# SPECTRAL SUBSAMPLING MCMC FOR MULTIVARIATE TIME SERIES

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ABSTRACT. Whatevs

## 1. INTRODUCTION

Spectral subsampling MCMC was proposed in Salomone et al. (2020) to accelerate MCMC for long stationary univariate time series.

## 2. MODEL

Let $\mathbf{X}_t \in \mathbb{R}^d$ be a $d$-variate zero mean stationary time series with autocovariance function

$$\mathbb{C}_{\mathbf{X}}(\tau) = \mathrm{Cov}(\mathbf{X}_t, \mathbf{X}_{t-u}) \tag{2.1}$$

The spectral density matrix is

$$f_{\mathbf{X}}(\omega) = \frac{1}{2\pi} \sum_{\tau=-\infty}^{\infty} \mathbb{C}_{\mathbf{X}}(\tau) \exp(-i\omega\tau), \tag{2.2}$$

with the diagonal elements is the usual spectral density for each time series and the off-diagonal elements are cross-spectral densities

$$f_{jk}(\omega) = \sum_{\tau=-\infty}^{\infty} \mathbb{C}_{jk}(\tau) \exp(-i\omega\tau). \tag{2.3}$$

The Discrete Fourier Transform (DFT) of $\mathbf{X}_t$ is

$$J_T(\omega) = \sum_{t=0}^{T-1} \mathbf{X}_t \exp(-i\omega t) \tag{2.4}$$

for $\omega \in [-\pi/2, \pi/2]$

The elements of the the DFT $J_T(\omega)$ at the Fourier frequencies for $\Omega_T = \{2\pi k/T\}_{k=0}^{[T/2]}$ are asymptotically independent complex multivariate normal (Brillinger, 2001)

$$J_T(\omega_k) \sim \mathrm{CN}(0, 2\pi T f_{\mathbf{X}}(\omega_k)) \text{ as } T \to \infty. \tag{2.5}$$

The periodogram ordinates $I_T(\omega) = T^{-1} J(\omega) \bar{J}(\omega)^\top$, where $\bar{J}(\omega)^\top$ is the conjugate transpose, are therefore asymptotically independent Complex Wishart distributed with one degree of freedom $I_T(\omega) \sim CW(1, f_{\mathbf{X}}(\omega))$

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The Whittle log-likelihood is therefore

$$\ell_{\mathcal{W}}(\theta) \stackrel{c}{=} -\sum_{\omega \in \Omega_T}^{T} \left( \log |f_{\mathbf{X}}(\omega_k)| + \operatorname{tr}\left[ f_{\mathbf{X}}(\omega_k)^{-1} I_T(\omega) \right] \right)$$

## 3. Experiments

## 4. Conclusions

## References

Brillinger, D. R. (2001). *Time series: data analysis and theory*. SIAM.

Salomone, R., Quiroz, M., Kohn, R., Villani, M., and Tran, M.-N. (2020). Spectral subsampling mcmc for stationary time series. *International Conference on Machine Learning (ICML2020)*.

## Appendix A. Additional results or proofs

Stuff goes in here.