K-11045 DYNAMIIKKA

Kaavakokoelma

Luku 2. Partikkelin kinematiikka

$v = \frac{ds}{dt} = \dot{s}$ $\quad a = \frac{dv}{dt} = \frac{d^2s}{dt^2} = \dot{v} = \ddot{s}$ $\quad v\,dv = a\,ds$

$v = v_0 + a(t - t_0)$ $\quad v^2 = v_0^2 + 2a(s - s_0)$ $\quad s = s_0 + v_0(t - t_0) + \frac{1}{2}a(t - t_0)^2$

$\bar{v} = \frac{d\bar{r}}{dt} = \dot{\bar{r}}$ $\quad \bar{a} = \frac{d\bar{v}}{dt} = \dot{\bar{v}} = \frac{d^2\bar{r}}{dt^2} = \ddot{\bar{r}}$

$\bar{r} = x\bar{i} + y\bar{j}$ $\quad \bar{v} = \dot{\bar{r}} = \dot{x}\bar{i} + \dot{y}\bar{j}$ $\quad \bar{a} = \dot{\bar{v}} = \ddot{\bar{r}} = \ddot{x}\bar{i} + \ddot{y}\bar{j}$

$\bar{v} = v\bar{e}_t = \rho\dot{\beta}\bar{e}_t$ $\quad \bar{a} = \dot{v}\bar{e}_t + \frac{v^2}{\rho}\bar{e}_n$

$v = r\dot{\theta} = r\omega$ $\quad a_t = r\ddot{\theta} = r\alpha$ $\quad a_n = v^2/r = r\omega^2$

$\bar{v} = \dot{r}\bar{e}_r + r\dot{\theta}\bar{e}_\theta$ $\quad \bar{a} = (\ddot{r} - r\dot{\theta}^2)\bar{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\bar{e}_\theta$

$\vec{r}_A = \vec{r}_B + \vec{r}_{A/B}$ $\quad \vec{v}_A = \vec{v}_B + \vec{v}_{A/B}$ $\quad \vec{a}_A = \vec{a}_B + \vec{a}_{A/B}$

Luku 3. Partikkelin kinetiikka

$\vec{R} = m\bar{a} = m\ddot{\bar{r}}$ $\quad R_r = ma_r = m(\ddot{r} - r\dot{\theta}^2)$ $\quad R_\theta = ma_\theta = m(r\ddot{\theta} + 2\dot{r}\dot{\theta})$

$R_x = ma_x = m\ddot{x}$ $\quad R_y = ma_y = m\ddot{y}$ $\quad R_t = ma_t = m\dot{v}$ $\quad R_n = ma_n = mv^2/\rho$

$W = \int_{s_A}^{s_B} \vec{F} \cdot d\bar{r}$ $\quad W = F_t\,\Delta s$ $\quad T = \frac{1}{2}mv^2$

$V_g = mgh$ $\quad V_g = -mgR^2/r$ $\quad V_e = \frac{1}{2}kx^2$

$W_{1-2} = \Delta T$ $\quad T_1 + W_{1-2} = T_2$ $\quad W'_{1-2} = \Delta T + \Delta V_g + \Delta V_e$

$\vec{p} = m\vec{v}$ $\quad$ $\vec{R} = m\dot{\vec{v}} = \dot{\vec{p}}$ $\quad$ $\vec{I}_R = \int_{t_1}^{t_2} \vec{R}\,dt$ $\quad$ $\int_{t_1}^{t_2} \vec{R}\,dt = \vec{p}_2 - \vec{p}_1$

$\vec{L}_O = \vec{r} \times \vec{p}$ $\quad$ $\vec{M}_O = \dot{\vec{L}}_O$ $\quad$ $\vec{I}_{MO} = \int_{t_1}^{t_2} \vec{M}_O\,dt$ $\quad$ $\int_{t_1}^{t_2} \vec{M}_O\,dt = \vec{L}_{O2} - \vec{L}_{O1}$

Luku 4. Partikkelisysteemin kinetiikka

$m\vec{r}_G = \sum_{i=1}^{n} m_i \vec{r}_i$ $\quad$ $\vec{R} = m\vec{a}_G$ $\quad$ $R_x = ma_{Gx}$ $\quad$ $R_y = ma_{Gy}$ $\quad$ $R_z = ma_{Gz}$

$W = \Delta T$ $\quad$ $W' = \Delta T + \Delta V_g + \Delta V_e$ $\quad$ $T = \frac{1}{2} m v_G^2 + \sum_{i=1}^{n} \frac{1}{2} m_i v_{i/G}^2$

$\vec{p} = m\vec{v}_G$ $\quad$ $\vec{R} = \dot{\vec{p}}$ $\quad$ $\int_{t_1}^{t_2} \vec{R}\,dt = \vec{p}_2 - \vec{p}_1$

$\vec{M}_O = \dot{\vec{L}}_O$ $\quad$ $\vec{L}_O = \sum_{i=1}^{n} \vec{r}_i \times m_i \vec{v}_i$ $\quad$ $\vec{M}_G = \dot{\vec{L}}_G$ $\quad$ $\vec{L}_G = \sum_{i=1}^{n} \vec{r}_{i/G} \times m_i \vec{v}_i$

