In[2908]:=
```
Clear["Global`*"]
f[x_, y_] := μ * x + y - x^2;
g[x_, y_] := -x + μ * y + 2 x^2;
sol = Solve[f[x, y] == 0 && g[x, y] == 0, {x, y}]
J[x_, y_] := ( D[f[x, y], x]  D[f[x, y], y]
               D[g[x, y], x]  D[g[x, y], y] );
μ = 0;
J[x, y] /. x → 0 // MatrixForm
eval = J[x, y] /. sol[[2]] // Eigenvalues
evec = J[x, y] /. sol[[2]] // Eigenvectors;
```

Out[2911]= $\left\{\{x \to 0, y \to 0\}, \left\{x \to \frac{1+\mu^2}{2+\mu}, y \to \frac{1-2\mu+\mu^2-2\mu^3}{(2+\mu)^2}\right\}\right\}$

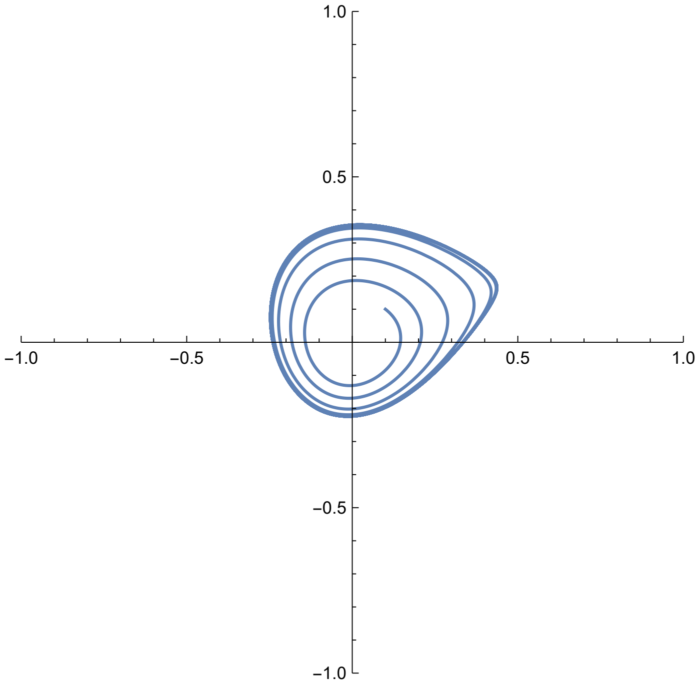
Out[2914]//MatrixForm=

$$\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

Out[2915]= $\left\{\frac{1}{2} \times \left(-1-\sqrt{5}\right), \frac{1}{2} \times \left(-1+\sqrt{5}\right)\right\}$

In[∘]:=
```
s = NDSolve[{x'[t] == μ * x[t] + y[t] - x[t]^2, y'[t] == -x[t] + μ * y[t] + 2 x[t]^2,
    x[0] == y[0] == 0.1} /. μ → 0.060, {x, y}, {t, 0, 100}];
ParametricPlot[Evaluate[{x[t], y[t]} /. s], {t, 0, 100}, PlotRange → {{-1, 1}, {-1, 1}}]
```

Out[∘]=



In[2954]:=
```
s =.
sol = DSolve[
   {x'[t] == u * x[t], y'[t] == s * y[t], x[0] == γ, y[0] == 1}, {x[t], y[t]}, t] // Flatten
```

Out[2955]= $\{x[t] \to e^{t u} \gamma, y[t] \to e^{s t}\}$

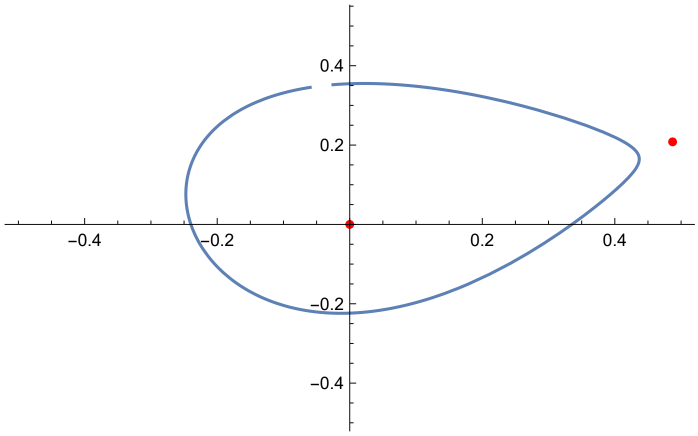
In[∘]:=
```
Solve[x[t] == 1 /. sol, t]
```

$$\left\{\left\{t \to \boxed{\frac{2\, i\, \pi\, c_1 + \text{Log}\left[\frac{1}{\gamma}\right]}{u} \text{ if } c_1 \in \mathbb{Z}}\right\}\right\}$$

Out[∘]= $$\left\{\left\{t \to \boxed{\frac{2\, i\, \pi\, c_1 + \text{Log}\left[\frac{1}{\gamma}\right]}{u} \text{ if } c_1 \in \mathbb{Z}}\right\}\right\}$$

