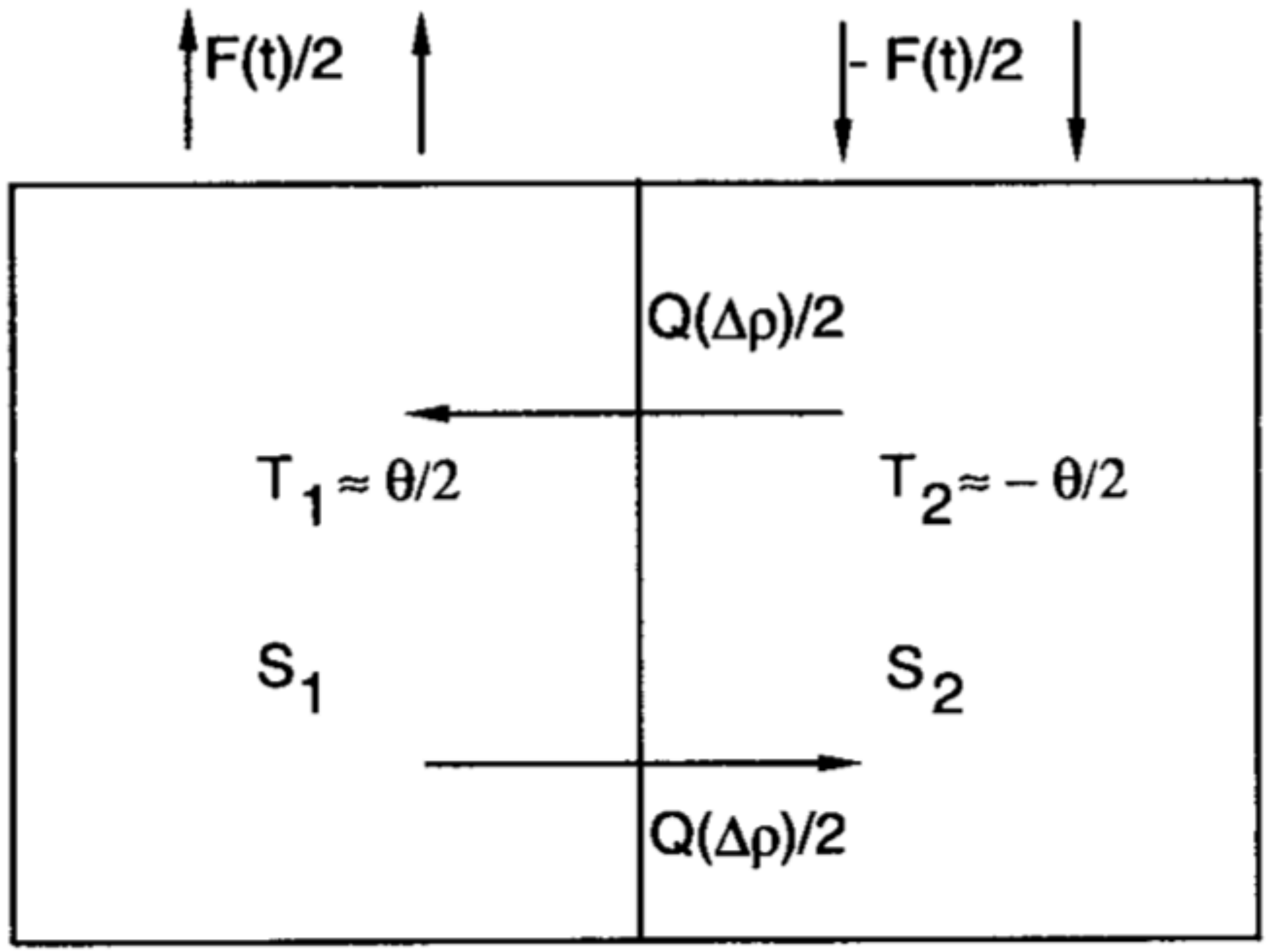
2. AMOC modelling