$\vec{M}_Q = \dot{\vec{L}}_G + \vec{r}_{G/Q} \times m\vec{a}_G$ $\quad$ $\vec{L}_Q = \sum_{i=1}^{n} \vec{r}_{i/Q} \times m_i \vec{v}_i$ $\quad$ $\vec{L}_Q = \vec{L}_G + \vec{r}_{G/Q} \times m\vec{v}_G$

$\int_{t_1}^{t_2} \vec{M}_G\,dt = \vec{L}_{G2} - \vec{L}_{G1}$ $\quad$ $\int_{t_1}^{t_2} \vec{M}_O\,dt = \vec{L}_{O2} - \vec{L}_{O1}$

Luku 5. Jäykän kappaleen tasokinematiikka

$\omega = \frac{d\theta}{dt} = \dot{\theta}$ $\quad$ $\alpha = \frac{d\omega}{dt} = \dot{\omega} = \frac{d^2\theta}{dt^2} = \ddot{\theta}$ $\quad$ $\omega\,d\omega = \alpha\,d\theta$

$\omega = \omega_0 + \alpha(t - t_0)$ $\quad$ $\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$ $\quad$ $\theta = \theta_0 + \omega_0(t - t_0) + \frac{1}{2}\alpha(t - t_0)^2$

$v_P = r_{P/O}\,\omega$ $\quad$ $a_{Pt} = r_{P/O}\,\alpha$ $\quad$ $a_{Pn} = r_{P/O}\,\omega^2 = v_P^2 / r_{P/O}$

$\vec{v}_P = \dot{\vec{r}}_{P/O} = \vec{\omega} \times \vec{r}_{P/O}$ $\quad$ $\vec{a}_P = \ddot{\vec{r}}_{P/O} = \vec{a}_{Pn} + \vec{a}_{Pt} = \vec{\omega} \times (\vec{\omega} \times \vec{r}_{P/O}) + \vec{\alpha} \times \vec{r}_{P/O}$

$\Delta\vec{r}_A = \Delta\vec{r}_B + \Delta\vec{r}_{A/B}$ $\quad$ $\vec{v}_A = \vec{v}_B + \vec{v}_{A/B}$ $\quad$ $v_{A/B} = r_{A/B}\,\omega$ $\quad$ $\vec{v}_A = \vec{v}_B + \vec{\omega} \times \vec{r}_{A/B}$

$$\vec{a}_A = \vec{a}_B + \vec{a}_{A/B} \qquad \vec{a}_A = \vec{a}_B + \vec{a}^t_{A/B} + \vec{a}^n_{A/B}$$

$$a^t_{A/B} = \dot{v}_{A/B} = r_{A/B}\,\alpha \qquad a^n_{A/B} = v^2_{A/B}/r_{A/B} = r_{A/B}\,\omega^2$$

$$\vec{a}_A = \vec{a}_B + \vec{\alpha}\times\vec{r}_{A/B} + \vec{\omega}\times(\vec{\omega}\times\vec{r}_{A/B}) = \vec{a}_B + \vec{\alpha}\times\vec{r}_{A/B} - \omega^2\,\vec{r}_{A/B}$$

$$\dot{\vec{i}} = \vec{\omega}\times\vec{i} \qquad \dot{\vec{j}} = \vec{\omega}\times\vec{j} \qquad \vec{v}_A = \vec{v}_B + \vec{\omega}\times\vec{r}_{A/B} + \vec{v}_{rel}$$

$$\vec{a}_A = \vec{a}_B + \vec{\alpha}\times\vec{r}_{A/B} + \vec{\omega}\times(\vec{\omega}\times\vec{r}_{A/B}) + 2\vec{\omega}\times\vec{v}_{rel} + \vec{a}_{rel}$$

Luku 6. Jäykän kappaleen tasokinetiikka

$$I_G = \sum_{i=1}^{n} r^2_{i/G}\,m_i \qquad \vec{R} = m\vec{a}_G \qquad M_G = I_G\alpha$$

$$M_Q = I_G\alpha \pm m a_G d \qquad M_O = I_O\alpha$$

$$W = \int_{\theta_1}^{\theta_2} M\,d\theta \qquad W = M\Delta\theta \qquad T = \frac{1}{2}mv^2_G + \frac{1}{2}I_G\,\omega^2 \qquad T = \frac{1}{2}I_O\,\omega^2$$

$$W = \Delta T \qquad W' = \Delta T + \Delta V_g + \Delta V_e$$

$$\int_{t_1}^{t_2} \vec{R}\,dt = m(\vec{v}_{G2} - \vec{v}_{G1})$$

$$\int_{t_1}^{t_2} M_G\,dt = I_G(\omega_2 - \omega_1) \qquad \int_{t_1}^{t_2} M_O\,dt = I_O(\omega_2 - \omega_1)$$

$$\int_{t_1}^{t_2} M_Q\,dt = L_{Q2} - L_{Q1} \qquad L_Q = I_G\,\omega \pm m v_G\,d$$