In[3243]:=
```
Clear["Global`*"]
μc = 0.066;
f[x_, y_] := μ x + y - x^2;
g[x_, y_] := -x + μ y + 2 x^2;
μ = 0.060;
tMin = 165;
tMax = tMin + 9.8624007122786 - 0.1;
fp = Solve[{f[x, y] == 0, g[x, y] == 0}, {x, y}];
data = {{fp[[1, 1, 2]], fp[[1, 2, 2]]}, {fp[[2, 1, 2]], fp[[2, 2, 2]]}};
line = Fit[data, {1, x}, x];
p1 = ListPlot[data, PlotStyle → Red];
s = NDSolve[{x'[t] == μ * x[t] + y[t] - x[t]^2,
    y'[t] == -x[t] + μ * y[t] + 2 x[t]^2, x[0] == y[0] == 0.1}, {x, y}, {t, tMin, tMax}];
p3 = ParametricPlot[Evaluate[{x[t], y[t]} /. s], {t, tMin, tMax},
   PlotRange → {{-1, 1}, {-1, 1}}] ;
Show[p1, p3, PlotRange → {{-0.5, 0.5}, {-0.5, 0.5}}]
```

Solve: Solve was unable to solve the system with inexact coefficients. The answer was obtained by solving a corresponding exact system and numericizing the result.

Out[3256]=


In[3257]:=
```
(*μ, γ*)
tIntersect = Solve[{y[t] == fp[[2, 2, 2]]} /. s[[1, 2]], t][[1, 1, 2]];
xIf = s[[1, 1, 2]];

γ = xIf[tIntersect] (* x intersect *)
xVal = Log[Abs[μ - μc]]
yVal = Log[γ]
```

Solve: Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information.

InterpolatingFunction: Input value {164.312} lies outside the range of data in the interpolating function. Extrapolation will be used.

Out[3259]= -0.21211

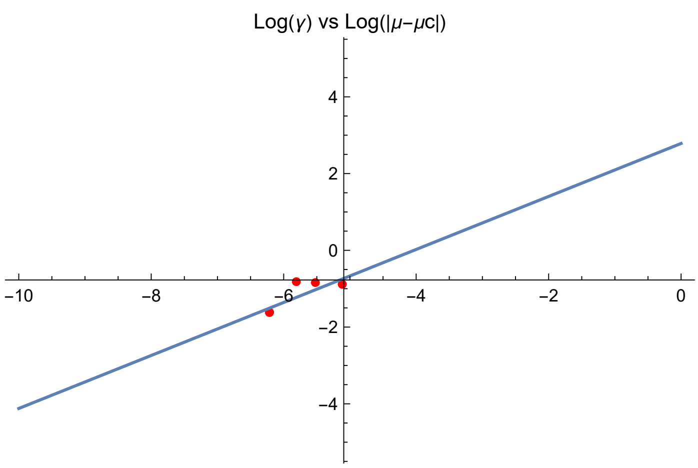
Out[3260]= -5.116

Out[3261]= -1.55065 + 3.14159 i

In[3116]:=
```
plotData = {{-5.115995809754081, -0.8831824175751058},
   {-5.115995809754081, -0.8831824175751058}, {-5.521460917862245,
    -0.8397348233958074}, {-5.809142990314027, -0.8169247714703272},
   {-6.2146080984221905, -1.6161665232963962}, {-6.2146080984221905,
    -1.6161665232963962}, {-6.2146080984221905, -1.6161665232963962}};
line = Fit[plotData, {1, x}, x]
gammaPlot = Show[ListPlot[plotData, PlotStyle → Red], Plot[line, {x, -10, 0}],
  PlotRange → {{-10, 0}, {-5, 5}}, PlotLabel → "Log(γ) vs Log(|μ-μc|)"]
```

Out[3117]= 2.78486 + 0.690574 x

Out[3118]=



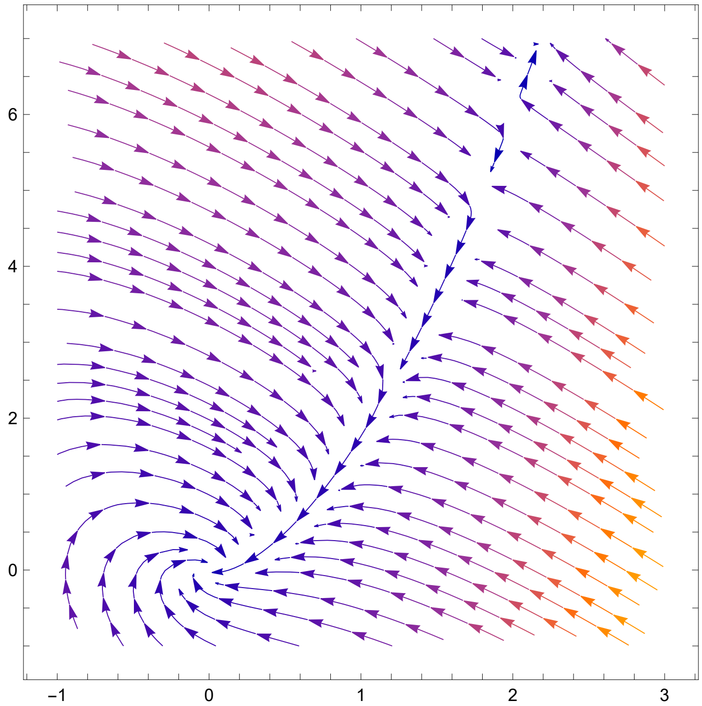
In[2172]:=
```
(* b *)
Clear["Global`*"]
μc = 0.066;
f[x_, y_] := μ * x + y - x^2;
g[x_, y_] := -x + μ * y + 2 x^2;
sol = Solve[f[x, y] == 0 && g[x, y] == 0, {x, y}];
```

