In[]:=
```
f = -x^3;
g = 2y^3;
```

In[]:=
```
f = -x^2;
g = 2x^2;
```

In[]:=
```
Solve[16 a == D[f, x, x, x] + D[f, x, y, y] + D[g, x, x, y] + D[g, y, y, y] +
    1 / ω (D[f, x, y] * (D[f, x, x] + D[f, y, y]) - D[g, x, y] * (D[g, x, x] + D[g, y, y]) -
       D[f, x, x] * D[g, x, x] + D[f, y, y] * D[g, y, y]), a] /. ω → -1
```

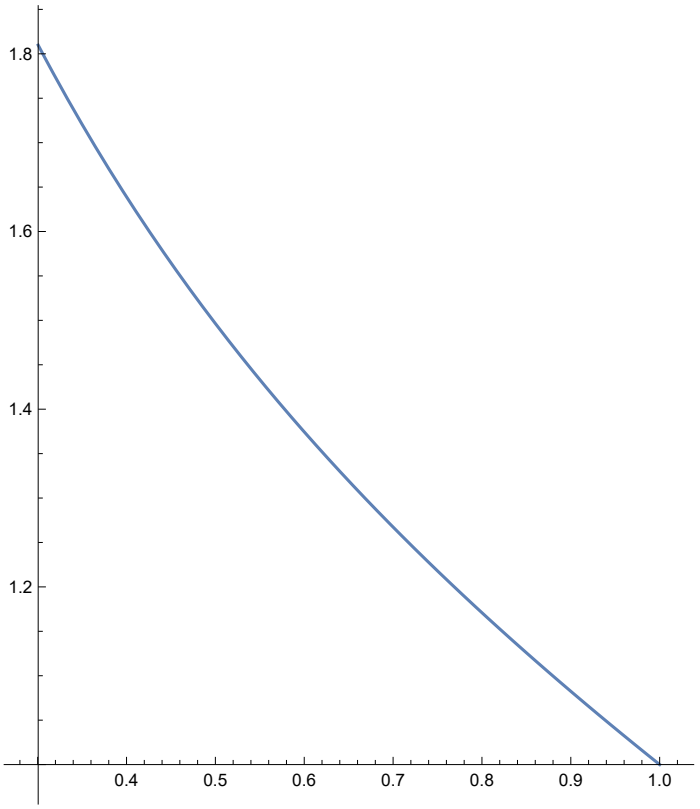
Out[]=
$$\left\{\left\{a \to -\frac{1}{2}\right\}\right\}$$

In[]:=
```
s = NDSolve[{x'[t] == μ * x[t] - 3 y[t] - x[t]^3,
    y'[t] == 3 x[t] + μ * y[t] + 2 y[t]^3, x[0] == y[0] == 1} /. μ → -1, {x, y}, {t, 0, 10}]

ParametricPlot[Evaluate[{x[t], y[t]} /. s], {t, 0, 0.139}]
```

NDSolve: At t == 0.22039904532598636`, step size is effectively zero; singularity or stiff system suspected.

Out[]=
$$\left\{\left\{x \to \text{InterpolatingFunction}\left[\text{Domain: } \{\{0., 0.22\}\} \text{ Output: scalar}\right],\right.\right.$$
$$\left.\left.y \to \text{InterpolatingFunction}\left[\text{Domain: } \{\{0., 0.22\}\} \text{ Output: scalar}\right]\right\}\right\}$$

