Imperial College London

Smart Rods (Hardware) Interim Report

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Chapter 1

Project Specification

Cuisenaire® Rods [1] are an educational tool used in primary school mathematics classes to aid in the learning of, among other things, 'number bonds'. These are simple addition sums typically resulting in a round number like 10 or 20, such as $7 + 3 = 10$, which children learn as a foundation for more complex mathematical relationships. These are an important part of the Key Stage One (ages 5-7) curriculum in the UK, as recommended by the British Government [2].

Figure 1.1: A set of Cuisenaire® Rods in use [3]

A set of rods consists of different-coloured cuboids, each of a different length, the smallest of which represents one unit. The rest of the rods are sized in multiples of this unit up to the longest rod, usually of size 10. Pupils are taught how different sized rods can be placed end-to-end in order to reach the target summation result, and are encouraged to find several alternate combinations of rods that add to the result, as demonstrated in Figure 1.1. This is a physical representation of number bonds, as the pupil can learn to associate a 7-rod (a rod of length 7) and a 3-rod forming a line of length 10 units with the mathematical concept that $7 + 3 = 10$.

The current disadvantage of the use of these rods is that in a class of dozens of students, it can be difficult for a teacher to monitor and track student progress. The teacher can survey the classroom, but is not capable of giving every student constant attention, so inevitably, poor progress sometimes goes unnoticed and students may fall behind without extra support. Additionally, students need to make a record of the work they have done, which can be difficult with the existing rods, as their use does not necessarily involve any written work. The aim of this project is to design and build a product that resolves this, by using a technologically-enhanced version of the rods which will allow the teacher to detect struggling pupils and provide assistance to them, as well as keeping an electronic record of all the students' sessions with the rods. They should do so by gathering data about the way the students are using the enhanced rods, such as time spent on an exercise, how many answers were found, and perhaps even more complex information like detecting patterns in which solutions were discovered. This data can be analysed to provide an overview of how the entire class is coping with the task.

The project is split into two halves: the software, including processing of data and user interfaces, is being completed by another student (Pierre Azalbert). The hardware, including the design and production of the rods, is the focus of this report.

Making the correct design choices will greatly affect the efficacy of the product. The final product will likely be used by publicly-funded schools, so costs should be kept low. The aim of the product is to make the teaching process easier for the teacher and to improve the learning experience for the pupils, so care should be taken to make the product as accessible as possible for both parties. This means keeping setup steps simple for teachers, and using human-computer interface (HCI) principles to ensure students find the product easy to use.

Chapter 2

Background

2.1 Cuisenaire® Philosophy

Background research began with discovering how Cuisenaire® rods are used and why they are useful. John V. Trivett, in his article, *The Cuisenaire Rods—Numbers in Colour*, from the Journal of the Association of Teachers of Mathematics [4] talks about how abstracting the notation from the teaching of mathematics allows students to form conceptual instincts about ratios between values. He argues this is more valuable than committing equations and expressions to memory to be regurgitated during examinations. This idea is reinforced by Tony Wing in his article in the same journal, *Working Towards Mental Arithmetic... And (Still) Counting* [5], where he states that it is important for children to learn to play with the rods "before conventional number names and symbols are attached to them".

This is something to keep in mind throughout the design process: we want to adhere to that principle of abstraction and not construct the rods in a way that would cause the children to treat them as numbers, for example, by having each rod's length written on it.

2.2 Teacher Advice

Further research was carried out by contacting primary school maths teachers who have made use of Cuisenaire® rods, to ask them about the benefits of the rods and what could be improved. A criticism of the rods is that teachers need to record proof of students' work for administrative purposes, and the rods do not lend themselves to this very well because the children are required to draw diagrams of their solutions in workbooks, which takes considerable time from the exercise. This is a problem that our product will likely be able to solve, as an electronic system will be able to record every attempt a student makes at an answer, meaning that our records will be more detailed than what the children could draw themselves.

