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[illegible]

Keywords: EEG data, brain network, connectivity graph, graph indices, motif analysis, community detection

Generally we worked in `python`, so in order to read EEG data, that are stored in EDF format we used the `pyedflib` library²

I. CONNECTIVITY GRAPHS

Graph theory provides powerful means to assess topology and organization of brain connectivity networks

A network based approach is especially useful when it's important to comprehend and highlight non obvious interactions between agents in a system, their nature depending on the case study. In computational neuroscience we measure the neuronal activity of different brain regions through sensors applied on top of the scalp. Functional connectivity can be estimated using methods based on Granger causality, such as multivariate autoregressive models (MVAR).

A multivariate autoregressive model of order p can be defined as

$$X(t) = \sum_{r=1}^p A(r)X(t-r) + E(t) \quad (1)$$

In our case, $X(t)$ is a vector that contains measurements from 64 signals at time t ; $A(r)$ $r \in \{1, 2, ..p\}$ are matrices in $\mathbb{R}^{N \times N}$ that contain the parameters (to be estimated) that describe the dependence between the entries in $X(t)$ and $X(t - r)$; $E(t)$ is a vector of random variables and denotes the uncorrelated Gaussian process with zero mean.

The formula says that values at time t linearly depend from the last p values taken from our 64 time series through an appropriate set of $N \cdot N \cdot p$ coefficients.

It is possible to select the best value of p with respect to the Akaike information criterion. This selection required us to spend too much time on that, and since we considered this not crucial for our purposes, we set the order of the model to $p = 3$. Further investigations can surely be made in future studies.³

What is written above can be rewritten also like.

$$\sum_{r=0}^p A(r)X(t-r) = E(t) \quad (2)$$

if $A(0) = \mathbb{I}$, and $A(r)$ $r \in \{1, 2, 3\}$ are estimated by solving the Yule Walker equations.

EEG signals are both defined in time as well as in frequency. We are interested in the reconstruction of the direction of information flows in the network.

Applying the Fourier Transform to both sides, hence passing to the frequency domain we have these two results

$$A(f)X(f) = E(f) \quad (3)$$

$$X(f) = A^{-1}(f)E(f) = H(f)E(f) \quad (4)$$

Partial Directed Coherence - PDC - is defined as

$$\pi_{ij}(f) = \frac{|A_{ij}(f)|^2}{\sum_{m=1}^N |A_{mi}(f)|^2} \quad (5)$$

Directed Fourier Transform - DTF - is defined as

$$\theta_{ij}(f) = \frac{|H_{ij}(f)|^2}{\sum_{m=1}^N |H_{mi}(f)|^2} \quad (6)$$

Once we calculated these estimators, choosing a particular frequency f^* we can effectively start using a graph

formalism: in fact what we did is to construct a graph from using either $\pi(f^*)$ or $\theta(f^*)$.

We conduct the analysis below selecting the topology that arises choosing $f^* = 10$ Hz; they basically corresponds to the alpha waves (8 – 12.5 Hz), that predominantly originate from the occipital lobe during wakeful relaxation with closed eyes. Alpha waves are reduced with open eyes, drowsiness and sleep⁶.

We also generate the graphs relative to $f^* = 28$ Hz, i.e. to high beta waves (20.5–28 Hz); beta states are the states associated with normal waking consciousness⁷.

II. GRAPH THEORY INDICES

III. MOTIF ANALYSIS

IV. COMMUNITY DETECTION

V. REFERENCES

¹Goldberger AL, Amaral LAN, Glass L, Hausdorff JM, Ivanov PCh, Mark RG, Mietus JE, Moody GB, Peng C-K, Stanley HE. PhysioBank, PhysioToolkit, and PhysioNet: Components of a New Research Resource for Complex Physiologic Signals. Circulation 101(23):e215-e220 [Circulation Electronic Pages; <http://circ.ahajournals.org/cgi/content/full/101/23/e215>]; 2000 (June 13). Dataset at <https://physionet.org/physiobank/database/eegmldb/>

²pyedflib module to read EDF files in python. <https://github.com/holgern/pyedflib>

³<https://www.ncbi.nlm.nih.gov/pmc/articles/PMC4873517/>

⁴https://en.wikipedia.org/wiki/Brain_connectivity_estimators

⁵Foster, J. J., Sutterer, D. W., Serences, J. T., Vogel, E. K., Awh, E. (2017). Alpha-Band Oscillations Enable Spatially and Temporally Resolved Tracking of Covert Spatial Attention. Psychological Science, 28(7), 929–941. <https://doi.org/10.1177/0956797617699167>

⁶New vistas for α -frequency band oscillations, Satu Palva, J. Matias Palva, Trends in Neurosciences, Elsevier, April 2007

⁷https://en.wikipedia.org/wiki/Beta_wave

Appendix A: Figures

Appendix B: Tables

Task	Class
1.1	mandatory
1.2	A
1.3	A
1.5	C
1.6	B
2.1	mandatory
2.2	D
2.4	C
2.5	B
2.7	C
3.1	mandatory
3.2	C
3.3	C
4.1	mandatory
4.2	B
4.3	C

Appendix C: First-level heading

When commands are referred to in this example file, they are always shown with their required arguments, using normal \TeX format. In this format, `#1`, `#2`, etc. stand for required author-supplied arguments to commands. For example, in `\section{#1}` the `#1` stands for the title text of the author's section heading, and in `\title{#1}` the `#1` stands for the title text of the paper.