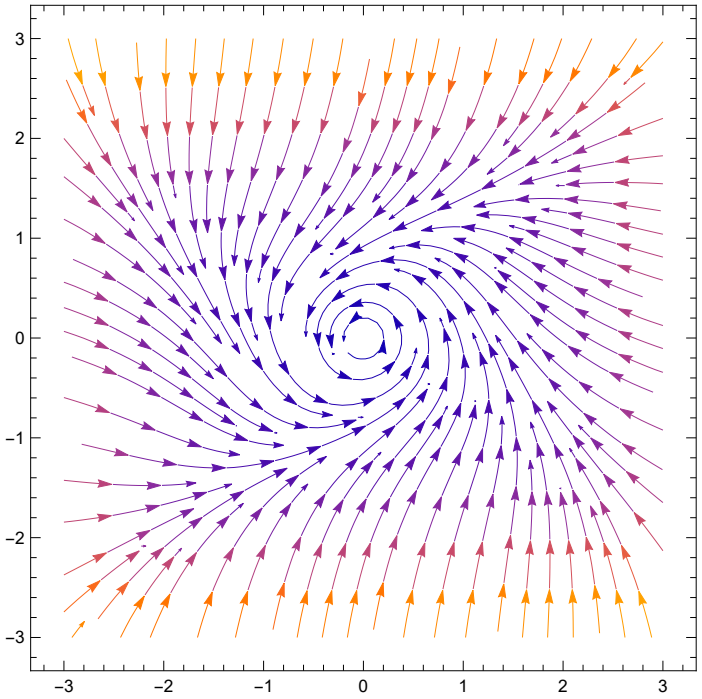
Out[]=



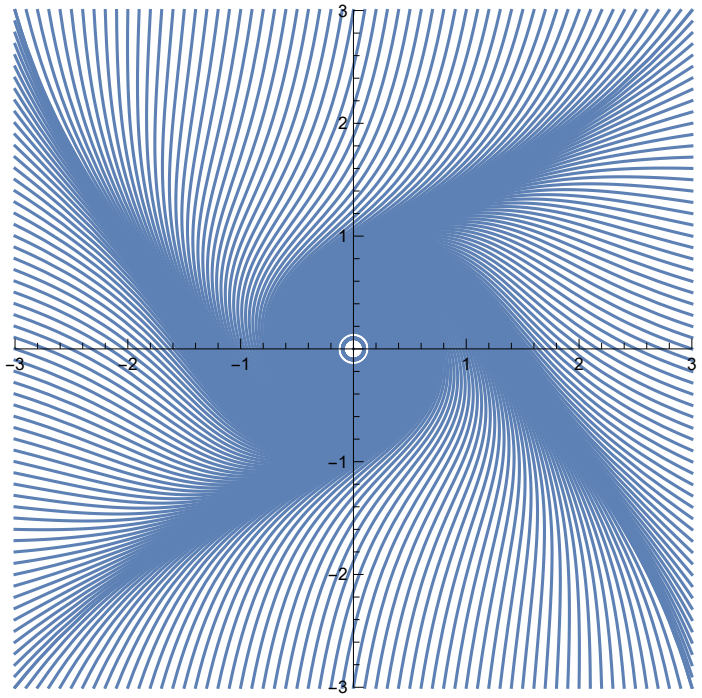
```
(*1 a > 0 subcritical*)
μ = .;
f[x_, y_] = μ * x - 3 y - x^3;
g[x_, y_] = 3 x + μ * y - 2 y^3;

μ = -1
StreamPlot[{f[x, y], g[x, y]} /. μ → -1, {x, -3, 3}, {y, -3, 3}]
minx = -3;
maxx = 3;
miny = -3;
maxy = 3;
sol[x0_ , y0_] := NDSolve[{x'[t] == μ * x[t] - 3 * y[t] - x[t]^3,
   y'[t] == 3 * x[t] + μ * y[t] - 2 * y[t]^3 , x[0] == x0 , y[0] == y0}, {x, y} , {t, 0, 10}]
initialCondition = Join[Table[{0, y}, {y , miny , maxy , 0.1}] ,
   Table[{minx, y}, {y , miny , maxy , 0.1}] , Table[{maxx, y}, {y , miny , maxy, 0.1}] ,
   Table[{x, miny}, {x , minx , maxx , 0.1}] , Table[{x, maxy}, {x , minx , maxx, 0.1}]];
Show[Table[ParametricPlot[Evaluate[{x[t], y[t]} /.
     sol[initialCondition[[i, 1]] , initialCondition[[i, 2]]]], {t, 0, 10},
   PlotRange → {{minx, maxx}, {miny, maxy}}] , {i, Length[initialCondition]} ]]

μ = 0
StreamPlot[{f[x, y], g[x, y]} /. μ → 0, {x, -3, 3}, {y, -3, 3}]
minx = -3;
maxx = 3;
miny = -3;
maxy = 3;
sol[x0_ , y0_] := NDSolve[{x'[t] == μ * x[t] - 3 * y[t] - x[t]^3,
   y'[t] == 3 * x[t] + μ * y[t] - 2 * y[t]^3 , x[0] == x0 , y[0] == y0}, {x, y} , {t, 0, 10}]
initialCondition = Join[Table[{minx, y}, {y , miny , maxy , 0.1}] ,
   Table[{maxx, y}, {y , miny , maxy, 0.1}] , Table[{x, miny}, {x , minx , maxx , 0.1}] ,
   Table[{x, maxy}, {x , minx , maxx, 0.1}]];
Show[Table[ParametricPlot[Evaluate[{x[t], y[t]} /.
     sol[initialCondition[[i, 1]] , initialCondition[[i, 2]]]], {t, 0, 10},
   PlotRange → {{minx, maxx}, {miny, maxy}}] , {i, Length[initialCondition]} ]]
```

