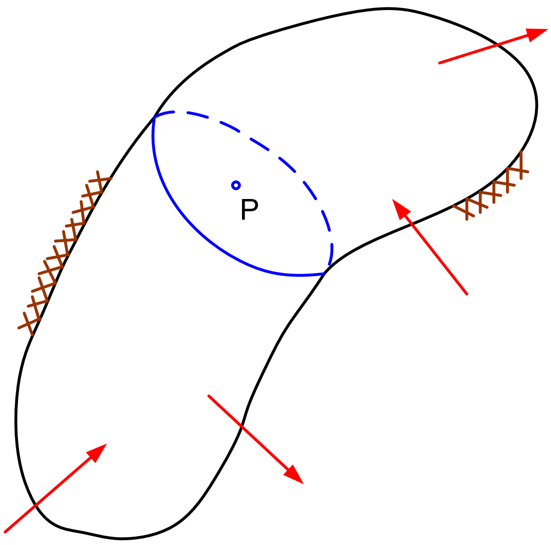
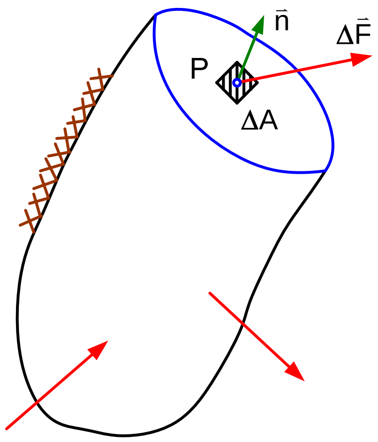
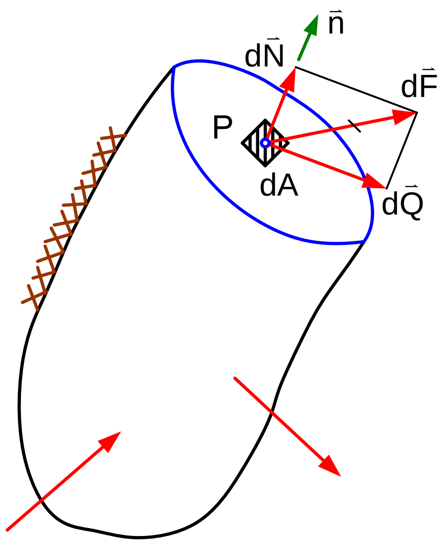
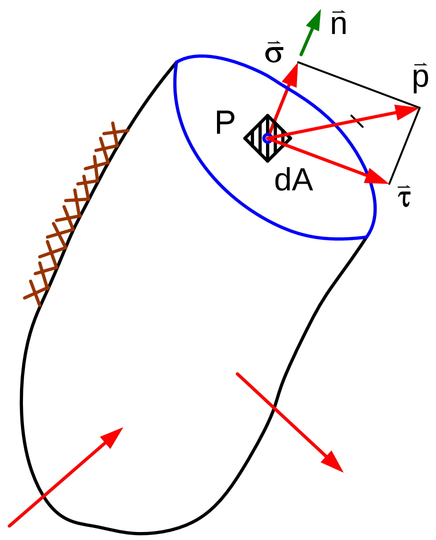
JÄNNITYSTILAN KÄSITE





$$\vec{p} = \lim_{\Delta A \to 0} \frac{\Delta \vec{F}}{\Delta A} = \frac{d\vec{F}}{dA}$$

