Multiple equilibria and abrupt changes of the oceanic thermohaline circulation

Personal work • LPHYS2162 – Academic year 2023–2024

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The Atlantic Meridional Overturning Circulation (AMOC) transports heat northward in the Atlantic, contributing to the relatively mild temperature in some regions of the North Atlantic and adjacent continents. This circulation has displayed in the past abrupt changes leading to rapid climate variations. The possibility of a future collapse of the AMOC in response to anthropogenic forcing has been the subject of several studies, as that may lead to severe cooling of some regions in Europe. The goal of this personal work is to investigate, using a simple box model, the processes responsible for such abrupt changes. The reference paper is:

> Cessi, P., 1994: A simple box model of stochastically forced thermoaline flow, Journal of Physical Oceanography, 24(9), 1911–1920.

Model description

A low-dimensional description of the AMOC is given by the classic box model of Stommel (1961), which describes the dynamics of meridional temperature and salinity gradients in the Atlantic. Here we consider the Stommel-like two-box model presented in Cessi (1994). The starting equations are (formula 2.3 in Cessi 1994)

$$\frac{d}{dt}\Delta T = -\frac{\Delta T - \theta}{\tau_r} - Q(\Delta\rho)\Delta T \quad (1)$$

$$\frac{d}{dt}\Delta S = \frac{F(t)}{H} S_0 - Q(\Delta\rho)\Delta S \quad (2)$$

where ΔT, ΔS and $\Delta\rho$ are respectively the meridional temperature, salinity and density differences between the two boxes, θ and S_0 are reference values of temperature and salinity, τ_r is the temperature relaxation time scale, $F(t)$ is the freshwater flux, H is the depth of the ocean, and $Q(\Delta\rho)$ is a function of the density difference representing the mass transport between the two boxes. The system is closed by the equation of state

$$\Delta\rho = \alpha_S \Delta S - \alpha_T \Delta T \quad (3)$$

where α_S and α_T are empirical parameters, and by specifying the functional forms of the forcing term $F(t)$ and of the mass transport $Q(\Delta\rho)$. See Cessi (1994) for a full description of the model and standard values of the parameters based on observations.

For certain choices of $F(t)$ and $Q(\Delta\rho)$ this model allows two stable equilibria, corresponding to active or collapsed AMOC states (see Goosse 2015). Transitions between states can be triggered by changes in the freshwater flux term. Physically, this is due to the fact that changes in freshwater fluxes can change the density of surface waters in the North Atlantic and therefore the magnitude of deep water formation. The original Stommel model is purely deterministic, and internal variability can not trigger transitions if the external conditions do not change. Cessi (1994) includes a stochastic part in $F(t)$ representing short-term chaotic variability of freshwater fluxes, which can generate noise-induced transitions between states. The project consists of an exploration of the properties of this model.

Instructions

The work should be carried out in pairs, composed of students with different backgrounds if possible. You are expected to prepare a report in English, and to submit it two weeks before the date of the exam. There are no limits to the number of pages allowed, but an extensive discussion is expected. The project is divided in two parts. Part 1 does not need knowledge of stochastic processes, so it is possible to start working on it without having to wait for the classes on stochastic modelling.

Part 1

As explained in Cessi (1994), the classic Stommel model is obtained assuming that the freshwater flux is a constant $F(t) = \bar{F}$, and that the mass transport as a function of the density difference is given by

$$Q(\Delta\rho) = \frac{1}{\tau_d} + \frac{q}{V}|\Delta\rho| \quad (4)$$

where τ_d is a diffusive timescale, V is the volume of one box, and q is an hydrodynamic parameter. You have seen this case in class (with a different notation). The first modification that Cessi (1994) introduces to the model is to consider a different mass transport

$$Q(\Delta\rho) = \frac{1}{\tau_d} + \frac{q}{V}\Delta\rho^2 \tag{5}$$

which is meant to be slightly more realistic than the original version of Stommel.

1/ Consider the model with fixed $F(t) = \bar{F}$ and this choice of Q. With all the other parameters fixed at the values presented in Cessi (1994), show that depending on the value of $\bar{F}$ two stationary solutions can coexist, replicating the procedure seen in class for the Stommel model (see Goosse 2015 as refefence). Note that you may not be able to obtain a closed analytical formula for the solutions like for the Stommel model, but you can show the existence of multiple equilibria graphically (how?). Start considering $\bar{F} = 3.25\ m\ year^{-1}$.

2/ Cessi (1994) proposes $\bar{F} = 2.3\ m\ year^{-1}$ as best estimate from observations. Is the system bistable in this case? Explain how the system can develop an hysteresis behavior, and estimate in a simple way the critical values of $\bar{F}$ beyond which only a single state is stable.

3/ Write a numerical code to integrate the equations of the model in time and present plots of numerical solutions to support your explanation (practical suggestion: you can use a time step of one year once the model is reduced to a single equation for salinity).

Part 2

Consider now the freshwater flux term to be composed of a deterministic part $\bar{F}$ and a stochastic part $F'(t)$

$$F = \bar{F} + F'(t) \tag{6}$$

where $F'(t)$ is a Gaussian white noise with zero mean and standard deviation σ_F. According to Cessi (1994), if $F'(t)$ represents noisy fluctuations at seasonal scale a reasonable choice is to take $\sigma_F \approx 3\bar{F}$. In this second part of the project you should explore the properties of this stochastic version of the model.

1/ Compute and plot the potential of the model once it is reduced to a single equation for salinity, for different choices of $\bar{F}$ aimed at showing the presence or lack of multistability. Note that Cessi (1994) reduces the model to nondimensional form before computing the potential, so figures like Figure 2 are in nondimensional units. You can work with the nondimensional model in your code if you want, but when you present your own figures plot physical quantities in dimensional units.

2/ Write a code to simulate the time evolution of the system. The minimal goal is to obtain a figure like Figure 5 in Cessi (1994), again in dimensional units. Start with $\bar{F} = 3.25\ m\ year^{-1}$. How long do you need to run the model to see transitions? What does this model tell you about the typical waiting time of spontaneaous transitions between the two states? Try different values of $\bar{F}$ to showcase the properties of the time evolution of the system in different conditions.

3/ If you have time, consider a case in which you have both a stochastic component different from zero, and variations in the mean freshwater fluxes. Design experiments to give examples of what happens to the likelihood of noise-induced transitions when the mean freshwater fluxes change, and discuss your results.

References

Cessi, P. (1994). A simple box model of stochastically forced thermohaline flow. *Journal of Physical Oceanography*, 24(9):1911 – 1920.

Goosse, H. (2015). *Climate system dynamics and modelling*. Cambridge University Press.

Stommel, H. (1961). Thermohaline convection with two stable regimes of flow. *Tellus*, 13(2):224–230.