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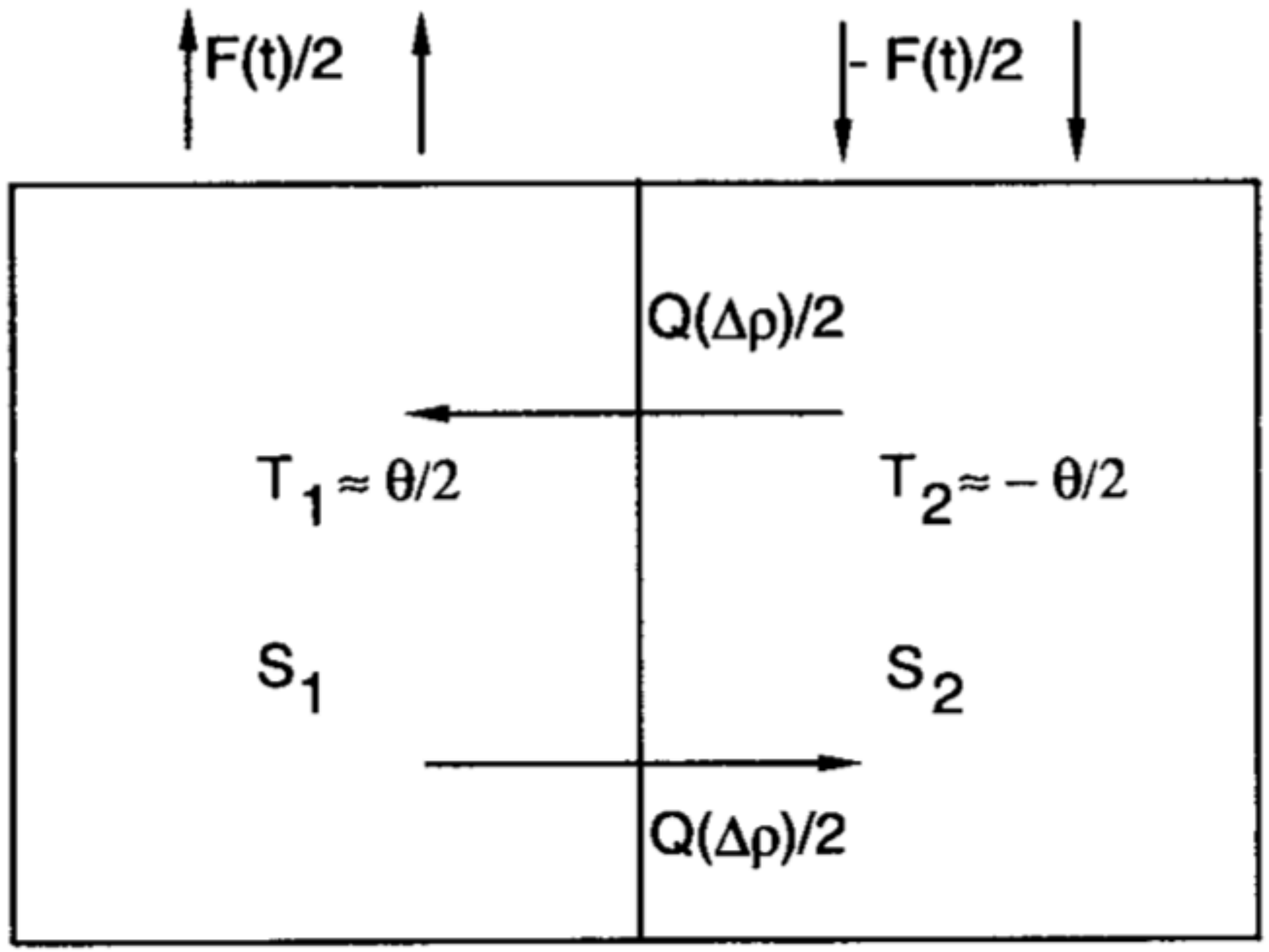


