Last time:

Conditional prob. / Bayes' rule: $p(x|y)$ or $p(x)\,p(y|x)$

↑ posterior — "whatever knowledge after y"

↑ prior — what we knowledge before y

likelihood — how y related to x

- Examples and some basics.

Monte Carlo: Suppose η r.v. $\eta \sim p_\eta(\cdot)$

$$E[f(\eta)] \approx \underbrace{\frac{1}{n}\sum_{i=1}^{n} f(\eta_i)}_{\hat{f}} \qquad \eta_i \text{ are independent samples of } \eta$$

Error: from Chebyshev.

$$\Pr\left(|\hat{f} - E[f(\eta)]| > \frac{\kappa \sqrt{\mathrm{Var}(f(\eta))}}{\sqrt{n}}\right) < \frac{1}{\kappa^2}$$

→ we need many samples.

→ M.C. is not a good way to compute integrals!

→ why would anybody do M.C.?

Try quadrature in 8-dimensions..

10 grid points in each dimension

How do we draw samples from a given r.v.?

1) $x \sim U[0,1]$ is "easy" on a computer.

$y = (b-a)x + a \rightarrow y \sim U[a,b]$ is easy.

Note: what computer gives you is not random.
It is a sequence of #s that is not easy
distinguishable from truly random numbers.

2) Gaussians: x_1, x_2 are $U[0,1]$, independent

$$y_1 = \sqrt{-2\sigma \log x_1}\cos(2\pi x_2)$$
$$y_2 = \sqrt{-2\sigma \log x_1}\sin(2\pi x_2)$$

~~$p(y)$~~ $y \sim N(0, \Sigma)$ $\Sigma = \begin{pmatrix} \sigma_1 & \\ & \sigma_2 \end{pmatrix}$

$z = \mu + Ly \rightarrow z \sim N(\mu + L\Sigma L^T)$

3) Everything related to Gaussians:

log-normal.

x is log-normal if $\exp(x)$ is Gaussian.

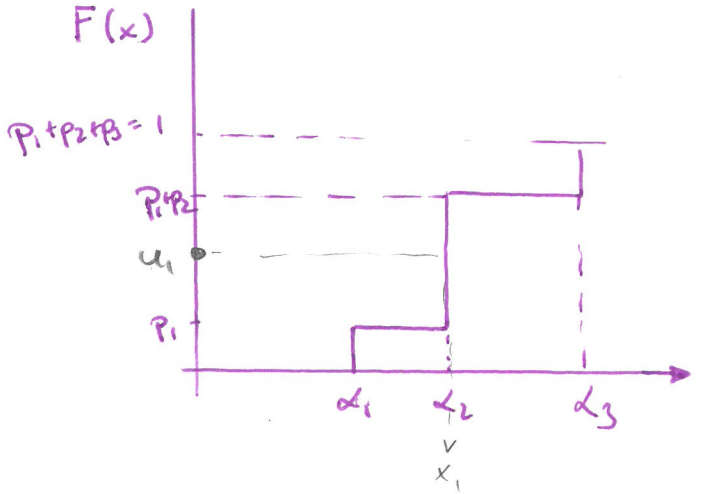
x ~ [illegible]

What about the general case:

Example:

$$X = \begin{cases} x_1 & \text{with prob } p_1 \\ x_2 & \text{with prob } p_2 \\ x_3 & \text{with prob } p_3 \end{cases}$$

$$F(x) = P(\{\omega \in \Omega : X(\omega) \leq x\})$$



Draw $u_1 \sim U[0,1]$

Solve $F(y) = u_1$ for y

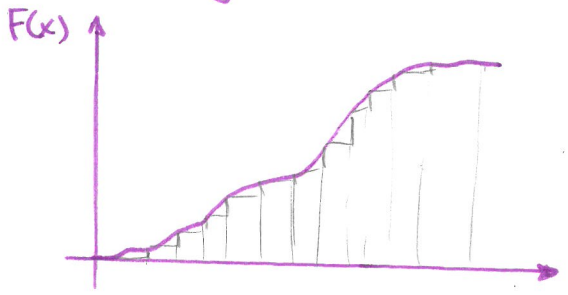
$\rightsquigarrow y_1 = F^{-1}(u_1)$

Repeat. $y_2 = F^{-1}(u_2)$

Random numbers generated in this way have the desired PDF ($\rightsquigarrow$ pdf)

$$y = \begin{cases} x_1 & \text{with prob } p_1 \\ x_2 & \text{with prob } p_2 \\ x_3 & \text{with prob } p_3 \end{cases}$$

General strategy:



- Construct $F(x)$
- Approximate by step fct.
- Use above procedure.