Out[1854]= 0

Out[1855]=



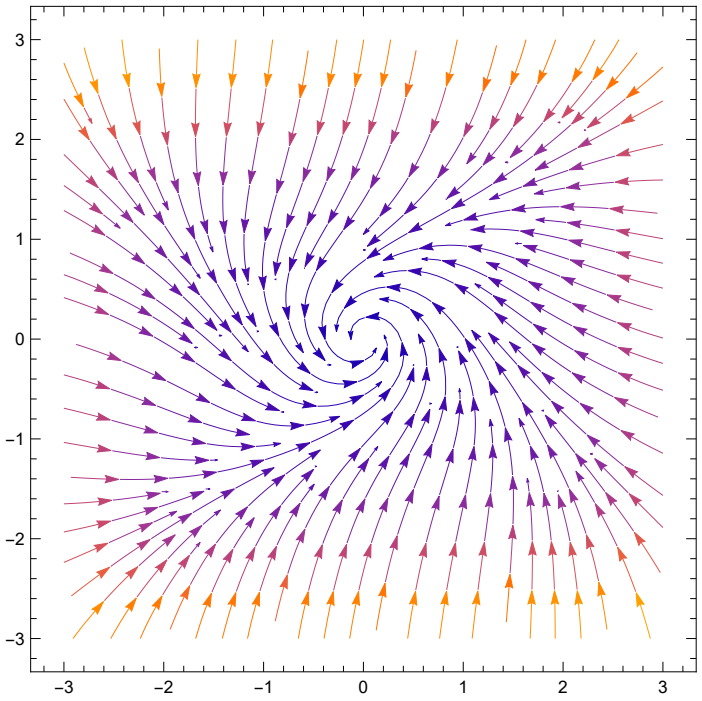
Out[1862]=



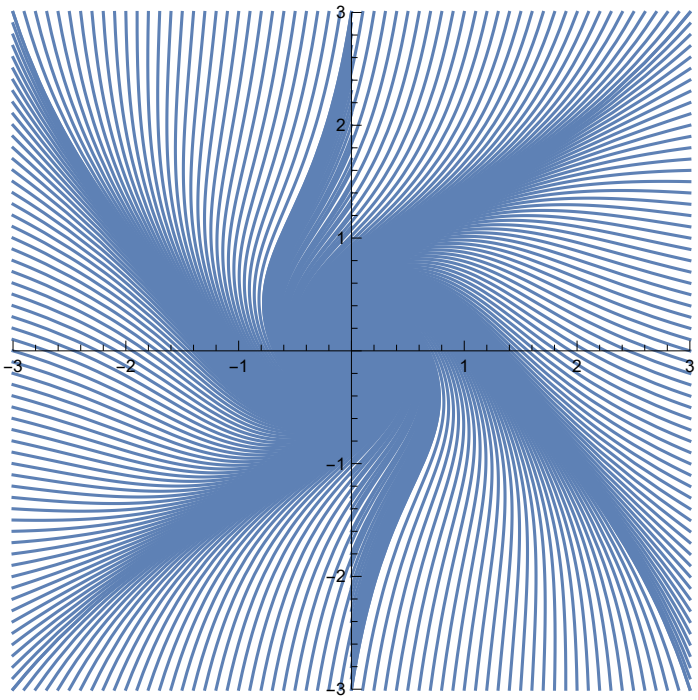
```
μ = 1
StreamPlot[{f[x, y], g[x, y]} /. μ → 1, {x, -3, 3}, {y, -3, 3}]
minx = -3;
maxx = 3;
miny = -3;
maxy = 3;
sol[x0_, y0_] := NDSolve[{x'[t] == μ * x[t] - 3 * y[t] - x[t]^3,
   y'[t] == 3 * x[t] + μ * y[t] - 2 * y[t]^3, x[0] == x0, y[0] == y0}, {x, y}, {t, 0, 10}]
initialCondition = Join[Table[{0, y}, {y, miny, maxy, 0.1}],
   Table[{minx, y}, {y, miny, maxy, 0.1}], Table[{maxx, y}, {y, miny, maxy, 0.1}],
   Table[{x, miny}, {x, minx, maxx, 0.1}], Table[{x, maxy}, {x, minx, maxx, 0.1}]];
Show[Table[ParametricPlot[Evaluate[{x[t], y[t]} /.
     sol[initialCondition[[i, 1]], initialCondition[[i, 2]]], {t, 0, 10},
   PlotRange → {{minx, maxx}, {miny, maxy}}], {i, Length[initialCondition]}]]
```

