ESIMERKKI: Vakiovenymän tetraedrielementti

Yksiköt: N,mm $\quad$ ORIGIN := 1

Solmukoordinaatit:

$x_1 := -40 \quad x_2 := 0 \quad x_3 := 20 \quad x_4 := 10$

$y_1 := 20 \quad y_2 := 10 \quad y_3 := 0 \quad y_4 := 60$

$z_1 := 0 \quad z_2 := 40 \quad z_3 := -30 \quad z_4 := 10$

$$V := \left| \begin{pmatrix} 1 & x_1 & y_1 & z_1 \\ 1 & x_2 & y_2 & z_2 \\ 1 & x_3 & y_3 & z_3 \\ 1 & x_4 & y_4 & z_4 \end{pmatrix} \right| \cdot \frac{1}{6} \qquad V = 32833.333$$

Interpolointifunktiot:

$N_1(\xi,\eta,\zeta) := 1-\xi-\eta-\zeta \qquad N_2(\xi,\eta,\zeta) := \xi \qquad N_3(\xi,\eta,\zeta) := \eta \qquad N_4(\xi,\eta,\zeta) := \zeta$

Geometrian kuvaus:

$$x(\xi,\eta,\zeta) := N_1(\xi,\eta,\zeta)\cdot x_1 + N_2(\xi,\eta,\zeta)\cdot x_2 + N_3(\xi,\eta,\zeta)\cdot x_3 + N_4(\xi,\eta,\zeta)\cdot x_4$$

$$y(\xi,\eta,\zeta) := N_1(\xi,\eta,\zeta)\cdot y_1 + N_2(\xi,\eta,\zeta)\cdot y_2 + N_3(\xi,\eta,\zeta)\cdot y_3 + N_4(\xi,\eta,\zeta)\cdot y_4$$

$$z(\xi,\eta,\zeta) := N_1(\xi,\eta,\zeta)\cdot z_1 + N_2(\xi,\eta,\zeta)\cdot z_2 + N_3(\xi,\eta,\zeta)\cdot z_3 + N_4(\xi,\eta,\zeta)\cdot z_4$$

$$N(\xi,\eta,\zeta) := \begin{pmatrix} 1-\xi-\eta-\zeta & 0 & 0 & \xi & 0 & 0 & \eta & 0 & 0 & \zeta & 0 & 0 \\ 0 & 1-\xi-\eta-\zeta & 0 & 0 & \xi & 0 & 0 & \eta & 0 & 0 & \zeta & 0 \\ 0 & 0 & 1-\xi-\eta-\zeta & 0 & 0 & \xi & 0 & 0 & \eta & 0 & 0 & \zeta \end{pmatrix}$$

$$\gamma_1 := -\left|\begin{pmatrix} 1 & y_2 & z_2 \\ 1 & y_3 & z_3 \\ 1 & y_4 & z_4 \end{pmatrix}\right| \qquad \beta_1 := \left|\begin{pmatrix} 1 & x_2 & z_2 \\ 1 & x_3 & z_3 \\ 1 & x_4 & z_4 \end{pmatrix}\right| \qquad \alpha_1 := -\left|\begin{pmatrix} 1 & x_2 & y_2 \\ 1 & x_3 & y_3 \\ 1 & x_4 & y_4 \end{pmatrix}\right|$$

$$\gamma_2 := \left|\begin{pmatrix} 1 & y_1 & z_1 \\ 1 & y_3 & z_3 \\ 1 & y_4 & z_4 \end{pmatrix}\right| \qquad \beta_2 := -\left|\begin{pmatrix} 1 & x_1 & z_1 \\ 1 & x_3 & z_3 \\ 1 & x_4 & z_4 \end{pmatrix}\right| \qquad \alpha_2 := \left|\begin{pmatrix} 1 & x_1 & y_1 \\ 1 & x_3 & y_3 \\ 1 & x_4 & y_4 \end{pmatrix}\right|$$