In[2808]:=
```
μ = -1
sol = Solve[f[x, y] == 0 && g[x, y] == 0, {x, y}]
StreamPlot[{f[x, y], g[x, y]}, {x, -1, 3}, {y, -1, 7}]
minx = -1; maxx = 3; miny = -1;  maxy = 7;
s[x0_, y0_] := NDSolve[{x'[t] == μ * x[t] + y[t] - x[t]^2,
    y'[t] == -x[t] + μ * y[t] + 2 x[t]^2, x[0] == x0, y[0] == y0}, {x, y}, {t, 0, 10}];
initialCondition = Join[Table[{minx, y}, {y, miny, maxy, 0.1}],
   Table[{maxx, y}, {y, miny, maxy, 0.1}], Table[{x, miny}, {x, minx, maxx, 0.1}],
   Table[{x, maxy}, {x, minx, maxx, 0.1}]];
Show[Table[ParametricPlot[Evaluate[{x[t], y[t]} /.
      s[initialCondition[[i, 1]], initialCondition[[i, 2]]]], {t, 0, 10},
   PlotRange → {{minx, maxx}, {miny, maxy}}], {i, Length[initialCondition]}],
 ListPlot[{{0, 0}}, PlotStyle → {Red}, PlotMarkers → {Automatic, 6},
  PlotLegends → {"Sink"}],
 ListPlot[{{sol[[2, 1, 2]], sol[[2, 2, 2]]}}, PlotStyle → {Green},
  PlotMarkers → {Automatic, 6}, PlotLegends → {"Saddle"}], PlotLabel → "μ<0"]
```