Out[1791]= -1

Out[1792]=

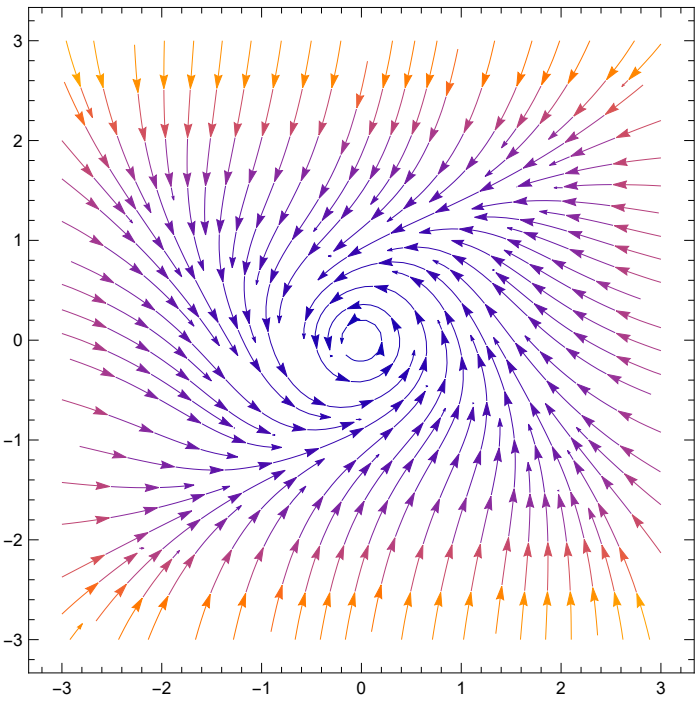


Out[1799]=

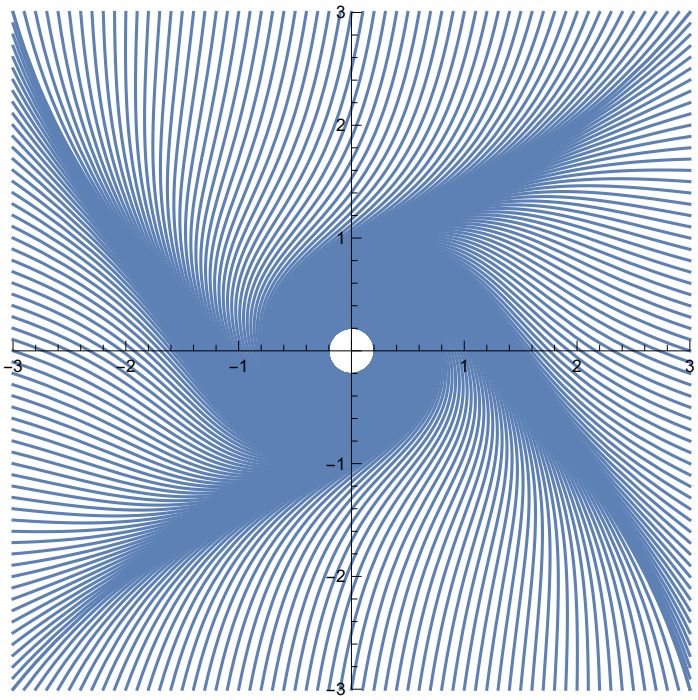


Out[1800]= 0

Out[1801]=

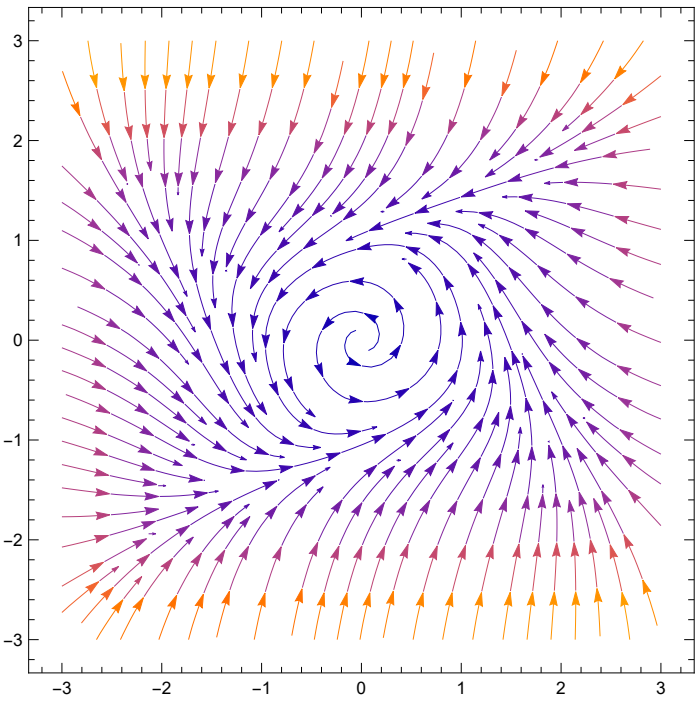


Out[1808]=

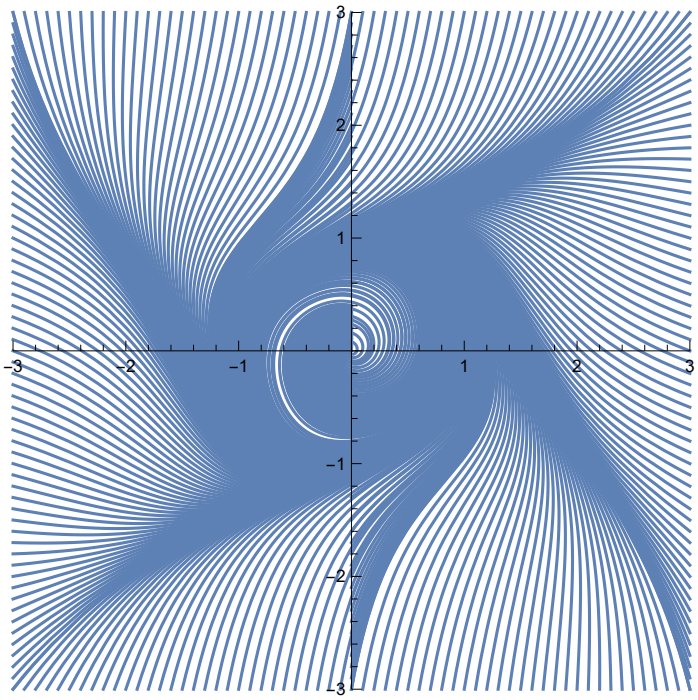


Out[1809]= 1

Out[1810]=



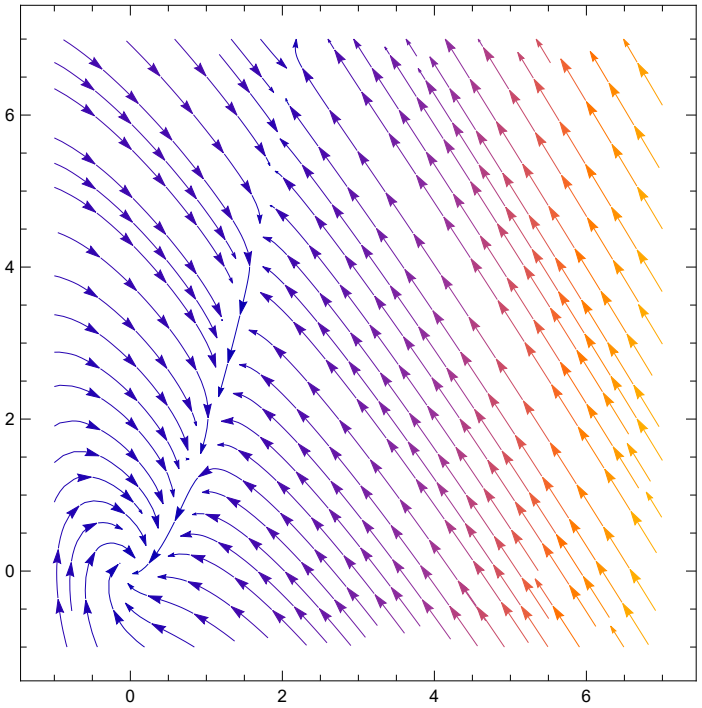
Out[1817]=



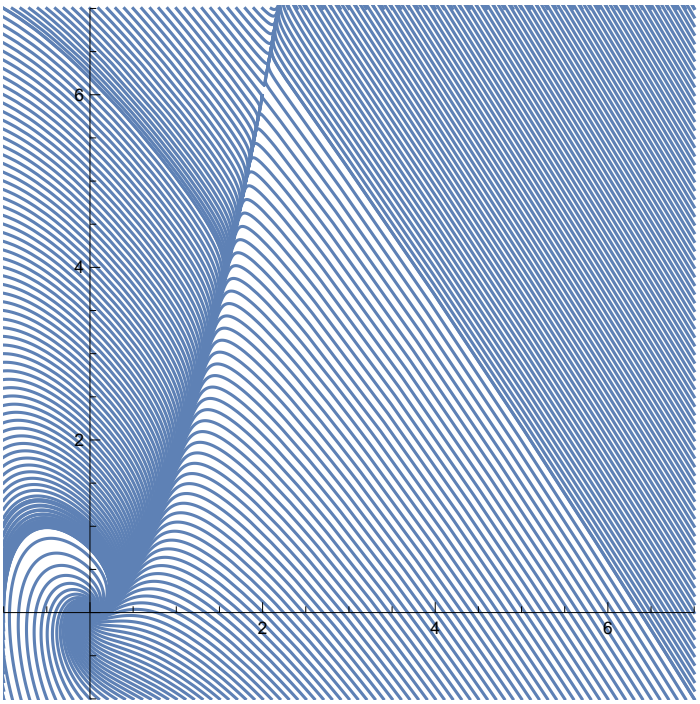
In[1660]:=
```
(*2 a < 0 supercritical*)
Clear["Global`*"]
μ = .;
f[x_, y_] = μ * x + y - x^2;
g[x_, y_] = -x + μ * y + 2 x^2;

μ = -1
Solve[f[x, y] == 0 && g[x, y] == 0, {x, y}];
StreamPlot[{f[x, y], g[x, y]}, {x, -1, 7}, {y, -1, 7}]
minx = -1;
maxx = 7;
miny = -1;
maxy = 7;
sol[x0_ , y0_] := NDSolve[{x'[t] == μ * x[t] + y[t] - x[t]^2,
   y'[t] == -x[t] + μ * y[t] + 2 * x[t]^2 , x[0] == x0 , y[0] == y0}, {x, y} , {t, 0, 10}]
initialCondition = Join[Table[{minx, y}, {y , miny , maxy , 0.1}] ,
   Table[{maxx, y}, {y , miny , maxy, 0.1}] , Table[{x, miny}, {x , minx , maxx , 0.1}] ,
   Table[{x, maxy}, {x , minx , maxx, 0.1}]];
Show[Table[ParametricPlot[Evaluate[{x[t], y[t]} /.
     sol[initialCondition[[i, 1]] , initialCondition[[i, 2]]]], {t, 0, 10},
   PlotRange → {{minx, maxx}, {miny, maxy}}] , {i, Length[initialCondition]} ]]
```

