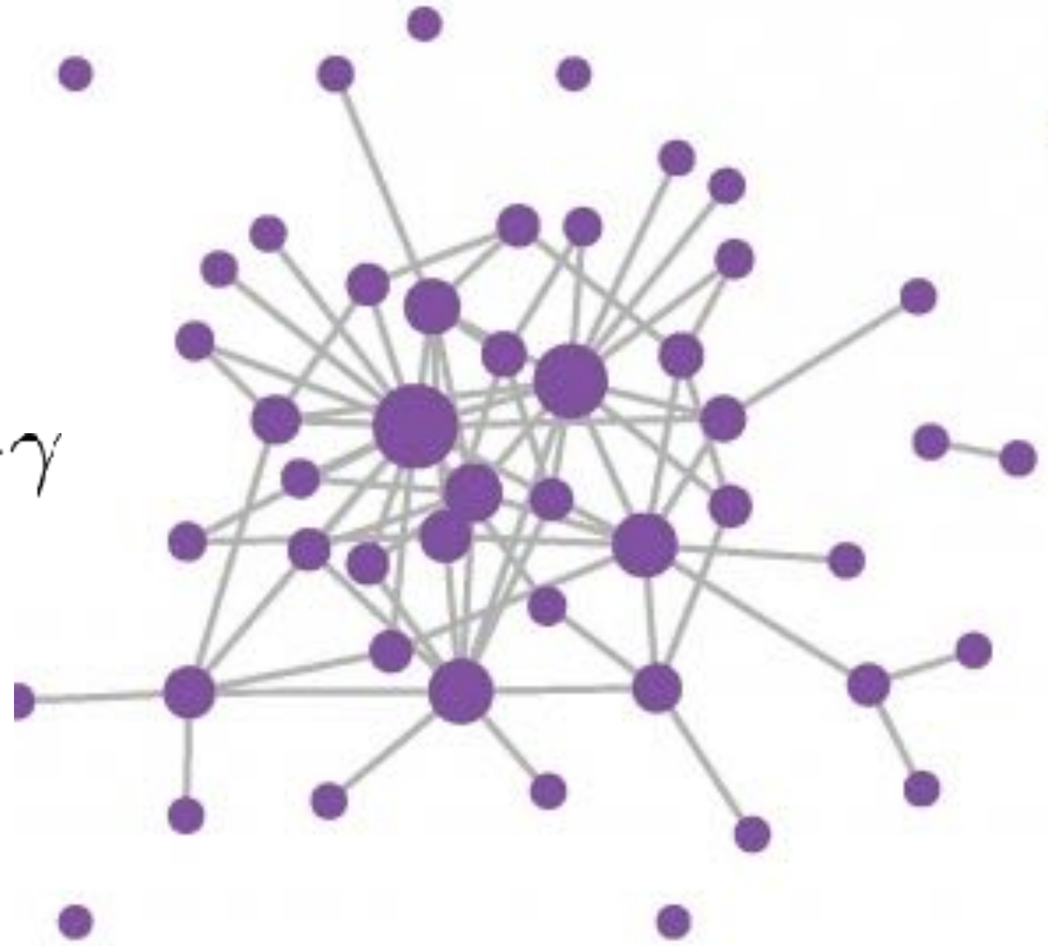


$\langle k \rangle$ average contacts (degree) per individual

$\langle k^2 \rangle$ 2nd moment of degree

$$\begin{cases} \text{If } \gamma > 2 \\ \langle k^2 \rangle \to \infty \end{cases}$$

$$P(k) \propto k^{-\gamma}$$



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Epidemic threshold in heterogeneous networks

One population

each individual has its own *k* contacts -> heterogeneous network

The epidemic threshold becomes $$\lambda_c = \frac{\langle k \rangle}{\langle k^2 \rangle - \langle k \rangle}$$

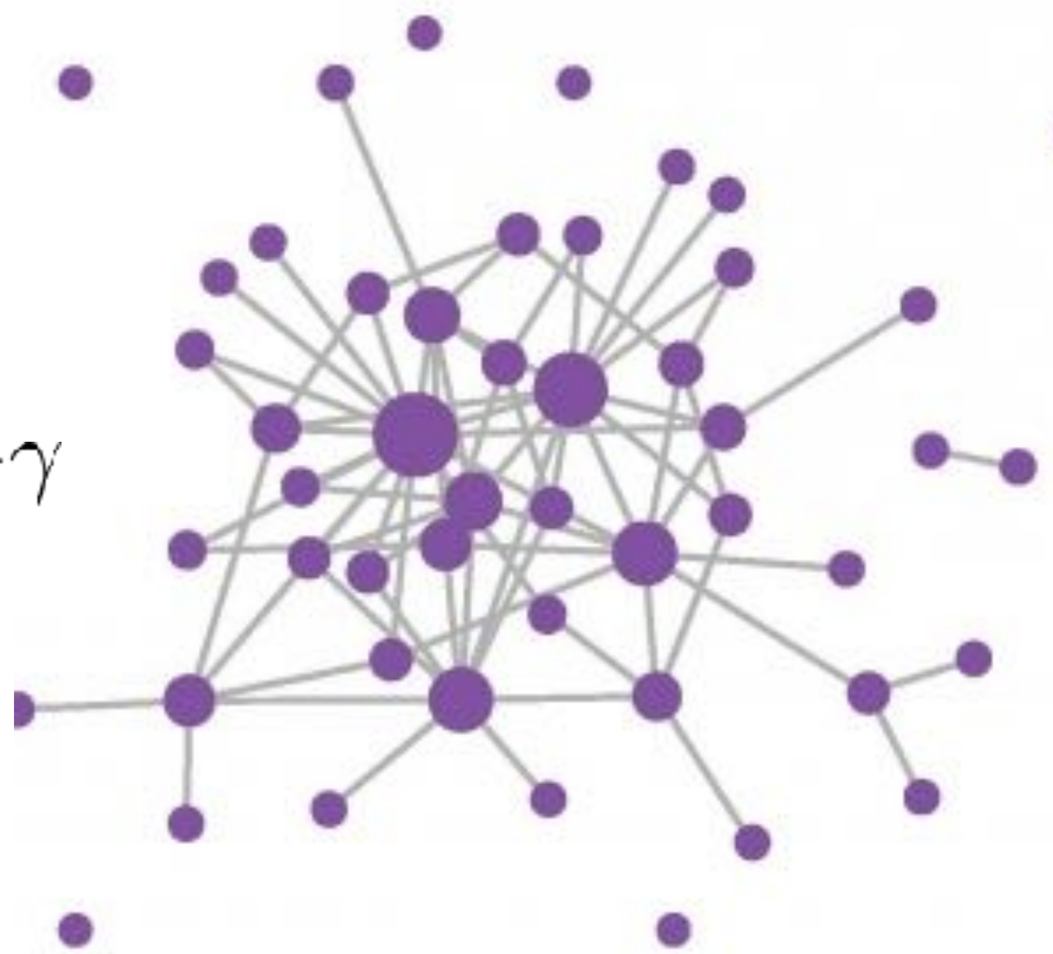
$\langle k \rangle$ average contacts (degree) per individual

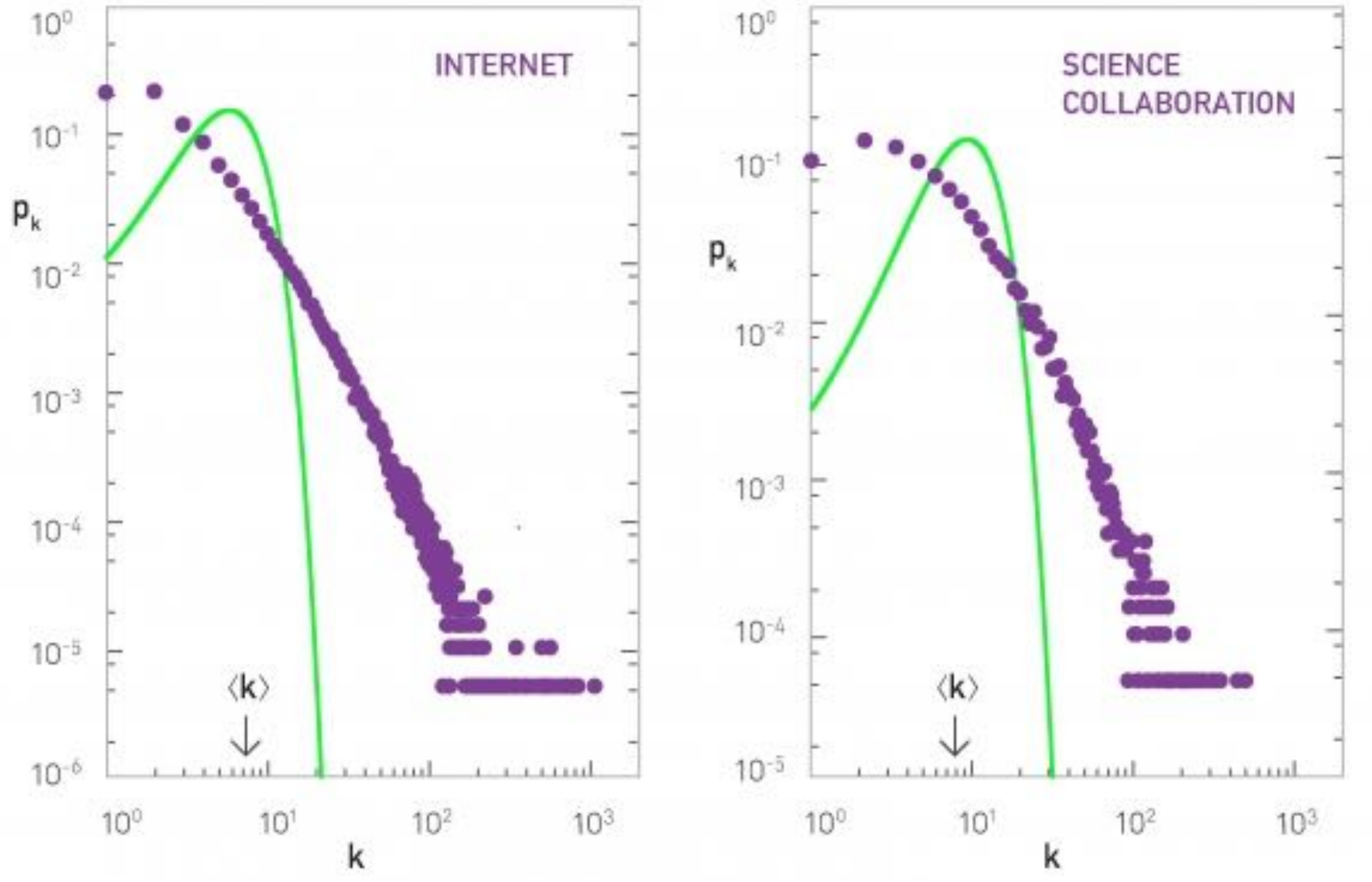
$\langle k^2 \rangle$ 2nd moment of degree distribution

$$\begin{cases} \text{If } \gamma > 2 \\ \langle k^2 \rangle \to \infty \\ \lambda_c \to 0 \end{cases}$$

$$P(k) \propto k^{-\gamma}$$

$$\lambda = \frac{\beta}{\mu}$$





"Scale-free networks are very vulnerable to epidemics"

