CHALMERS, GÖTEBORGS UNIVERSITET

EXAM for
COMPUTATIONAL BIOLOGY A

COURSE CODES: **FFR 110, FIM740GU, PhD**

Time:	August 29, 2018, at $08^{30} - 12^{30}$
Place:	Johanneberg
Teachers:	Kristian Gustafsson, 070-050 2211 (mobile), visits once around 10^{00}
Allowed material:	Mathematics Handbook for Science and Engineering
Not allowed:	any other written material, calculator

Maximum score on this exam: 12 points (need 5 points to pass).
Maximum score for homework problems: 18 points (need 7 points to pass).
CTH $\geq$15 grade 3; $\geq$20 grade 4; $\geq$25 grade 5,
GU $\geq$15 grade G; $\geq$ 23 grade VG.

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1. Short questions [3 points] For each of the following questions give a concise answer within a few lines per question.

a) Give two examples of how discrete dynamical systems can be obtained from continuous ones.

b) Explain how a discrete dynamical system of dimensionality one can be visualised using a cobweb plot. Sketch the cobweb plot for a system that in the long run show oscillations with period 2.

c) Explain the difference between the microscopic view of diffusion (Brownian motion) and a macroscopic view (Fick's law).

d) Explain why travelling wave solutions of reaction-diffusion equations (for example Fisher's equation) typically spread quicker than pure diffusive spread (law of diffusion) in the diffusion equation.

e) Sketch the wave fronts of the spiral wave with the phase

$$\phi(r,\theta) = 4\theta + r^2\,,$$

where r and θ are radial and angular coordinates.

f) Explain the long-term behaviour of the SIR model:

$$\dot{S} = -rSI\,, \qquad \dot{I} = rSI - \alpha I\,, \qquad \dot{R} = \alpha I\,.$$

How does an epidemic die out in the SIR-model? You do not need to do/show any calculations for this problem.

g) Explain the difference between stochastic and deterministic models for disease spreading. Under which circumstances is it better to use a stochastic model?

h) Give two examples of systems for which the Kuramoto model may be a reasonable model.

2. Delay differential model for whales [2.5 points] The following delay equation is a model for the population $N(t)$ of sexually mature blue whales

$$\frac{\mathrm{d}N}{\mathrm{d}t} = -dN(t) + N(t-T)\left[d + b\left\{1 - \left(\frac{N(t-T)}{K}\right)^z\right\}\right]. \tag{1}$$

Here d is a death rate, b a birth rate, K a carrying capacity, z models a non-linear per capita growth rate, and T is a delay time. Assume that all parameters are positive, $d > 0$, $b > 0$, $K > 0$, $z > 0$, and $T > 0$.

a) For the case $z = 1$, give a plausible motivation of the delay time T and the forms of the two terms on the right-hand side in Eq. (1). Why are there two terms proportional to the death rate, $-dN(t) + dN(t-T)$?

b) Find all steady states ($N(t) = \text{const.}$) of the delay equation (1).

c) Show that close to the most positive steady state, the dynamics of a small perturbation η can be approximated by

$$\frac{\mathrm{d}\eta}{\mathrm{d}t} \approx -d\eta(t) + (d - bz)\eta(t-T)\,. \tag{2}$$

d) Using the ansatz $\eta(t) = Ae^{\lambda t}$ in Eq. (2), derive an equation for λ.

e) By analyzing the equation for λ, contrast the dynamics for the two special cases $T = 0$ and very large T (compared to all other time scales of the problem). Explain the relevant time scale of the dynamics in the two cases.

3. Effect of spruce budworms on a forest [2.5 points] A model for the effect of a constant spruce budworm population of size P on a forest with average tree size $S(t)$ (surface area of brances) and 'energy reserve' $E(t)$ (health of the trees) is given by

$$\begin{aligned} \dot{S} &= r_S S\left(1 - \frac{S}{K_S}\frac{K_E}{E}\right) \\ \dot{E} &= r_E E\left(1 - \frac{E}{K_E}\right) - P\frac{B}{S} \end{aligned}, \tag{3}$$

where r_S, r_E, K_S, K_E, and P are positive parameters.

a) Interpret all the terms in Eq. (3) from a biological viewpoint.

b) Change to suitable dimensionless units and rewrite the system (3) in terms of two dimensionless parameters.

c) Using a graphical method, show that your dimensionless system in subtask b) has two biologically relevant fixed points if B is small and no relevant fixed point if B is large.

d) What is the critical level of B (in terms of the other dimensional parameters) above which no biologically relevant steady state exists.

e) Given that one of the biologically relevant fixed points is stable if it exists, discuss possible effects of refuge and outbreaks of spruce budworms on the forest.

4. A linear reaction-diffusion equation [2.5 points] Consider the reaction-diffusion equation

$$\begin{aligned} \frac{\partial u}{\partial t} &= u + 3v - 4 + \frac{\partial^2 u}{\partial x^2} \\ \frac{\partial v}{\partial t} &= -u - 2v + 3 + 8\frac{\partial^2 v}{\partial x^2} \end{aligned}. \tag{4}$$

a) Find the homogeneous steady state (u^*, v^*) of the system (4) and determine its stability.

b) By making an ansatz $(u, v) = (u^*, v^*) + e^{\lambda t + ikx}(\delta u_0, \delta v_0)$ where $(\delta u_0, \delta v_0)$ are (possibly complex) constants, show that λ is related to k by

$$\lambda(k) = \frac{1}{2}\left(-1 - 9k^2 \pm \sqrt{49k^4 + 42k^2 - 3}\right).$$

c) Show that the homogeneous steady state becomes unstable to linear perturbations for a range of wave numbers $k^2_{\text{min}} < k^2 < k^2_{\text{max}}$. Determine k^2_{min} and k^2_{max}.

d) Assume that the spatial part of the system (4) is constrained to zero at $x = 0$ and $x = 3\pi$. Which k-values in the ansatz in subtask b) are relevant for this constrained dynamics? Sketch how a small perturbation in this system evolves in time.

5. Noise in time series [1.5 points] Consider a general time series x_0, x_1, ... generated by a map F:

$$x_{n+1} = F(x_n)\,.$$

Measurement noise in a time series is defined as an error in the observations of the values of x_n. Dynamical noise is defined as an inherent disturbance to the dynamics: at each time step an error is added to the map. In what follows, you can assume the measurement and dynamical noises to be Gaussian white noise with variance σ^2.

a) Discuss and contrast the effects of measurement noise and dynamical noise on a time series which is generated by linear (Malthus) decay (negative growth rate).

b) Assume that you are given noisy time series data and that you know that the data is generated by an underlying linear (Malthus) decay map. Discuss how the (negative) growth rate of the underlying map can be recovered from the time series for the two cases of measurement noise and of dynamical noise. For which case is it easier to reconstruct the underlying growth rate?