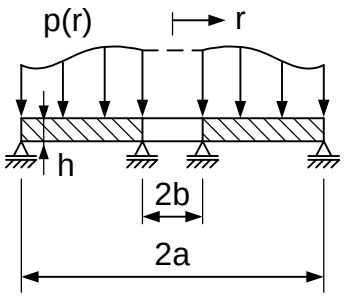


III.10 Laadi Mathcad-dokumentti, joka ratkaisee ulko- ja sisäreunaltaan niveltuetun rengaslaatan statiikan (w, M_r, M_θ, Q_r), kun dokumentissa annettava kuormitusfunktio p(r) on rotaatiosymmetrinen. Dokumentin tulee myös piirtää suureiden w, M_r, M_θ, Q_r kuvaajat ja etsiä niiden ääriarvot. Sovella dokumenttia kuormitusfunktioihin a) $p(r)=p_0$ ja b) $p(r)=\frac{p_0}{a-b}(a-r)$ tapauksessa $E=210\ \text{GPa}$, $\nu=0{,}3$, $a=200\ \text{mm}$, $b=100\ \text{mm}$, $h=5\ \text{mm}$ ja $p_0=0{,}1\ \text{MPa}$.

Ratkaisu:

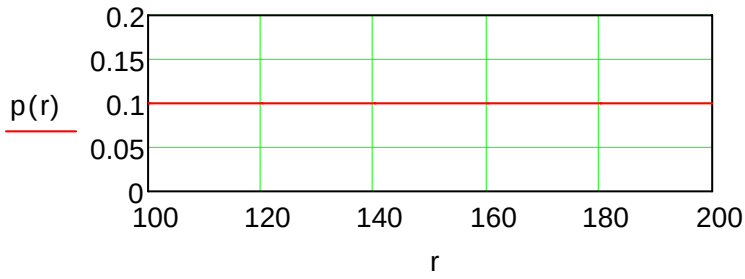
Kimmomoduuli: E := 210000 Poissonin vakio: $\nu := 0.3$ Yksiköt: N,mm

Laatan ulkosäde: a := 200 Laatan sisäsäde: b := 100 Laatan paksuus: h := 5

Kuormituksen määrittely:

a) $p_0 := 0.1$ $p(r) := p_0$

$$D := \frac{E\cdot h^3}{12\cdot\left(1-\nu^2\right)}$$



$$w_y(r) := \left[\int \frac{1}{r}\cdot\left[\int r\cdot\left[\int \frac{1}{r}\cdot\left(\int \frac{r\cdot p(r)}{D}\,dr\right)dr\right]dr\right]dr\right]\ \text{float},10 \rightarrow 6.5\text{e-}10\cdot r^4$$

$$W\left(r,a_0,b_0,c_0,d_0\right) := a_0 + b_0\cdot\ln(r) + c_0\cdot r^2 + d_0\cdot r^2\cdot\ln(r) + 6.5\text{e-}10\cdot r^4$$

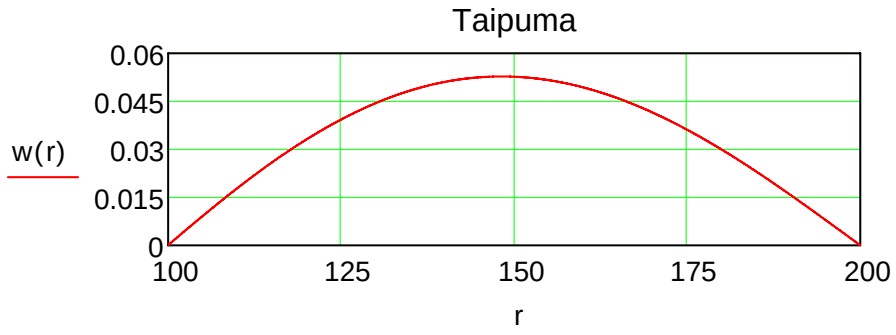
$$M_r\left(r,a_0,b_0,c_0,d_0\right) := -D\cdot\left[\frac{d^2}{dr^2}W\left(r,a_0,b_0,c_0,d_0\right) + \frac{\nu}{r}\cdot\left(\frac{d}{dr}W\left(r,a_0,b_0,c_0,d_0\right)\right)\right]$$