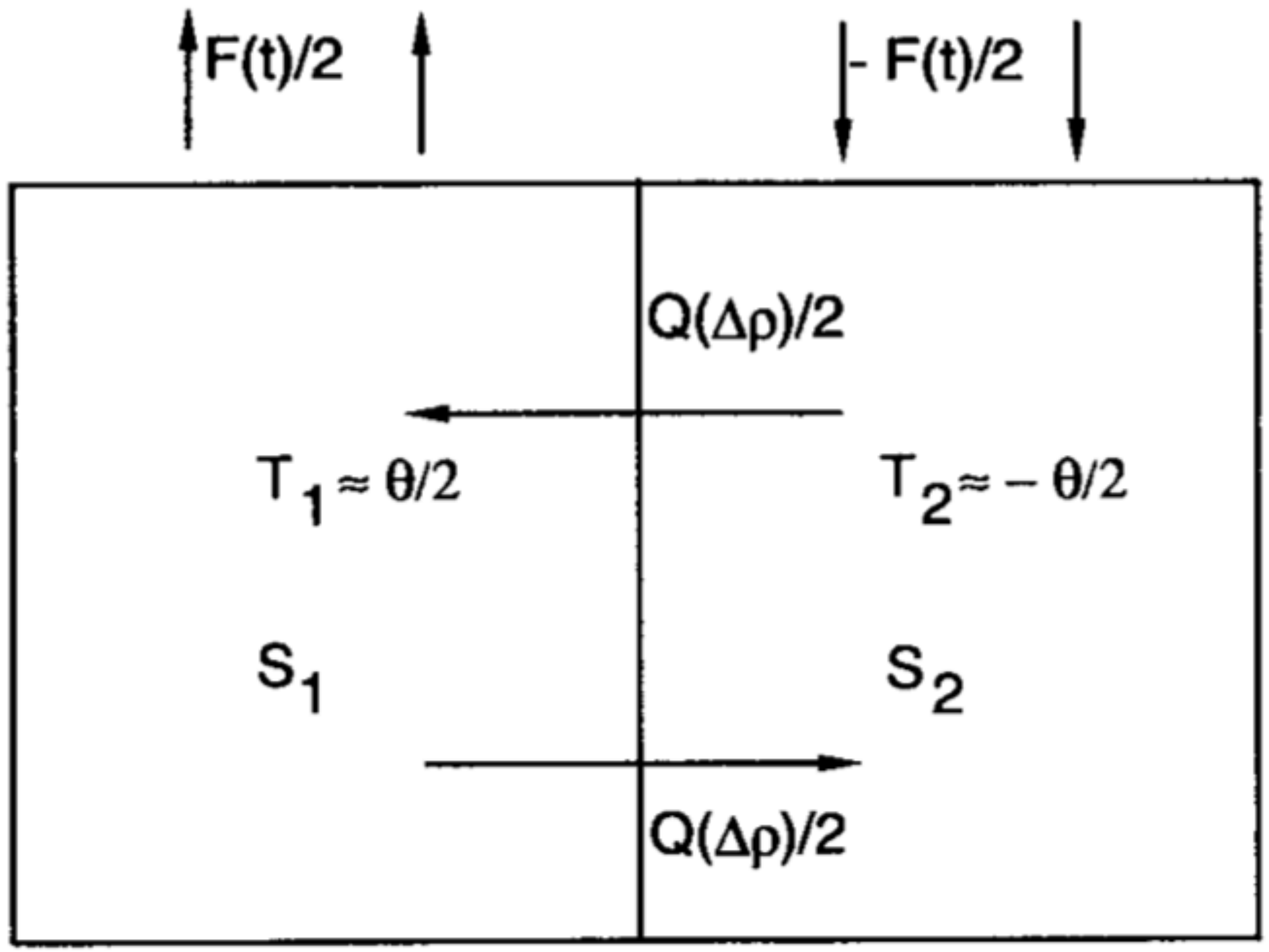
$$\Delta T = T_1 - T_2, \quad \Delta S = S_1 - S_2$$

$$\frac{d\Delta T}{dt} = -\frac{\Delta T - \theta}{\tau_r} - Q(\Delta\rho)\Delta T$$

$$\frac{d\Delta S}{dt} = \frac{F(t)}{H} S_0 - Q(\Delta\rho)\Delta S$$

$$\Delta\rho = \alpha_S \Delta S - \alpha_T \Delta T$$

2. AMOC modelling



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Stommel model

$$Q(\Delta\rho) = \frac{1}{\tau_d} + \frac{q\,|\,\Delta\rho\,|}{V}$$

Cessi version

$$Q(\Delta\rho) = \frac{1}{\tau_d} + \frac{q\Delta\rho^2}{V}$$

2. AMOC modelling

The equations for the Cessi version of the Stommel model are

$$\frac{d\Delta T}{dt} = -\frac{\Delta T - \theta}{\tau_r} - \frac{\Delta T}{\tau_d} + \frac{q\Delta\rho^2}{V}\Delta T$$