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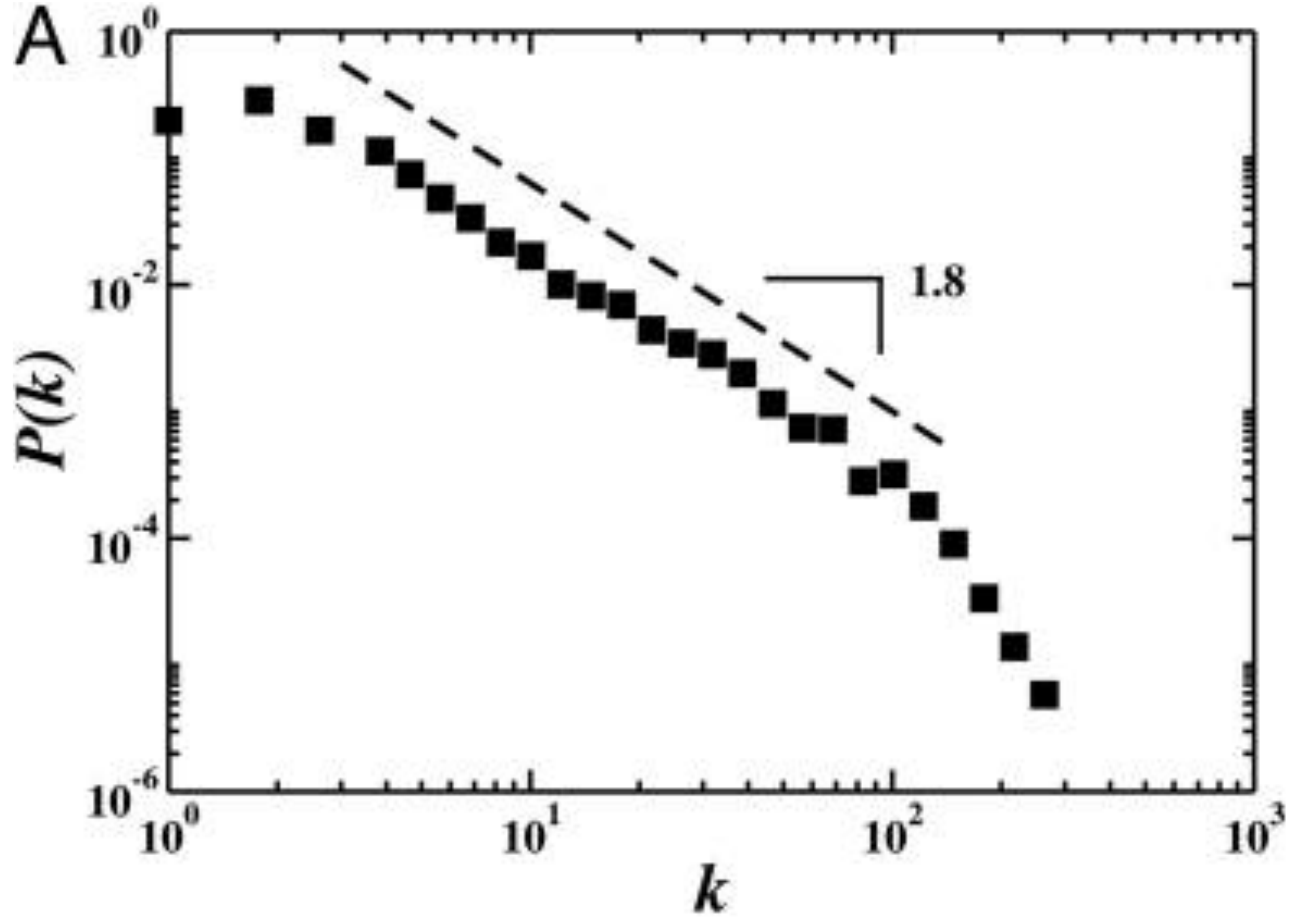
The air transportation network



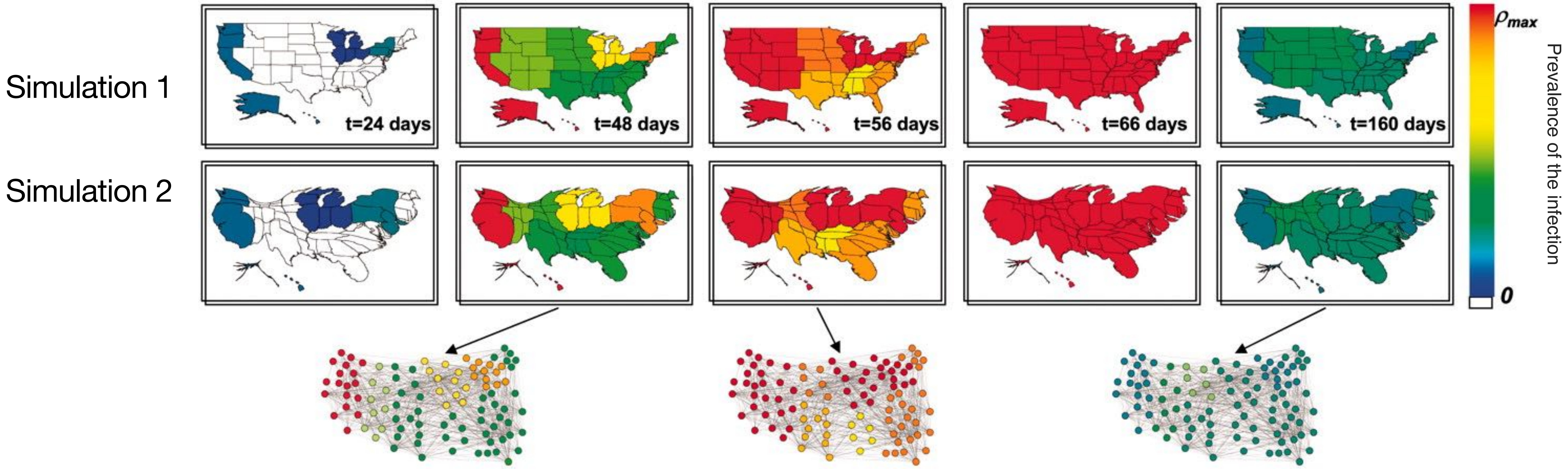
The air transportation network is vulnerable to epidemics

Colizza, Vittoria, et al. "The role of the airline transportation network in the prediction and predictability of global epidemics." *PNAS* 103.7 (2006)

Pathways of spatial invasion

Simulating epidemic starting in Hong Kong on US States
Similarity of outcome affected by:

- stochasticity of the model
- heterogeneity of the air transportation network



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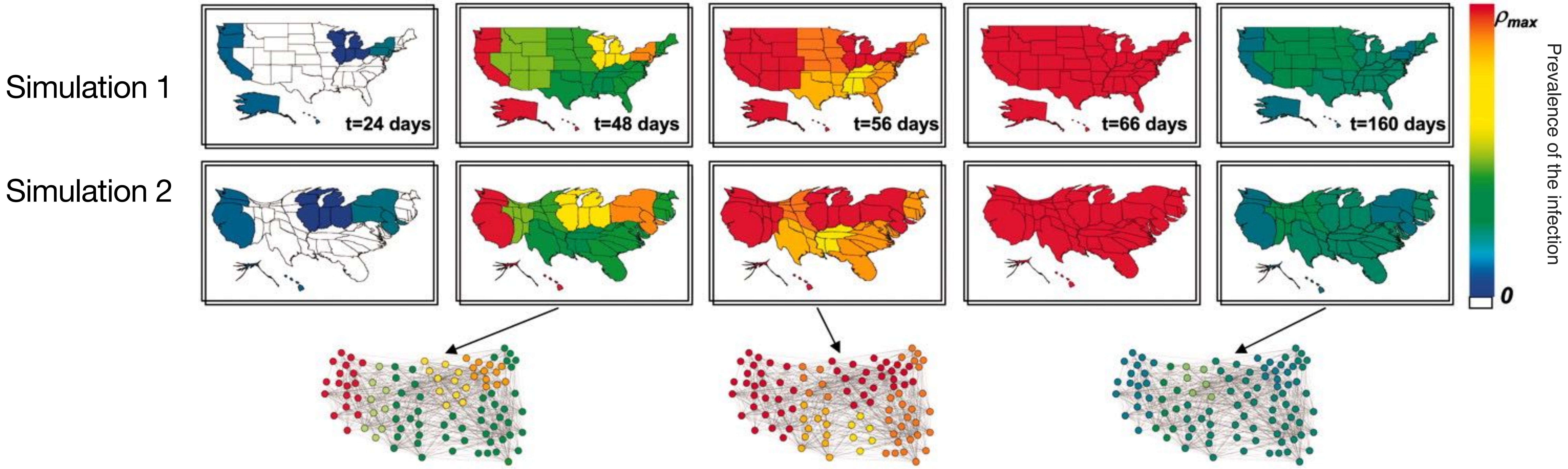
Pathways of spatial invasion

$\Theta(t)$ Overlap function: measures similarity between prevalence at time t in all cities i