Another complaint was that, often, neither students nor teachers know how to properly make use of the rods when presented with them. This creates, for us, a task in human-computer interaction, as we will have to design a system that is intuitive to use with little mental effort required by the user. This includes being intuitive to set up (for teachers) and intuitive to operate (for children).

It was warned that students may not treat the product with care and so it should be able to withstand bending or general mistreatment at the hands of the children. This will inform our choice of material and construction method once we begin production.

2.3 Similar Products

Online searching yielded very few internet-enabled educational devices for the KS1 demographic, excluding educational applications for tablets and computers. This means that we will not be able to compare the hardware aspect of our product to existing designs, and will be setting precedents with some of our design choices. While this gives us the opportunity to explore and discover new methods of integrating technology into teaching, it does mean that the chance of our ideas being unsuccessful are increased, as we do not have a foundation of work to build upon. This effect can be mitigated via consultation of our contacts in schools and by working closely with our supervisor who can help guide our path.

Chapter 3

Design Choices

3.1 Where should electronic complexity lie?

Some form of electronic device has to be used to identify the relative positions of the rods, in order to ascertain how the student has arranged them, but we have some choice in where this device exists. If we want to keep our product to be simply a set of rods, then the only place for the device is within the rods themselves. The rods would each need to be able to communicate with each other, to discern relative position, and also communicate with a server for further processing.

Unfortunately, this solution brings several impracticalities. Firstly, the size of the rods has to be kept small, not only for storage purposes but also to allow the children to comfortably pick them up and work with them. We will try to keep our rods as close as possible to the original rods, which are about 1cm in diameter. This constrains the size of battery we could embed into them. They will have to be capable of transmitting and receiving wireless messages, and even perhaps perform some amount of processing, so their power consumption will be too high for a battery of that size. Charging the batteries would also be impracticable; consider a classroom of up to 30 students, each with a set of dozens of rods, and it becomes clear that the hassle of having to charge each device individually would outweigh any benefit of the product to a teacher.

Another issue concerns the identification of rods and their assignment to students: remember again that each student needs their own set of dozens of rods, and that any information these rods transmit will be attached to that child's profile. The teacher would have to perform the frustrating and repetitive task of assigning each rod to a student, causing a considerable amount of inconvenience. Additionally, we must keep in mind that our target users are children, who may have the tendency to take rods from another student, misplace their own rods, or perhaps even throw rods around the classroom – all of these actions could cause sets of rods to be mixed together, leading to incorrect data being sent to the server, as progress from one student will be attached to the profile of another.

The last issue is economic: each of these rods will require several electronic components, including a processing unit and wireless transceivers, driving the cost of each unit up considerably. This is especially relevant as the smaller rods are easily lost in the classroom, so replacements would likely need to be purchased, increasing ongoing costs further.

An alternative solution to putting complexity in the rods is to design a playing board with which the rods can interact, and embed the majority of the electronics in that instead. The 10x10 board would resemble a chess board, with grid squares guiding the placement of rods. The students arrange their rods on the board, which can identify the positions of the rods through some means, and communicate that data to the server. This alleviates all of the problems detailed above. The board is large and can house a larger battery than the rods, so there would be no power problem. There would only be one board per child so there are fewer devices to charge and much less work involved in assigning devices to children, also meaning that mixing of rods is not an issue, as it is the board which is unique to the child, not the rods. The cost to replace rods would be much lower as they would not contain as much electronics, and the overall cost would be lower as only the boards would require processing and communications equipment, as opposed to fitting one to every rod.

During an interview with a primary school teacher, we were informed that having a board may actually aid in the students' learning, as having a grid could help them conceptualise how the rods fit together.

3.2 How should the board detect rod positions?

Having settled on the use of a board, the next design choice to be made is what technology it will make use of to detect where rods have been placed. The chosen method needs to have the following characteristics to be viable: it must be able to *reliably* detect rods and their positions, as any errors may mislead and confuse the child, weakening the product's effectiveness as an educational tool. It should keep *cost* low, as there could be up to hundreds of these products in use at an institution, and since they will likely be using public funding they will be under budget constraints. It should keep *power consumption* to a minimum to maximise the amount of use between charges. Lastly, it should remove as much *complexity* from the rods as possible, for the reasons outlined in section 3.1.