$$\frac{d\Delta S}{dt} = \frac{F(t)}{H}S_0 - \frac{\Delta S}{\tau_d} + \frac{q\Delta\rho^2}{V}\Delta S$$

$$\Delta\rho = \alpha_S \Delta S - \alpha_T \Delta T$$

The equations can be simplified by noting that the temperature relaxation time scale is much shorter than the diffusive time scale

$$\alpha = \frac{\tau_d}{\tau_r} \gg 1$$

From observations $\tau_r \approx 25$ days, $\tau_d \approx 219$ years. This implies that the relaxation term in the temperature equation dominates over all the others, and the temperature gradient can be considered to be fixed $\Delta T \approx \theta$

2. AMOC modelling

The equations for the Cessi version of the Stommel model are then

$$\Delta T = \theta$$

$$\frac{d\Delta S}{dt} = \frac{F(t)}{H}S_0 - \frac{\Delta S}{\tau_d} + \frac{q\Delta\rho^2}{V}\Delta S$$

$$\Delta\rho = \alpha_S \Delta S - \alpha_T \Delta T$$

Which means that we have only one equation

$$\frac{d\Delta S}{dt} = \frac{F(t)}{H}S_0 - \frac{\Delta S}{\tau_d} + \frac{q}{V}(\alpha_S \Delta S - \alpha_T \theta)^2 \Delta S$$

The right hand side is cubic in ΔS, which means that the potential will be quartic, which means that it can show bistability

2. AMOC modelling

Project

$$\frac{d\Delta S}{dt} = \frac{F(t)}{H}S_0 - \frac{\Delta S}{\tau_d} + \frac{q}{V}(\alpha_S \Delta S - \alpha_T \theta)^2 \Delta S$$

Part 1

Consider F(t)=F=constant and study the multistability of the model

1.1 Take F=3.24 m year^{-1}. Show graphically that the system is bistable.

1.2 Cessi proposes F=2.3 m year^{-1}. Is the system bistable in this case? Estimate numerically the critical value of F at which the system changes the number of stable solutions

1.3 Write a code to integrate in time the equation and show plots of time series of the solutions to support what you found in points 1 and 2. You can take a timestep of 1 year.

2. AMOC modelling

Project

$$\frac{d\Delta S}{dt} = \frac{F(t)}{H}S_0 - \frac{\Delta S}{\tau_d} + \frac{q}{V}(\alpha_S \Delta S - \alpha_T \theta)^2 \Delta S$$

Part 2

Consider F(t) as a stochastic forcing and study the multistability

1.1 Take F=3.24 m year^{-1} and plot the potential of the model

1.2 Consider a stochastic forcing

$$F(t) = \bar{F} + \sigma_F \xi(t), \qquad \sigma_F = 3\bar{F}\sqrt{\Delta t}$$

write a code and perform simulations, discussing the results. Try different values of $\bar{F}$ and σ_F

1.3 Optional: play with the forcing, for example what if $\bar{F}$ depends on time?

2. AMOC modelling

Cessi (1994) analyses the equation in non-dimensional form, after rescaling the variables. I suggest to work instead with the full dimensional form

$$\frac{d\Delta S}{dt} = \frac{F(t)}{H} S_0 - \frac{\Delta S}{\tau_d} + \frac{q}{V}(\alpha_S \Delta S - \alpha_T \theta)^2 \Delta S$$

With values of the parameters:

$$V = L\, H\, \sigma_w \qquad \theta = 20\, C$$

$$L = 8250 \cdot 10^3\, m \qquad S_0 = 35\, psu$$

$$H = 4500\, m \qquad \alpha_T = 0.17 \cdot 10^{-3} C^{-1}$$

$$\sigma_w = 300 \cdot 10^3\, m \qquad \alpha_S = 0.75 \cdot 10^{-3} psu^{-1}$$

$$\tau_d = 219\, years \qquad q = 3.27 \cdot 10^{19}\, m^3 years^{-1}$$