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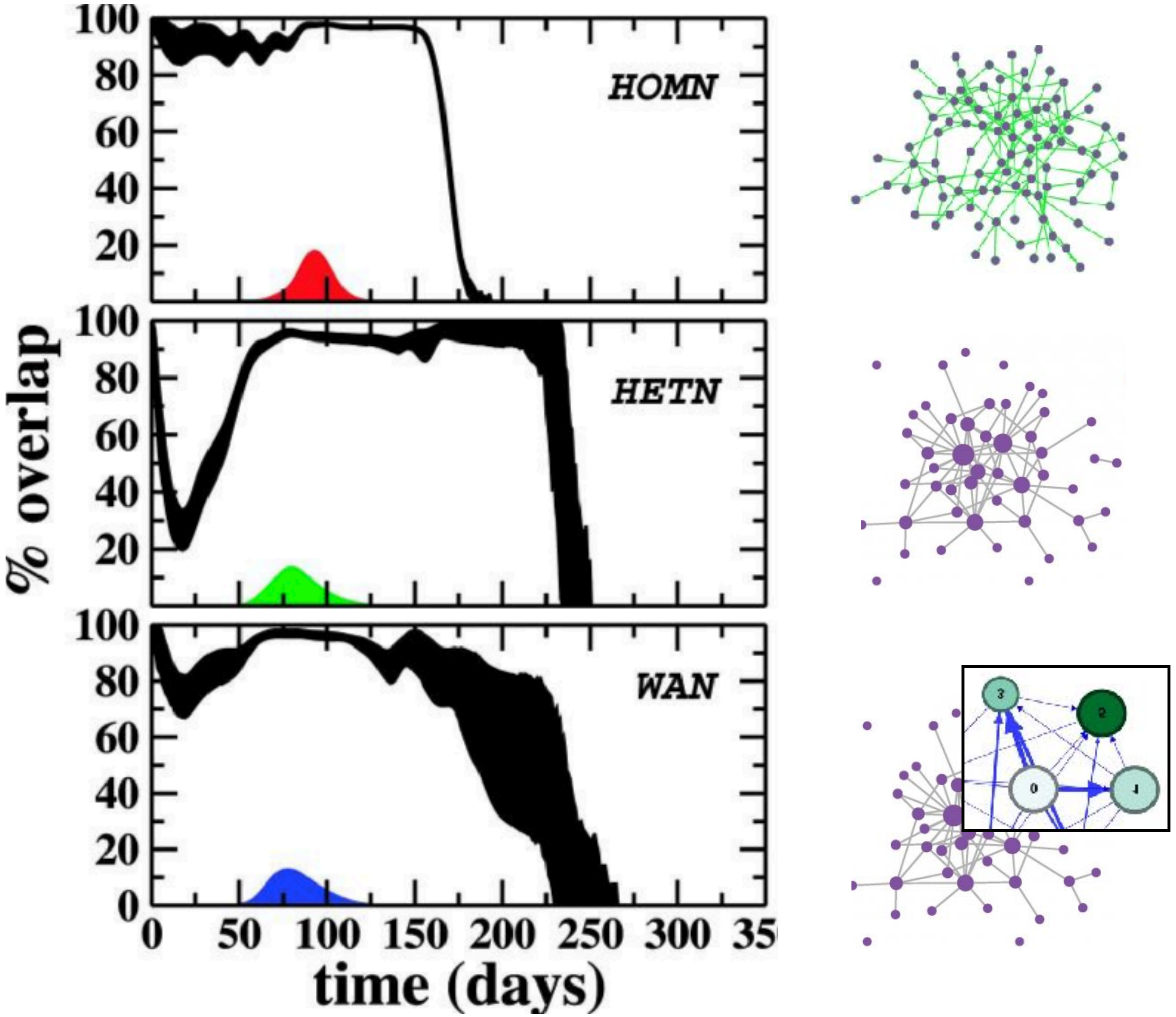

