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Citation: The Journal of the Acoustical Society of America **142**, 3748 (2017); doi: 10.1121/1.5018342
View online: https://doi.org/10.1121/1.5018342
View Table of Contents: https://asa.scitation.org/toc/jas/142/6
Published by the Acoustical Society of America

Calculation of the acoustical properties of triadic harmonies

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(Received 29 June 2017; revised 20 November 2017; accepted 2 December 2017; published online 26 December 2017)

The author reports that the harmonic "tension" and major/minor "valence" of pitch combinations can be calculated directly from acoustical properties without relying on concepts from traditional harmony theory. The capability to compute the well-known types of harmonic triads means that their perception is not simply a consequence of learning an arbitrary cultural "idiom" handed down from the Italian Renaissance. On the contrary, for typical listeners familiar with diatonic music, attention to certain, definable, acoustical features underlies the perception of the valence (modality) and the inherent tension (instability) of three-tone harmonies.

https://doi.org/10.1121/1.5018342

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I. INTRODUCTION

The nature/nurture debate in human psychology is as robust as ever. In music psychology, it has taken the form of arguments for and against the importance of the acoustical signal in the understanding and appreciation of diatonic music, in general, and the major and minor harmonies, in particular. On the one hand, the traditional view of Western diatonic music has been that specific pitch structures have implicit affective implications that are heard by all normal listeners after a minimal period of acculturation. The contrary view is that there is no inherent affect in the acoustical signal and that the emotional interpretations are merely adherence to a cultural idiom passed down since the Italian Renaissance. There is vociferous support for both views. Leonard Bernstein[1] famously declared that the "physical universe" has provided us with the beauty of diatonic music, but clearly an appreciation of music develops throughout childhood and is enhanced by musical training,[2] suggesting the importance of learning.

In fact, numerical algorithms have been developed to explain the perception of certain acoustical properties. The most successful theoretical models have been concerned with interval consonance/dissonance, but the perception of more complex pitch structures has received far less attention by psychophysicists. As discussed below, we have devised algorithms for explaining the perception of various types of triadic harmonies. While such numerical techniques do not resolve the nature/nurture debate, they do indicate the acoustical properties that are perceived by typical listeners to distinguish among so-called tonal and atonal chords, and to hear the affective modality of major and minor chords.

II. INTERVAL DISSONANCE

Discussion of the consonance and dissonance of pitch intervals dates back to Plato in classical Greece, but modern-era psychoacoustical research did not begin until Helmholtz's work[3] in the mid-1800s. Mathematical modeling by Plomp and Levelt[4] in the 1960s and more recent work by Parncutt,[5] Sethares,[6] and Huron[7] have clarified the important role of higher harmonics (overtones or upper partials) in the perception of the consonance/dissonance of pitch dyads.

The gist of such research can be illustrated as in Fig. 1. Stated briefly, certain pitch intervals—when played in isolation without any musical context—are perceived as dissonant and mildly unpleasant. It has been found that quite simple theoretical models of the dissonance of two auditory frequencies separated by a small interval can explain the full spectrum of interval perception, provided only that the simultaneous effects of higher harmonics are included in the calculations. Specifically, the model curve shown in Fig. 1(A) indicates a peak of dissonance at an interval of about 1 semitone, but also implies somewhat weaker peaks of dissonance at 6 and 11 semitones [Fig. 1(B)], due to the close proximity of upper partials to one another at higher frequencies.

Those peaks of relative dissonance are empirically well known from more than a century of psychological research, but individual differences are typically large and learning (especially, musical training) effects have also been found, so that the relative contributions of nature and nurture—even for pitch dyads—cannot be definitively stated. While further psychophysical research on interval perception is still ongoing, the approximate reproduction of the empirical data using the quantitative model of Eq. (1) is already a strong indication that dissonance perception is influenced by pitch intervals between fundamental frequencies *and* among their upper partials.

In a thorough examination of the acoustics of pitch intervals, Sethares[6] has defined dissonance as

$$\text{dissonance} = \beta_3\left[\exp\left(-\beta_1 x_{12}^{\gamma}\right) - \exp\left(-\beta_2 x_{12}^{\gamma}\right)\right], \qquad (1)$$

where the interval size is indicated by variable x_{12}, which is the difference in frequency (f) of two tones (in Hertz): $x_{12} = \log(f_2/f_1)$. The parameters $\boldsymbol{\beta}_1$ (=0.80), $\boldsymbol{\beta}_2$ (=1.60), $\boldsymbol{\beta}_3$ (=4.00), and $\boldsymbol{\gamma}$ (=1.25) are set to give a dissonance score of