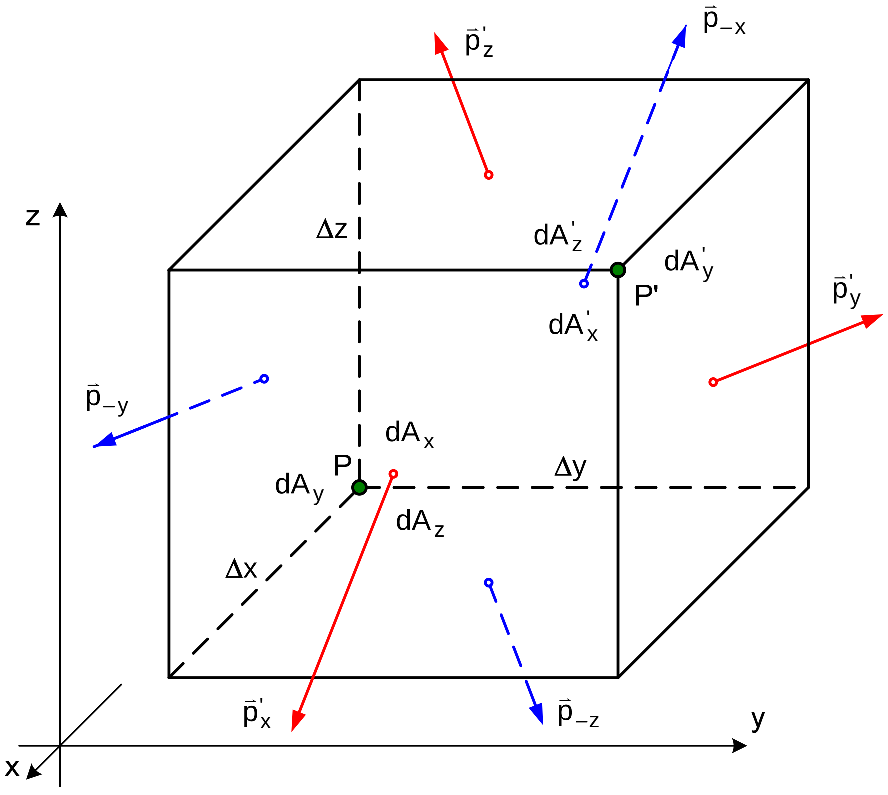


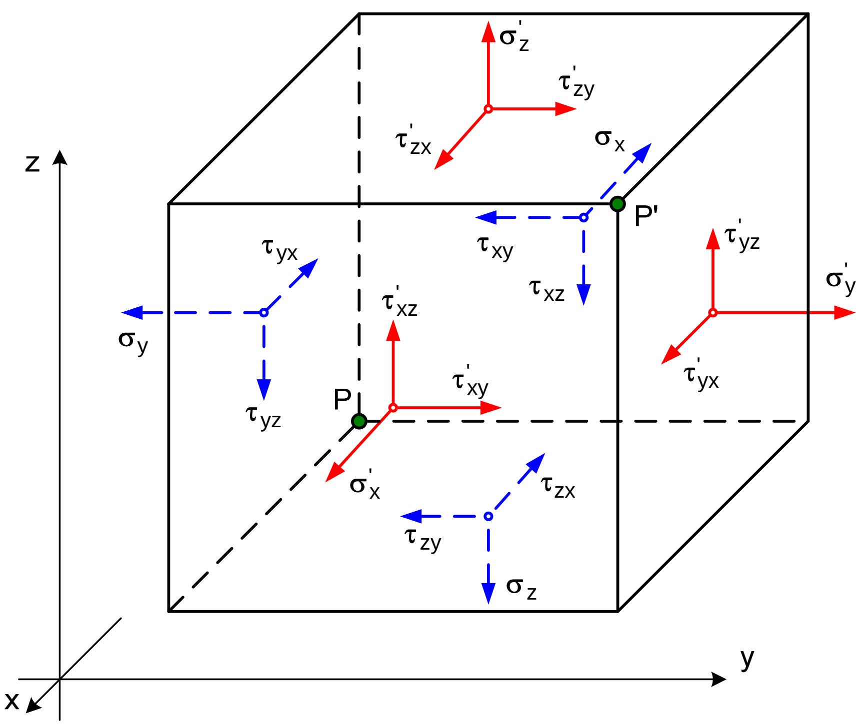


$$d\vec{F} = d\vec{N} + d\vec{Q}$$

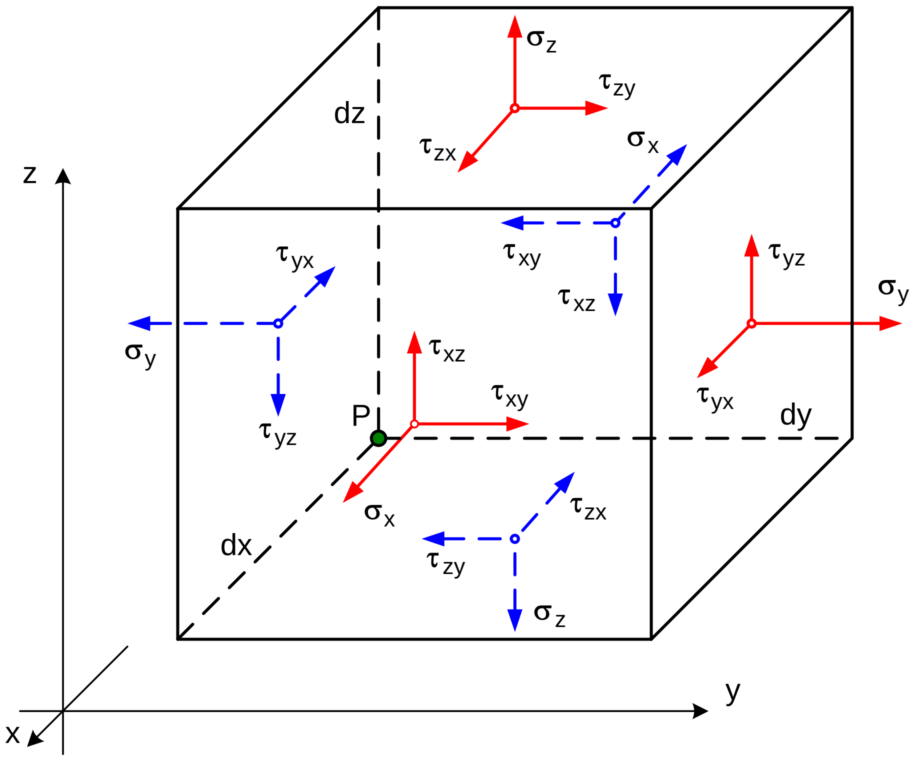
$$\vec{p} = \frac{d\vec{N}}{dA} + \frac{d\vec{Q}}{dA} = \vec{\sigma} + \vec{\tau}$$

JÄNNITYSTLAN KOMPONENTIT





JÄNNITYSELEMENTTI



JÄNNITYSMATRIISI

$$
\begin{array}{cc}
 & \text{sivun normaali} \\
[S] = \begin{bmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_y & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_z \end{bmatrix} & \begin{matrix} x \\ y \\ z \end{matrix}
\end{array}
$$

TASAPAINOYHTÄLÖT

Tasojännitystila:

Momenttitasapaino:

$$\tau_{xy} = \tau_{yx}$$

Voimatasapaino:

$$\sigma_{x,x} + \tau_{xy,y} + f_x = 0 \qquad \tau_{xy,x} + \sigma_{y,y} + f_y = 0$$

Yleinen jännitystila:

Momenttitasapaino:

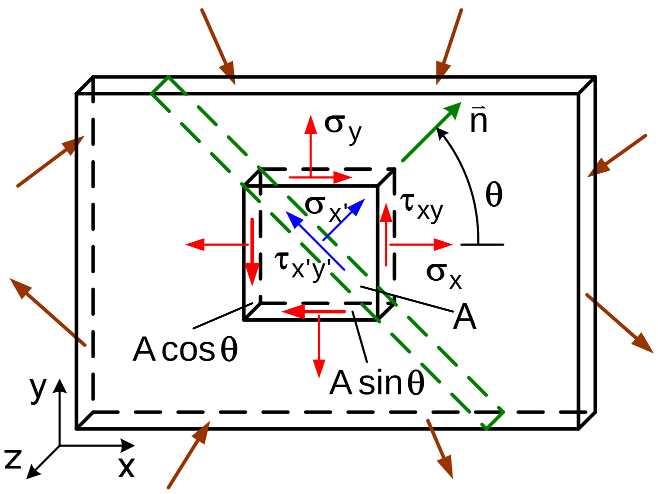
$$\tau_{yz} = \tau_{zy} \qquad \tau_{xz} = \tau_{zx} \qquad \tau_{xy} = \tau_{yx}$$