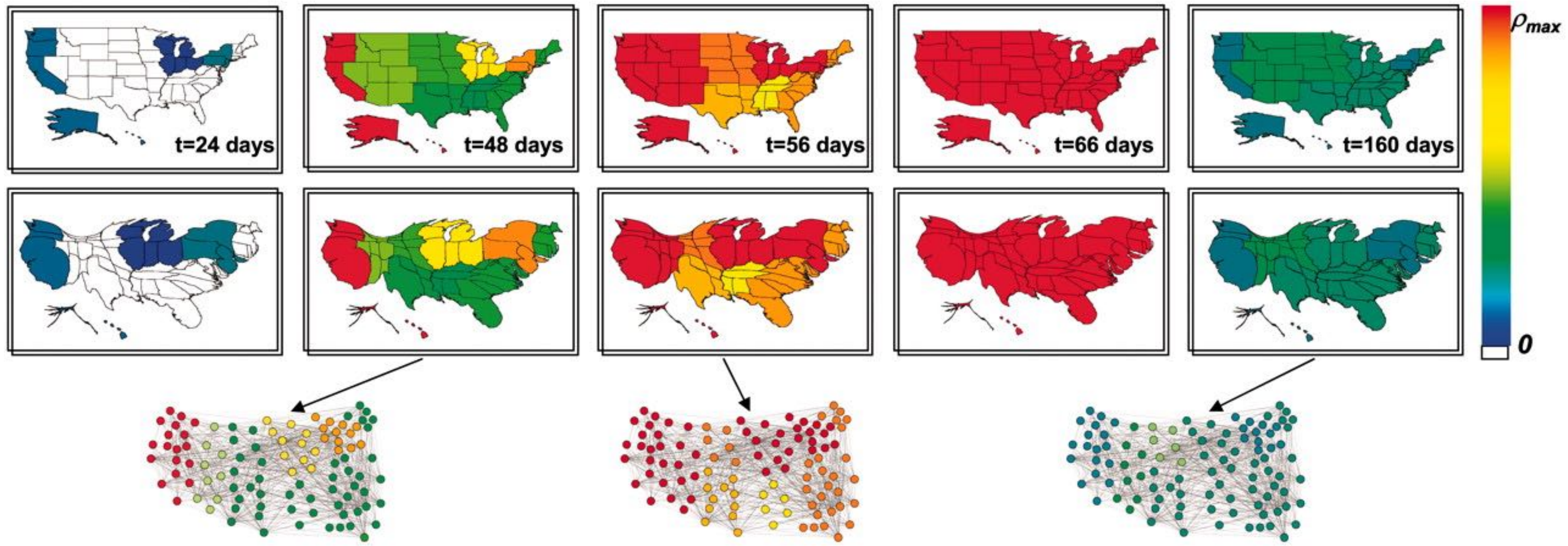

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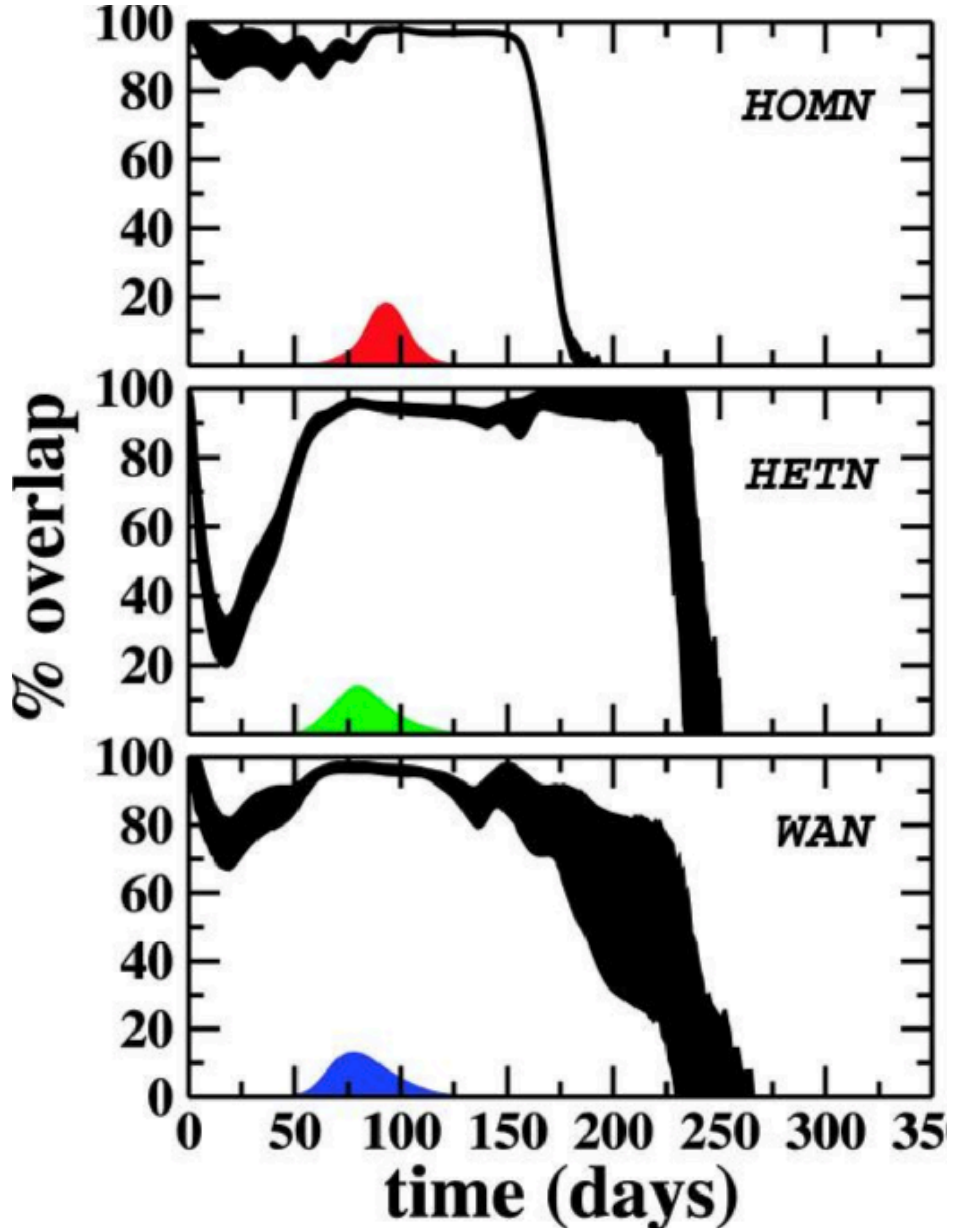
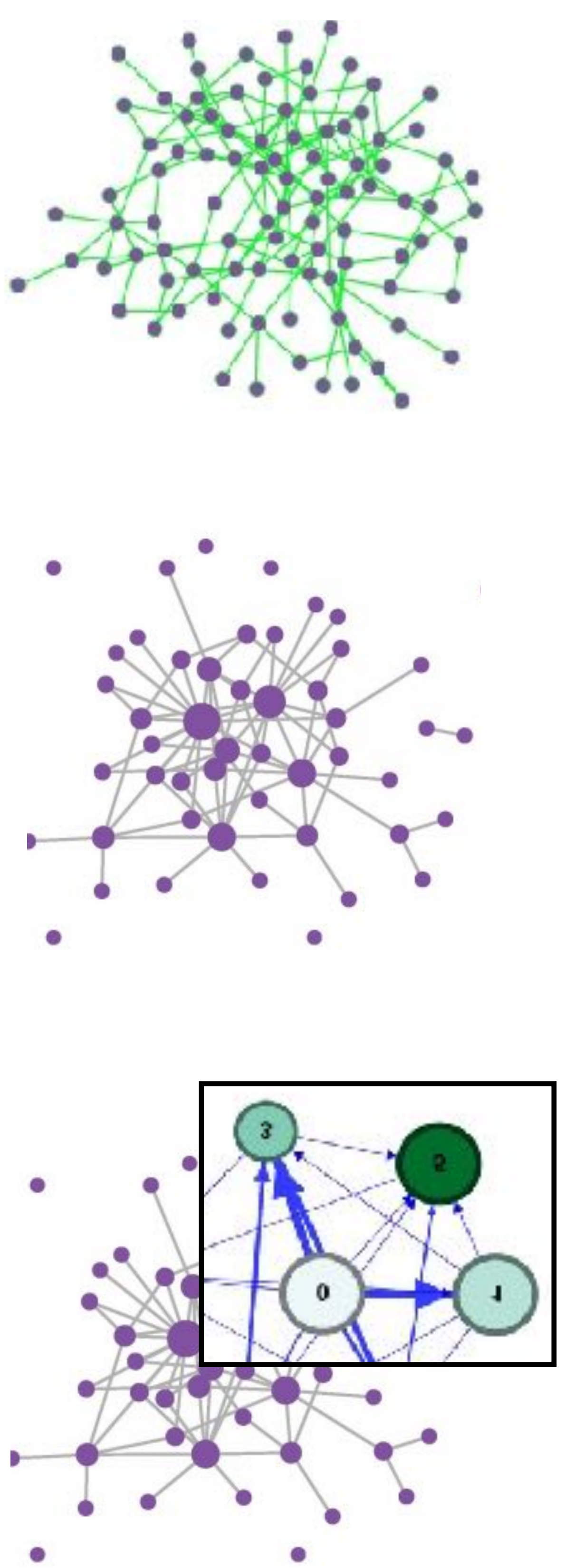
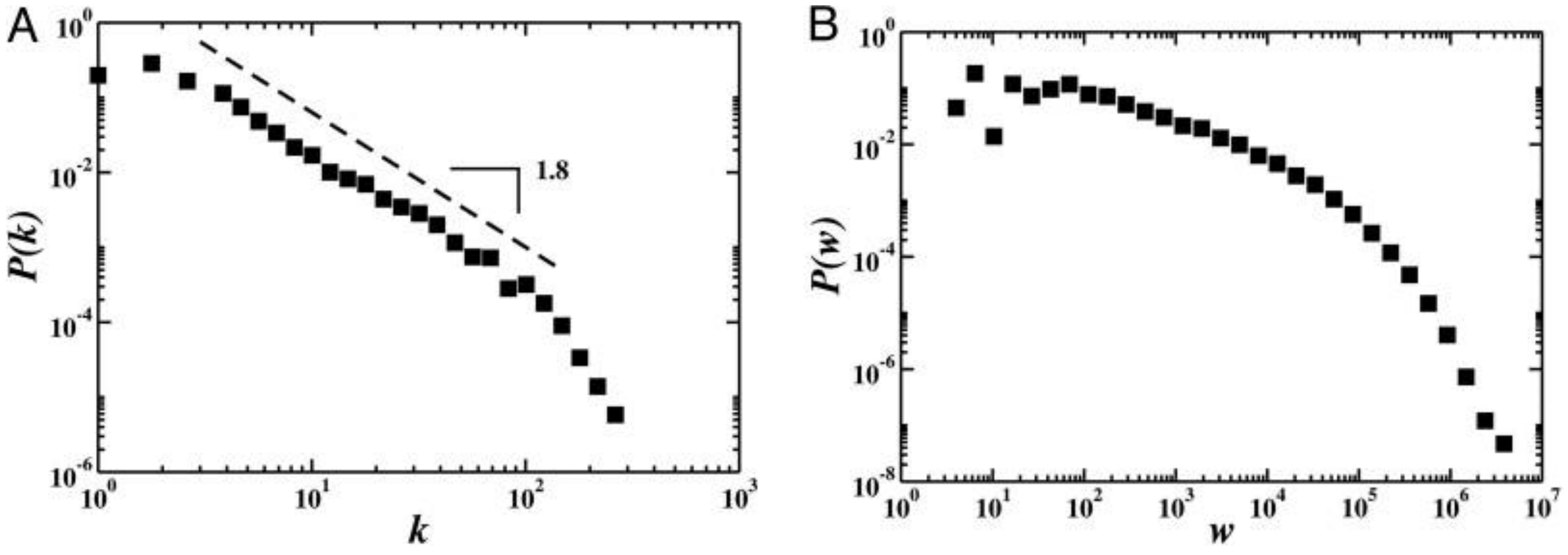






Two dynamics at play:

Degree heterogeneity
lowers predictability of spatial invasion

Weight heterogeneity
makes certain pathways more probable overall

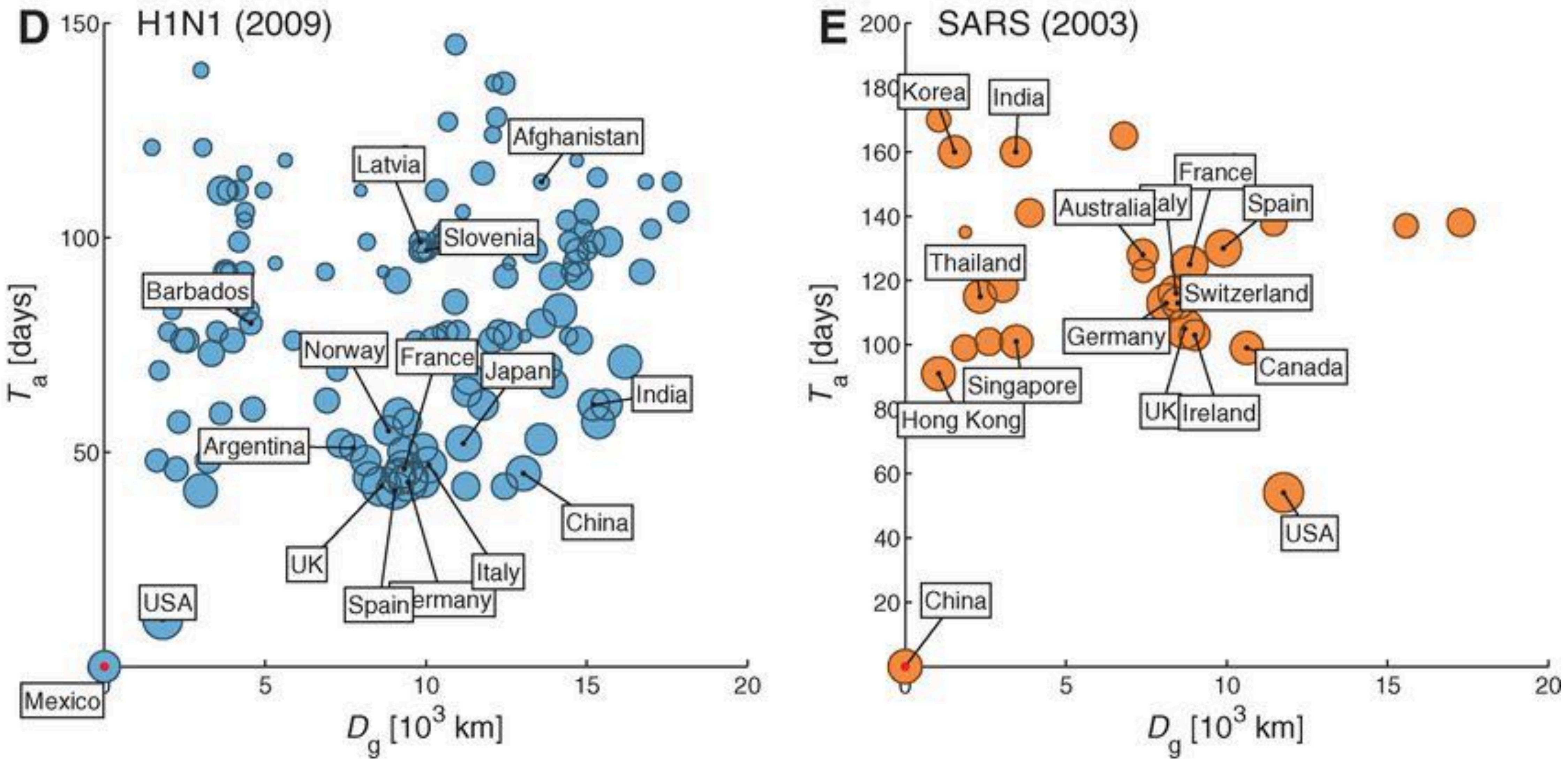
High heterogeneity at large times due to different lifetimes of epidemics in each simulation

Colizza, Vittoria, et al. "The role of the airline transportation network in the prediction and predictability of global epidemics." *PNAS* 103.7 (2006)