Out[1664]= −1

Out[1666]=



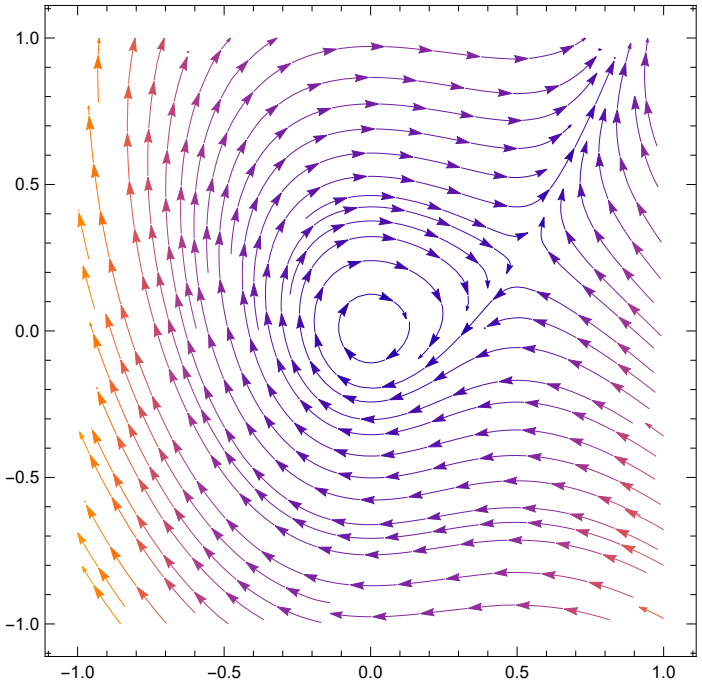
Out[1673]=



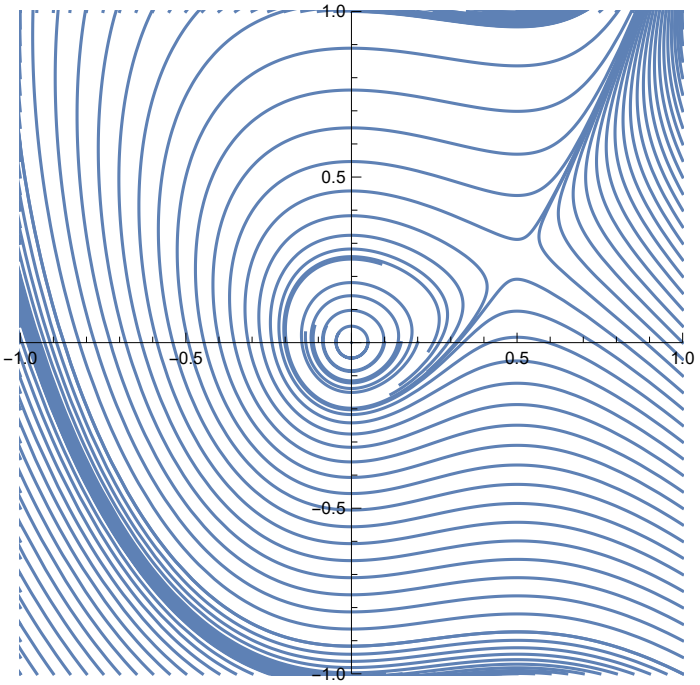
In[1778]:=
```
μ = 0
Solve[f[x, y] == 0 && g[x, y] == 0, {x, y}];
StreamPlot[{f[x, y], g[x, y]}, {x, -1, 1}, {y, -1, 1}]
minx = -1;
maxx = 1;
miny = -1;
maxy = 1;
sol[x0_, y0_] := NDSolve[{x'[t] == μ * x[t] + y[t] - x[t]^2,
   y'[t] == -x[t] + μ * y[t] + 2 * x[t]^2, x[0] == x0, y[0] == y0}, {x, y}, {t, 0, 10}]
initialCondition = Join[Table[{minx, y}, {y, miny, maxy, 0.05}],
   Table[{x, 0}, {x, 0, 0.2, 0.05}], Table[{maxx, y}, {y, miny, maxy, 0.05}],
   Table[{x, miny}, {x, minx, maxx, 0.05}],
   Table[{x, maxy}, {x, minx, maxx, 0.05}]];
Show[Table[ParametricPlot[Evaluate[{x[t], y[t]} /.
     sol[initialCondition[[i, 1]], initialCondition[[i, 2]]]], {t, 0, 10},
