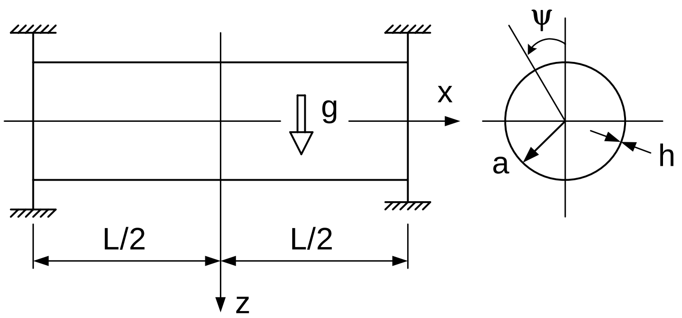
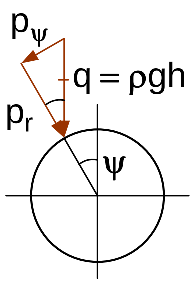


IV.8. Kuvan mukaista vaaka-asennossa olevaa sylinterikuorta rasittaa sen oma painovoima $q = \rho g h$. Määritä kuoren kalvojännitykset, kun sen päissä on (a) tuennat, joiden kohdalla $N_x = 0$ ja (b) tuennat, jotka estävät kuoren pituudenmuutoksen.

Ratkaisu:



$$p_x = 0 \qquad p_\psi = q\sin\psi \qquad p_r = -q\cos\psi \qquad r = a \qquad q = \rho g h$$

$$N_\psi = -qa\cos\psi \qquad \Rightarrow \qquad \sigma_\psi = -\frac{qa}{h}\cos\psi$$

$$N_{x\psi} = -\int \left(q\sin\psi + \frac{1}{a} qa\sin\psi \right) dx + f_1(\psi) = -2qx\sin\psi + f_1(\psi)$$

Reunaehto: $N_{x\psi}(x=0) = 0 \quad \Rightarrow \quad f_1(\psi) = 0 \quad \Rightarrow \quad N_{x\psi} = -2qx\sin\psi$

$$\Rightarrow \qquad \tau_{x\psi} = -\frac{2q}{h} x\sin\psi$$

$$N_x = -\int \frac{1}{a}(-2qx\cos\psi)\,dx + f_2(\psi) = \frac{q}{a}x^2\cos\psi + f_2(\psi)$$

a) Reunaehto: $N_x(x = \pm L/2) = 0 \quad \Rightarrow \quad \frac{q}{a}\frac{L^2}{4}\cos\psi + f_2(\psi) = 0 \quad \Rightarrow \quad f_2(\psi) = -\frac{qL^2}{4a}\cos\psi$

$$\Rightarrow \qquad N_x = -\frac{q}{4a}(L^2 - 4x^2)\cos\psi \qquad \Rightarrow \qquad \sigma_x = -\frac{q}{4ah}(L^2 - 4x^2)\cos\psi$$

b)

$$\varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_\psi) = \frac{1}{Eh}\left(\frac{q}{a}x^2\cos\psi + f_2(\psi) + \nu qa\cos\psi\right)$$

$$\Delta L = \int_{-L/2}^{L/2} \varepsilon_x\,dx = \frac{1}{Eh}\Big/_{-L/2}^{L/2}\left(\frac{q}{3a}x^3\cos\psi + f_2(\psi)\cdot x + \nu qax\cos\psi\right) \qquad \Rightarrow$$

$$\Delta L = \frac{2}{Eh}\left(\frac{q}{3a}\frac{L^3}{8}\cos\psi + f_2(\psi)\cdot\frac{L}{2} + \nu qa\frac{L}{2}\cos\psi\right)$$

Reunaehto: $\Delta L = 0 \quad \Rightarrow \quad f_2(\psi) = -\nu qa\cos\psi - \frac{qL^2}{12a}\cos\psi$

$$\Rightarrow \qquad N_x = \frac{q}{a}\left(x^2 - \frac{L^2}{12} - \nu a^2\right)\cos\psi \qquad \Rightarrow \qquad \sigma_x = \frac{q}{ah}\left(x^2 - \frac{L^2}{12} - \nu a^2\right)\cos\psi$$