Line breaks in section headings at all levels can be introduced using `\\`. A blank input line tells \TeX that the paragraph has ended.

1. Second-level heading: Formatting

This file may be formatted in both the `preprint` (the default) and `reprint` styles; the latter format may be used to mimic final journal output. Either format may be used for submission purposes; however, for peer review and production, AIP will format the article using the `preprint` class option. Hence, it is essential that authors check that their manuscripts format acceptably under `preprint`. Manuscripts submitted to AIP that do not format correctly under the `preprint` option may be delayed in both the editorial and production processes.

The `widetext` environment will make the text the width of the full page, as on page 4. (Note the use the `\pageref{#1}` to get the page number right automatically.) The width-changing commands only take effect in `twocolumn` formatting. It has no effect if `preprint` formatting is chosen instead.

a. Third-level heading: Citations and Footnotes

Citations in text refer to entries in the Bibliography; they use the commands `\cite{#1}` or `\onlinecite{#1}`. Because \LaTeX uses the `natbib` package of Patrick Daly, its entire repertoire of commands are available in your document; see the `natbib` documentation for further details. The argument of `\cite` is a comma-separated list of *keys*; a key may consist of letters and numerals.

By default, citations are numerical;[?] author-year citations are an option. To give a textual citation, use `\onlinecite{#1}`: (Refs. ? ? ?). \LaTeX “collapses” lists of consecutive numerical citations when appropriate. \LaTeX provides the ability to properly punctuate textual citations in author-year style; this facility works correctly with numerical citations only with `natbib`'s `compress` option turned off. To illustrate, we cite several together^{? ? ? ?}, and once again (Refs. ? ? ? ?). Note that, when numerical citations are used, the references were sorted into the same order they appear in the bibliography.

A reference within the bibliography is specified with a `\bibitem{#1}` command, where the argument is the ci-

tation key mentioned above. `\bibitem{#1}` commands may be crafted by hand or, preferably, generated by using \BibTeX . The AIP styles for \LaTeX 4 include \BibTeX style files `aipnum.bst` and `aipauth.bst`, appropriate for numbered and author-year bibliographies, respectively. \LaTeX 4 will automatically choose the style appropriate for the document's selected class options: the default is numerical, and you obtain the author-year style by specifying a class option of `author-year`.

This sample file demonstrates a simple use of \BibTeX via a `\bibliography` command referencing the `aipsamp.bib` file. Running \BibTeX (in this case `bibtex aipsamp`) after the first pass of \LaTeX produces the file `aipsamp.bbl` which contains the automatically formatted `\bibitem` commands (including extra markup information via `\bibinfo` commands). If not using \BibTeX , the `thebibliography` environment should be used instead.

a. *Fourth-level heading is run in.* Footnotes are produced using the `\footnote{#1}` command. Numerical style citations put footnotes into the bibliography[?]. Author-year and numerical author-year citation styles (each for its own reason) cannot use this method. Note: due to the method used to place footnotes in the bibliography, *you must re-run BibTeX every time you change any of your document's footnotes.*

Appendix D: Math and Equations

Inline math may be typeset using the `$` delimiters. Bold math symbols may be achieved using the `bm` package and the `\bm{#1}` command it supplies. For instance, a bold α can be typeset as `$\bm{\alpha}$` giving α . Fraktur and Blackboard (or open face or double struck) characters should be typeset using the `\mathfrak{#1}` and `\mathbb{#1}` commands respectively. Both are supplied by the `amssymb` package. For example, `\mathbb{R}` gives \mathbb{R} and `\mathfrak{G}` gives \mathfrak{G} .

In \LaTeX there are many different ways to display equations, and a few preferred ways are noted below. Displayed math will center by default. Use the class option `fleqn` to flush equations left.

Below we have numbered single-line equations, the most common kind:

$$\chi_+(p) \lesssim [2|\mathbf{p}|(|\mathbf{p}| + p_z)]^{-1/2} \left(\frac{|\mathbf{p}| + p_z}{px + ip_y} \right), \quad (\text{D1})$$

$$\left\{ 1234567890abc123\alpha\beta\gamma\delta1234556\alpha\beta\frac{1\sum_b^a}{A^2} \right\}. \quad (\text{D2})$$

Note the open one in Eq. (D2).

