Regressions- och tidsserieanalys

Föreläsning 11 - Multipel logistisk regression.

Mattias Villani

Statistiska institutionen
Stockholms universitet

Institutionen för datavetenskap
Linköpings universitet

mattiasvillani.com @matvil mattiasvillani

Översikt

- Multipel logistisk regression
- Estimation av logistisk regression

Multipel logistisk regression

- **Multipel logistisk regression** med k förklarande variabler:

$$P(y = 1|x_1, \ldots, x_k) = \frac{\exp(\beta_0 + \beta_1 x_1 + \ldots + \beta_k x_k)}{1 + \exp(\beta_0 + \beta_1 x_1 + \ldots + \beta_k x_k)}$$

- **Odds**

$$\text{Odds}(y = 1|\text{alla } x) = \frac{P(y = 1|\text{alla } x)}{P(y = 0|\text{alla } x)} = \exp(\beta_0 + \beta_1 x_1 + \ldots + \beta_k x_k)$$

- **Oddskvot**

$$OR_j = \frac{\text{Odds}(y = 1|x_j + 1, \text{allt annat lika})}{\text{Odds}(y = 1|x_j, \text{allt annat lika})} = \exp(\beta_j)$$

Vilka överlevde Titanic? Multipel logistisk

- $n = 891$ personer på Titanic, varav 342 överlevande.
- Responsvariabel: $y = 1$ om överlevde, annars $y = 0$.
- Förklarande variabler:
  - Age
  - Sex (1=Kvinna, 0 = Man)
  - FirstClass (1=Första klass, 0 = Ej första klass)

```
> library(regkurs)
> fit <- glm(survived ~ age + sex + firstclass, data = titanic, family = binomial)
> logisticregsummary(fit, conf_intervals = T)

Parameter estimates
-----------------------------------------------
               Estimate Std. Error z value   Pr(>|z|)     2.5 %    97.5 %
(Intercept)    -1.190302  0.2168513 -5.4890 4.0416e-08 -1.621056 -0.769945
age            -0.027371  0.0068935 -3.9705 7.1724e-05 -0.041084 -0.014028
sexfemale       2.586333  0.1815696 14.2443 4.8637e-46  2.236692  2.949135
firstclassTRUE  1.958678  0.2274770  8.6104 7.2779e-18  1.520081  2.412879

Odds ratio estimates
-----------------------------------------------
              Estimate Std. Error z value   Pr(>|z|)   2.5 %   97.5 %
(Intercept)    0.30413     1.2422 -5.4890 4.0416e-08 0.19769  0.46304
age            0.97300     1.0069 -3.9705 7.1724e-05 0.95975  0.98607
sexfemale     13.28098     1.1991 14.2443 4.8637e-46 9.36231 19.08943
firstclassTRUE 7.08995     1.2554  8.6104 7.2779e-18 4.57259 11.16607
```

Vilka överlevde Titanic?

```
Parameter estimates
------------------------
                 Estimate
(Intercept)    -1.190302
age            -0.027371
sexfemale       2.586333
firstclassTRUE  1.958678
```

```
Odds ratio estimates
----------------------
                 Estimate
(Intercept)      0.30413
age              0.97300
sexfemale       13.28098
firstclassTRUE   7.08995
```

- Logistisk regression - Odds version

$$\text{Odds}(y = 1|x) = \exp(\beta_0 + \beta_1 \cdot \texttt{Age} + \beta_2 \cdot \texttt{Sex} + \beta_3 \cdot \texttt{FirstClass})$$

- Interceptet β_0 - Oddset överleva, nyfödd pojke, ej första klass:

$$\text{Odds}(y = 1|\texttt{Age} = 0, \texttt{Sex} = 0, \texttt{First} = 0) = \exp(\beta_0) = \exp(-1.190302) = 0.30413$$

$$\text{P}(y = 1|\texttt{Age} = 0, \texttt{Sex} = 0, \texttt{First} = 0) = \frac{\text{Odds}}{1 + \text{Odds}} = \frac{0.30413}{1 + 0.30413} = 0.23321$$

- Nyfödd flicka, ej i första klass:

$$\begin{aligned}\text{Odds}(y = 1|\texttt{Age} = 0, \texttt{Sex} = 1, \texttt{First} = 0) &= \exp(\beta_0 + \beta_2) = \exp(\beta_0)\exp(\beta_2) \\ &= 0.30413 \cdot 13.28098 = 4.039144\end{aligned}$$

$$\text{P}(y = 1|\texttt{Age} = 0, \texttt{Sex} = 1, \texttt{First} = 0) = \frac{4.039144}{1 + 4.039144} = 0.8015536$$