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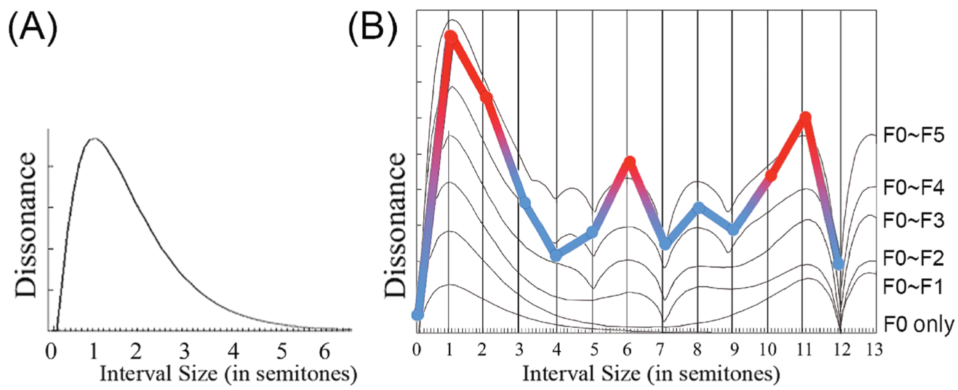

FIG. 1. (A) The dissonance model of Sethares (Ref. 6, p. 345). (B) The predicted relative dissonance when higher harmonics are included. The theoretical dissonance curves become progressively more complex with the addition of overtones (F1–F5). Reproduction of the empirical curve (color) by the theoretical curves (black) is imperfect, but the three dominant peaks of relative dissonance (red) and several troughs of relative consonance—typically found experimentally (Refs. 18 and 33)—are obtained when three or more upper partials are included in the dissonance calculations.

1.0 at an interval of one semitone (see Ref. 6). By means of "stretching" and "shrinking" upper partial frequencies, Sethares has developed pre-computer age techniques and demonstrated how traditionally "consonant" intervals can be made dissonant, and "dissonant" intervals made consonant. Such effects unambiguously illustrate the importance of upper partials in interval perception.

III. TRIADIC HARMONIES

In spite of the successes of the dissonance model [Eq. (1) and similar curves proposed by Plomp and Levelt,[4] Kameoka and Kuriyagawa,[8] and others] in predicting the general psychological trends of *interval* perception, the more complex topic of the "consonance" of pitch *triads* (more often referred to as "sonority," "harmoniousness," or "euphony") has generally remained beyond quantification even when the effects of higher harmonics are considered. Specifically, as shown in Table I, the predicted rank order of the overall consonance of the common triads does *not* agree with the incidence of their usage in classical or popular Western music[13,14] *nor* with empirical results from laboratory studies (e.g., Refs. 15–17).

Model predictions vary somewhat with the number of upper partials and their relative amplitudes, but no combination of factors leads to the prediction that the major and minor triads are consonant, while the diminished, augmented, and suspended triads are dissonant. Theoretical anomalies among the triad inversions are numerous,[18] but the most notable shortcoming is the prediction of a relatively high consonance of the augmented triad or the suspended triads, despite their low sonority, as evaluated by both musicians and nonmusicians. At the very least, factors in addition to dyadic dissonance effects are needed to explain the perception of triadic harmonies.

As a consequence of the inability to predict harmonic sonority on the basis of interval consonance, the modern consensus among music psychologists is that: (i) pitch combinations in 3-tone harmonies must consist of more-or-less consonant intervals (in light of the acoustical explanation of interval dissonance), but (ii) the characteristic sonority and emotional valence of such harmonies is a learned "tradition" *without* deep acoustical roots. In recent years that view on the presumed cultural origins of harmony perception has been supported by numerous editorials in *Nature*. For example, "It may even be that acclimatization to a convention can completely override [the] acoustic facts."[19] "Our emotional response to particular scales or chords seems likely to be acquired from exposure to a particular culture."[20] "The objective organization of sounds is only loosely related to how minds interpret those sounds."[21] "Scales and harmonic

TABLE I. The relative sonority of the common harmonic triads. Data from Eberlein (Ref. 13; English translation, Ref. 14) are the relative incidences (percentages) of 1538 triads from samples of music by Bach, Handel, Mozart, Beethoven, and Mendelssohn. The rank order in both classical music and in psychological studies [Roberts (Ref. 15) and Fujisawa (Ref. 16)] is ***not*** predicted by any of the dissonance models. Note that neither Parncutt (Ref. 5) nor Bidelman and colleagues (Refs. 9–12) calculated the total consonance of the suspended chords, but, in the latter case, such calculations can be done on the basis of their own empirical data (Ref. 9) on "auditory nerve pitch salience."