The hidden geometry of epidemic spread, predicting arrival times

Predicting disease arrival times at country scale

From distance to the effective distance



Geographic distance

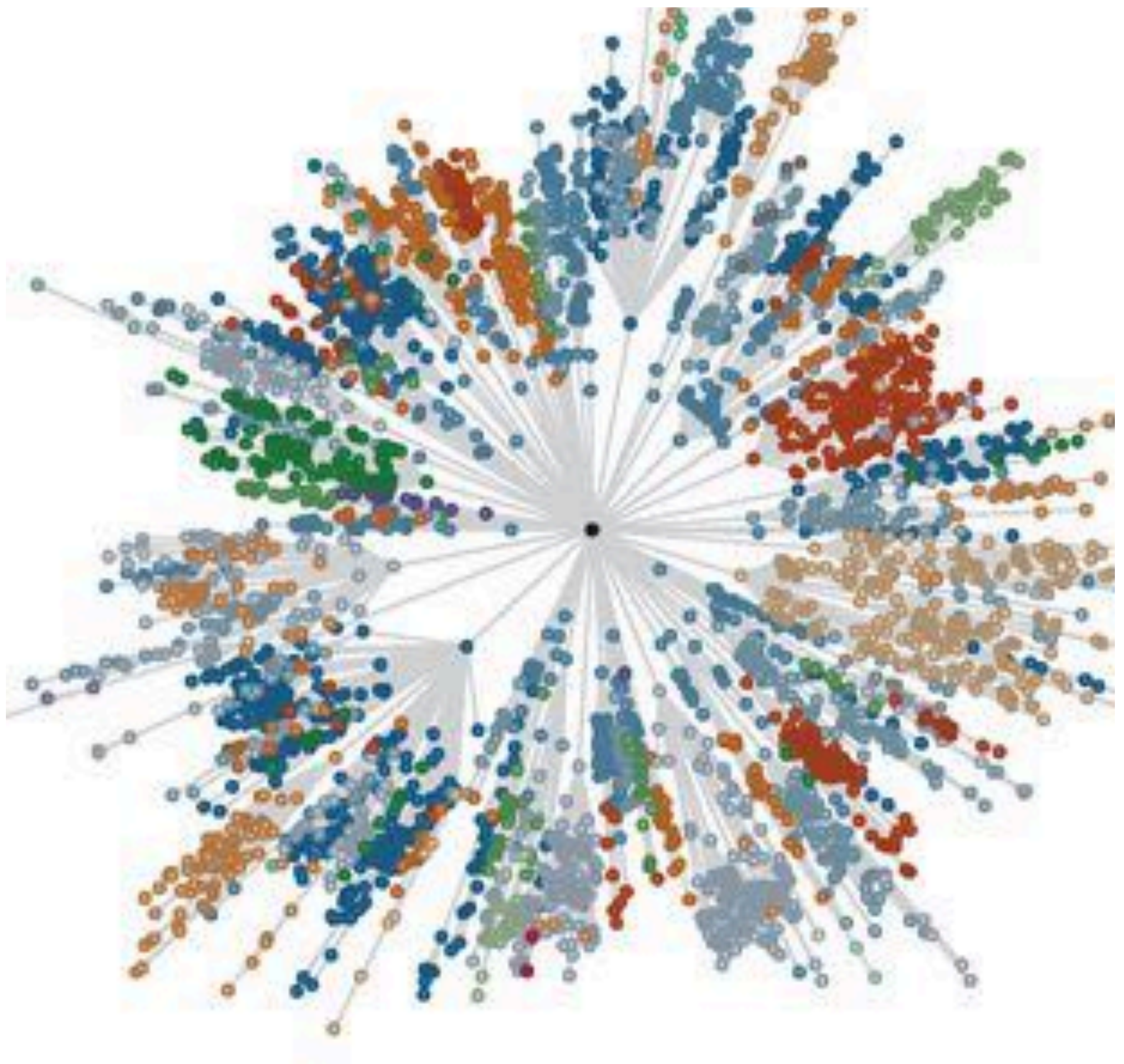
Dirk Brockmann, Dirk Helbing ,The Hidden Geometry of Complex, Network-Driven Contagion Phenomena.Science342,1337-1342(2013)

The hidden geometry of epidemic spread, predicting arrival times

$$d_{mn} = (1 - log P_{mn})$$

Flow fraction

$$D_{mn} = min_{\Gamma} \lambda(\Gamma)$$



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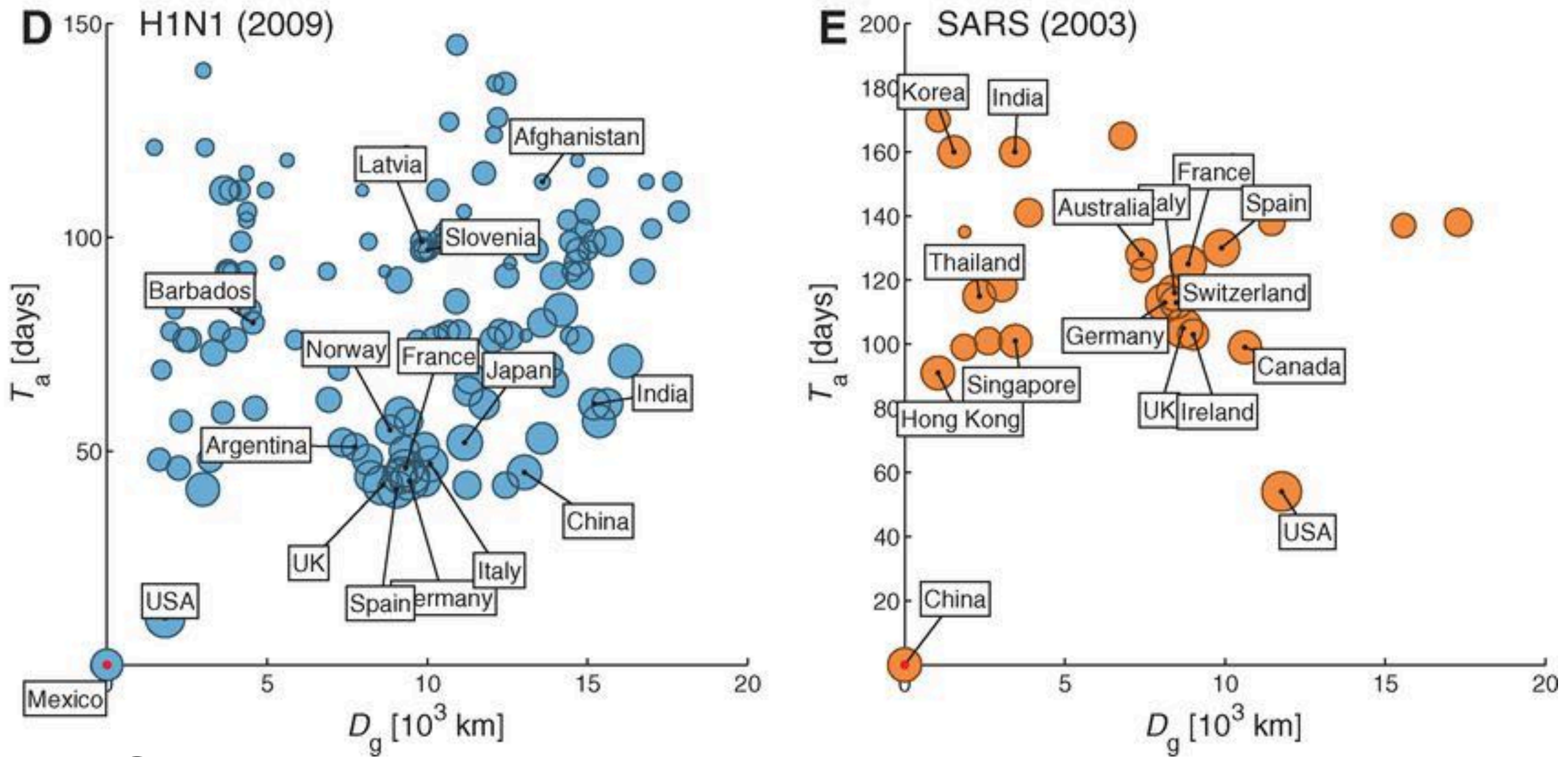

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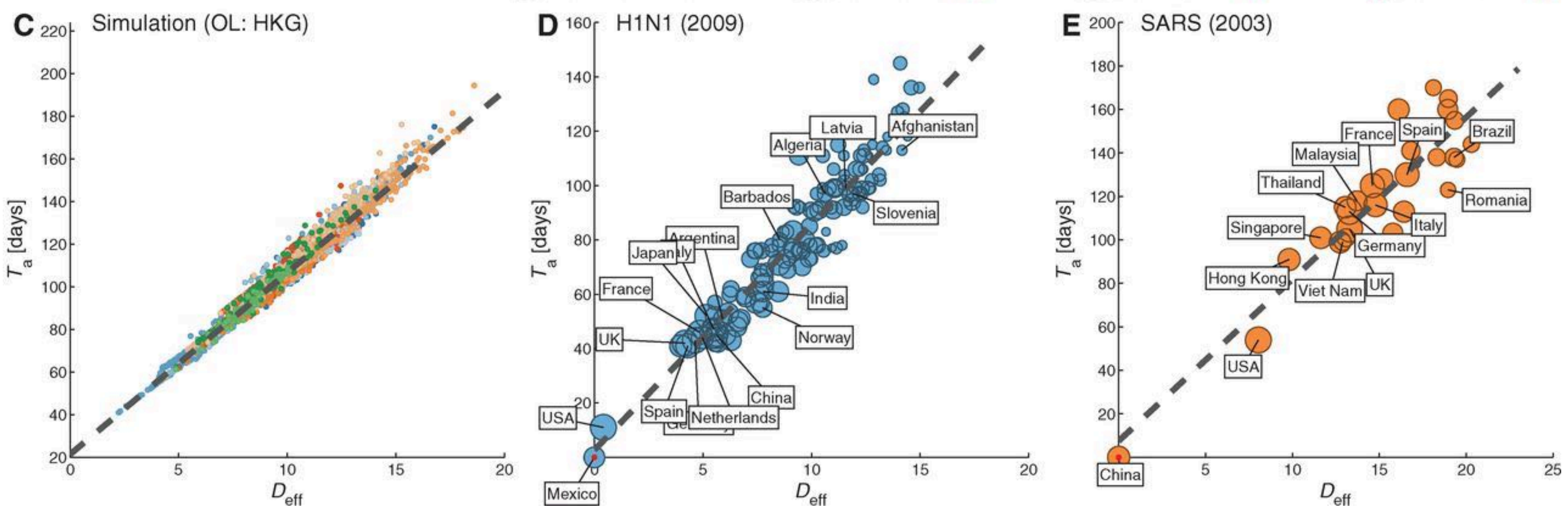

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The hidden geometry of epidemic spread, predicting arrival times

Dirk Brockmann, YouTube

Wave-like diffusion is still there, but now it is projected in another space!