Vilka överlevde Titanic?

```
Parameter estimates
------------------------
                 Estimate
(Intercept)    -1.190302
age            -0.027371
sexfemale       2.586333
firstclassTRUE  1.958678
```

```
Odds ratio estimates
----------------------
                 Estimate
(Intercept)       0.30413
age               0.97300
sexfemale        13.28098
firstclassTRUE    7.08995
```

- Nyfödd flicka, första klass:

$$\begin{aligned}\text{Odds}(y = 1|\texttt{Age} = 0, \texttt{Sex} = 1, \texttt{FirstClass} = 1) &= \exp(\beta_0 + \beta_2 + \beta_3) \\ &= \exp(\beta_0)\exp(\beta_2)\exp(\beta_3) = 4.039144 \cdot 7.08995 = 28.63733\end{aligned}$$

$$\text{P}(y = 1|\texttt{Age} = 0, \texttt{Sex} = 1, \texttt{FirstClass} = 1) = \frac{28.63733}{1 + 28.63733} = 0.9662588$$

- 1-årig flicka, första klass:

$$\begin{aligned}\text{Odds}(y = 1|\texttt{Age} = 1, \texttt{Sex} = 1, \texttt{FirstClass} = 1) &= \exp(\beta_0 + \beta_1 \cdot 1 + \beta_2 + \beta_3) \\ &= \exp(\beta_0)\exp(\beta_1)\exp(\beta_2)\exp(\beta_3) = 28.63733 \cdot 0.973 = 27.86412\end{aligned}$$

$$\text{P}(y = 1|\texttt{Age} = 1, \texttt{Sex} = 1, \texttt{FirstClass} = 1) = \frac{27.86412}{1 + 27.86412} = 0.9653549$$

Vilka överlevde Titanic?

```
Parameter estimates
------------------------
                 Estimate
(Intercept)     -1.190302
age             -0.027371
sexfemale        2.586333
firstclassTRUE   1.958678
```

```
Odds ratio estimates
------------------------
                 Estimate
(Intercept)      0.30413
age              0.97300
sexfemale       13.28098
firstclassTRUE   7.08995
```

- 2-årig flicka, första klass:

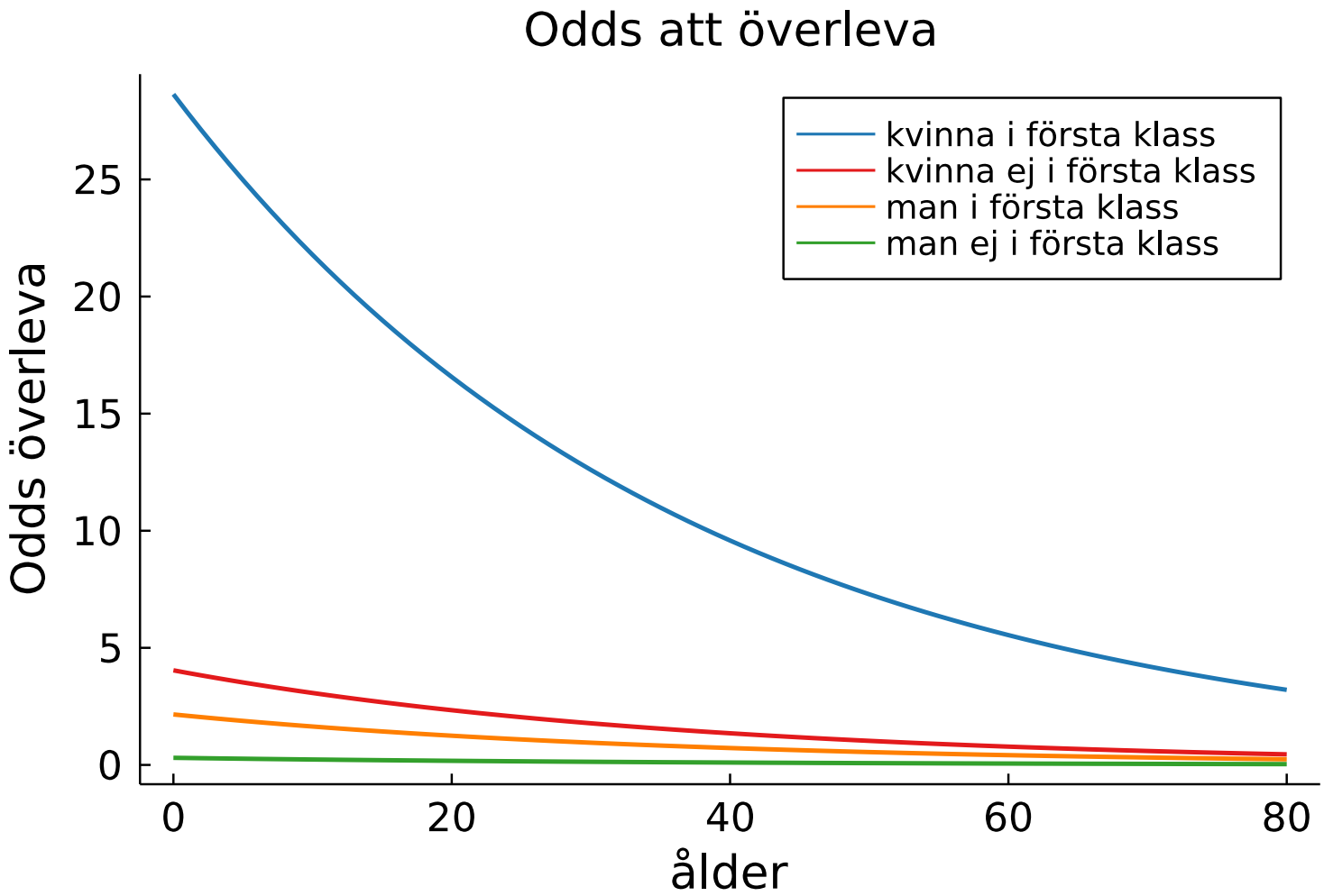
$$\begin{aligned}\text{Odds}(y = 1|\texttt{Age} = 2, \texttt{Sex} = 1, \texttt{FirstClass} = 1) &= \exp(\beta_0 + \beta_1 \cdot 2 + \beta_2 + \beta_3) \\ &= \exp(\beta_0 + \beta_1 + \beta_1 + \beta_2 + \beta_3) \\ &= \exp(\beta_0)\exp(\beta_1)\exp(\beta_1)\exp(\beta_2)\exp(\beta_3) = 27.86412 \cdot 0.973 = 27.11179\end{aligned}$$

$$\text{P}(y = 1|\texttt{Age} = 2, \texttt{Sex} = 1, \texttt{FirstClass} = 1) = \frac{27.11179}{1 + 27.11179} = 0.9644277$$

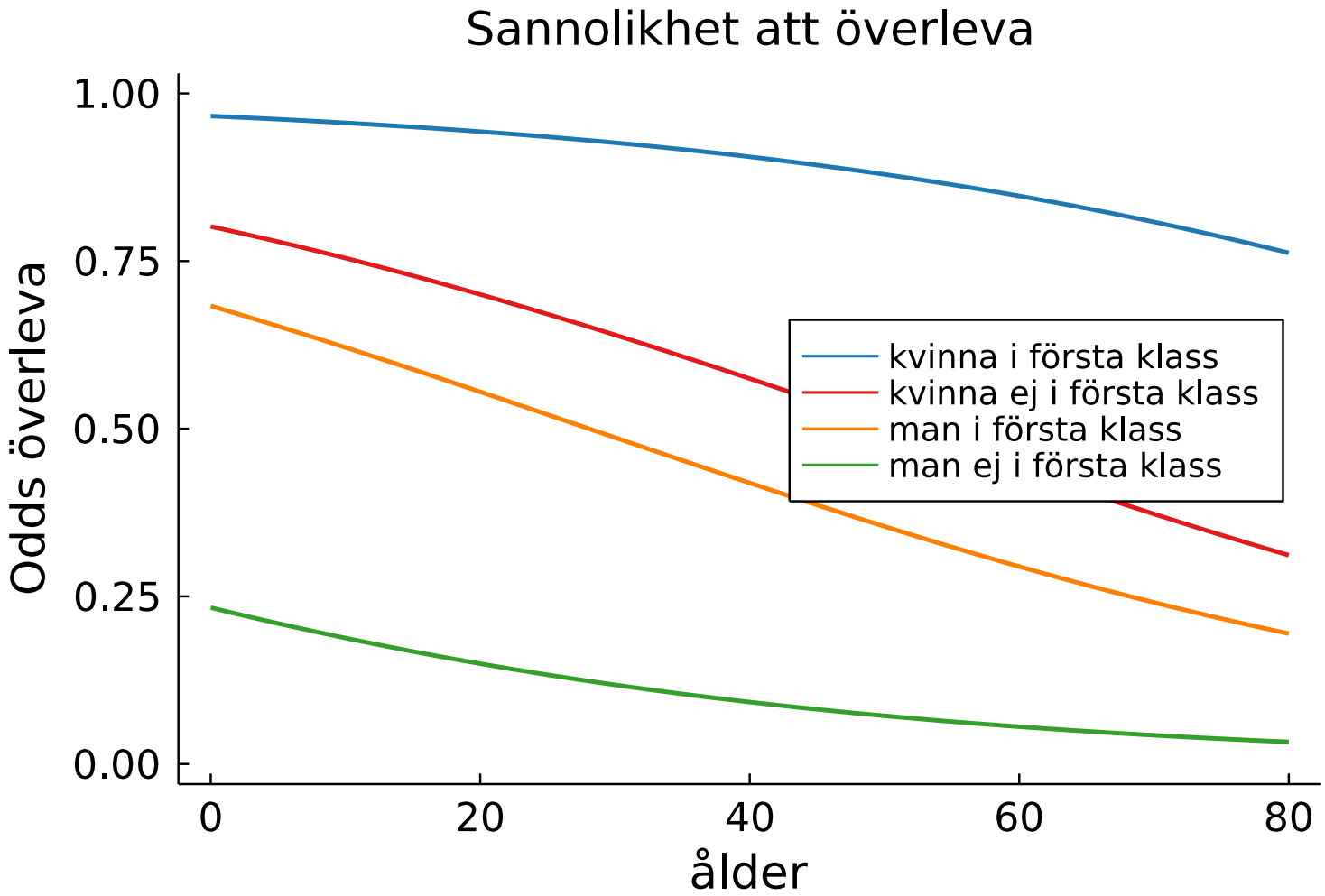
- Kontroll:

$$\begin{aligned}&\text{P}(y = 1|\texttt{Age} = 2, \texttt{Sex} = 1, \texttt{FirstClass} = 1) \\ &= \frac{\exp(-1.190302 - 0.027371 \cdot 2 + 2.586333 + 1.958678)}{1 + \exp(-1.190302 - 0.027371 \cdot 2 + 2.586333 + 1.958678)} = 0.9644277\end{aligned}$$

Vilka överlevde Titanic? Odds



Vilka överlevde Titanic? Sannolikhet



Skatta en logistisk regression

- Datamaterial med tre **oberoende** datapunkter ($n = 3$):

$$y_1 = 0, y_2 = 1, y_3 = 0.$$

- Varje y_i observeras tillsammans med en förklarande variabel

$$x_1, x_2, x_3$$

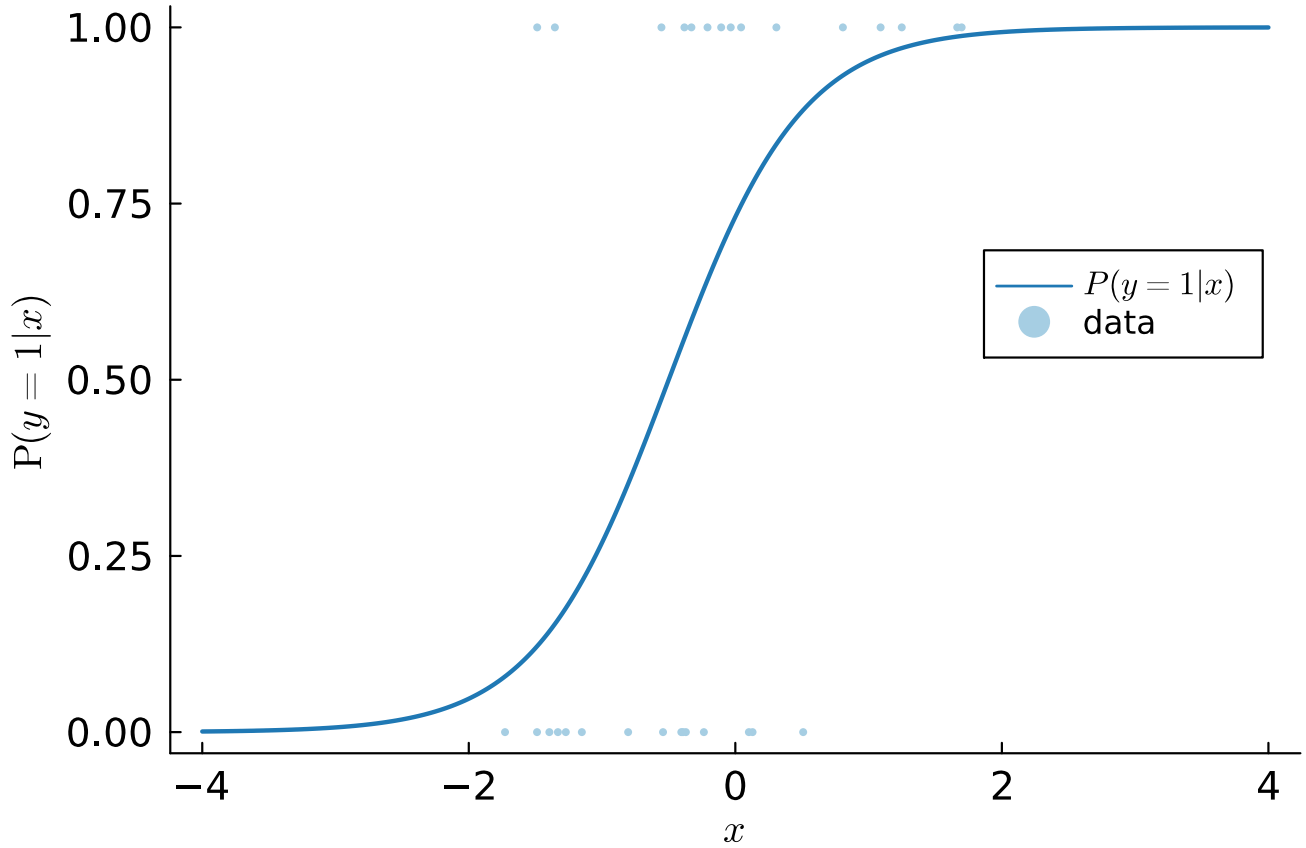
- **Sannolikheten för just detta datamaterial**:

$$\underbrace{\frac{1}{1+\exp(\beta_0+\beta_1 x_1)}}_{P(y_1=0)} \cdot \underbrace{\frac{\exp(\beta_0+\beta_1 x_2)}{1+\exp(\beta_0+\beta_1 x_2)}}_{P(y_2=1)} \cdot \underbrace{\frac{1}{1+\exp(\beta_0+\beta_1 x_3)}}_{P(y_3=0)}$$

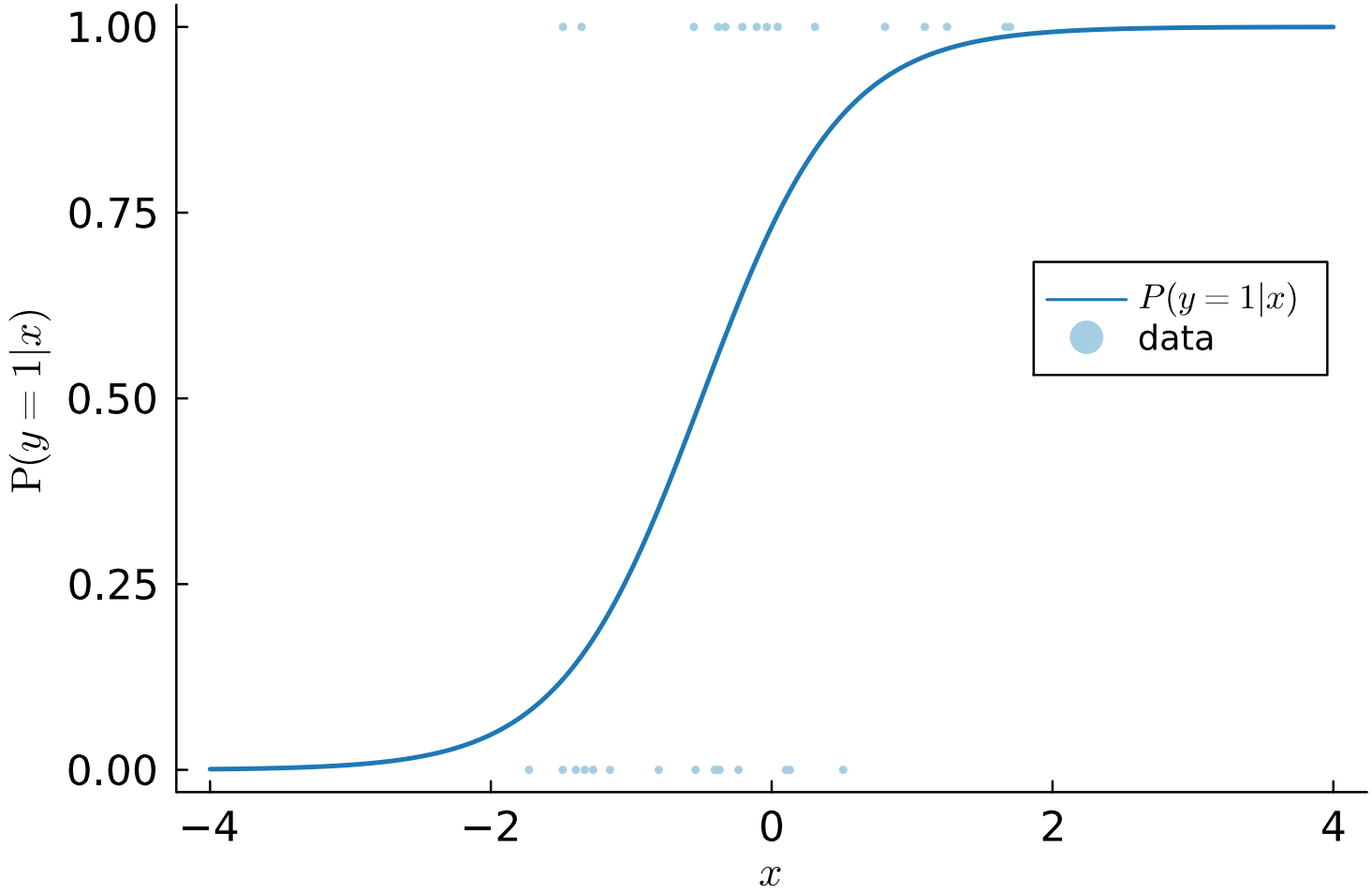
- **Maximum likelihood**: välj de parametervärden β_0 och β_1 som maximerar sannolikheten för det observerade datamaterialet.

