I.2. Osoita, että jännityskomponentit (A, B ja C ovat vakioita)

$$\sigma_x = A zx^2 - Bx^3/3 \qquad \sigma_y = -Bxy^2 + Cy^3/3 \qquad \sigma_z = Cyz^2 + Az^3/3$$

$$\tau_{xy} = Bx^2y \qquad \tau_{xz} = -Az^2x \qquad \tau_{yz} = -Cy^2z$$

toteuttavat jännityskomponenttien kolmiulotteiset tasapainoehdot, kun tilavuusvoimat f_x, f_y ja f_z ovat nollia.

Ratkaisu:

$$\sigma_{x,x} = 2Azx - Bx^2 \qquad \tau_{xy,y} = Bx^2 \qquad \tau_{xz,z} = -2Azx \qquad f_x = 0$$

$$\Rightarrow \qquad \sigma_{x,x} + \tau_{xy,y} + \tau_{xz,z} + f_x = 0 \qquad \text{OK}$$

$$\tau_{xy,x} = 2Bxy \qquad \sigma_{y,y} = -2Bxy + Cy^2 \qquad \tau_{yz,z} = -Cy^2 \qquad f_y = 0$$

$$\Rightarrow \qquad \tau_{xy,x} + \sigma_{y,y} + \tau_{yz,z} + f_y = 0 \qquad \text{OK}$$

$$\tau_{xz,x} = -Az^2 \qquad \tau_{yz,y} = -2Cyz \qquad \sigma_{z,z} = 2Cyz + Az^2 \qquad f_z = 0$$

$$\Rightarrow \qquad \tau_{xz,x} + \tau_{yz,y} + \sigma_{z,z} + f_z = 0 \qquad \text{OK}$$

Ratkaisu Mathcadilla:

$$\sigma_x(x,y,z,A,B,C) := A\cdot z\cdot x^2 - \frac{1}{3}\cdot B\cdot x^3 \qquad \sigma_y(x,y,z,A,B,C) := -B\cdot x\cdot y^2 + \frac{1}{3}\cdot C\cdot y^3$$

$$\sigma_z(x,y,z,A,B,C) := C\cdot y\cdot z^2 + \frac{1}{3}\cdot A\cdot z^3 \qquad \tau_{xy}(x,y,z,A,B,C) := B\cdot x^2\cdot y$$

$$\tau_{xz}(x,y,z,A,B,C) := -A\cdot z^2\cdot x \qquad \tau_{yz}(x,y,z,A,B,C) := -C\cdot y^2\cdot z$$

Tilavuusvoimat ovat nollia $\qquad f_x := 0 \qquad f_y := 0 \qquad f_z := 0$

$$\frac{d}{dx}\sigma_x(x,y,z,A,B,C) + \frac{d}{dy}\tau_{xy}(x,y,z,A,B,C) + \frac{d}{dz}\tau_{xz}(x,y,z,A,B,C) + f_x \to 0$$

$$\frac{d}{dx}\tau_{xy}(x,y,z,A,B,C) + \frac{d}{dy}\sigma_y(x,y,z,A,B,C) + \frac{d}{dz}\tau_{yz}(x,y,z,A,B,C) + f_y \to 0$$

$$\frac{d}{dx}\tau_{xz}(x,y,z,A,B,C) + \frac{d}{dy}\tau_{yz}(x,y,z,A,B,C) + \frac{d}{dz}\sigma_z(x,y,z,A,B,C) + f_z \to 0$$

Tasapainoyhtälöt toteutuvat, joten jännityskomponentit ovat mahdollisia.