KU Leuven

1D finite difference 1st order upwind HD solver

Ileyk El Mellah & Jon Sundqvist

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Aim : implement a finite difference 1st order upwind scheme to solve the one-dimensional hydrodynamics equations. Validate the implementation by convergence tests.

Steps Before all, perform the analytic pen and paper investigation to identify, for the equations of hydrodynamics, the explicit form of the objects you will need (e.g. the Jacobian, its eigenvalues, etc). Then, proceed to the numerical implementation :

- define a 1D domain divided in equidistant cells
- set fixed or periodic boundary conditions (resp. for first and second test cases)
- which data structure for solution array?
- write an upwind solver
- specify the explicit time advancement and define a stable time step
- save data (parameters, log and data files)
- validate on the 2 elementary tests below

Global Inputs/Outputs (I/O) Inputs (parameters to be changed by the user) :

- which initial conditions : shock tube or acoustics?
- which boundary conditions? initial state or periodic?
- save conditions (either time step or number of snapshots)?
- do you want to perform a convergence analysis?

Inputs (not to be changed often) :

- how many cells? how many ghost cells?

Outputs :

- data files with primitive variables
- if convergence analysis, EOC info

- total time taken to compute

N.B. : use and adapt if needed the Python script provided to plot the EOC as a function of the number of cells, and to make animated GIF with your data files.

Possible code structure

See Doxygen documentation provided.

Validation

- compute, as a function of the number of grid points, the maximal error and the empirical order of convergence.
- evaluate the departure from pure conservation.

Test case # 1 : the Sod shock tube problem

On a domain from -0.5 to 0.5, set up the following initial conditions for $x < 0$:

$$\rho = 8 \quad \mathbf{v} = \mathbf{0} \quad P = 8/\gamma \tag{1}$$

and for $x > 0$:

$$\rho = 1 \quad \mathbf{v} = \mathbf{0} \quad P = 1 \tag{2}$$

with $\gamma = 1.4$ and up to run up to $t_{\text{max}} = 0.2$. As boundary conditions, simply force the ghost cells to the initial state above (since the front shock and the rarefaction wave will not have time to reach the edge of the simulation space within 0.2, it is safe). Visualize the output and experience with various time/spatial steps. What happens at the discontinuity? Why? How could we solve it?

Here, we have an analytic solution to be compared to (see 10.2.2 in this document).

Test case # 2 : Isothermal acoustic wave

On a domain from 0 to 1, set up the smooth following initial conditions :

$$\rho = \gamma P \quad \mathbf{v} = \mathbf{0} \quad P = 0.1 + 0.001 \exp\left[-\frac{(x-0.5)^2}{0.1^2}\right] \tag{3}$$

and work with the adiabatic index γ of a diatomic gas, a suitable CFL parameter. Set up periodic boundary conditions. Run up to $t_{\text{max}} = 3$. Do you retrieve what you expect from the exact solution for acoustic waves (in the linear regime)? In particular, do you retrieve the expected periodicity? What about a bigger splash, with an amplitude for the Gaussian of the order of the background level (non-linear regime)?

Possible extensions

- connect to the Physics by making physical equations dimensionless
- convert into finite volume formulation (session 1).
- implement and test different 1^{st} order solvers.
- solve other sets of conservation laws (e.g. magneto-hydrodynamics and divergence-cleaning methods).
- add viscosity to Euler equations.
- extend to higher dimensions and other geometries (cylindrical & spherical).
- go 2^{nd} order accurate in space and time.
- ...