$$\gamma_3 := -\left|\begin{pmatrix} 1 & y_1 & z_1 \\ 1 & y_2 & z_2 \\ 1 & y_4 & z_4 \end{pmatrix}\right| \qquad \beta_3 := \left|\begin{pmatrix} 1 & x_1 & z_1 \\ 1 & x_2 & z_2 \\ 1 & x_4 & z_4 \end{pmatrix}\right| \qquad \alpha_3 := -\left|\begin{pmatrix} 1 & x_1 & y_1 \\ 1 & x_2 & y_2 \\ 1 & x_4 & y_4 \end{pmatrix}\right|$$

$$\gamma_4 := \left|\begin{pmatrix} 1 & y_1 & z_1 \\ 1 & y_2 & z_2 \\ 1 & y_3 & z_3 \end{pmatrix}\right| \qquad \beta_4 := -\left|\begin{pmatrix} 1 & x_1 & z_1 \\ 1 & x_2 & z_2 \\ 1 & x_3 & z_3 \end{pmatrix}\right| \qquad \alpha_4 := \left|\begin{pmatrix} 1 & x_1 & y_1 \\ 1 & x_2 & y_2 \\ 1 & x_3 & y_3 \end{pmatrix}\right|$$

$$B := \frac{1}{6\cdot V}\cdot\begin{pmatrix} \gamma_1 & 0 & 0 & \gamma_2 & 0 & 0 & \gamma_3 & 0 & 0 & \gamma_4 & 0 & 0 \\ 0 & \beta_1 & 0 & 0 & \beta_2 & 0 & 0 & \beta_3 & 0 & 0 & \beta_4 & 0 \\ 0 & 0 & \alpha_1 & 0 & 0 & \alpha_2 & 0 & 0 & \alpha_3 & 0 & 0 & \alpha_4 \\ \beta_1 & \gamma_1 & 0 & \beta_2 & \gamma_2 & 0 & \beta_3 & \gamma_3 & 0 & \beta_4 & \gamma_4 & 0 \\ \alpha_1 & 0 & \gamma_1 & \alpha_2 & 0 & \gamma_2 & \alpha_3 & 0 & \gamma_3 & \alpha_4 & 0 & \gamma_4 \\ 0 & \alpha_1 & \beta_1 & 0 & \alpha_2 & \beta_2 & 0 & \alpha_3 & \beta_3 & 0 & \alpha_4 & \beta_4 \end{pmatrix}$$

Tilavuusvoimakuormitus: Rotaatio z-akselin ympäri.

$$\rho := 7850 \cdot 10^{-12} \qquad \omega := 200 \qquad g(\xi,\eta,\zeta) := \begin{pmatrix} \rho\cdot\omega^2\cdot x(\xi,\eta,\zeta) \\ \rho\cdot\omega^2\cdot y(\xi,\eta,\zeta) \\ 0 \end{pmatrix}$$

$$i := 1 .. 12$$

$$r_i := 6\cdot V\cdot\int_0^1\int_0^{1-\xi}\int_0^{1-\xi-\eta} N(\xi,\eta,\zeta)^{\langle i\rangle}\cdot g(\xi,\eta,\zeta)\,d\zeta\,d\eta\,d\xi$$

r =

	1
1	-25.774
2	56.703
3	0.000
4	-5.155
5	51.548
6	0.000
7	5.155
8	46.394
9	0.000
10	0.000
11	77.322
12	0.000

Pintavoimakuormitus: Taholla 123 lineaarinen pintakuormitus x-suuntaan.

$$p_{x1} := 1 \qquad p_{x2} := 2 \qquad p_{x3} := 4 \qquad A_{xy} := \frac{1}{2}\cdot\left|\begin{pmatrix} 1 & x_1 & y_1 \\ 1 & x_2 & y_2 \\ 1 & x_3 & y_3 \end{pmatrix}\right| \qquad A_{xy} =$$

$$a_1 := \left|\begin{pmatrix} p_{x1} & x_1 & y_1 \\ p_{x2} & x_2 & y_2 \\ p_{x3} & x_3 & y_3 \end{pmatrix}\right| \qquad a_2 := \left|\begin{pmatrix} 1 & p_{x1} & y_1 \\ 1 & p_{x2} & y_2 \\ 1 & p_{x3} & y_3 \end{pmatrix}\right| \qquad a_3 := \left|\begin{pmatrix} 1 & x_1 & p_{x1} \\ 1 & x_2 & p_{x2} \\ 1 & x_3 & p_{x3} \end{pmatrix}\right|$$