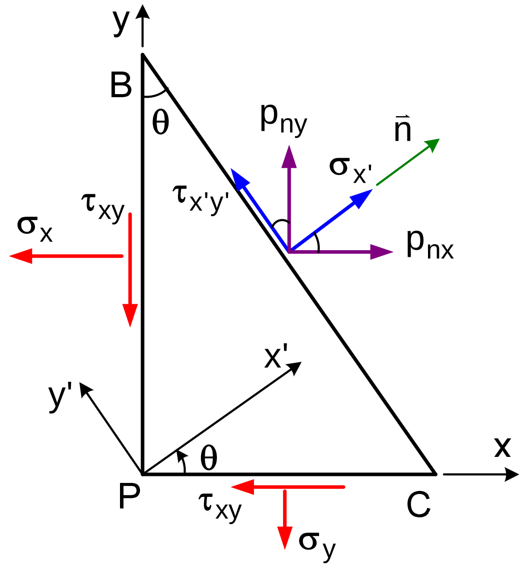
Voimatasapaino:

$$\sigma_{x,x} + \tau_{xy,y} + \tau_{xz,z} + f_x = 0$$
$$\tau_{xy,x} + \sigma_{y,y} + \tau_{yz,z} + f_y = 0$$
$$\tau_{xz,x} + \tau_{yz,y} + \sigma_{z,z} + f_z = 0$$

JÄNNITYSTEN TRANSFORMOINTI

Tasojännitystila:





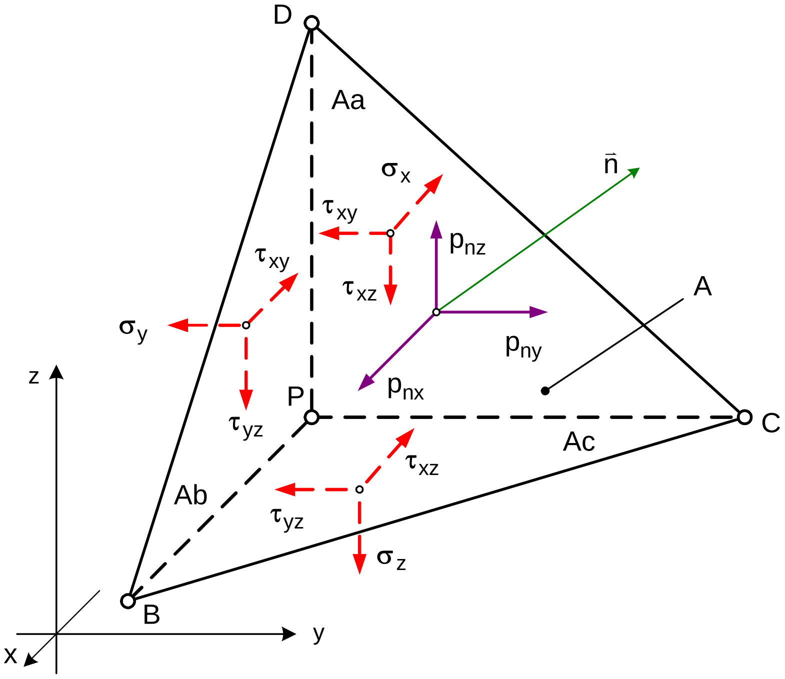
$$\vec{p}_n = (\sigma_x a + \tau_{xy} b)\,\vec{i} + (\tau_{xy} a + \sigma_y b)\,\vec{j}$$

$$\sigma_{x'} = \frac{1}{2}(\sigma_x + \sigma_y) + \frac{1}{2}(\sigma_x - \sigma_y)\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\tau_{x'y'} = -\frac{1}{2}(\sigma_x - \sigma_y)\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$\sigma_{y'} = \frac{1}{2}(\sigma_x + \sigma_y) - \frac{1}{2}(\sigma_x - \sigma_y)\cos 2\theta - \tau_{xy}\sin 2\theta$$

Yleinen jännitystila:



$$\boxed{\begin{aligned}\vec{p}_n &= (\sigma_x a + \tau_{xy} b + \tau_{xz} c)\,\vec{i} \\ &\quad + (\tau_{xy} a + \sigma_y b + \tau_{yz} c)\,\vec{j} \\ &\quad + (\tau_{xz} a + \tau_{yz} b + \sigma_z c)\,\vec{k}\end{aligned}}$$

$$\boxed{\sigma_n = \sigma_x a^2 + \sigma_y b^2 + \sigma_z c^2 + 2(\tau_{xy} ab + \tau_{yz} bc + \tau_{xz} ac)}$$