Given $W(a, a_0, b_0, c_0, d_0) = 0$ $W(b, a_0, b_0, c_0, d_0) = 0$

$M_r(a, a_0, b_0, c_0, d_0) = 0$ $M_r(b, a_0, b_0, c_0, d_0) = 0$

$$\begin{pmatrix} a_0 \\ b_0 \\ c_0 \\ d_0 \end{pmatrix} := \text{Find}(a_0, b_0, c_0, d_0)\ \text{float}, 10 \rightarrow \begin{pmatrix} 2.710076176 \\ -0.8119298862 \\ 0.0006452173936 \\ -0.0001191742017 \end{pmatrix}$$

$w(r) := W(r, a_0, b_0, c_0, d_0)$



$w(a) = 0.000$

$w(b) = 0.000$

Alkuarvaus: $r_1 := \frac{a + b}{2}$

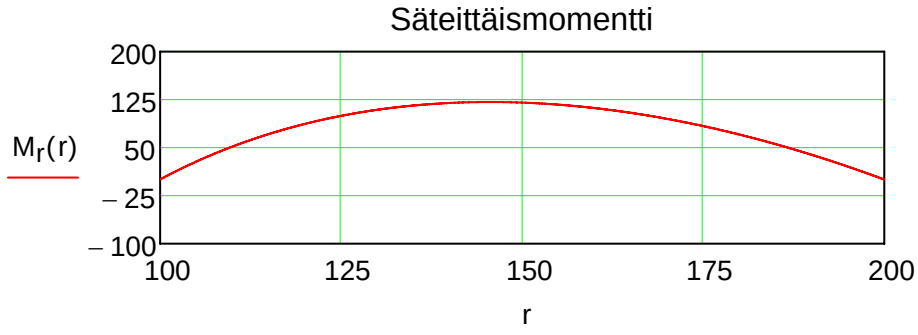
Given $b \le r_1 \le a$ $p := \text{Maximize}(w, r_1)$

Given $b \le r_1 \le a$ $q := \text{Minimize}(w, r_1)$

Maksimitaipuma: $p = 148.194$ $w(p) = 0.053$

Minimitaipuma: $q = 200.000$ $w(q) = 0.000$

$$M_r(r) := -D \cdot \left[\frac{d^2}{dr^2} w(r) + \frac{\nu}{r} \cdot \left(\frac{d}{dr} w(r) \right) \right]$$



$M_r(a) = -0.000$

$M_r(b) = 0.000$

Alkuarvaus: $r_1 := \frac{a+b}{2}$

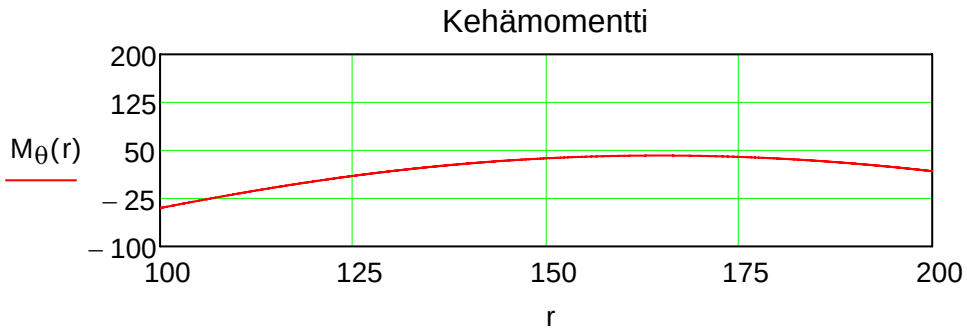
Given $b \le r_1 \le a$ $p := \text{Maximize}(M_r, r_1)$