		Empirical sonority (rank order)			Theoretical consonance (rank order)				
Chord class	Interval structures	Eberlein (Ref. 8) (1994)	Roberts (Ref. 14) (1986)	Fujisawa (Ref. 15) (2006)	P&L (Ref. 3) (1965)	K&K (Ref. 33) (1969)	Parncutt (Ref. 4) (1989)	Sethares (Ref. 5) (2005)	B&H (Ref. 9) (2011)
Major	4–3, 3-5, 5-4	1 (51%)	1	1	2	2	2	2	2
Minor	3-4, 4-5, 5-3	2 (37%)	2	2	2	2	3	2	3
Diminished	3-3, 3-6, 6-3	3 (9%)	3	4	5	4	4	4	4
Suspended	5-5, 5-2, 2-5	4 (2%)	—	3	1	1	—	1	1
Augmented	4-4	5 (<1%)	4	5	4	5	1	5	5

structures depend on learning."[22] And, "It is even possible that music's emotive power is itself in part merely a tacit cultural consensus."[23]

The cultural argument noted above is a source of particular confusion, because popular notions clearly indicate: a distinction between the inherently stable, "sonorous" (major and minor) triads and the inherently unstable, "tense" (diminished, augmented, and suspended) triads, and, moreover, an affective dichotomy between major and minor keys (scales, harmonies, and melodies). While laboratory studies of people exposed to diatonic music (both musicians and non-musicians, Easterners and Westerner, adults and children)[15–17,24–32] differentiate among (i) the bright, positive, "upbeat" affect of major keys, (ii) the dark, negative, "downbeat" affect of minor keys, and (iii) the tense, unresolved, "up-in-the-air" feeling of atonal harmonies, the current view in music psychology is that these distinctions are "cultural constructs," and passed along generation-after-generation through the unrelenting repetition of this "Western idiom" in all forms of the modern media.

IV. TRIADIC ALGORITHMS

Contrary to the view that harmony perception is a "learned idiom," however, we have developed three-pitch algorithms through which both the perceived tension of pitch combinations [Eq. (2)] and their positive/negative affective valence [Eq. (3)] can be calculated solely on the basis of acoustical properties.[16,18,30–36] Note that the term "tension" is used in various ways in music perception. The acoustical tension inherent to the timbre of musical instruments has recently been addressed by Farbood and colleagues,[37,38] whereas the diatonic tension inherent to melodic phrases, harmonic cadences, and larger musical structures has traditionally been a topic central to music theory—notably, as discussed by Hugo Riemann and his contemporaries (summarized in Ref. 39). The psychology of tonal tension is normally discussed in relation to musical expectations and how they are realized, thwarted, delayed, and/or eventually resolved.[7,40–42] Distinct from both considerations of timbre and harmonic cadences, the present work is concerned with the in-between phenomenon of the harmonic tension of isolated pitch-triads—and their perception as inherently resolved (sonorous) or unresolved (tense). Specifically, without employing any of the concepts of traditional harmony theory (e.g., the special role of scales, keys, roots, inversions, tonic, dominant, and subdominant tones, etc.), the acoustical calculations allow for predictions that are concordant both with common perceptions (global and intergenerational, if perhaps not universal). The harmonic tension of pitch triads can be defined as

$$\text{tension} = \exp\left[-\left(\frac{x_{12}-x_{23}}{\alpha}\right)^2\right], \tag{2}$$

where interval sizes are indicated by variables x_{12} and x_{23}, defined as the difference in frequency (f) (in Hertz) of two tones: the lower interval $x_{12} = \log(f_2/f_1)$ and the upper interval $x_{23} = \log(f_3/f_2)$. Parameter α (0.60) determines the steepness of the fall from maximal tension and is essentially the factor that specifies the breadth of the peak of triadic tension. A sharp peak indicates a narrow range of frequencies where the triad is perceived as categorically atonal, as distinct from a gradual shift from tonal to atonal harmonies (see Refs. 14, 31, and 32 for further discussion).

Consistent with usage in the diatonic tradition, modal valence is defined as

$$\text{valence} = -\left[\frac{2(x_{12}-x_{23})}{\varepsilon}\right]\exp\left\{-\left[\frac{-(x_{12}-x_{23})^4}{4}\right]\right\}, \tag{3}$$

where, again, the interval sizes are indicated by variables x_{12} and x_{23}. Parameter ε (1.558) is set to give a positive modality score of 1.0 for the major chord in root position and a negative modality score of -1.0 for the minor chord in root position. Note that the curves for both tension and valence are "generic" in reproducing the perceptions of typical listeners acculturated to tonal music. In principle, these parameters could be adjusted to fit the empirical data for individual listeners, but such psychophysical modeling will not be addressed here. Equations (2) and (3) lead to the model curves shown in Fig. 2.