   PlotRange → {{minx, maxx}, {miny, maxy}}], {i, Length[initialCondition]}]]
```

Out[1778]= 0

Out[1780]=



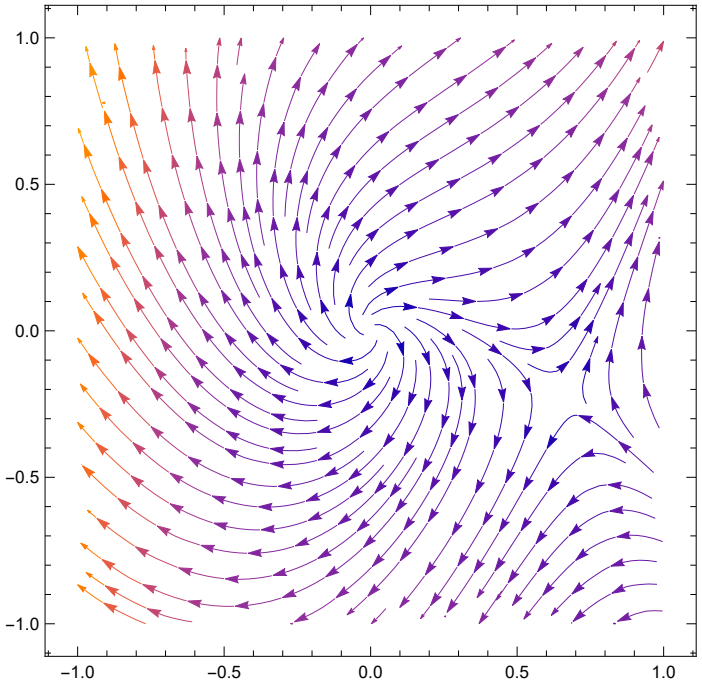
Out[1787]=



In[1746]:=
```
μ = 1
Solve[f[x, y] == 0 && g[x, y] == 0, {x, y}];
StreamPlot[{f[x, y], g[x, y]}, {x, -1, 1}, {y, -1, 1}]
minx = -1;
maxx = 1;
miny = -1;
maxy = 1;
sol[x0_, y0_] := NDSolve[{x'[t] == μ * x[t] + y[t] - x[t]^2,
   y'[t] == -x[t] + μ * y[t] + 2 * x[t]^2, x[0] == x0, y[0] == y0}, {x, y}, {t, 0, 10}]
initialCondition = Join[Table[{0, y}, {y, miny, maxy, 0.01}],
   Table[{x, 0}, {x, minx, maxx, 0.01}], Table[{1, y}, {y, 0, -1, -0.01}]];
Show[Table[ParametricPlot[Evaluate[{x[t], y[t]} /.
     sol[initialCondition[[i, 1]], initialCondition[[i, 2]]]], {t, 0, 10},
   PlotRange → {{minx, maxx}, {miny, maxy}}], {i, Length[initialCondition]}]]
```