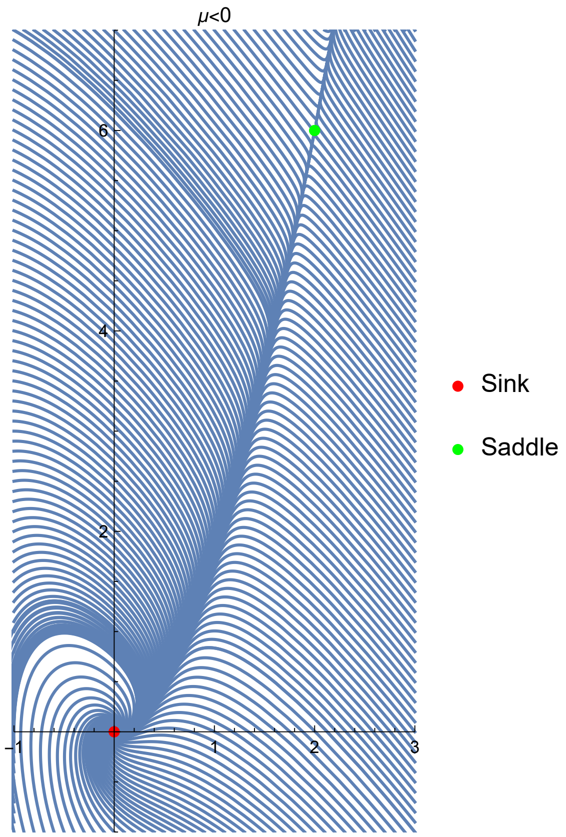
Out[2808]= -1

Out[2809]= {{x → 0, y → 0}, {x → 2, y → 6}}

Out[2810]=



Out[2814]=

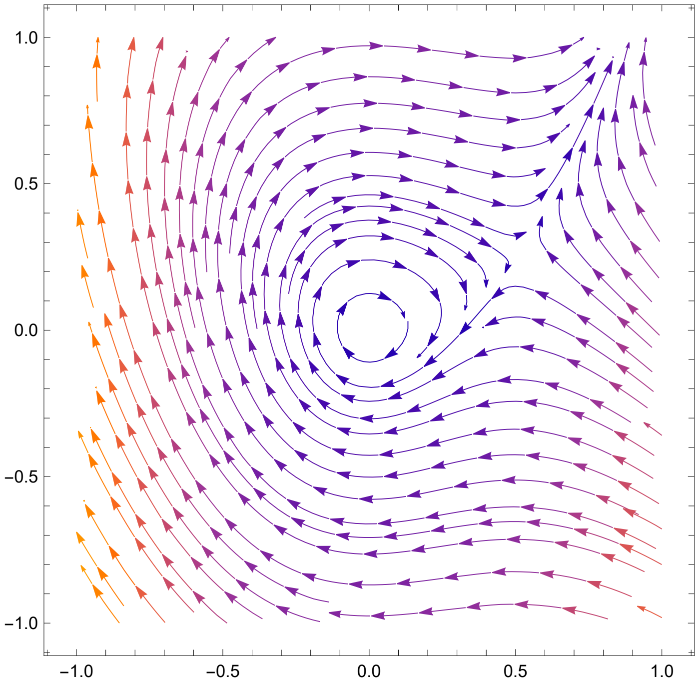


In[2931]:=
```
μ = 0
sol = Solve[f[x, y] == 0 && g[x, y] == 0, {x, y}]
StreamPlot[{f[x, y], g[x, y]}, {x, -1, 1}, {y, -1, 1}]
minx = -1; maxx = 1; miny = -1;  maxy = 1;
s[x0_, y0_] := NDSolve[{x'[t] == μ * x[t] + y[t] - x[t]^2,
    y'[t] == -x[t] + μ * y[t] + 2 x[t]^2, x[0] == x0, y[0] == y0}, {x, y}, {t, 0, 100}];
initialCondition = Join[Table[{minx, y}, {y, miny, maxy, 0.1}],
   Table[{maxx, y}, {y, miny, maxy, 0.1}], Table[{x, miny}, {x, minx, maxx, 0.1}],
   Table[{x, maxy}, {x, minx, maxx, 0.1}]];
Show[Table[ParametricPlot[Evaluate[{x[t], y[t]} /.
     s[initialCondition[[i, 1]], initialCondition[[i, 2]]]], {t, 0, 20},
   PlotRange → {{minx, maxx}, {miny, maxy}}], {i, Length[initialCondition]}],
 ListPlot[{{0, 0}}, PlotStyle → {Red}, PlotMarkers → {Automatic, 6},
  PlotLegends → {"Center"}],
 ListPlot[{{sol[[2, 1, 2]], sol[[2, 2, 2]]}}, PlotStyle → {Green},
  PlotMarkers → {Automatic, 6}, PlotLegends → {"Saddle"}],
 ParametricPlot[Evaluate[{x[t], y[t]} /. s[0.2, 0]],
  {t, 0, 100}, PlotRange → {{minx, maxx}, {miny, maxy}}],
 ParametricPlot[Evaluate[{x[t], y[t]} /. s[0.05, 0.05]], {t, 0, 10},
  PlotRange → {{minx, maxx}, {miny, maxy}}, PlotStyle → {Orange},
  PlotLegends → {"Closed Orbit"}], PlotLabel → "μ=0"]
```

Out[2931]= 0

Out[2932]= $\left\{\{x \to 0, y \to 0\}, \left\{x \to \frac{1}{2}, y \to \frac{1}{4}\right\}\right\}$

Out[2933]=



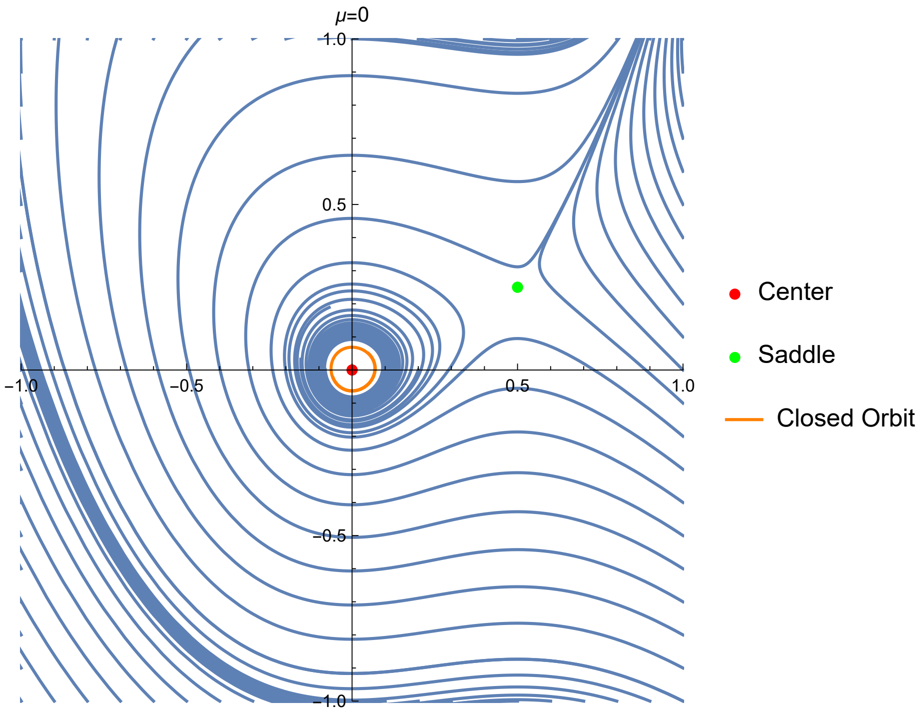
NDSolve: Error test failure at t == 25.121436837220145`; unable to continue.

NDSolve: Error test failure at t == 25.217988069241958`; unable to continue.

NDSolve: Error test failure at t == 25.303752841232345`; unable to continue.

General: Further output of NDSolve::nderr will be suppressed during this calculation.