The acoustical algorithms for triadic tension and modal valence are based on the difference in magnitude of the two smaller intervals contained within any triad of pitches. For example, when expressed as the difference in interval size, the fundamental frequencies of all major and minor triads exhibit interval differences of $|1|$ or $|2|$ semitones (e.g., CEG: $4–3 = 1$, BDG: $3–5 = -2$, and CFA: $5–4 = 1$ for, respectively, the tonic, dominant, and subdominant triads in the key of C major and CEbG: $3–4 = -1$, BbDG: $4–5 = -1$, and CFAb $5–3 = 2$ in C minor). In contrast, the diminished (e.g., CEbGb, CEbA, CGbA), augmented (e.g., CEG#), and suspended triads (e.g., Csus4 = CFG, Csus2 = CDG, and C7sus4 = CFA#) exhibit interval differences of 0 or $|3|$

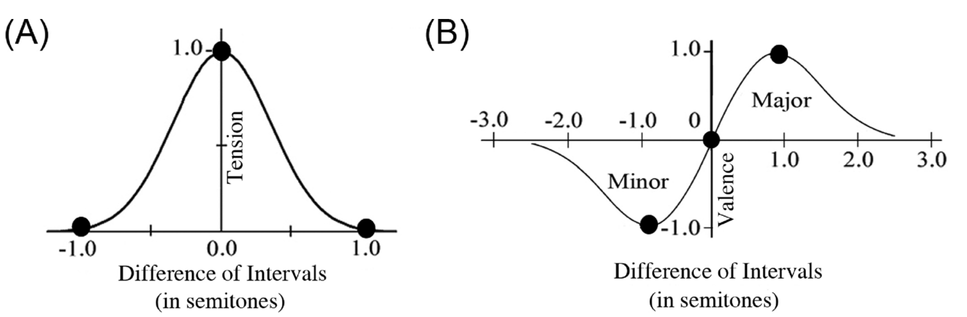

FIG. 2. The three-pitch acoustical models for (A) harmonic tension and (B) modal valence. Both are based on the difference of interval size (lower interval minus upper interval), as shown in Eqs. (2) and (3). The black dots indicate the locations of important tonal and atonal triads created in 12-tone scales.

semitones. Using those interval differences, the overall tension and valence of any triad (with tones lying on or between the scalar tones of the 12-tone equitempered scales) can be calculated while including any number of upper partials.

As shown in Table II, regardless of the number of upper partials and their relative amplitudes, major and minor chords consistently exhibit relatively low tension scores. This result is a consequence of the fact that, among any set of higher harmonics, major and minor chords contain no consecutive intervals of the same magnitude among their upper partials. In contrast, the diminished, augmented, and suspended triads exhibit numerous consecutive intervals of the same size, and consequently have high tension scores. It is noteworthy that with or without consideration of the small differences in total dissonance [Eq. (1)] among the intervals within these chords, the tension effect alone [Eq. (2)] is sufficient to distinguish between the "resolved" major and minor chords, on the one hand, and the inherently "unresolved" tension chords, on the other. Similarly, the valence calculations distinguish between major and minor chords, unrelated to dissonance effects (Table II).

We have previously argued that the essential difference between dyadic "dissonance" and triadic tension is the difference between two- and three-body perceptions.[18,33] Dissonance is a consequence of the frequency difference between ***two*** tones (and their upper partials), whereas tension is a consequence of a "difference of differences," the magnitude of the interval difference among *three* tones (and their upper partials). Both dissonant intervals and tense triads produce a perception of unsettled instability that is often evaluated as mildly "unpleasant," but the acoustical sources of dissonance and tension differ and their locations of neocortical activation also differ.[33,43] Significantly, by distinguishing between dyadic dissonance and triadic tension, the need for the dubious distinction between "sensory dissonance" and "musical dissonance" that is sometimes made in the psychoacoustics literature[44] is entirely avoided.

In most musical traditions worldwide, both dissonance and tension are followed by harmonic resolution. In contrast, jazz, some modern classical, and the Gamelan music of Bali, for example, emphasize the unsettled dissonance/tension itself—often minimizing or eliminating resolved harmonies. While music critics may indulge in value judgments on the desirability, beauty, or "musicality" of these musical genres,[45] the more pertinent topic concerns the acoustical definition of both triadic tension and "sonority." If they can be defined on an acoustical basis, their effectiveness in musical composition can be discussed rigorously without reliance on arguments of familiarity and subjective preference.