Spatial invasion

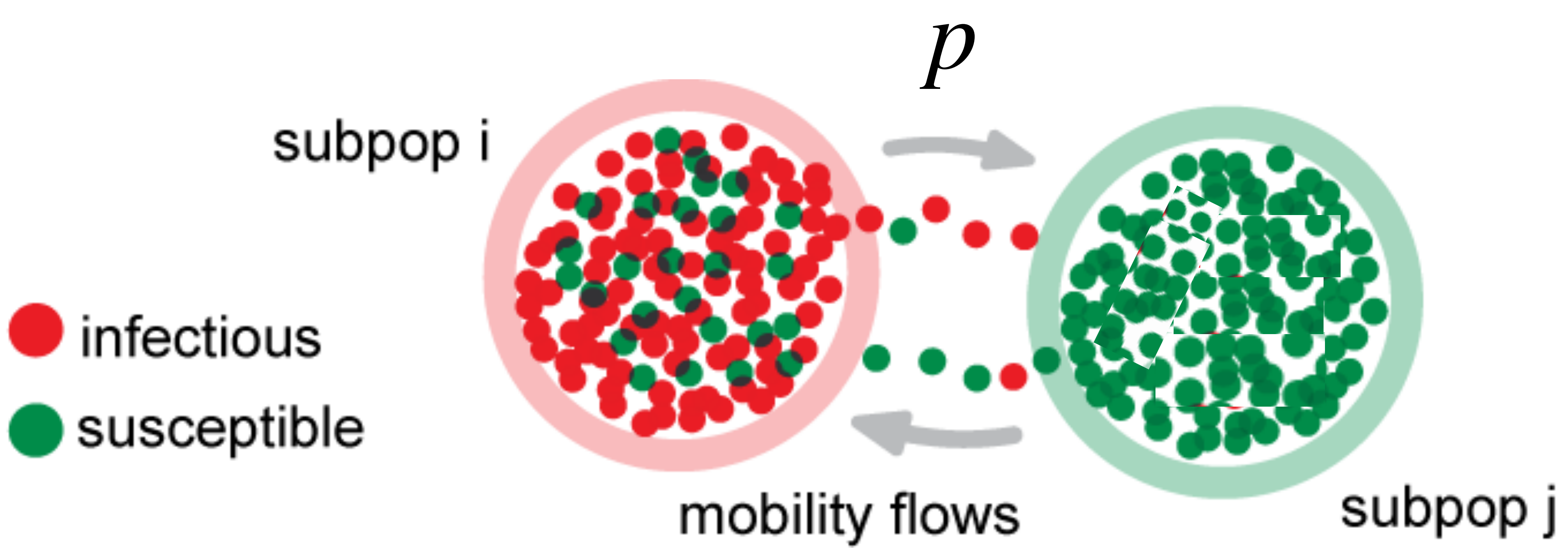
Context: 2 cities i and j, 1 infected in i, j fully susceptible

Probability of traveling from i to j: p

Probability of not traveling from i to j: $1 - p$

Probability of no infected of i traveling to j: $(1 - p)^{I(t\Delta t)}$

Probability of invasion in j after time t: $1 - (1 - p)^{I(t)}$



Gautreau, Aurélien, Alain Barrat, and Marc Barthelemy. "Global disease spread: statistics and estimation of arrival times." *Journal of theoretical biology* 251.3 (2008): 509-522.

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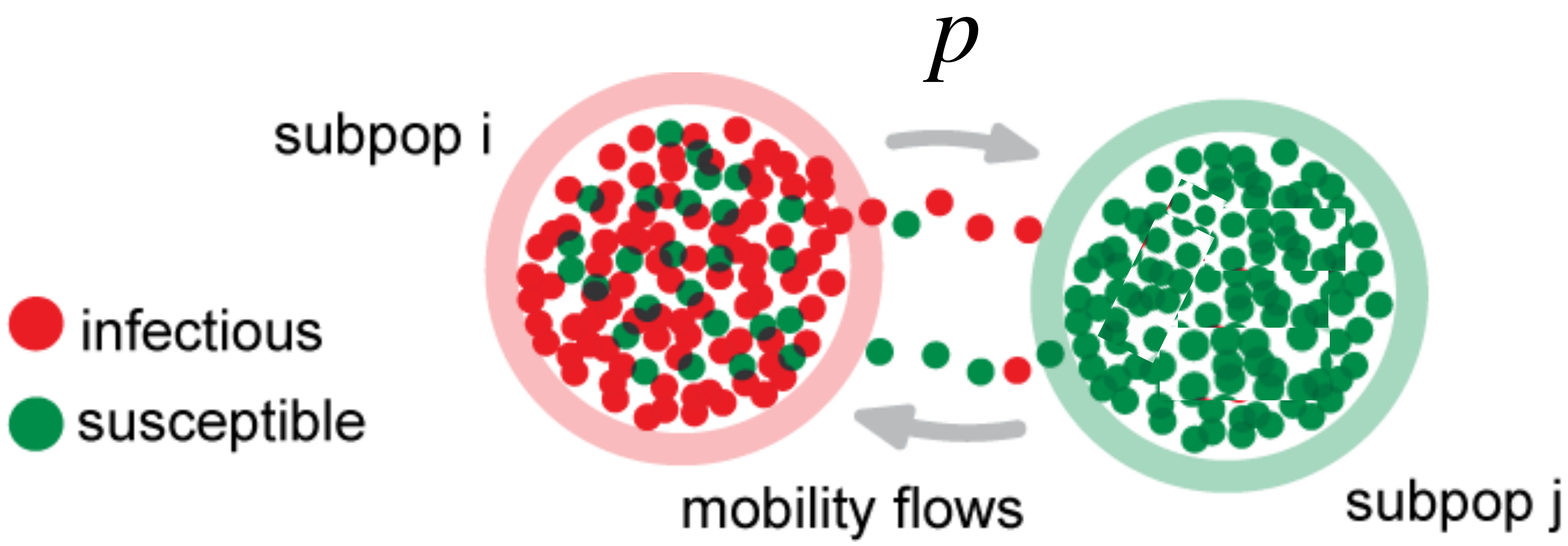

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Probability of not traveling from i to j: $1 - p$

Probability of no infected of i traveling to j: $(1-p)^{I(t\Delta t)}$

Probability of invasion in j after time t: $1 - (1-p)^{I(t)}$

Probability of invasion exactly at time t: $p(t_{arrival} = n\Delta t) = \prod_{s=0}^{n-1} (1-p)^{I(s\Delta t)} \times [1 - (1-p)^{I(n\Delta t)}]$

If travellers are negligible wrt total population of origin: $p \to 0$

Expansion

$$p(t_{arrival} = t) = pI(t)e^{-p\sum_{s=0}^{t} I(s\Delta t)}$$

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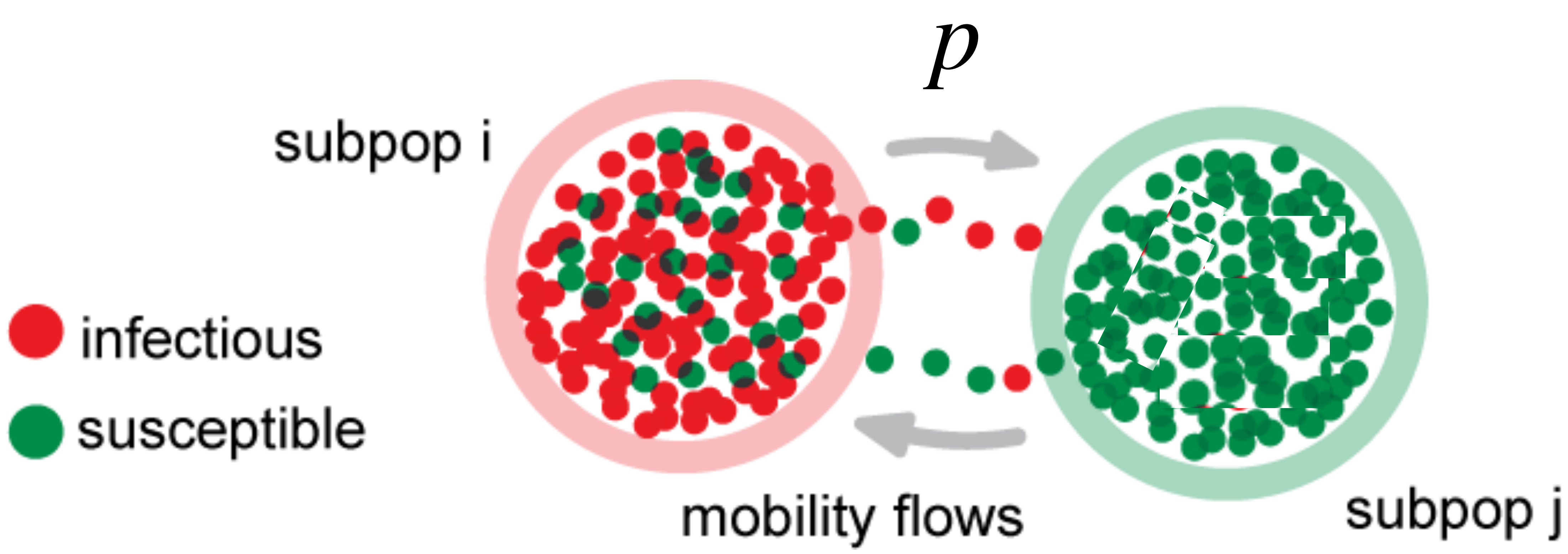
Spatial invasion

Probability of invasion exactly at time t, given $p \to 0$

$$p(t_{arrival} = t) = pI(t)e^{-p\sum_{s=0}^{t} I(s\Delta t)}$$



$$\begin{cases} I(t) \sim I(0)e^{(\beta-\mu)\Delta t} = I(0)e^{\mu(R_0-1)\Delta t} \\ I(0) = 1 \\ a = \mu(R_0 - 1) \end{cases}$$

$p(t_{arrival} = t) = pe^{at}e^{-pae^{at}} \implies$ Gumbel distribution $\langle t_{arrival} \rangle \simeq \frac{1}{a} log(\frac{a}{p})$

Gautreau, Aurélien, Alain Barrat, and Marc Barthelemy. "Global disease spread: statistics and estimation of arrival times." *Journal of theoretical biology* 251.3 (2008): 509-522.