3.2.1 Weight

Rods could be given different weights, which are sensed by load sensors on the board. Each weight would be mapped to a different length of rod.

Reliability: It may be difficult to distinguish between the weight of a rod and the weight from the child touching the board, which could lead to false readings. However, under good conditions it should be possible to distinguish between rods well.

Cost: The cost of the rods will likely be low, as they will just require weighting, however the board will require a load sensor in every grid square. The load sensor would need to be small enough to fit into a square around $2cm^2$ in area, and be sensitive enough to detect weighted rods light enough for a child to safely play with. A preliminary search revealed that components meeting those specifications would cost tens of pounds [6]. Considering we are using a 10x10 board, purchasing 100 of these components would increase the cost to unacceptable levels.

Power Consumption: The datasheet of a suitable sensor [7] states a typical voltage requirement of $10V$, and an input resistance of $5k\Omega$. Using $P = \frac{V^2}{R}$ we can determine a power usage of $0.02W$. 100 of these components would then draw 2W, which is relatively high, but could be acceptable depending on the battery used.

Rod Complexity: The rods will not need any electronics, just some weighted material, so the complexity is quite low.

3.2.2 Magnetic Fields

Magnets of varying strength could be inserted into rods to be detected by Hall effect sensors in the board. Each strength would be mapped to a different length of rod.

Reliability: In terms of magnetic stability, a system using magnets could be quite reliable, as rare earth magnets can be used for several years without their strength diminishing [8]. The limiting factor would be the operation range of the sensors, as too high a range would mean it could mistakenly detect a field from a neighbouring rod. The activation distance of a representative sensor was found to be up to 1-2cm [9], which is slightly too high for our purposes. There is also the problem that since dipole magnetic fields follow the inverse-cube law [10], the strength of the field as detected by the sensor will increase as the magnetic rod approaches the sensor, so the sensor may mistake the rod for one of a weaker strength as it is being placed.

Cost: A search for the least expensive sensor that meets the specifications most closely yielded a result [11] which is priced at over £1 per unit, making it too expensive for our purposes, as 100 would be required for our 10x10 board.

Power Consumption: The datasheet of a representative sensor [9] suggests a power usage of 0.04-0.24W per unit (4-24V $\cdot$ 10mA) and with 100 units, that would bring power usage to 4-24W, which is too high to maintain.

Rod Complexity: The rods would be kept quite simple, as they would just require a magnet inserted into them. However, it would be important for the strength of the rods to be precise to distinguish rods from each other.

3.2.3 RFID

Passive RFID chips could be inserted into the rods, and RFID sensors into the board. Each RFID chip would contain identifying information about the rod it is in. When a rod is placed on the board, its RFID chip is powered and the data is read, and sent to the server.

Reliability: Our requirements are such that the range of detection of a rod should be very small, no more than a centimetre, as otherwise neighbouring grid squares could erroneously detect nearby rods and send false data to the server. A search for RFID products that operate in this range found no suitable results, which indicates RFID is not tailored to the precision detection needed for this project.

Cost: An example of one of the shortest range components of the correct dimensions retails at between £5-14 per unit [12]; while this is much cheaper than the load sensor described above, it would still raise the price of the product to an unacceptable level given the quantity of boards/rods needed for a classroom.

Power Consumption: Since the components in the rod are passive, they would draw a minimal amount of power, powered by the reader. A suitable reader [13] is listed to draw 0.558-1.705W. 100 of these would draw an unmanageable amount of power.

Rod Complexity: Since this solution requires inserting electronics into the rods, the complexity is quite high. This increases the chance that something may fail and rods will need to be replaced. Another problem is that readings will be inaccurate until the fault is discovered, which may take some time.

3.2.4 RGB Light

Each grid square of the board could contain an RGB LED and a photodiode; the LED would flash periodically and the photodiode would measure the frequency of the reflected light. Since each rod is of a distinctly different colour, each would produce a different reflected frequency.

