In[•]:=
$$\lambda = \text{Eigenvalues}\left[\begin{pmatrix} \sigma+1 & 3 \\ -2 & \sigma-1 \end{pmatrix}\right]$$

Out[•]=
$$\left\{-i\sqrt{5}+\sigma,\ i\sqrt{5}+\sigma\right\}$$

In[•]:=
$$V = \text{Eigenvectors}\left[\begin{pmatrix} \sigma+1 & 3 \\ -2 & \sigma-1 \end{pmatrix}\right]$$

$$\left\{\left\{\frac{1}{2}\left(-1+i\sqrt{5}\right),1\right\},\left\{\frac{1}{2}\left(-1-i\sqrt{5}\right),1\right\}\right\}$$

In[•]:=
$$\{\{A,\ B\}\} = \text{Solve}[C1 * V[[1]] + C2 * V[[2]] == \{u,\ v\},\ \{C1,\ C2\}]$$

Out[•]=
$$\left\{\left\{C1 \to -\frac{1}{10}\,i\left(2\sqrt{5}\,u+5\,i\,v+\sqrt{5}\,v\right),\ C2 \to \frac{1}{10}\,i\left(2\sqrt{5}\,u-5\,i\,v+\sqrt{5}\,v\right)\right\}\right\}$$

In[•]:=
$$C1 = -\frac{1}{10}\,i\left(2\sqrt{5}\,u+5\,i\,v+\sqrt{5}\,v\right)$$

Out[•]=
$$-\frac{1}{10}\,i\left(2\sqrt{5}\,u+5\,i\,v+\sqrt{5}\,v\right)$$

In[•]:=
$$C2 = \frac{1}{10}\,i\left(2\sqrt{5}\,u-5\,i\,v+\sqrt{5}\,v\right)$$

Out[•]=
$$\frac{1}{10}\,i\left(2\sqrt{5}\,u-5\,i\,v+\sqrt{5}\,v\right)$$

In[•]:=
$$S = C1 * \text{Exp}[\lambda[[1]] * t] * V[[1]] + C2 * \text{Exp}[\lambda[[2]] * t] * V[[2]]$$

Out[•]=
$$\left\{\frac{1}{20}\,i\left(-1-i\sqrt{5}\right)e^{t\left(i\sqrt{5}+\sigma\right)}\left(2\sqrt{5}\,u-5\,i\,v+\sqrt{5}\,v\right)-\frac{1}{20}\,i\left(-1+i\sqrt{5}\right)e^{t\left(-i\sqrt{5}+\sigma\right)}\left(2\sqrt{5}\,u+5\,i\,v+\sqrt{5}\,v\right),\ \frac{1}{10}\,i\,e^{t\left(i\sqrt{5}+\sigma\right)}\left(2\sqrt{5}\,u-5\,i\,v+\sqrt{5}\,v\right)-\frac{1}{10}\,i\,e^{t\left(-i\sqrt{5}+\sigma\right)}\left(2\sqrt{5}\,u+5\,i\,v+\sqrt{5}\,v\right)\right\}$$

In[•]:=
$$S = \text{Simplify}[\text{ComplexExpand}[S]]$$

Out[•]=
$$\left\{\frac{1}{5}\,e^{t\,\sigma}\left(5\,u\,\text{Cos}\left[\sqrt{5}\,t\right]+\sqrt{5}\,(u+3\,v)\,\text{Sin}\left[\sqrt{5}\,t\right]\right),\ \frac{1}{5}\,e^{t\,\sigma}\left(5\,v\,\text{Cos}\left[\sqrt{5}\,t\right]-\sqrt{5}\,(2\,u+v)\,\text{Sin}\left[\sqrt{5}\,t\right]\right)\right\}$$

In[•]:=
$$S$$

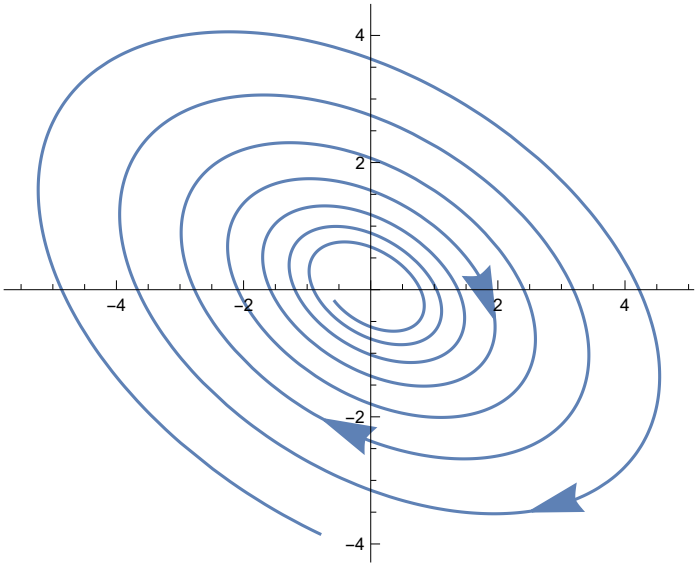
Out[•]=
$$\left\{\frac{1}{5}\,e^{t\,\sigma}\left(5\,u\,\text{Cos}\left[\sqrt{5}\,t\right]+\sqrt{5}\,(u+3\,v)\,\text{Sin}\left[\sqrt{5}\,t\right]\right),\ \frac{1}{5}\,e^{t\,\sigma}\left(5\,v\,\text{Cos}\left[\sqrt{5}\,t\right]-\sqrt{5}\,(2\,u+v)\,\text{Sin}\left[\sqrt{5}\,t\right]\right)\right\}$$