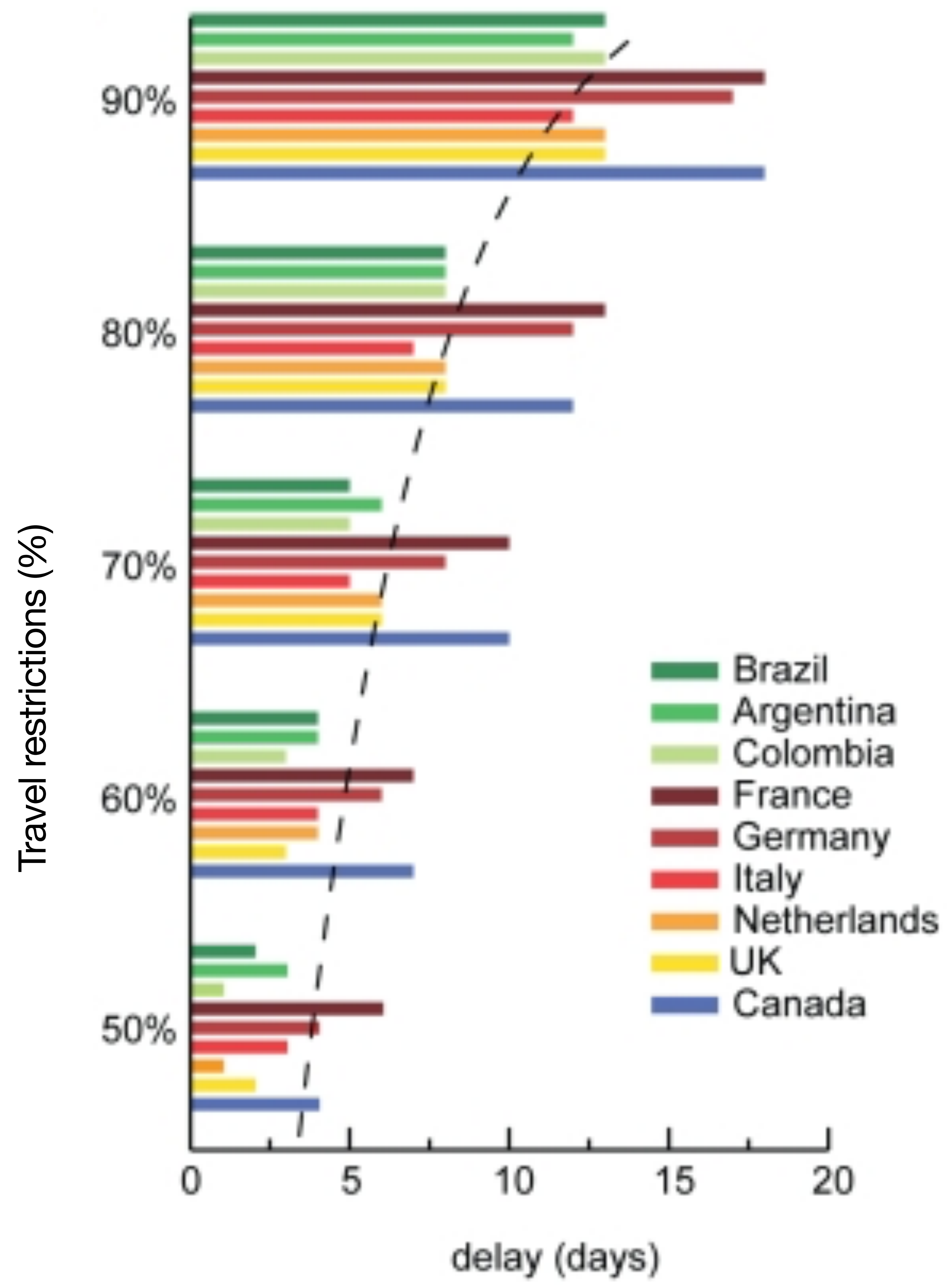
Public health interventions on spatial invasion: air travel bans

Only with two areas

Impose travel restriction (TR), cut travels from infected origin by w = 50%

$$\langle t_{arr,TR} \rangle - \langle t_{arrival} \rangle \simeq \frac{1}{a} log \left(\frac{a}{(1-w)p} \right) - \frac{1}{a} log \left(\frac{a}{p} \right)$$

$$= \frac{1}{a} log \left(\frac{a}{p} \right) - \frac{1}{a} log \left(\frac{a}{p} \right) - \frac{1}{a} log(1-w) = -\frac{1}{a} log(1-w)$$

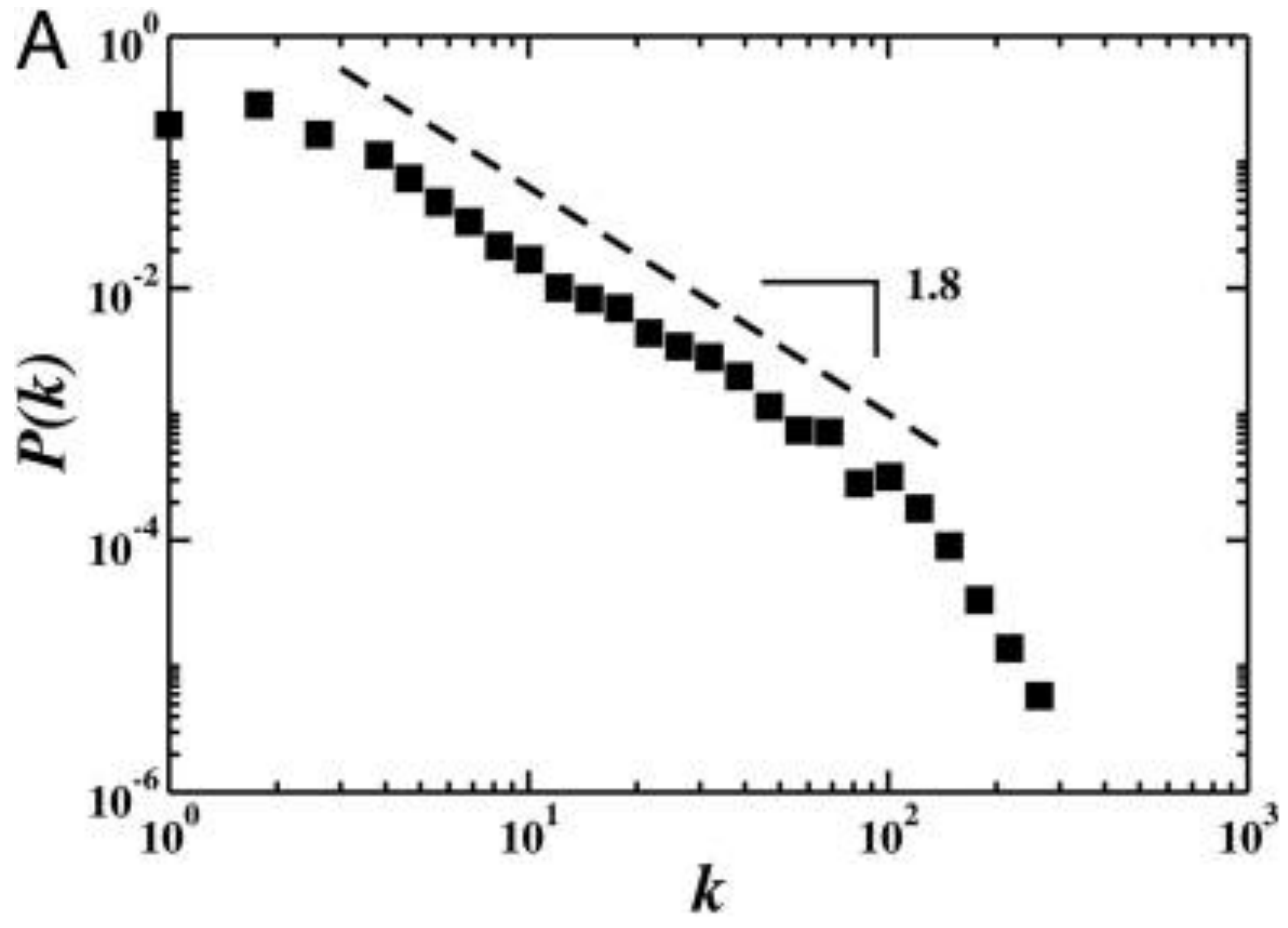


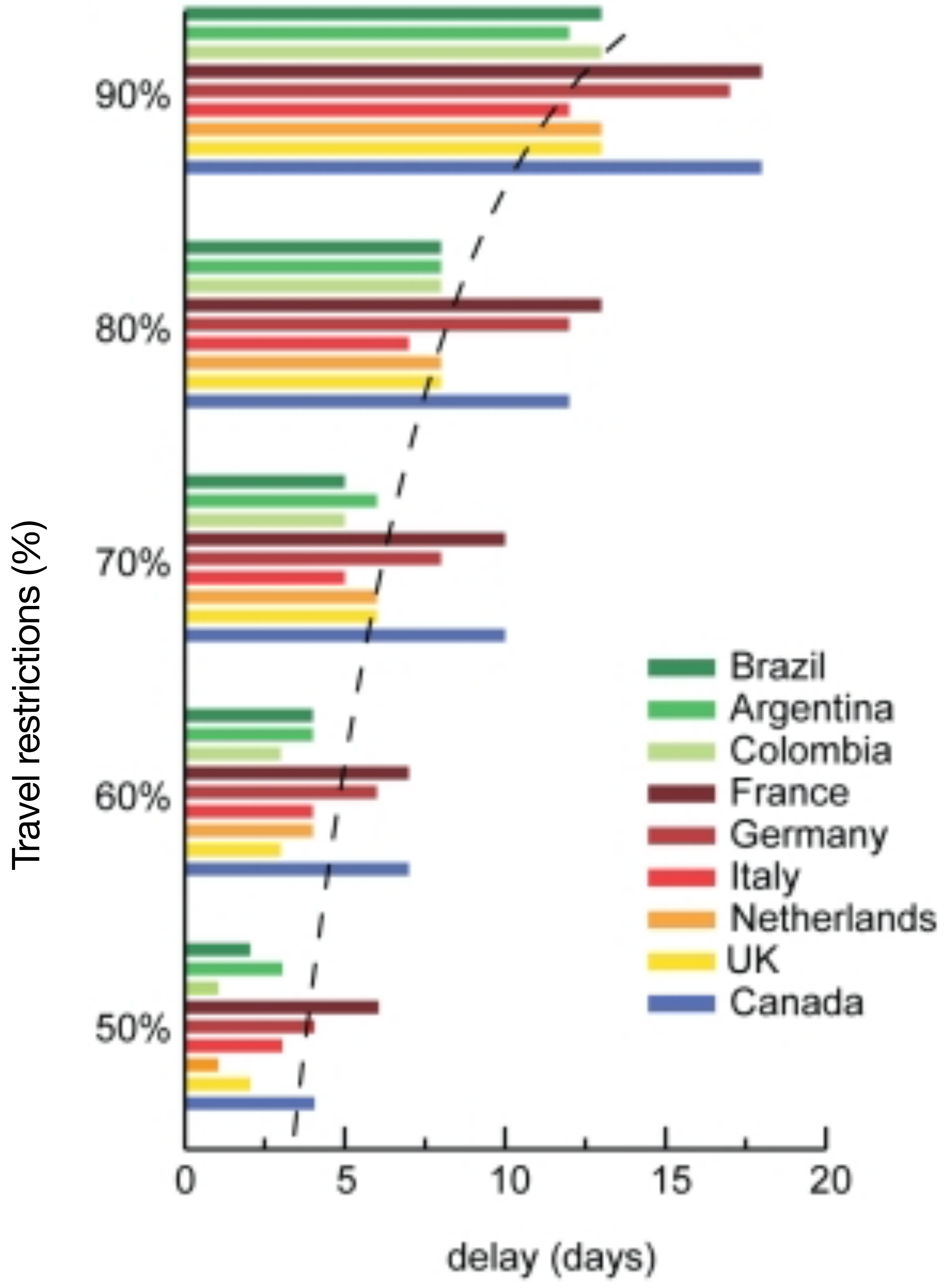
Bajardi P, Poletto C, Ramasco JJ, Tizzoni M, Colizza V, Vespignani A. Human mobility networks, travel restrictions, and the global spread of 2009 H1N1 pandemic. PLoS One. 2011
Colizza V, Barrat A, Barthélemy M, Vespignani A. The role of the airline transportation network in the prediction and predictability of global epidemics. Proc Natl Acad Sci U S A. 2006
Scalia Tomba G, Wallinga J. A simple explanation for the low impact of border control as a countermeasure to the spread of an infectious disease. Math Biosci. 2008