Out[2937]=



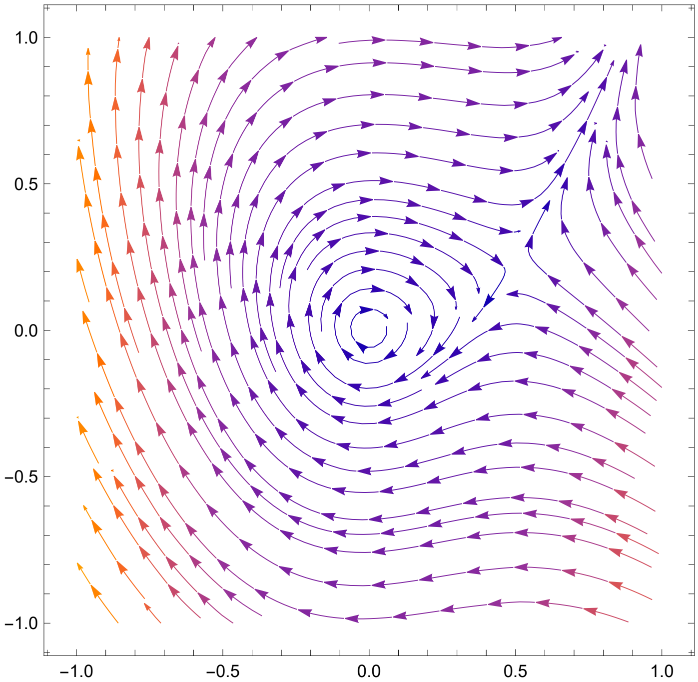
In[2656]:=
```
μ = μc / 2
sol = Solve[f[x, y] == 0 && g[x, y] == 0, {x, y}]
StreamPlot[{f[x, y], g[x, y]}, {x, -1, 1}, {y, -1, 1}]
minx = -1; maxx = 1; miny = -1; maxy = 1;
s[x0_, y0_] := NDSolve[{x'[t] == μ * x[t] + y[t] - x[t]^2,
    y'[t] == -x[t] + μ * y[t] + 2 x[t]^2, x[0] == x0, y[0] == y0}, {x, y}, {t, 0, 100}];
initialCondition = Join[Table[{minx, y}, {y, miny, maxy, 0.1}],
   Table[{maxx, y}, {y, miny, maxy, 0.1}], Table[{x, miny}, {x, minx, maxx, 0.1}],
   Table[{x, maxy}, {x, minx, maxx, 0.1}]];
Show[Table[ParametricPlot[Evaluate[{x[t], y[t]} /.
      s[initialCondition[[i, 1]], initialCondition[[i, 2]]]], {t, 0, 20},
   PlotRange → {{minx, maxx}, {miny, maxy}}], {i, Length[initialCondition]}],
 ListPlot[{{0, 0}}, PlotStyle → {Red}, PlotMarkers → {Automatic, 6},
  PlotLegends → {"Spiral Source"}],
 ListPlot[{{sol[[2, 1, 2]], sol[[2, 2, 2]]}}, PlotStyle → {Green},
  PlotMarkers → {Automatic, 6}, PlotLegends → {"Saddle"}],
 ParametricPlot[Evaluate[{x[t], y[t]} /. s[-0.2, 0]],
  {t, 0, 100}, PlotRange → {{minx, maxx}, {miny, maxy}},
  PlotStyle → {Orange}, PlotLegends → {"Limit Cycle"}],
 ParametricPlot[Evaluate[{x[t], y[t]} /. s[0.01, 0.01]], {t, 0, 100},
  PlotRange → {{minx, maxx}, {miny, maxy}}], PlotLabel → "0<μ<μc"]
```

Out[2656]= 0.033

Solve: Solve was unable to solve the system with inexact coefficients. The answer was obtained by solving a corresponding exact system and numericizing the result.

Out[2657]= {{x → 0., y → 0.}, {x → 0.49242, y → 0.226227}}

Out[2658]=



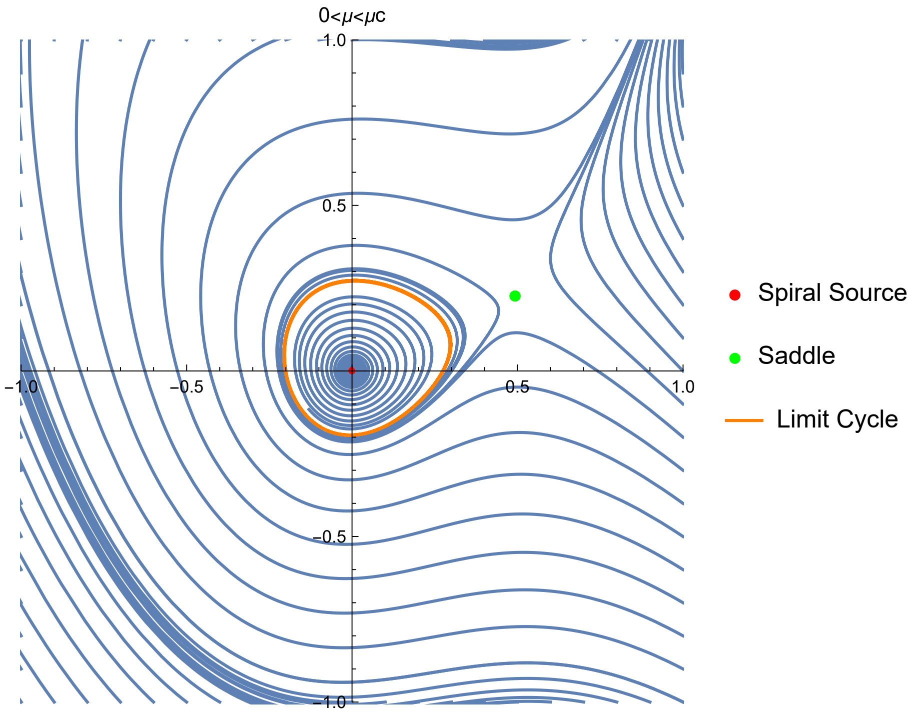
NDSolve: Error test failure at t == 24.64683271099191`; unable to continue.