TABLE II. Tension and valence scores for the 13 common triads, as calculated with Eqs. (2) and (3). Despite gradual changes with the addition of higher harmonics ($F1$, $F2$, $F3$,…), the overall pattern indicates: (i) low tension scores for the resolved (major and minor) triads, and (ii) high tension scores for the unresolved (diminished, augmented, and suspended) triads. Moreover, there is a clear pattern of (iii) strongly positive valence scores for the major triads, (iv) strongly negative scores for the minor triads, and (v) valence scores closer to zero for the high tension atonal triads. This pattern holds true regardless of the number of upper partials (1 through 8) and the relative amplitudes of the upper partials (e.g., $F0 = F1 = F2 = F3$,…; $F0 = 1$, $F1 = 1/2$, $F2 = 1/3$,…). Values shown in the table were calculated using slowly decreasing partial amplitudes ($F0 = 1.0$, $F1 = 0.88*F0$, $F2 = 0.88*F1$, $F3 = 0.88*F2$,…) (see Ref. 36 for related software).

	Tension			Valence		
	$F0$–$F1$	$F0$–$F2$	$F0$–$F3$	$F0$–$F1$	$F0$–$F2$	$F0$–$F3$
Major Root	0.20	0.23	0.75	3.15	3.61	5.51
1st Inversion	0.21	0.23	1.29	3.23	2.68	4.12
2nd Inversion	0.21	0.87	2.08	3.37	3.34	5.92
Minor Root	0.21	0.24	0.87	−3.37	−3.38	−5.10
1st Inversion	0.20	0.79	1.03	−3.15	−3.04	−3.35
2nd Inversion	0.21	0.24	0.42	−3.37	−3.38	−3.00
Diminished Root	1.68	2.15	3.72	0.00	0.01	−0.09
1st Inversion	1.55	1.59	2.48	0.00	0.62	−0.24
2nd Inversion	1.76	1.81	2.94	0.00	−0.62	−1.65
Augmented Root	4.99	5.46	8.84	0.00	0.00	0.07
Suspended Root	1.68	2.23	3.38	0.00	−0.07	−1.06
1st Inversion	1.55	1.55	2.39	0.00	0.07	−0.30
2nd Inversion	1.76	1.76	2.92	0.00	0.02	−0.46

V. IMPLICATIONS

The calculation of modal valence is the most interesting result of the triadic calculations [Eq. (3) and Fig. 2(b)]. A positive sum (for the difference in magnitude between the lower and upper intervals in the triad) indicates a chord with major-like sonority (e.g., $4–3 = 1$ and $5–4 = 1$ for the major triads in root and second inversion) and a negative sum indicates a chord with minor-like valence ($3–4 = -1$ and $4–5 = -1$ for the minor triads in root and first inversion). Inclusion of any number of upper partials [at least up to F8 (Ref. 35)] consistently produces positive (negative) valence scores for all of the major (minor) triads. In contrast, the valence scores of the atonal, tension triads are near zero and remain relatively low as upper partials are included in the calculations. By considering interval differences among the pitches in the higher harmonics, the calculations become numerous, but the numerical results show a pattern of relatively low valence scores and relatively high tension scores due to repeating intervals of the same size among upper partials for all three types of atonal chords (Table II).

Chords consisting of two intervals of the same number of semitones are, in the diatonic tradition, considered to be inherently tense, ambiguous, unresolved, and not generally sonorous. Historically, the notion that triads containing "two successive consonances of the same size do not produce a consonance" was stated already by Vincenzo Galilei[46] (the astronomer's father) in 1588 and explained in terms of Gestalt psychology ("intervallic equidistance") by Meyer[40] in 1956. Remarkably, rising or falling semitone movement of any of the three tones of a tension chord reveals regularity inherent to the harmonic phenomena of 12-tone scales. That is, an upward semitone step from atonal tension leads to minor chords, and a downward semitone step from atonal tension leads to major chords (Fig. 3).

Without entering into the complexities of the creation and resolution of harmonic tension in tonal[40–42] or atonal[47] music, it is remarkable that, from the perspective of the