Logistisk regression - maximum likelihood

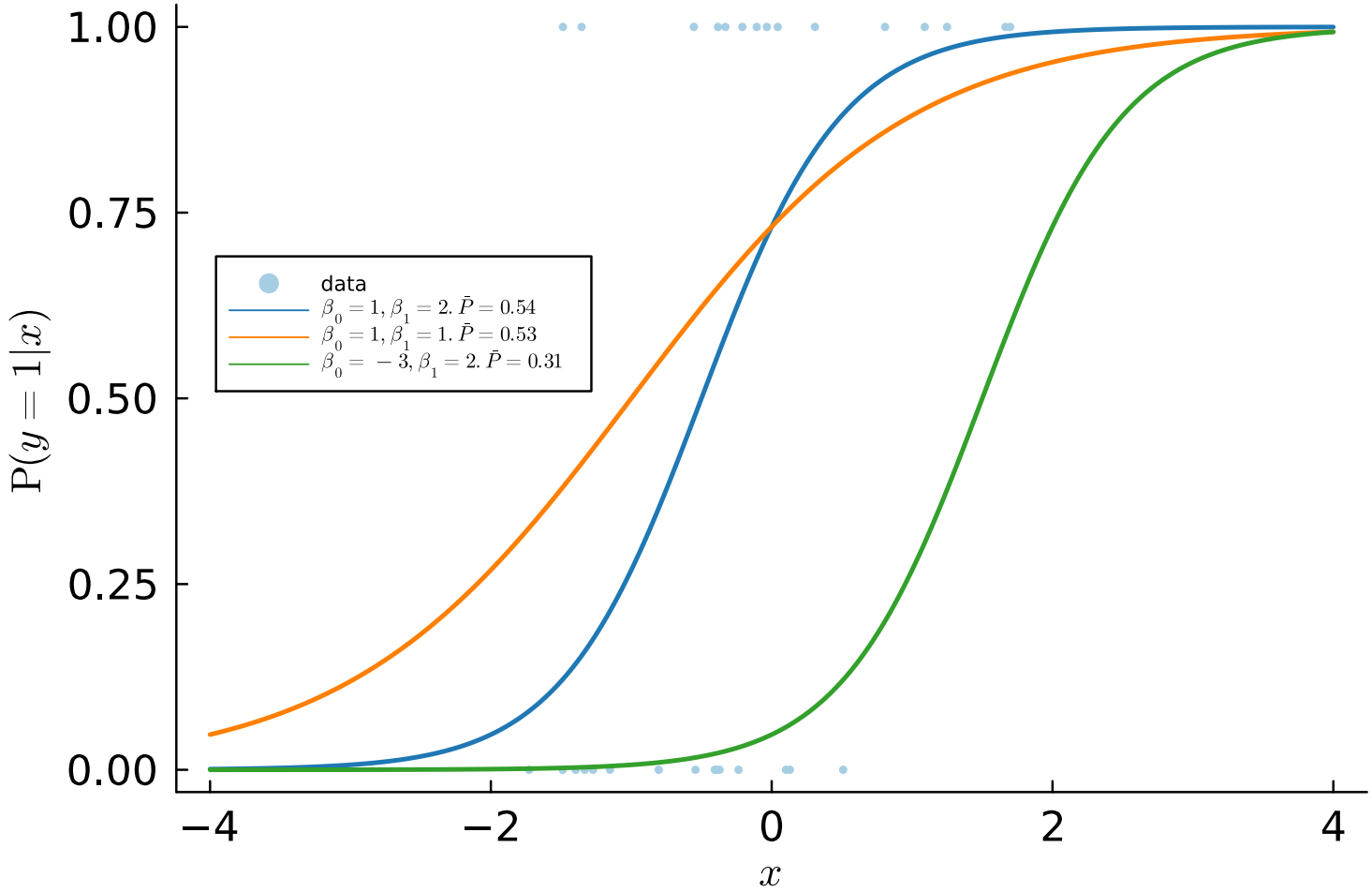
- Data $(x_1, y_1), \ldots, (x_n, y_n)$ simulerat från logistisk regression.
  - $\beta_0 = 1$ och $\beta_1 = 2$
  - $n = 30$