Reliability: This method has the potential for very good reliability, as there are few enough different sizes of rod such that each can have a distinct colour, which reduces the possibility of a false reading. However, it is unclear how such a system might respond to a different object like a child's hand being placed on the board.

Cost: The cost of the components for this design is quite high: 100 pairs of a suitable photodiode [14] and LED [15] would cost over £150.

Power Consumption: Taking the typical operating voltages of each colour of the RGB LED (1.95V. 3.3V and 3.3V for red, green and blue, respectively) at the typical current draw (20mA), constant powering of an LED would require $(1.95 + 3.3 + 3.3) \cdot 0.02 = 0.171$W.

Assuming a 25% duty cycle, since the LEDs will be flashing, this is reduced to 0.04275W. 100 of these would then draw 4.275W, which is relatively high.

Rod Complexity: This method requires adding nothing at all to the rods: since the solution works by sensing colour, the rods would be simply coloured as they are in the original set of rods.

3.2.5 Shorting Resistor Chains

Each row on the board could be linked to a long series chain of resistors of known values, and the rods themselves could contain a wire connecting two contacts, on either end of the rod. Each row would have its own chain, connected in parallel to the other chains, with a contact on each grid square connected to a node along the chain. When the rod is placed on the board, its contacts would connect to contacts on the board, shorting a number of resistors in the board relative to the rod's length. The change in voltage at the different nodes in the line could be used to detect if and where a rod is placed. Figure 3.3 demonstrates more clearly how this could work.

Reliability: This solution will be very reliable as long as the contacts are designed in such a way that a child could not short resistors without a rod, which would produce a false reading. Another requirement is that the voltage level be high enough so that every node in the chain of resistors is sufficiently different and distinguishable.

Cost: The only component required is resistors, which are orders of magnitude cheaper [16] than the resources required for the alternative solutions.

Power Consumption: The power consumption will be low: assuming the board will be controlled by a microcontroller with a 5V rail powering 10 parallel chains of resistors each totalling $2K\Omega$ (meaning the overall resistance is 200Ω), a $P = \frac{V^2}{R}$ calculation reveals a power usage of $0.125W$, which is quite acceptable.

Rod Complexity: The rods will be kept relatively simple, as they will just need a wire between the two contacts inside them, however the design of the contact could increase complexity.

Table 3.1 summarises each method's adherence to the specification with colour, green representing a strong adherence, red representing a weak adherence, and yellow representing a moderate adherence.

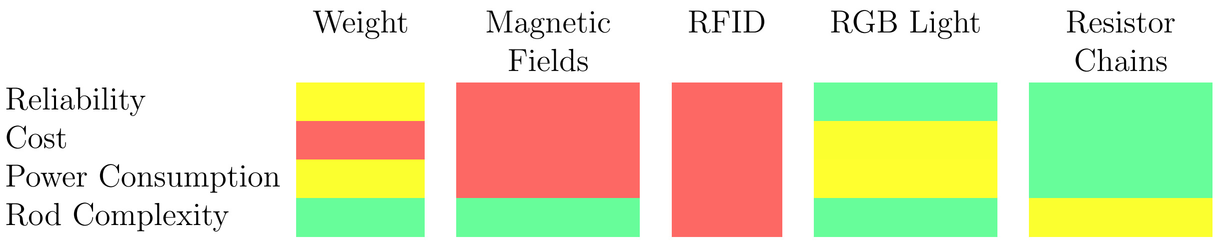

Table 3.1: Method Comparison Summary

Considering the above comparison, the best solution is the resistor chains; it is comparatively inexpensive, uses comparatively low power, and does not rely on any complex technologies so has high reliability. The complexity of the rods is slightly increased due to the need for contacts, but faults with these should be more apparent than faults with electronics embedded in a rod, and in the event of a fault, replacement rods will not incur significant cost.