Public health interventions on spatial invasion: air travel bans

Slowing down the international spread of diseases

Impossible to stop spread without suppressing > 99% of flights
Effect is due to the topology of the air travel network
Best you can do is delaying arrival of disease to anticipate and better prepare response





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Colizza V, Barrat A, Barthélemy M, Vespignani A. The role of the airline transportation network in the prediction and predictability of global epidemics. Proc Natl Acad Sci U S A. 2006

The hidden geometry of epidemic spread, predicting arrival times

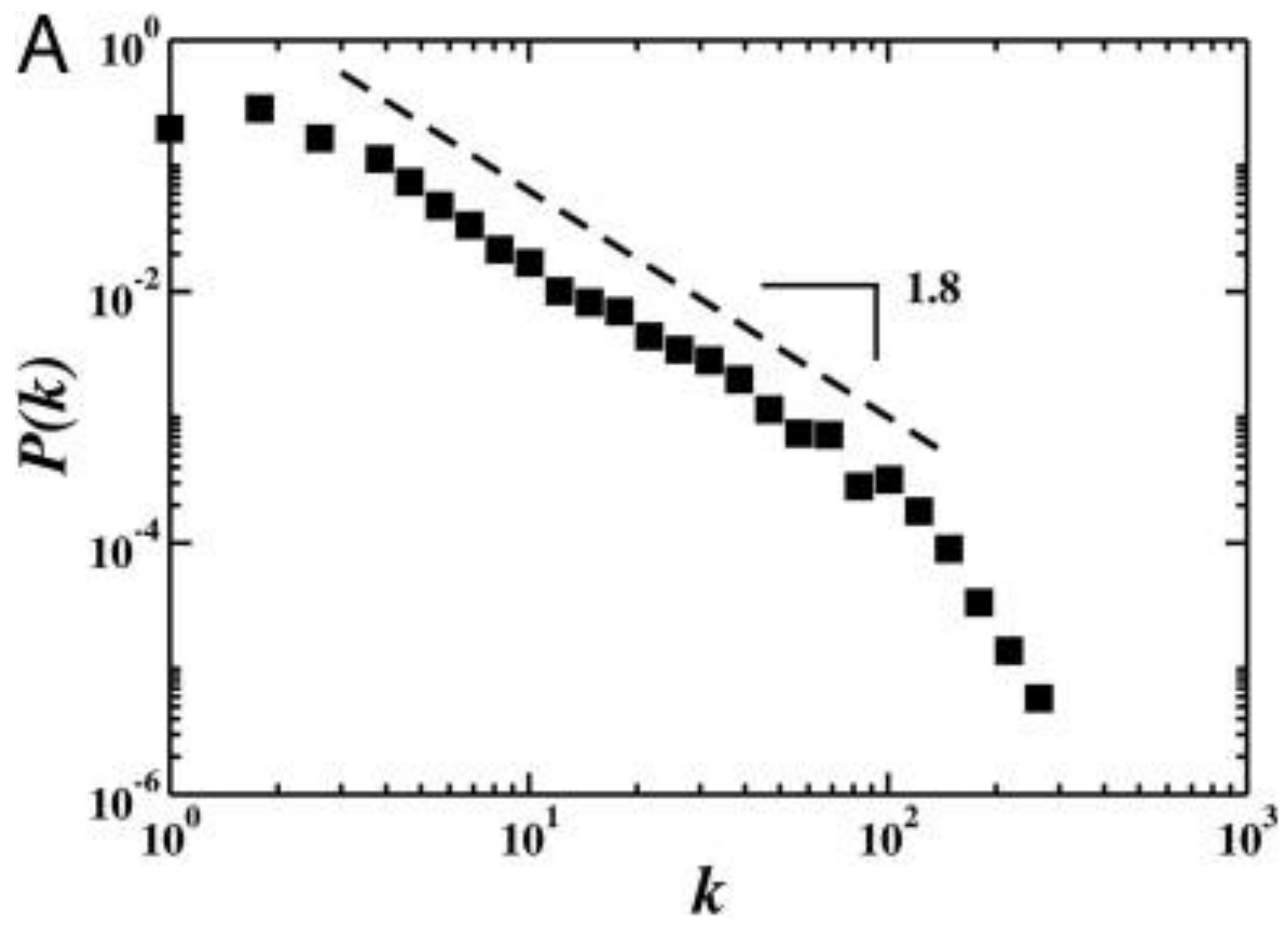
Dirk Brockmann, YouTube

HANDS ON SESSION
Go to https://github.com/mattiamazzoli/Teaching/arrival_times.ipynb

Public health interventions on spatial invasion: air travel bans

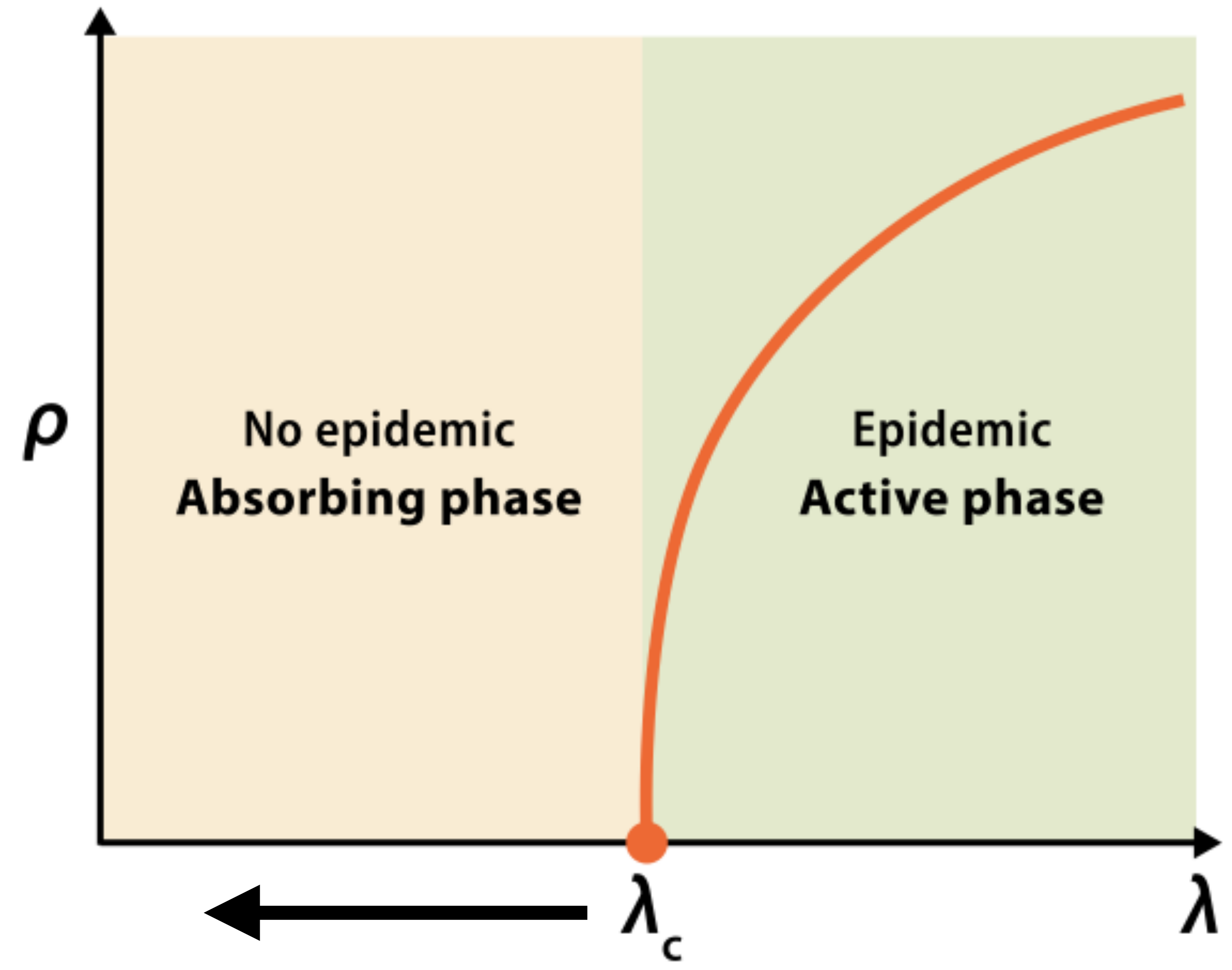
Why is it so hard?

Epidemic threshold affected by 2nd moment of degree distribution of the air travel network



$$\lambda_c = \frac{\langle k \rangle}{\langle k^2 \rangle - \langle k \rangle}$$

$$\begin{cases} \text{If } \gamma > 2 \\ \langle k^2 \rangle \to \infty \\ \lambda_c \to 0 \end{cases}$$



HANDS ON SESSION

Try this yourself! Go to page: https://epirisk.net/
Play with travel restrictions (reach ~99%), check the effect on the n of importated cases

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