$$\boxed{\begin{aligned}\tau_n^2 &= (\sigma_x a + \tau_{xy} b + \tau_{xz} c)^2 + (\tau_{xy} a + \sigma_y b + \tau_{yz} c)^2 \\ &\qquad + (\tau_{xz} a + \tau_{yz} b + \sigma_z c)^2 - \sigma_n^2\end{aligned}}$$

$$\boxed{\{p_n\} = \begin{bmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{xy} & \sigma_y & \tau_{yz} \\ \tau_{xz} & \tau_{yz} & \sigma_z \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = [S]\{n\}}$$

$$\boxed{[S]' = \begin{bmatrix} \sigma_{x'} & \tau_{x'y'} & \tau_{x'z'} \\ \tau_{x'y'} & \sigma_{y'} & \tau_{y'z'} \\ \tau_{x'z'} & \tau_{y'z'} & \sigma_{z'} \end{bmatrix} = [Q]^T [S] [Q]}$$

$$\boxed{[Q] = \begin{bmatrix} \cos(x,x') & \cos(x,y') & \cos(x,z') \\ \cos(y,x') & \cos(y,y') & \cos(y,z') \\ \cos(z,x') & \cos(z,y') & \cos(z,z') \end{bmatrix}}$$

PÄÄJÄNNITYKSET JA -SUUNNAT

Tasojännitystila:

Pääjännitykset:

$$\sigma_{1,2} = \frac{1}{2}(\sigma_x + \sigma_y) \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

Pääsuunnat:

$$\theta_1 = \frac{1}{2}\arctan\frac{2\tau_{xy}}{\sigma_x - \sigma_y} \quad \text{ja} \quad \theta_2 = \theta_1 + \pi/2$$

Leikkausjännityksen ääriarvot:

$$\tau_1 = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \quad \text{kun} \quad \theta = \theta_1 - \pi/4$$

$$\tau_2 = -\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \quad \text{kun} \quad \theta = \theta_1 + \pi/4$$

Yleinen jännitystila:

Pääsuunnat:

$$\begin{cases} (\sigma_x - \sigma_p)a + \tau_{xy}b + \tau_{xz}c = 0 \\ \tau_{xy}a + (\sigma_y - \sigma_p)b + \tau_{yz}c = 0 \\ \tau_{xz}a + \tau_{yz}b + (\sigma_z - \sigma_p)c = 0 \end{cases}$$

$$a^2 + b^2 + c^2 = 1$$

Pääjännitykset:

$$\begin{vmatrix} \sigma_x - \sigma_p & \tau_{xy} & \tau_{xz} \\ \tau_{xy} & \sigma_y - \sigma_p & \tau_{yz} \\ \tau_{xz} & \tau_{yz} & \sigma_z - \sigma_p \end{vmatrix} = 0$$

$$\sigma_p^3 - I_1\sigma_p^2 + I_2\sigma_p - I_3 = 0$$

$$I_1 = \sigma_x + \sigma_y + \sigma_z \qquad I_2 = \sigma_x\sigma_y + \sigma_y\sigma_z + \sigma_z\sigma_x - \tau_{xy}^2 - \tau_{yz}^2 - \tau_{xz}^2$$

$$I_3 = \det[S] = \begin{vmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{xy} & \sigma_y & \tau_{yz} \\ \tau_{xz} & \tau_{yz} & \sigma_z \end{vmatrix}$$

Leikkausjännityksen ääriarvot:

$$\pm\tau_1 = \pm\frac{\sigma_2 - \sigma_3}{2} \qquad \pm\tau_2 = \pm\frac{\sigma_3 - \sigma_1}{2} \qquad \pm\tau_3 = \pm\frac{\sigma_1 - \sigma_2}{2}$$

$$\tau_{max} = \frac{\sigma_I - \sigma_{III}}{2}$$

JÄNNITYSKOMPONENTTIEN REUNAEHDOT

Tasojännitystila:

$$\boxed{t_x = \sigma_x a + \tau_{xy} b \qquad t_y = \tau_{xy} a + \sigma_y b}$$

Yleinen jännitystila:

$$\boxed{\begin{aligned} t_x &= \sigma_x a + \tau_{xy} b + \tau_{xz} c \\ t_y &= \tau_{xy} a + \sigma_y b + \tau_{yz} c \\ t_z &= \tau_{xz} a + \tau_{yz} b + \sigma_z c \end{aligned}}$$