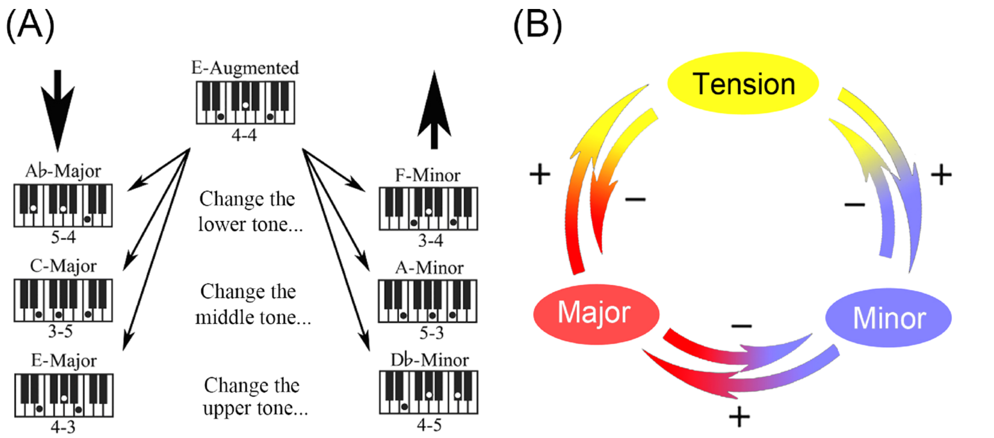

FIG. 3. (Color online) (A) The effects of semitone rises or falls from the augmented chord. (B) The Cycle of Modes among the major, minor, and tension chords. The semitone transitions ("+" and "−") between major and minor triads are well known, but are only part of the more general cycle obtained when the diminished, augmented, and suspended triads are considered as a third tension mode. Because a rise or fall of any tone in an augmented triad results in a minor or major triad, it provides the best example of the Cycle of Modes. Diminished or suspended triads show the same pattern except when interval dissonance is created.

Cycle of Modes, the unresolved ambiguity of atonal tension can be resolved in two (and only two) ways through semitone steps: toward major or toward minor [Fig. 3(A)]. Similarly, a semitone fall (rise) from a major (minor) triad can resolve to a minor (major) triad, but never vice versa (i.e., major chord resolution through a semitone rise from tension or minor chord resolution through a semitone fall from tension does not occur). Because the tension chords, in general, and the augmented triad, in particular, did not play an important role in the development of Renaissance harmony theory, the Cycle of Modes is not mentioned in any of the classic textbooks on harmony, but is a noteworthy statement of the affective regularities inherent to the common triads. Specifically, in-between the resolved sonority of major chords and the resolved sonority of minor chords, there lays the tonally-ambiguous, unresolved tension of the atonal chords [Fig. 3(B)]. Moving away from tension, semitone pitch rises or falls lead inevitably (i) to the mild dissonance of a triad containing a whole-tone interval or (ii) to minor/major resolution.

VI. THE TRIADIC GRID

As illustrated in Figs. 4 and 5, the locations of all possible major and minor triads within one octave can be displayed on a "triadic grid" onto which the theoretical tension and valence scores are mapped. Provided that at least the first set of upper partials are included in the valence calculations, positive peaks of major modality arise at *all* of the locations known to correspond to major chords, and negative valleys of minor modality arise at *all* of the locations of the minor chords. Adding further higher harmonics gives more complex topology (often with small peaks and valleys at locations off of the tones of the 12-tone equitempered scales), but *always* with peaks and valleys at the diatonic major and minor chords, respectively.

Note that the results of the calculations do not rely on any specific tuning system. The black grid lines are drawn in accord with the equitempered 12-tone scale, but the tension and valence calculations themselves were done at intervals of 100 cents (1/10th of a semitone). It is found that, despite the