Not all numbered equations will fit within a narrow column this way. The equation number will move down automatically if it cannot fit on the same line with a

one-line equation:

$$\left\{ ab12345678abc123456abcdef\alpha\beta\gamma\delta1234556\alpha\beta\frac{1\sum_b^a}{A^2} \right\}. \quad (\text{D3})$$

When the `\label{#1}` command is used [cf. input for Eq. (D2)], the equation can be referred to in text without knowing the equation number that T_EX will assign to it. Just use `\ref{#1}`, where `#1` is the same name that used in the `\label{#1}` command.

Unnumbered single-line equations can be typeset using the `\[, \]` format:

$$g^+g^+ \rightarrow g^+g^+g^+g^+ \dots, \quad q^+q^+ \rightarrow q^+g^+g^+ \dots$$

1. Multiline equations

Multiline equations are obtained by using the `eqnarray` environment. Use the `\nonumber` command at the end of each line to avoid assigning a number:

$$\begin{aligned} \mathcal{M} = & ig_Z^2(4E_1E_2)^{1/2}(l_i^2)^{-1}\delta_{\sigma_1,-\sigma_2}(g_{\sigma_2}^e)^2\chi_{-\sigma_2}(p_2) \\ & \times [\epsilon_j l_i \epsilon_i]_{\sigma_1} \chi_{\sigma_1}(p_1), \end{aligned} \quad (\text{D4})$$

$$\begin{aligned} \sum |M_g^{\text{viol}}|^2 = & g_S^{2n-4}(Q^2) N^{n-2}(N^2-1) \\ & \times \left(\sum_{i<j} \right) \sum_{\text{perm}} \frac{1}{S_{12}} \frac{1}{S_{12}} \sum_{\tau} c_{\tau}^f. \end{aligned} \quad (\text{D5})$$

Note: Do not use `\label{#1}` on a line of a multiline equation if `\nonumber` is also used on that line. Incorrect cross-referencing will result. Notice the use `\text{#1}` for using a Roman font within a math environment.

To set a multiline equation without *any* equation numbers, use the `\begin{eqnarray*}`, `\end{eqnarray*}` format:

$$\begin{aligned} \sum |M_g^{\text{viol}}|^2 = & g_S^{2n-4}(Q^2) N^{n-2}(N^2-1) \\ & \times \left(\sum_{i<j} \right) \left(\sum_{\text{perm}} \frac{1}{S_{12}S_{23}S_{n1}} \right) \frac{1}{S_{12}}. \end{aligned}$$

$$\mathcal{R}^{(\text{d})} = g_{\sigma_2}^e \left(\frac{[\Gamma^Z(3,21)]_{\sigma_1}}{Q_{12}^2 - M_W^2} + \frac{[\Gamma^Z(13,2)]_{\sigma_1}}{Q_{13}^2 - M_W^2} \right) + x_W Q_e \left(\frac{[\Gamma^\gamma(3,21)]_{\sigma_1}}{Q_{12}^2 - M_W^2} + \frac{[\Gamma^\gamma(13,2)]_{\sigma_1}}{Q_{13}^2 - M_W^2} \right). \quad (\text{D7})$$

This is typed to show the output is in wide format. (Since

To obtain numbers not normally produced by the automatic numbering, use the `\tag{#1}` command, where `#1` is the desired equation number. For example, to get an equation number of (2.6'),

$$g^+g^+ \rightarrow g^+g^+g^+g^+ \dots, \quad q^+q^+ \rightarrow q^+g^+g^+ \dots \quad (2.6')$$

A few notes on `\tag{#1}`. `\tag{#1}` requires `amsmath`. The `\tag{#1}` must come before the `\label{#1}`, if any. The numbering set with `\tag{#1}` is *transparent* to the automatic numbering in REV_TE_X; therefore, the number must be known ahead of time, and it must be manually adjusted if other equations are added. `\tag{#1}` works with both single-line and multiline equations. `\tag{#1}` should only be used in exceptional case - do not use it to number all equations in a paper.

Enclosing single-line and multiline equations in `\begin{subequations}` and `\end{subequations}` will produce a set of equations that are “numbered” with letters, as shown in Eqs. (D6a) and (D6b) below:

$$\left\{ abc123456abcdef\alpha\beta\gamma\delta1234556\alpha\beta\frac{1\sum_b^a}{A^2} \right\}, \quad (\text{D6a})$$

$$\begin{aligned} \mathcal{M} = & ig_Z^2(4E_1E_2)^{1/2}(l_i^2)^{-1}(g_{\sigma_2}^e)^2\chi_{-\sigma_2}(p_2) \\ & \times [\epsilon_i]_{\sigma_1} \chi_{\sigma_1}(p_1). \end{aligned} \quad (\text{D6b})$$

Putting a `\label{#1}` command right after the `\begin{subequations}`, allows one to reference all the equations in a subequations environment. For example, the equations in the preceding subequations environment were Eqs. (D6).

a. Wide equations

The equation that follows is set in a wide format, i.e., it spans across the full page. The wide format is reserved for long equations that cannot be easily broken into four lines or less:

there is no input line between `\equation` and this paragraph, there is no paragraph indent for this paragraph.)