ME1 Computing- Session 9: Advanced Matrix Operations

Learning outcomes:

- Consolidate the use of numpy
- Consolidate basic matrix operations
- Being able to compute advanced matrix operations

Before you start

In your H drive create a folder *H:\ME1MCP\Session9* and work within it.

Task A: Defining and manipulating matrices

Generate a square matrix **S**, of dimension N x N, with the following pattern:

$$S = \begin{pmatrix} \mathbf{1} & 0 & \mathbf{1} & 0 & \mathbf{1} & 0 \\ 0 & \mathbf{1} & 0 & \mathbf{1} & 0 & \mathbf{1} \\ \mathbf{1} & 0 & \mathbf{1} & 0 & \mathbf{1} & 0 \\ 0 & \mathbf{1} & 0 & \mathbf{1} & 0 & \mathbf{1} \\ \mathbf{1} & 0 & \mathbf{1} & 0 & \mathbf{1} & 0 \\ 0 & \mathbf{1} & 0 & \mathbf{1} & 0 & \mathbf{1} \end{pmatrix}$$

Task B: Determinant of a matrix

To compute the determinant of a matrix, (pag. 105 of Maths notes) we proceed in two steps:

1) Write a function *Minor*, that receives a matrix A of dimension N x N, and two indices *i* and *j*.
   The function returns a matrix, of dimension (N-1) x (N-1), obtained by matrix A, after removing row *i* and column *j*.
2) Write a **recursive** function *Determinant*, that receives a matrix **A** and returns the value of its determinant, i.e. $|\boldsymbol{A}| = \sum_{k=1}^{N} a_{1k} A_{1k}$, where $A_{ij} = (-1)^{i+j} \left| M_{ij} \right|$ is the cofactor of a_{ij} and M_{ij} is the minor matrix of a_{ij}.

Task C: Inverse of a matrix

To compute the inverse of a matrix, (pag. 107 of Maths notes) we proceed in two steps:

1) Write a function *Adjoint*, that receives a matrix **A** of dimension N x N, and returns its adjointed matrix (*'the adjoint of a matrix is formed by replacing each element by its cofactor and transposing the result'*, cit. Maths notes).
2) Write a function *Inverse*, that receives a matrix A and returns its inverse, i.e. $\boldsymbol{A^{-1}} = \frac{1}{|\boldsymbol{A}|} adjoint(\boldsymbol{A})$.

Task D: System of linear equations

Given a set of linear equations, of your choice, in matrix form, $\boldsymbol{A} \cdot x = b$:

1) Determine the solution x, by inverting the matrix $\boldsymbol{A}$, i.e., $x = \boldsymbol{A^{-1}} \cdot b$
2) Determine the solution x, by applying Cramer's rule (pag. 108):

$$x_j = \frac{|\boldsymbol{B}_j|}{|\boldsymbol{A}|}$$

where the matrix $\boldsymbol{B}_j$ is obtained from matrix $\boldsymbol{A}$ by replacing the column j with array b.

Task E: Playing chess

A 8 x 8 integer matrix represents the chess board. In each cell there is either a zero, to represent an empty cell, or an integer number to represent a piece.
Black pieces are described by positive numbers, while white pieces by negative numbers. The Black King is represented by the integer 1.
Write a script to search for the Black King. Once found, check if there are enemy pieces surrounding him, in adjacent cells, and alert the player of a potential checkmate.
[Look at the slides on BB for some hints and inspiration].

A few set up games, for testing, are stored in files *Game1.txt*, *Game2.txt* and *Game3.txt*. Data are stored in sequential lines, as:
A_{00}
A_{01}
A_{02}, etc.