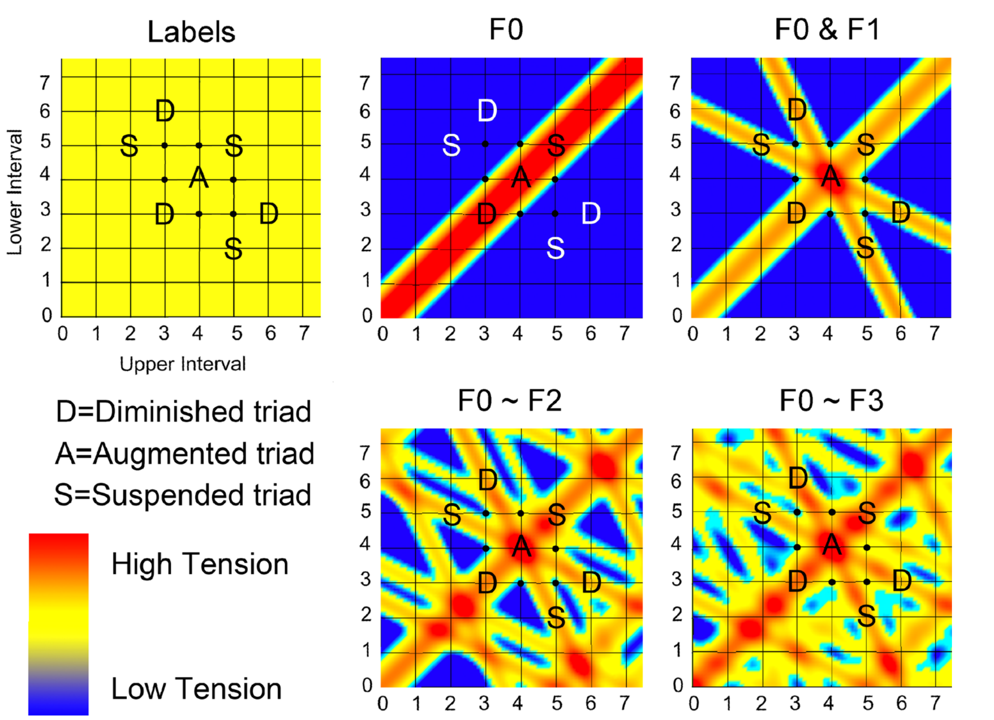

FIG. 4. (Color online) The "triadic grid" (Refs. 34 and 36) used to display the tension scores (calculated in steps of 100 cents) for all possible pitch triads within one octave. For all grids where upper partials are included in the calculations ($F0$ and $F1$, $F0$–$F2$, $F0$–$F3$), the atonal (diminished, augmented, and suspended) triads are located in regions of high tension (orange), whereas the tonal (major and minor) triads (black dots) are found in regions of low tension (blue). The diminished triads include a root chord with interval structure of 3–3 semitones, and inversions with interval structures of 3–6 and 6–3 semitones. The augmented chord in root position and its inversions maintain an interval structure of 4–4 semitones. The conventional description of suspended triads includes the suspended-fourth with an interval structure of 5–2 semitones, a suspended-second with 2–5 semitone structure, and a seven-chord with suspended-fourth and missing fifth, with an interval structure of 5–5 semitones.

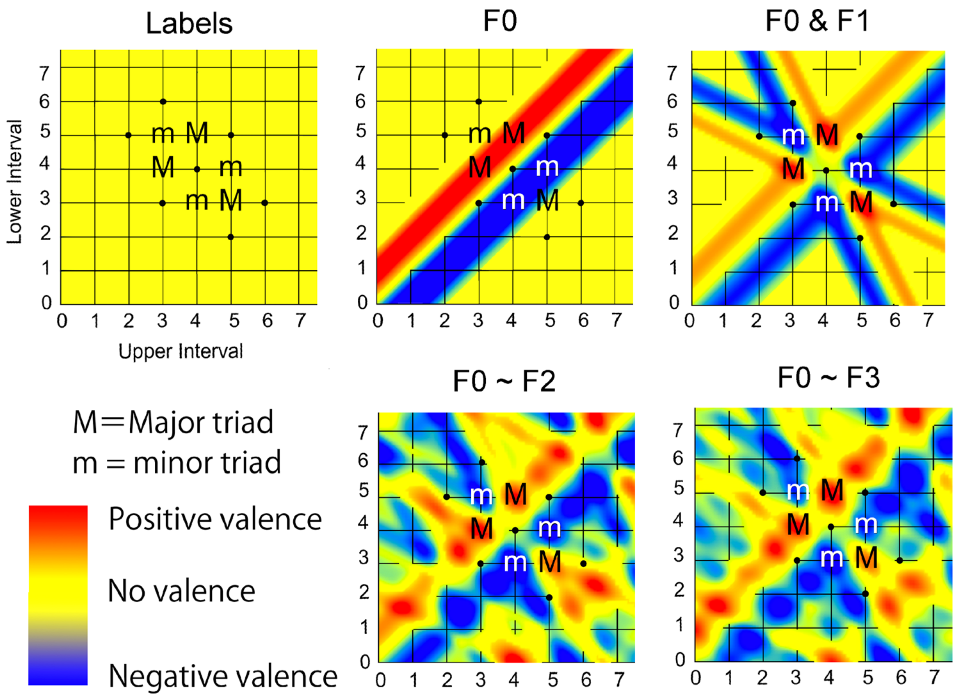

FIG. 5. (Color online) The "triadic grid" (Refs. 34 and 36) used to display the valence scores (calculated in steps of 100 cents) for all possible pitch triads within one octave. For all grids where upper partials are included in the calculations ($F0$ and $F1$, $F0$–$F2$, $F0$–$F3$), minor triads are located in regions of negative valence (blue) and major triads in regions of positive valence (orange). Here, the black dots indicate the locations of the atonal tension (diminished, augmented, and suspended) triads—consistently found in regions of negligible valence (yellow).

small deviations from equitempered tuning recommended in other tuning systems,[48] rather broad peaks or valleys of tension and valence are obtained at the localities of the major and minor triads, in particular. As a consequence, redrawing the grid lines in accordance with other tuning systems would not alter the main conclusions concerning the locality of the major, minor, and tension triads.