Out[1746]= 1

Out[1748]=



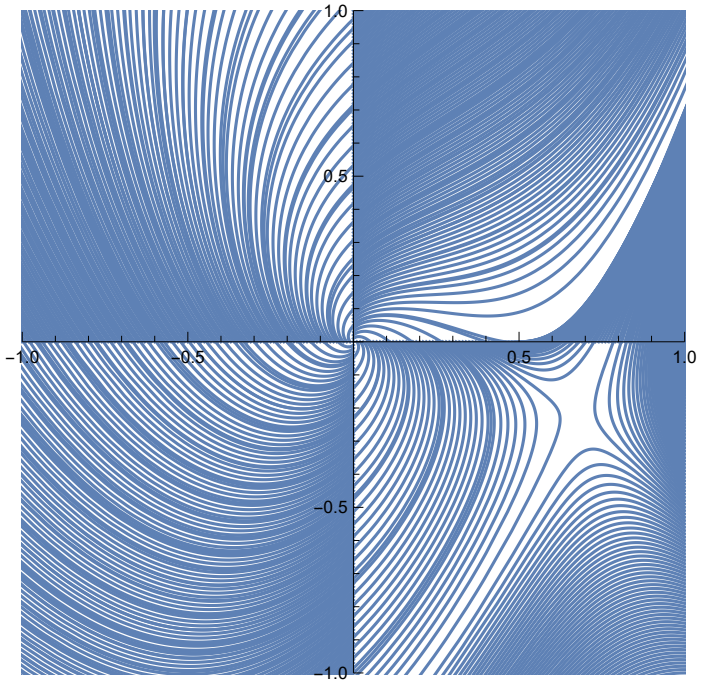
NDSolve: At t == 1.2547658246433089`, step size is effectively zero; singularity or stiff system suspected.

NDSolve: At t == 1.2604975447967204`, step size is effectively zero; singularity or stiff system suspected.

NDSolve: At t == 1.266320661779653`, step size is effectively zero; singularity or stiff system suspected.

General: Further output of NDSolve::ndsz will be suppressed during this calculation.

Out[1755]=



Out[1745]= {{1, 0.}, {1, −0.01}, {1, −0.02}, {1, −0.03}, {1, −0.04}, {1, −0.05}, {1, −0.06},
{1, −0.07}, {1, −0.08}, {1, −0.09}, {1, −0.1}, {1, −0.11}, {1, −0.12}, {1, −0.13},
{1, −0.14}, {1, −0.15}, {1, −0.16}, {1, −0.17}, {1, −0.18}, {1, −0.19}, {1, −0.2},
{1, −0.21}, {1, −0.22}, {1, −0.23}, {1, −0.24}, {1, −0.25}, {1, −0.26},
{1, −0.27}, {1, −0.28}, {1, −0.29}, {1, −0.3}, {1, −0.31}, {1, −0.32}, {1, −0.33},
{1, −0.34}, {1, −0.35}, {1, −0.36}, {1, −0.37}, {1, −0.38}, {1, −0.39}, {1, −0.4},
{1, −0.41}, {1, −0.42}, {1, −0.43}, {1, −0.44}, {1, −0.45}, {1, −0.46},
{1, −0.47}, {1, −0.48}, {1, −0.49}, {1, −0.5}, {1, −0.51}, {1, −0.52}, {1, −0.53},
{1, −0.54}, {1, −0.55}, {1, −0.56}, {1, −0.57}, {1, −0.58}, {1, −0.59}, {1, −0.6},
{1, −0.61}, {1, −0.62}, {1, −0.63}, {1, −0.64}, {1, −0.65}, {1, −0.66},
{1, −0.67}, {1, −0.68}, {1, −0.69}, {1, −0.7}, {1, −0.71}, {1, −0.72}, {1, −0.73},
{1, −0.74}, {1, −0.75}, {1, −0.76}, {1, −0.77}, {1, −0.78}, {1, −0.79}, {1, −0.8},
{1, −0.81}, {1, −0.82}, {1, −0.83}, {1, −0.84}, {1, −0.85}, {1, −0.86},
{1, −0.87}, {1, −0.88}, {1, −0.89}, {1, −0.9}, {1, −0.91}, {1, −0.92}, {1, −0.93},
{1, −0.94}, {1, −0.95}, {1, −0.96}, {1, −0.97}, {1, −0.98}, {1, −0.99}, {1, −1.}}