In[•]:=
$$S /. \{u \to 1,\ v \to 1,\ t \to 1,\ \sigma \to 0\}$$

Out[•]=
$$\left\{\frac{1}{5}\left(5\,\text{Cos}\left[\sqrt{5}\right]+4\sqrt{5}\,\text{Sin}\left[\sqrt{5}\right]\right),\ \frac{1}{5}\left(5\,\text{Cos}\left[\sqrt{5}\right]-3\sqrt{5}\,\text{Sin}\left[\sqrt{5}\right]\right)\right\}$$

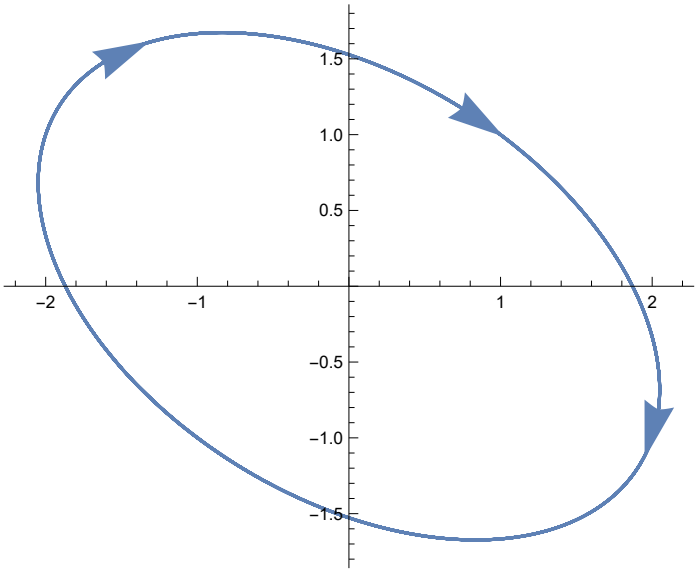
In[•]:=
```
ParametricPlot[S /. {u → 1, v → 1, σ → -1 / 10}, {t, -10, 10}] /.
  Line[x_] :> {Arrowheads[{0., 0.07, 0.07, 0.07, 0.}], Arrow[x]}
```

Out[ ]=



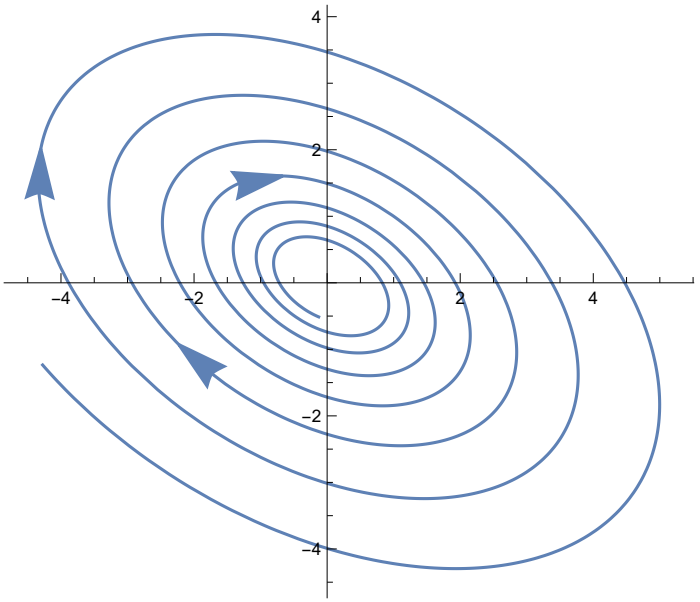
In[ ]:=
```
ParametricPlot[S /. {u → 1, v → 1, σ → 0}, {t, -10, 10}] /.
  Line[x_] :→ {Arrowheads[{0., 0.07, 0.07, 0.07, 0.}], Arrow[x]}
```

Out[ ]=



In[]:=
```
ParametricPlot[S /. {u → 1, v → 1, σ → 1 / 10}, {t, -10, 10}] /.
 Line[x_] :> {Arrowheads[{0., 0.07, 0.07, 0.07, 0.}], Arrow[x]}
```

Out[]=



In[]:=
```
FunctionPeriod[S /. {u → 1, v → 1, σ → 0}, t]
```

Out[]=
$$\frac{2\pi}{\sqrt{5}}$$

In[]:=
```
max = FindMaximum[Norm[S] /. {u → 1, v → 1, σ → 0}, t]
```

FindMaximum: The line search decreased the step size to within the tolerance specified by AccuracyGoal and PrecisionGoal but was unable to find a sufficient increase in the function. You may need more than MachinePrecision digits of working precision to meet these tolerances.

Out[]=
```
{2.25061, {t → 0.637691}}
```

```
min = FindMinimum[Norm[S] /. {u → 1, v → 1, σ → 0}, t]
```

FindMinimum: The line search decreased the step size to within the tolerance specified by AccuracyGoal and PrecisionGoal but was unable to find a sufficient decrease in the function. You may need more than MachinePrecision digits of working precision to meet these tolerances.

Out[]=
```
{4.0723, {t → 1.24179}}
```

In[]:=
```
max[[1]] / min[[1]]
```

Out[]=
```
1.61803
```

In[]:=
```
dir = S /. {u → 1, v → 1, σ → 0, max[[2]][[1]]}
```

Out[]=
```
{1.91448, -1.18322}
```

In[]:=
```
Normalize[dir]
```

Out[]=
```
{0.850651, -0.525731}
```