3.3 Prototyping

3.3.1 Controller

To control the reading of data from the resistor chains, it was decided to make use of an Arduino microcontroller board [17]. This was mainly because of its ease of use, and active community of users who can provide support during the prototyping process. The Arduino board provides an interface to external hardware via GPIO (General Purpose Input/Output) pins that can be used to read the voltages along the chain of resistors, and has a 5V power rail that can be used to power the prototype.

3.3.2 Determining Rod Placement

Prototyping began with the construction of two parallel simple resistor chains (shown in Figure 3.1) to learn to use the Arduino and to get some intuition on how rod placement could be ascertained from the measured voltages.

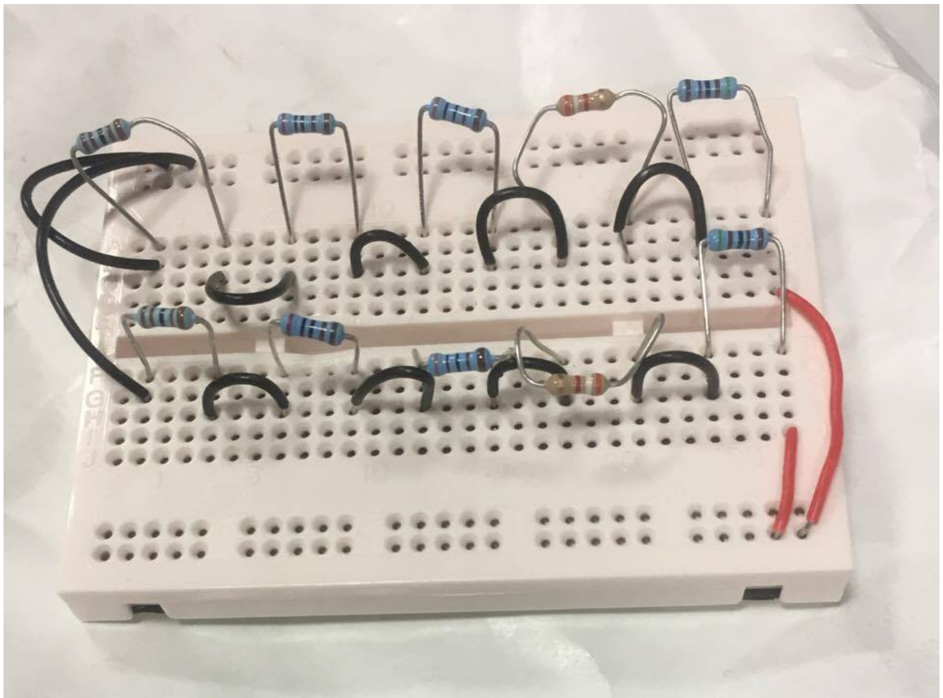

Figure 3.1: Parallel Resistor Chains

This first attempt incorporated the use of differing resistors down the chain, as in Figure 3.2 (although it was later realised that using equal resistors would be better suited as they would give uniform increments of voltage at each node along the chain). A wire was used to connect two of the labelled nodes A-E to represent the functionality of the rod shorting resistors. Voltage was measured at two points along the chain to see if each different position of the rod along a chain could produce a unique pair of voltages that could be used to identify that position.

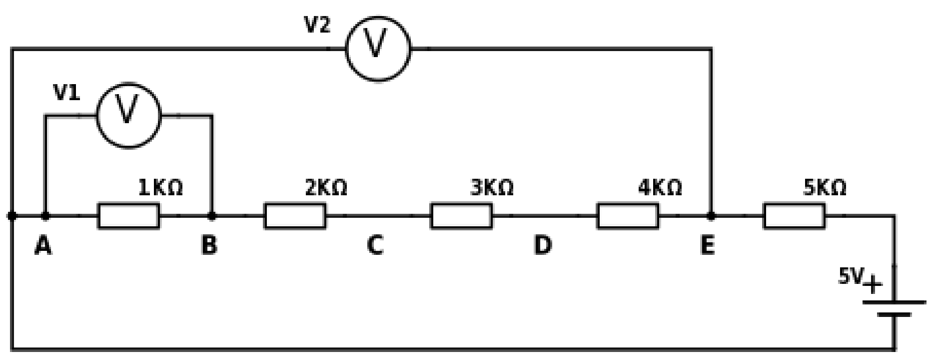

Figure 3.2: Simple prototype with labelled nodes

	A	B	C	D	E
A		0, 3.18	0, 2.88	0, 2.17	0, 0
B			0.41, 3.05	0.53, 2.47	0.87, 0.87
C				0.44, 2.89	0.66, 1.89
D					0.48, 2.73
E					