Given $b \le r_1 \le a$ $q := \text{Minimize}(M_r, r_1)$

Säteittäismomentin maksimiarvo: $p = 145.546$ $M_r(p) = 121.020$

Säteittäismomentin minimiarvo: $q = 200.000$ $M_r(q) = -0.000$

$$M_\theta(r) := -D \cdot \left(\nu \cdot \frac{d^2}{dr^2} w(r) + \frac{1}{r} \cdot \frac{d}{dr} w(r) \right)$$



$M_\theta(a) = 17.244$

$M_\theta(b) = -40.321$

Alkuarvaus: $r_1 := \frac{a+b}{2}$

Given $b \le r_1 \le a$ $p := \text{Maximize}(M_\theta, r_1)$

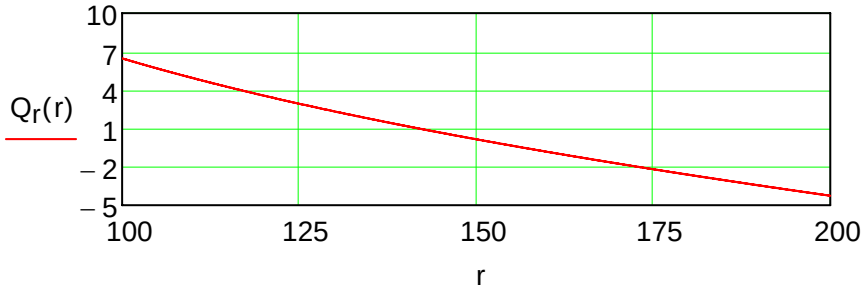
Given $b \le r_1 \le a$ $q := \text{Minimize}(M_\theta, r_1)$

Kehämomentin maksimiarvo: $p = 164.679$ $M_\theta(p) = 41.690677$

Kehämomentin minimiarvo: $q = 100.000$ $M_\theta(q) = -40.321391$

$$Q_r(r) := -D \cdot \frac{d}{dr} \left(\frac{d^2}{dr^2} w(r) + \frac{1}{r} \cdot \frac{d}{dr} w(r) \right)$$

Säteittäinen leikkausvoima



$Q_r(a) = -4.270$

$Q_r(b) = 6.459$

Alkuarvaus: $r_1 := \frac{a+b}{2}$

Given $b \le r_1 \le a$ $p := \text{Maximize}(Q_r, r_1)$

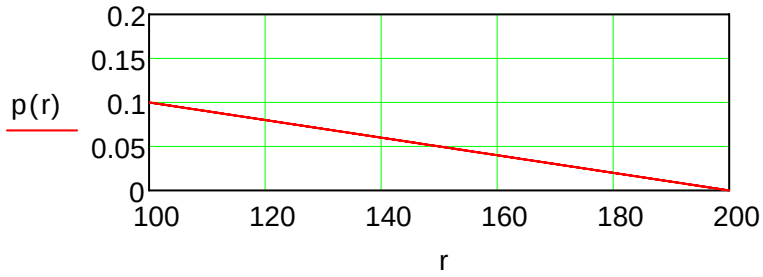
Given $b \le r_1 \le a$ $q := \text{Minimize}(Q_r, r_1)$

Säteittäisen leikkausvoiman maksimiarvo: $p = 100.000$ $Q_r(p) = 6.459$

Säteittäisen leikkausvoiman minimiarvo: $q = 200.000$ $Q_r(q) = -4.270$

Kuormituksen määrittely:

b) $p_0 := 0.1$ $p(r) := \frac{p_0}{a-b} \cdot (a - r)$



$$w_y(r) := \int \left[\frac{1}{r} \cdot \int \left[r \cdot \int \left[\frac{1}{r} \cdot \left(\int \frac{r \cdot p(r)}{D} \, dr \right) dr \right] dr \right] dr \right] dr$$

$w_y(r)\ \text{float}, 10 \rightarrow 1.3\text{e-}9 \cdot r^4 + -1.848888889\text{e-}12 \cdot r^5$

$$w_h(r, a_0, b_0, c_0, d_0) := a_0 + b_0 \cdot \ln(r) + c_0 \cdot r^2 + d_0 \cdot r^2 \cdot \ln(r)$$

$$W(r, a_0, b_0, c_0, d_0) := w_h(r, a_0, b_0, c_0, d_0) + 1.3e\text{-}9 \cdot r^4 + -1.848888889e\text{-}12 \cdot r^5$$