NDSolve: Error test failure at t == 24.770209737927193`; unable to continue.

NDSolve: Error test failure at t == 24.894668174090306`; unable to continue.

General: Further output of NDSolve::nderr will be suppressed during this calculation.

Out[2662]=



In[2712]:=

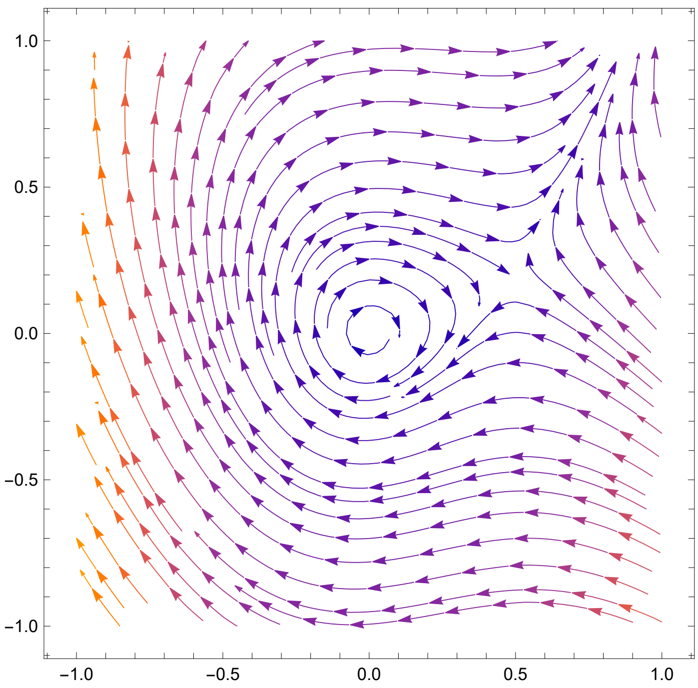
```
μ = μc
sol = Solve[f[x, y] == 0 && g[x, y] == 0, {x, y}]
StreamPlot[{f[x, y], g[x, y]}, {x, -1, 1}, {y, -1, 1}]
minx = -1; maxx = 1; miny = -1; maxy = 1;
s[x0_, y0_] := NDSolve[{x'[t] == μ * x[t] + y[t] - x[t]^2,
    y'[t] == -x[t] + μ * y[t] + 2 x[t]^2, x[0] == x0, y[0] == y0}, {x, y}, {t, 0, 100}];
initialCondition = Join[Table[{minx, y}, {y, miny, maxy, 0.1}],
   Table[{maxx, y}, {y, miny, maxy, 0.1}], Table[{x, miny}, {x, minx, maxx, 0.1}],
   Table[{x, maxy}, {x, minx, maxx, 0.1}]];
Show[Table[ParametricPlot[Evaluate[{x[t], y[t]} /.
     s[initialCondition[[i, 1]], initialCondition[[i, 2]]]], {t, 0, 20},
   PlotRange → {{minx, maxx}, {miny, maxy}}], {i, Length[initialCondition]}],
 ListPlot[{{0, 0}}, PlotStyle → {Red}, PlotMarkers → {Automatic, 6},
  PlotLegends → {"Spiral Source"}],
 ListPlot[{{sol[[2, 1, 2]], sol[[2, 2, 2]]}}, PlotStyle → {Green},
  PlotMarkers → {Automatic, 6}, PlotLegends → {"Saddle"}],
 ParametricPlot[Evaluate[{x[t], y[t]} /. s[0.01, 0.01]],
  {t, 0, 100}, PlotRange → {{minx, maxx}, {miny, maxy}}],
 ParametricPlot[Evaluate[{x[t], y[t]} /. s[0, -0.225]], {t, 0, 50},
  PlotRange → {{minx, maxx}, {miny, maxy}}, PlotStyle → {Orange},
  PlotLegends → {"Homoclinic orbit"}], PlotLabel → "μ=μc"]
```

Out[2712]= 0.066

Solve: Solve was unable to solve the system with inexact coefficients. The answer was obtained by solving a corresponding exact system and numericizing the result.

