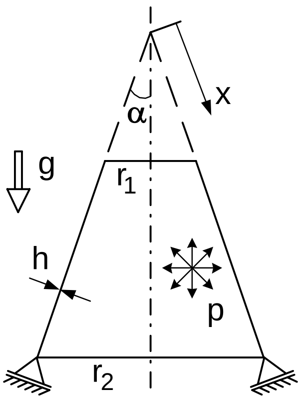


IV.5. Ratkaise oheisen katkaistun kartion kalvovoimien N_x ja N_θ lausekkeet ja niiden ääriarvot, kun kuormituksena on a) ylipaine p ja b) kuoren oma painovoima (tiheys on ρ). Tutki myös umpinainen kartio ($r_1 = 0$).

Ratkaisu:

a) $p_x = 0$ $\quad p_r = p \quad \Rightarrow \quad N_\theta = p\,x\tan\alpha$ (vetoa)

Yläreuna: $x_1 = \dfrac{r_1}{\sin\alpha}$ Alareuna: $x_2 = \dfrac{r_2}{\sin\alpha}$

$$\max N_\theta = p\frac{r_2}{\sin\alpha}\frac{\sin\alpha}{\cos\alpha} \quad \Rightarrow \quad \max N_\theta = p\frac{r_2}{\cos\alpha} \quad \text{(alareunassa)}$$

$$N_x = \frac{C}{x} + \frac{1}{x}\int p\,x\tan\alpha\,dx = \frac{C}{x} + \frac{1}{x}p\frac{x^2}{2}\tan\alpha = \frac{C}{x} + \frac{p\,x}{2}\tan\alpha$$

<u>Katkaistun kartion</u> reunaehto on: $N_x(x_1) = 0 \quad \Rightarrow \quad \dfrac{C}{x_1} + \dfrac{p\,x_1}{2}\tan\alpha = 0 \quad \Rightarrow$

$$C = -\frac{p\,x_1^2}{2}\tan\alpha \quad \Rightarrow \quad N_x = \frac{p}{2x}(x^2 - x_1^2)\tan\alpha \quad \text{(vetoa)}$$

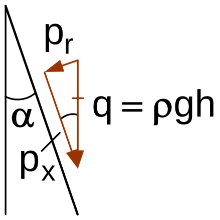
Koska funktiolla $\dfrac{1}{x}(x^2 - x_1^2)$ ei ole ääriarvoja välillä (x_1, x_2), on voimassa

$$\max N_x = \frac{p}{2x_2}(x_2^2 - x_1^2)\tan\alpha \quad \text{(alareunassa)}$$

<u>Umpinaisen kartion</u> $r_1 = x_1 = 0$ ja reunaehto on $N_x(0) \neq \infty \quad \Rightarrow \quad C = 0$

$$\Rightarrow \quad N_x = \frac{p\,x}{2}\tan\alpha = \frac{N_\theta}{2} \quad \Rightarrow \quad \max N_x = \frac{p\,x_2}{2}\tan\alpha \quad \text{(alareunassa)}$$

b) $p_x = q\cos\alpha \qquad p_r = -q\sin\alpha \qquad \Rightarrow$



$N_\theta = -qx\sin\alpha\,\tan\alpha \qquad \Rightarrow \qquad N_\theta = -qx\dfrac{\sin^2\alpha}{\cos\alpha}$ (puristus)

$\max|N_\theta| = q\dfrac{r_2}{\sin\alpha}\dfrac{\sin^2\alpha}{\cos\alpha} \qquad \Rightarrow \qquad \max|N_\theta| = qr_2\tan\alpha$ (alareunassa)

$$N_x = \frac{C}{x} + \frac{1}{x}\int\left(-qx\frac{\sin^2\alpha}{\cos\alpha} - qx\cos\alpha\right)dx = \frac{C}{x} - \frac{1}{x}q\frac{x^2}{2}\frac{\sin^2\alpha + \cos^2\alpha}{\cos\alpha} \qquad \Rightarrow$$

$$N_x = \frac{C}{x} - \frac{qx}{2\cos\alpha}$$

<u>Katkaistun kartion</u> reunaehto on: $N_x(x_1) = 0 \quad \Rightarrow \quad \dfrac{C}{x_1} - \dfrac{qx_1}{2\cos\alpha} = 0 \quad \Rightarrow \quad C = \dfrac{qx_1^2}{2\cos\alpha}$

$\Rightarrow \qquad N_x = -\dfrac{q}{2x\cos\alpha}(x^2 - x_1^2)$ (puristus)

$\Rightarrow \qquad \max|N_x| = \dfrac{q}{2x_2\cos\alpha}(x_2^2 - x_1^2)$ (alareunassa)

<u>Umpinaisen kartion</u> $r_1 = x_1 = 0$ ja reunaehto on $N_x(0) \neq \infty \quad \Rightarrow \quad C = 0$

$\Rightarrow \qquad N_x = -\dfrac{qx}{2\cos\alpha} \qquad \Rightarrow \qquad \max|N_x| = \dfrac{qx_2}{2\cos\alpha}$ (alareunassa)