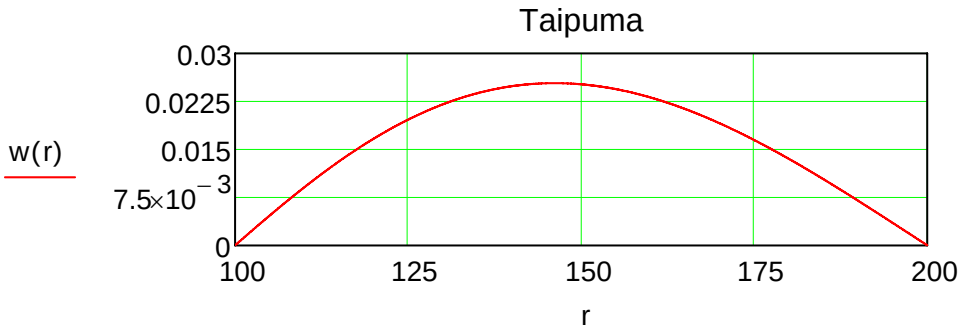
$$M_r(r, a_0, b_0, c_0, d_0) := -D \cdot \left[\frac{d^2}{dr^2} W(r, a_0, b_0, c_0, d_0) + \frac{\nu}{r} \cdot \left(\frac{d}{dr} W(r, a_0, b_0, c_0, d_0) \right) \right]$$

Given $\quad W(a, a_0, b_0, c_0, d_0) = 0 \quad W(b, a_0, b_0, c_0, d_0) = 0$

$\quad M_r(a, a_0, b_0, c_0, d_0) = 0 \quad M_r(b, a_0, b_0, c_0, d_0) = 0$

$$\begin{pmatrix} a_0 \\ b_0 \\ c_0 \\ d_0 \end{pmatrix} := \text{Find}(a_0, b_0, c_0, d_0) \text{ float}, 10 \rightarrow \begin{pmatrix} 2.262252115 \\ -0.6736194425 \\ 0.0005869108929 \\ -0.0001116297503 \end{pmatrix}$$

$$w(r) := W(r, a_0, b_0, c_0, d_0)$$



$w(a) = 0.000$

$w(b) = 0.000$

Alkuarvaus: $\quad r_1 := \frac{a + b}{2}$

Given $\quad b \le r_1 \le a \quad p := \text{Maximize}(w, r_1)$

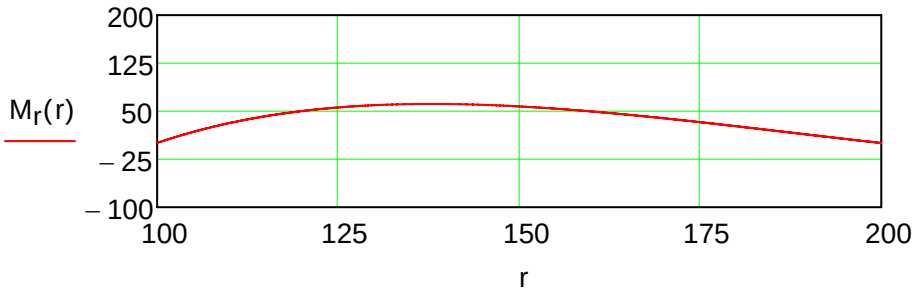
Given $\quad b \le r_1 \le a \quad q := \text{Minimize}(w, r_1)$

Maksimitaipuma: $\quad p = 146.155 \quad w(p) = 0.025$

Minimitaipuma: $\quad q = 200.000 \quad w(q) = 0.000$

$$M_r(r) := -D \cdot \left[\frac{d^2}{dr^2} w(r) + \frac{\nu}{r} \cdot \left(\frac{d}{dr} w(r) \right) \right]$$