Out[2713]= {{x → 0., y → 0.}, {x → 0.486136, y → 0.204243}}

Out[2714]=



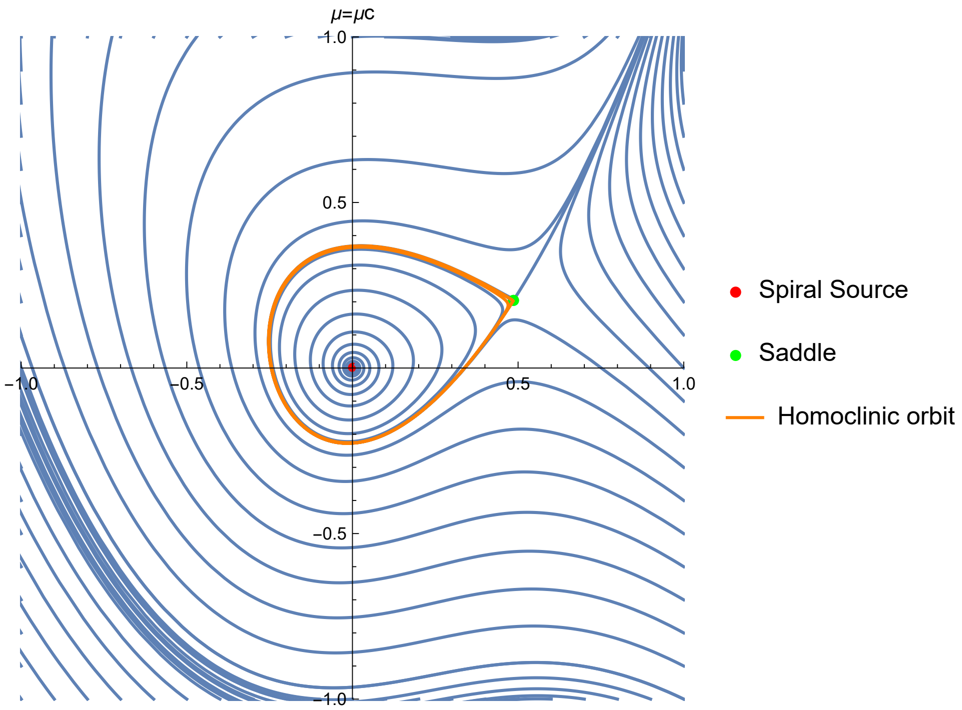
NDSolve: Error test failure at t == 24.114893559278727`; unable to continue.

NDSolve: Error test failure at t == 24.376055743810028`; unable to continue.

NDSolve: Error test failure at t == 24.442012341681597`; unable to continue.

General: Further output of NDSolve::nderr will be suppressed during this calculation.

Out[2718]=

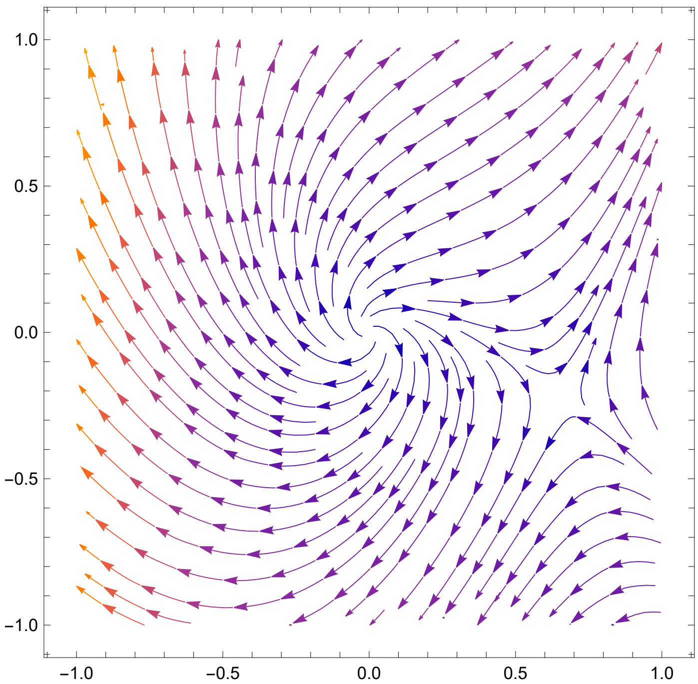


In[2801]:=
```
μ = 1
sol = Solve[f[x, y] == 0 && g[x, y] == 0, {x, y}]
StreamPlot[{f[x, y], g[x, y]}, {x, -1, 1}, {y, -1, 1}]
minx = -1; maxx = 1; miny = -1; maxy = 1;
s[x0_, y0_] := NDSolve[{x'[t] == μ * x[t] + y[t] - x[t]^2,
    y'[t] == -x[t] + μ * y[t] + 2 x[t]^2, x[0] == x0, y[0] == y0}, {x, y}, {t, 0, 10}];
initialCondition = Join[Table[{1, y}, {y, 0, miny, -0.05}],
   Table[{0, y}, {y, miny, maxy, 0.05}], Table[{x, 0}, {x, minx, maxx, 0.05}]];
Show[Table[ParametricPlot[Evaluate[{x[t], y[t]} /.
      s[initialCondition[[i, 1]], initialCondition[[i, 2]]]], {t, 0, 20},
   PlotRange → {{minx, maxx}, {miny, maxy}}], {i, Length[initialCondition]}],
 ListPlot[{{0, 0}}, PlotStyle → {Red}, PlotMarkers → {Automatic, 6},
  PlotLegends → {"Spiral Source"}],
 ListPlot[{{sol[[2, 1, 2]], sol[[2, 2, 2]]}}, PlotStyle → {Green},
  PlotMarkers → {Automatic, 6}, PlotLegends → {"Saddle"}], PlotLabel → "μ=1"]
```