Table 3.2: Voltages (V1, V2) when two nodes are connected by a wire

The values in Table 3.2 show the voltages at V1 and V2 when each pair of nodes is connected. It is apparent that every combination of connected nodes does produce a unique pair of voltages, which proves that it is possible, in principle, to identify rods based on the voltages of the nodes. This experiment was a proof-of-concept, as it only used 5 resistors, but the real prototype needs at least 10 resistors to detect rods of up to size 10.

3.3.3 The Problem with the Unit Rod

At the end of Section 3.3.2 we postured that we would need at least 10 resistors per chain to detect rods of up to size 10; while this is true, it overlooks the necessity to detect unit rods, of size 1. The assumption is that each node in the resistor chain is connected to a grid square on the board, and that rods connected to two nodes along a chain can short the resistors between those two nodes when connected to the board. However, a unit rod spans only one grid square, meaning that it is only connected to one node, and so cannot short any resistors. Figure 3.3 demonstrates why this is the case. Note that the representations of the wires connecting the rods and the resistor chain are symbolic and do not represent what the actual connections would look like.

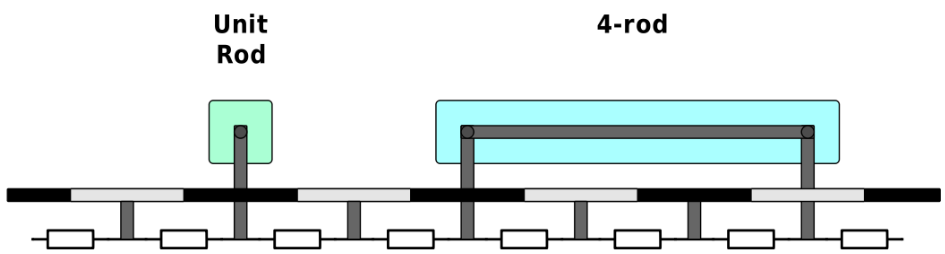

Figure 3.3: Cross-sectional view of board

The chosen solution to this problem was add a double connector to the unit rod instead of the unit one, so that it can connect to two nodes and short a resistor. This means that twice as many resistors are required along a chain, but allows us to detect unit rods without affecting the detection of the other sizes. Figure 3.4 demonstrates how doubling the number of resistors achieves this.

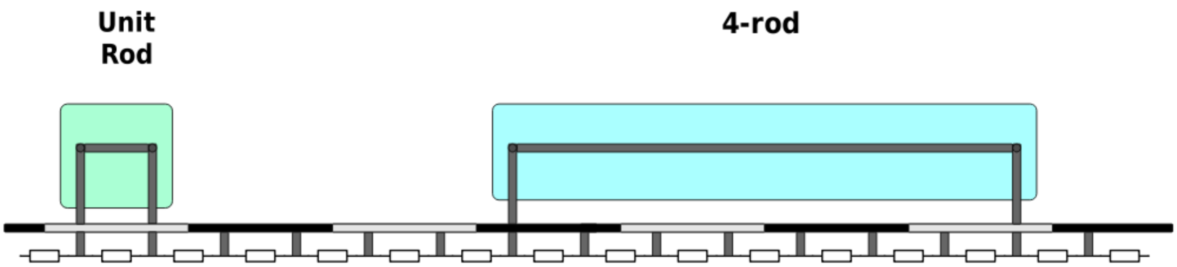

Figure 3.4: Cross-sectional view of board with double resistor chain length

3.3.4 Practicalities in Data Measurement

Taking only two voltage measurements worked well in the proof-of-concept from Section 3.3.2, but when the same method was applied to a resistor chain of size 20, the voltage values measured were much less distinguishable from each other. This is because when the number of resistors is increased, the voltage across each is smaller, so shorting one makes less of a difference to the measured voltage, and thus a distinction between states is less easily made.