Säteittäismomentti



$M_r(a) = -0.000$

$M_r(b) = 0.000$

Alkuarvaus: $r_1 := \frac{a+b}{2}$

Given $b \le r_1 \le a$ $p := \text{Maximize}(M_r, r_1)$

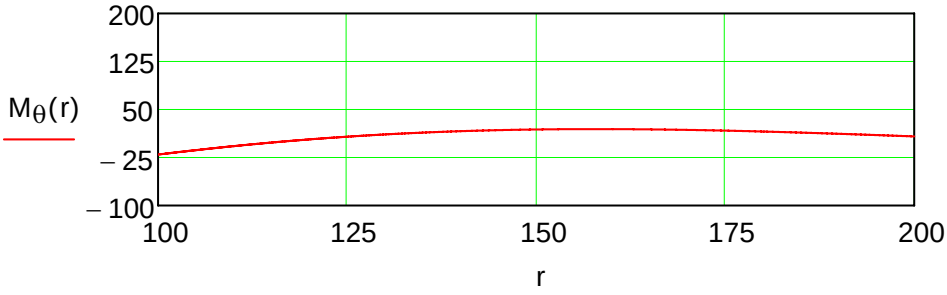
Given $b \le r_1 \le a$ $q := \text{Minimize}(M_r, r_1)$

Säteittäismomentin maksimiarvo: $p = 137.874$ $M_r(p) = 61.050$

Säteittäismomentin minimiarvo: $q = 200.000$ $M_r(q) = -0.000$

$$M_\theta(r) := -D \cdot \left(\nu \cdot \frac{d^2}{dr^2} w(r) + \frac{1}{r} \cdot \frac{d}{dr} w(r) \right)$$

Kehämomentti



$M_\theta(a) = 7.664$

$M_\theta(b) = -20.645$

Alkuarvaus: $r_1 := \frac{a+b}{2}$

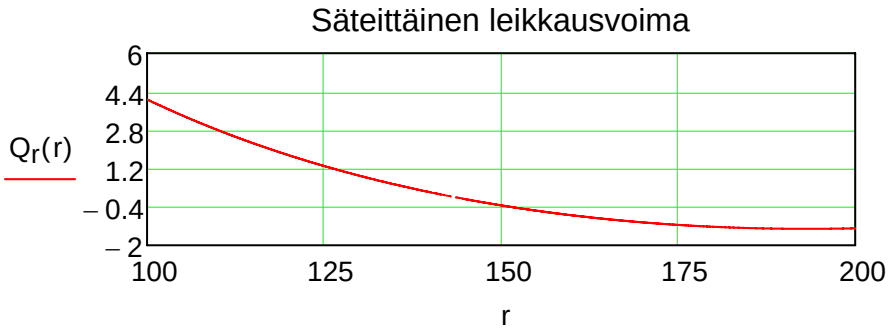
Given $b \le r_1 \le a$ $p := \text{Maximize}(M_\theta, r_1)$

Given $b \le r_1 \le a$ $q := \text{Minimize}(M_\theta, r_1)$

Kehämomentin maksimiarvo: $p = 157.856$ $M_\theta(p) = 19.099082$

Kehämomentin minimiarvo: $q = 100.000$ $M_\theta(q) = -20.644862$

$$Q_r(r) := -D \cdot \frac{d}{dr}\left(\frac{d^2}{dr^2} w(r) + \frac{1}{r} \cdot \frac{d}{dr} w(r)\right)$$



$Q_r(a) = -1.300$

$Q_r(b) = 4.067$

Alkuarvaus: $r_1 := \frac{a+b}{2}$

Given $b \le r_1 \le a$ $p := \text{Maximize}(Q_r, r_1)$

Given $b \le r_1 \le a$ $q := \text{Minimize}(Q_r, r_1)$

Säteittäisen leikkausvoiman maksimiarvo: $p = 100.000$ $Q_r(p) = 4.067$

Säteittäisen leikkausvoiman minimiarvo: $q = 193.155$ $Q_r(q) = -1.322$