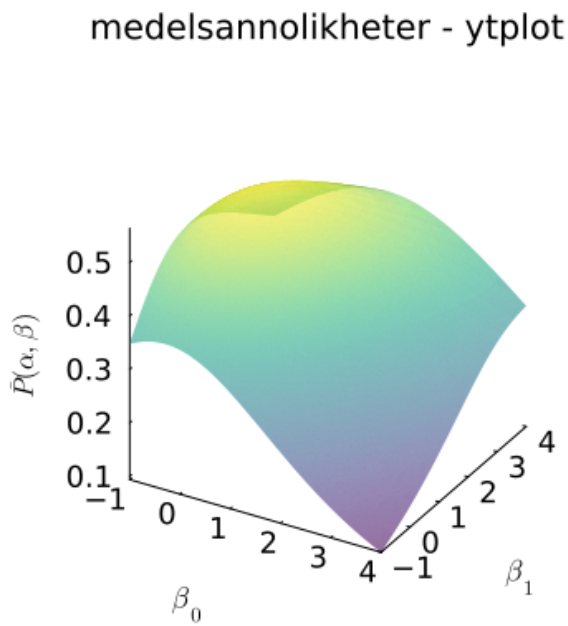
Skatta en logistisk regression

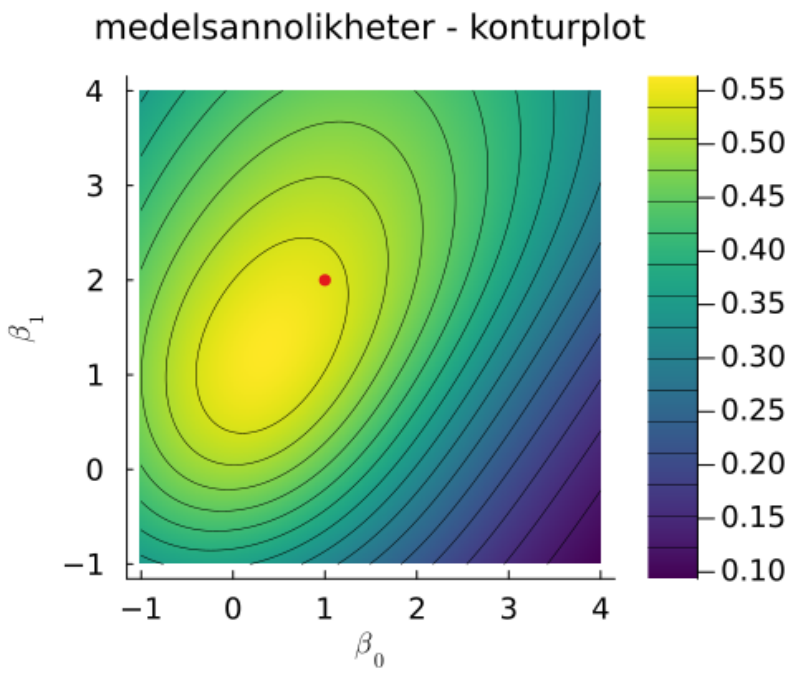


Skatta en logistisk regression



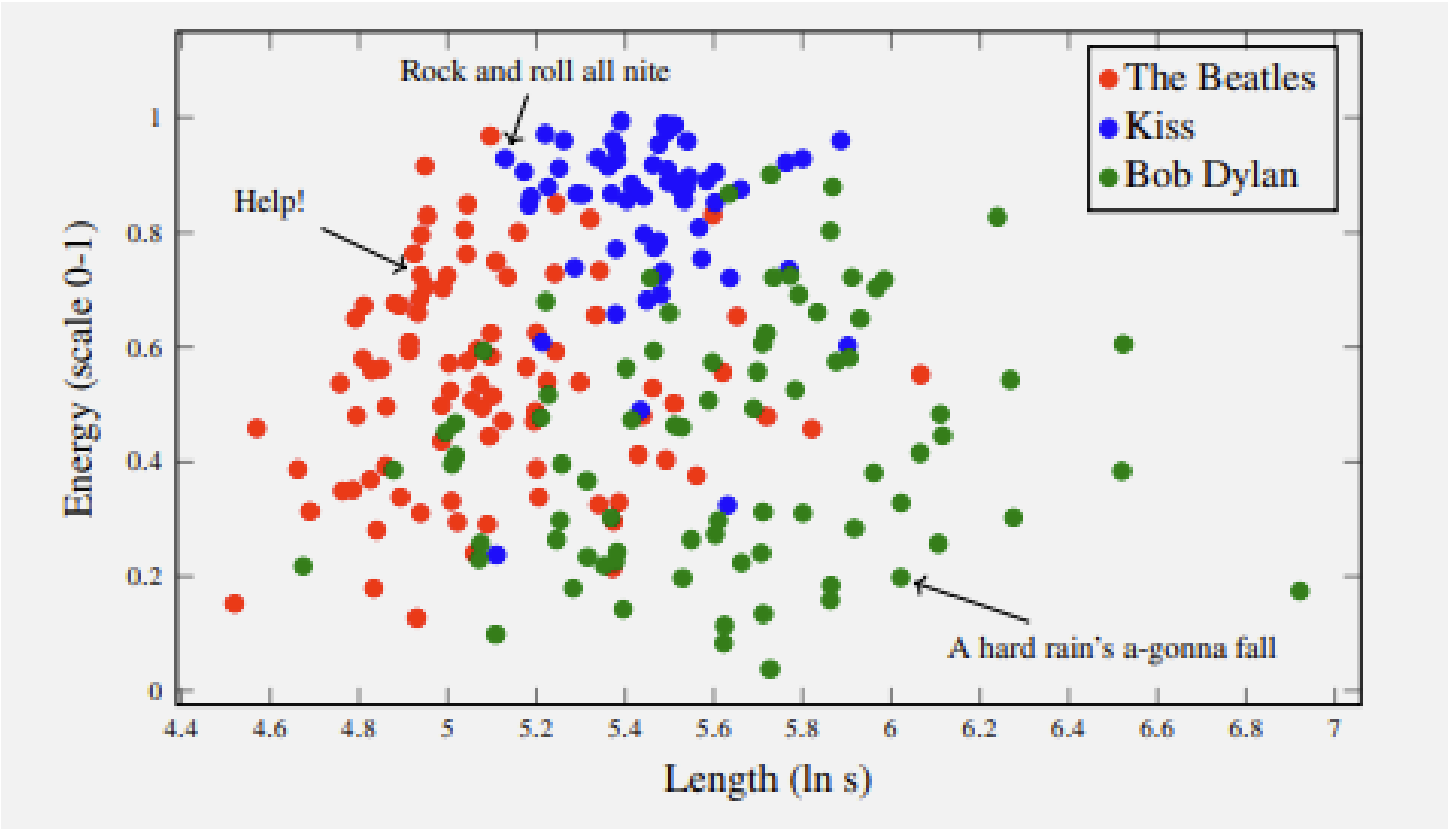
Skatta en logistisk regression





Multinomial logistisk regression

- Spotify-data från boken
  Machine Learning - A First Course for Engineers and Scientists



- Respons med fler än två kategorier (binärt). **Multinomial logistisk regression**.