Such modeling is of interest because the tension and affective valence of various triads is thereby quantified and can be compared with the evaluations of the emotional character of such chords, as reported by human listeners. Experiments of that kind have been undertaken by psychologists worldwide for many decades.[17,24–31] Although only a small minority of musicians with absolute pitch can readily identify chords by ear (e.g., E-flat major in second inversion), most normal music listeners are rather competent at indicating the emotional mood of chords and short melodies (when the effects of rhythms, lyrics, and the larger musical context are adequately controlled). For example, we have recently tested 98 Japanese undergraduates on their perception of the "brightness" or "darkness" of 54 three-chord cadenzas that differed only by ending in major or minor triads. Fully 87% of the listeners evaluated the major cadenzas as "brighter," on average, than the minor cadenzas. Moreover, intra-individual t-tests showed that distinction to be statistically significant ($p < 0.05$) in 41%, with no individuals showing a statistically significant reversal. Whether such perception is simply recognition of a learned cultural idiom or perception of something inherent to the acoustics of tone harmonies remains uncertain, but is well established as a cross-cultural psychological phenomenon.

The "reality" of the emotional valence of major and minor harmonies, in particular, is a strange phenomenon! From the perspective of hard-headed acoustical physics, the perception of any pitch is nothing more than the sensation of the oscillations of air molecules in the inner ear and yet, in the diatonic musical tradition, there is an inherent affective valence associated with various forms of sonority. The psychoacoustical question posed by the Cycle of Modes is as follows: Why are semitone *rises* in auditory frequency from the indecisive tension of an atonal chord traditionally interpreted as emotionally *negative* (minor), while a similar semitone step downward as emotionally *positive* (major)? Tonal duality has a long history in the diatonic tradition,[39] a tradition that has led both naive and sophisticated commentators to conclude that the affect of music, in general, and diatonic harmonies, in particular, is a consequence of acoustical physics. A more biological view is that pitch rises and falls have obtained inherent meaning from the long evolution of auditory signaling among animal organisms.[33] In either case, if the valence of major and minor modes is indeed entirely arbitrary and without an acoustical foundation, why are there no musical traditions where the affective valence of major and minor is reversed?

Such questions lead into the controversial field of the "sound symbolism" of auditory communications[49,50] and go well beyond the scope of the present report. Suffice it to say that the tension and valence algorithms discussed above suggest that perception of relative interval sizes in pitch triads forms the basis for both the tonal/atonal distinction and the major/minor distinction. The deeper psychological questions, however, remain unanswered: Why is the *symmetry* of interval equidistance perceived as tense and why is *asymmetry* with a larger interval below a smaller interval perceived as major, and vice versa for minor? These are topics that require further research, particularly in relation to the perception of individuals from non-diatonic musical cultures.

VII. CONCLUSION

Following upon the insights of Galilei and Meyer, we have shown that two characteristic features of triadic harmonies have quantifiable acoustical foundations. That is, their perceived sonority—as indicated by most normal listeners worldwide—is a function of the ratio of the two small intervals contained within the triad. Note that this perceptual fact is *not* an example of a hyper-complex "mathematical mind," that the Pythagoreans believed in and which Parncutt and Hair[51] have decisively rejected. On the contrary, distinguishing between auditory symmetry ("$=$") and asymmetry ("$\neq$") is the kind of "approximate" Gestalt psychology that Meyer[40] has emphasized. Moreover, the computed positive and negative valence of major and minor triads is consistent with the traditional characterization of such harmonies—with slight differences among the various inversions of these chords. Again, complex mathematical ratios appear *not* to be involved, but rather the basic inequality functions ("$<$" and "$>$") are involved.

The acoustical models discussed above do not end the nature/nurture debate in music psychology, but they do indicate that the triadic harmonies which have dominated Western classical music and much of popular music worldwide for many centuries have definable acoustical properties that correlate strongly with measurable psychological properties. Although the structure of scales and specific tuning systems are undoubtedly learned through exposure and training, the structure of the common diatonic triads produced in 12-tone scales can be characterized in terms of relative interval sizes. Without directly measuring the psychological factors that lead to evaluations of the tension or positive/negative valence of triadic harmonies, a parsimonious interpretation of the reported perceptual regularities is that, in attending to the acoustical phenomena of "intervallic equidistance," most listeners perceive an ambiguous tension that can be harmonically resolved by appropriate semitone changes to any of the three pitches. If small dissonant intervals are avoided, semitone rises from tension consistently resolve to minor triads and semitone falls resolve to major triads. Further experimental study of the perception of symmetrical and asymmetrical triads would be of interest in populations from diverse musical cultures.

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