Out[2801]= 1

Out[2802]= $\left\{\{x \to 0, y \to 0\}, \left\{x \to \frac{2}{3}, y \to -\frac{2}{9}\right\}\right\}$

Out[2803]=



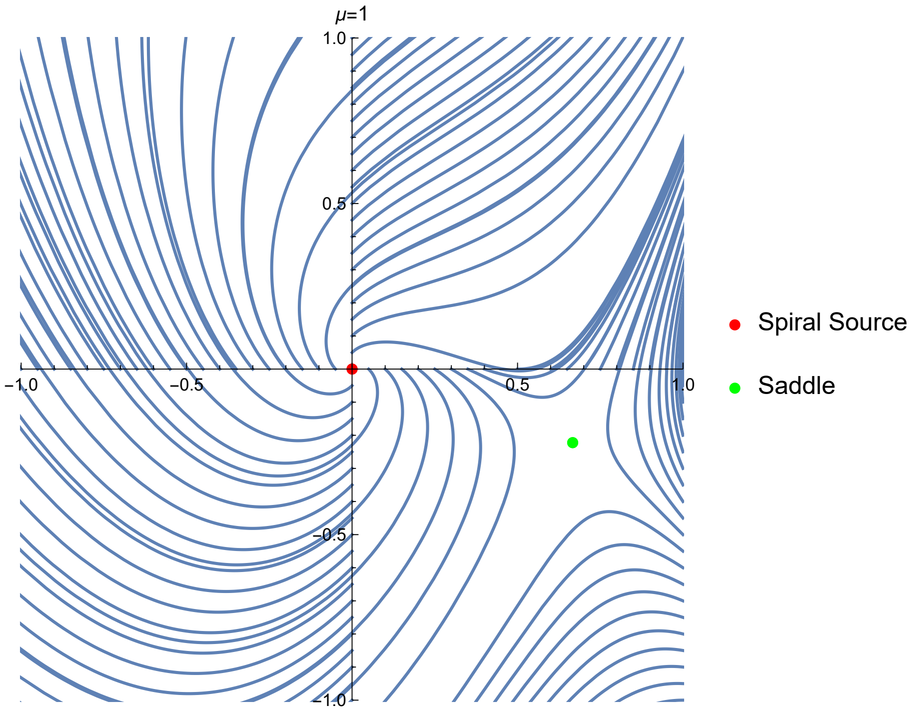
NDSolve: At t == 3.2402819504194933`, step size is effectively zero; singularity or stiff system suspected.

NDSolve: At t == 2.7568578671908086`, step size is effectively zero; singularity or stiff system suspected.

NDSolve: At t == 2.500943125669248`, step size is effectively zero; singularity or stiff system suspected.

General: Further output of NDSolve::ndsz will be suppressed during this calculation.

Out[2807]=



In[3217]:=

```
tConst = 0;
f = s⟦1, 1, 2⟧;
g = s⟦1, 2, 2⟧;
xStart = s⟦1, 1, 2⟧[tMin];
yStart = s⟦1, 2, 2⟧[tMin];
temp = FindRoot[f[t] + g[t] == xStart + yStart, {t, tMin + tConst}];

xStart
s⟦1, 1, 2⟧[temp⟦1, 2⟧]
yStart
s⟦1, 2, 2⟧[temp⟦1, 2⟧]
tPeriod = Abs[tMin - temp⟦1, 2⟧];
{Abs[μ - μc], tPeriod}
```

Out[3223]= 0.350527

Out[3224]= 0.350527

Out[3225]= 0.0186812

Out[3226]= 0.0186812

Out[3228]= {0.006, 0.}

In[3213]:=
```
plotDataPeriodTime =
  {{0.006000000000000005, 9.8624007122786}, {0.0050000000000000044, 10.13014722525466},
   {0.0040000000000000036, 10.457176976655632},
   {0.0030000000000000027, 10.87749191550094`}, {0.0020000000000000018,
    11.46649270087542}, {0.0010000000000000009, 12.45832661710071}};
logPlotDataPeriodTime = Log[plotDataPeriodTime];
line = Fit[logPlotDataPeriodTime, {1, x}, x]
periodPlot = Show[ListPlot[logPlotDataPeriodTime, PlotStyle → Red],
  Plot[line, {x, -10, 0}, PlotStyle → Orange],
  PlotRange → {{-10, 0}, {-5, 5}}, PlotLabel → "log(Tμ) vs. log|μ-μ_c|"]
```

Out[3215]= 1.62606 − 0.130313 x

Out[3216]=