**ME2 Computing- Tutorial 10: Root finding**

***Learning outcomes:***

- Being able to find roots of a function through bisection and Newton-Raphson methods.
- Being able to solve numerically systems of non-linear equations.
- Experience the basic knowledge of optimisation methods.

***Before you start***

In your H drive create a folder *H:\ME2CPT\Tutorial10* and work within it.

***Task A: Bisection method***

Write a Python function *mybisection*, to determine the root of the equation $f(x) = 0$, within a given interval $[a, b]$ and a specified accuracy $\varepsilon$, using the bisection method. $f(x)$ is a known function and might be implemented as a separate Python function.

Test your code by finding the root of the equation:

$$f(x) = x^2 + (x-2)^3 - 4 = 0$$

Find and compare the root with accuracies ε = 0.1, ε = 0.01 and ε = 0.001

***Task B: Newton-Raphson method***

Write a Python function *myNewton*, to determine the root of the equation $f(x) = 0$, given an initial guess $x_0$ and a specified accuracy $\varepsilon$, using the Newton-Raphson method. $f(x)$ is a known function and might be implemented as a separate Python function.

***Task C: Newton-Raphson method for systems of non-linear equations***

Write a script to determine the solutions of a system of non-linear equations, using the Newton-Raphson method.

**Task D: 2D optimization**

Write a script to determine the maximum of a two-dimensional function using a gradient-based method. Use numerical methods to evaluate the first and second order derivatives required. Test it out on the example given in the slides:

$$\max f(x, y) = 4xy - 2x^2 - 4y^2, \text{ starting from } (x_0, y_0) = (3,2)$$

**Task E (optional): Nonlinear optimization**

In your role investing in renewable energy technologies, you have been given a maximum of £100,000 to invest in two profitable projects. The amount you allocate to each project is denoted as $x_1$ and $x_2$ respectively, and you aim to maximize the total return.

The funding submission from the first project estimates a return of $20\%$ plus the square root of the initial money given, $\sqrt{x_1}$. As the second company is smaller, they will accept a maximum fund of £20,000 and return $10\%$ of the money invested.

Based on the technologies involved, the risk profile of the investment is represented by the function $x_1 + x_2^{0.7}$, which should not exceed half the amount you are able to invest.

a) Formulate the problem for the objective function and the constraints.

b) Find the amount to invest in each company and the expected consequent return.