$a_1 =$

$a_2 =$

$a_3 =$

$$p_x(\xi,\eta) := \frac{1}{2\cdot A_{xy}}\cdot\left(a_1 + a_2\cdot x(\xi,\eta,0) + a_3\cdot y(\xi,\eta,0)\right)$$

$$p(\xi,\eta) := \begin{pmatrix} p_x(\xi,\eta) \\ 0 \\ 0 \end{pmatrix} \qquad S1 := \begin{pmatrix} x_2 - x_1 \\ y_2 - y_1 \\ z_2 - z_1 \end{pmatrix} \qquad S2 := \begin{pmatrix} x_3 - x_1 \\ y_3 - y_1 \\ z_3 - z_1 \end{pmatrix} \qquad A_{123} := \frac{|S1 \times S2|}{2}$$

$$A_{123} = 1884.808$$

$$i := 1..12$$

$$r_i := 2 \cdot A_{123} \cdot \int_0^1 \int_0^{1-\eta} N(\xi,\eta,0)^{\langle i \rangle} \cdot p(\xi,\eta)\, d\xi\, d\eta$$

r =

	1
1	1256.538
2	0.000
3	0.000
4	1413.606
5	0.000
6	0.000
7	1727.740
8	0.000
9	0.000
10	0.000
11	0.000
12	0.000

<u>Esijännitystilakenttä:</u> x-suunnassa lineaarinen esijännitysvektori.

$$\sigma_0(\xi,\eta,\zeta) := \begin{pmatrix} 0.023 \\ -0.043 \\ -0.056 \\ 0.037 \\ -0.098 \\ 0.081 \end{pmatrix} \cdot x(\xi,\eta,\zeta) + \begin{pmatrix} 10 \\ -12 \\ 23 \\ -54 \\ -23 \\ 19 \end{pmatrix}$$

$$i := 1..12 \qquad r_i := -6 \cdot V \cdot \int_0^1 \int_0^{1-\xi} \int_0^{1-\xi-\eta} B^{\langle i \rangle} \cdot \sigma_0(\xi,\eta,\zeta)\, d\zeta\, d\eta\, d\xi$$

r =

	1
1	3026.708
2	-30614.167
3	-10482.458
4	-7694.958
5	-5798.875
6	-2741.042
7	-25205.958
8	18734.000
9	19558.917
10	29874.208
11	17679.042
12	-6335.417

```
    This is a Classroom License for instructional use only.
    Research and commercial use is prohibited.
>> clear all;
close all;
format compact;
%
% Lähtöarvot:
%
x0=[-40 0 20 10];
y0=[20 10 0 60];
z0=[0 40 -30 10];
E0=210000;
ny0=0.3;
%
% Kutsutaan funktiota tetra_k.m:
%
K=tetra_k(x0,y0,z0,E0,ny0);
>> K
K =
  1.0e+006 *
    3.5369   -0.0649    0.7141   -1.1787    0.8248   -1.3995   -1.3981    0.6348    0.6902   -0.9601   -1.3947   -0.0048
   -0.0649    1.0718   -0.0188    0.5555   -0.5655    0.1927    0.4329   -0.3218    0.0987   -0.9235   -0.1845   -0.2726
    0.7141   -0.0188    1.2768   -0.9956    0.2600   -1.1685    0.3536    0.1660    0.1001   -0.0721   -0.4073   -0.2084
   -1.1787    0.5555   -0.9956    1.3304   -0.3587    0.5808    0.1483   -0.4079    0.1797   -0.3000    0.2111    0.2351
    0.8248   -0.5655    0.2600   -0.3587    1.9130   -1.2197   -0.4753    0.4319    0.0803    0.0092   -1.7794    0.8794
   -1.3995    0.1927   -1.1685    0.5808   -1.2197    3.1344    0.4489   -0.2562   -1.3619    0.3697    1.2833   -0.6041
   -1.3981    0.4329    0.3536    0.1483   -0.4753    0.4489    1.1675   -0.4647   -0.6099    0.0823    0.5070   -0.1927
    0.6348   -0.3218    0.1660   -0.4079    0.4319   -0.2562   -0.4647    1.1111    0.5740    0.2378   -1.2211   -0.4838
    0.6902    0.0987    0.1001    0.1797    0.0803   -1.3619   -0.6099    0.5740    1.4271   -0.2600   -0.7530   -0.1654
   -0.9601   -0.9235   -0.0721   -0.3000    0.0092    0.3697    0.0823    0.2378   -0.2600    1.1777    0.6765   -0.0376
   -1.3947   -0.1845   -0.4073    0.2111   -1.7794    1.2833    0.5070   -1.2211   -0.7530    0.6765    3.1850   -0.1230
   -0.0048   -0.2726   -0.2084    0.2351    0.8794   -0.6041   -0.1927   -0.4838   -0.1654   -0.0376   -0.1230    0.9778
>>
```

```
function [K] = tetrak(xs,ys,zs,Emat,ny)
%
%
% Funktio muodostaa nelisolmuisen tetraedrielementin jäykkyysmatriisin.
% Parametreinä annetaan somujen koordinaattien vektorit, kimmomoduuli
% ja Poissonin vakio.
% 2001-02-15  Matti Lähteenmäki
%
%
J=[[1 1 1 1] ; xs ; ys ; zs]';
V=det(J)/6;
%
%
gam1=-det([1 ys(2) zs(2); 1 ys(3) zs(3); 1 ys(4) zs(4)]);
gam2=det([1 ys(1) zs(1); 1 ys(3) zs(3); 1 ys(4) zs(4)]);
gam3=-det([1 ys(1) zs(1); 1 ys(2) zs(2); 1 ys(4) zs(4)]);
gam4=det([1 ys(1) zs(1); 1 ys(2) zs(2); 1 ys(3) zs(3)]);
bet1=det([1 xs(2) zs(2); 1 xs(3) zs(3); 1 xs(4) zs(4)]);
bet2=-det([1 xs(1) zs(1); 1 xs(3) zs(3); 1 xs(4) zs(4)]);
bet3=det([1 xs(1) zs(1); 1 xs(2) zs(2); 1 xs(4) zs(4)]);
bet4=-det([1 xs(1) zs(1); 1 xs(2) zs(2); 1 xs(3) zs(3)]);
alf1=-det([1 xs(2) ys(2); 1 xs(3) ys(3); 1 xs(4) ys(4)]);
alf2=det([1 xs(1) ys(1); 1 xs(3) ys(3); 1 xs(4) ys(4)]);
alf3=-det([1 xs(1) ys(1); 1 xs(2) ys(2); 1 xs(4) ys(4)]);
alf4=det([1 xs(1) ys(1); 1 xs(2) ys(2); 1 xs(3) ys(3)]);
%
%
B=(1/6/V)*[gam1 0 0 gam2 0 0 gam3 0 0 gam4 0 0;
           0 bet1 0 0 bet2 0 0 bet3 0 0 bet4 0;
           0 0 alf1 0 0 alf2 0 0 alf3 0 0 alf4;
           bet1 gam1 0 bet2 gam2 0 bet3 gam3 0 bet4 gam4 0;
           alf1 0 gam1 alf2 0 gam2 alf3 0 gam3 alf4 0 gam4;
           0 alf1 bet1 0 alf2 bet2 0 alf3 bet3 0 alf4 bet4];
%
%
Evak=Emat/((1+ny)*(1-2*ny));
nyvak=(1-2*ny)/2;
E=Evak*[1-ny ny ny 0 0 0;
        ny 1-ny ny 0 0 0;
        ny ny 1-ny 0 0 0;
        0 0 0 nyvak 0 0;
        0 0 0 0 nyvak 0;
        0 0 0 0 0 nyvak;];
%
%
K=B'*E*B*V;
```