The solution to this was to measure the voltage at every node in the chain, and make use of the increased information to make a more accurate inference of the positions of rods. With so many nodes being measured, it would not be possible to connect each directly to the Arduino's limited GPIO (General Purpose Input/Output) pins. Instead, multiplexers are used to direct all measurements to a single pin. The CD4051BE 8:1 multiplexer [18] was chosen instead of available 16:1 multiplexers, because although more 8:1 units would be required than 16:1 units to meet our needs, its relatively low price meant it was still the more economic choice. The method for mapping the measured voltage values to rod placements is detailed in the Implementation Plan, in Section 4.4.

Chapter 4

Implementation Plan

The following are considerations for the remainder of the project.

4.1 Construction

Construction will likely involve 3D printing, as it is most convenient for prototyping. Guidance will be sought from Imperial College Robotics Society on how to operate a 3D printer, and how to use the computer-aided design tools to produce a prototype. Research will be made into the most appropriate material to use in the printer, with regards to ease of production process as well as properties of the printed product.

4.2 Circuit Board

The prototype that is built so far uses a breadboard and wires; for production, a custom-designed circuit board will need to be printed. The board will make use of the standalone Arduino microcontroller, rather than the hobbyist board being used in the prototype. The author does not have much experience in designing and printing circuit boards, so guidance will be sought from Imperial College Robotics Society and colleagues on how to do so.

4.3 Contacts and Wire

The prototypes used so far for testing have made use of a wire in place of the rod, replicating its functionality, but not addressing the tough design issue of actually constructing the rod. The design of the contacts that will be placed on either end of the rod (and on the grid squares) will need to strike a balance between durability, safety, accessibility, and ease of production.

Durability is a concern because the children using the product may not treat it delicately, so any fragile components of the design will likely fail quickly. Additionally, protrusions, sharp corners, or the possibility of splintered material could pose safety risks to the children, so such designs would not be suitable. The dexterity of the children should be kept in mind, since their motor skills may not be suited to working with a design that requires any considerable force or nimble movements to operate. Lastly, a complex design could drive production costs up, as well as making it more difficult to produce a good-quality prototype.

Inspiration can be drawn from related existing products: Cubelets® by Modular Robotics [19] are electronic cubes used to build modular robots. They make use of magnetic connectors, which are strong enough to keep the cubes together, but weak enough to allow a child to take them apart. The use of magnets in our rods may prove to be a useful tool.

It would also be best for the contacts to not be directly accessible by the children's hands, as a build-up of grease and dirt could impair their electrical conductivity.

The contacts should be designed such that a rod cannot be placed in an orientation other than that for which the board has been designed. Connecting nodes between chains will produce false or misleading data.

4.4 Mapping Voltages to Rod Placement

As explained in Section 3.3.2, we are able to detect the voltages at every node along the chain using multiplexers. To map those values to rod placements, we can use the fact that nodes which have been shorted by the same rod will be at the same voltage. The software running on the controller can sweep along each chain reading the voltages, and when it reads two or more nodes of the same value, it can conclude there is a rod in that position. Each rod spans a number of nodes equal to twice the rod's length; for example, as was seen in Figure 3.4, the 4-rod spans 8 nodes. This allows us to count the number of nodes that are of the same voltage, and divide that number by 2 to get the rod length. This information, as well as the positions of the nodes, is what will be sent to the server. The precise nature of the information sent to the server will depend on which configuration requires the least communication, as this will conserve battery power.

4.5 Wireless Communication

The board will need to transmit information about the rod placements (as well as some other auxiliary information such as remaining battery power) to the server to be processed by the software. This will likely be achieved using an ESP8266 Wi-Fi module, a popular choice for wireless communications in projects of this nature. The author is currently studying the Embedded Systems module under Dr. Edward Stott, during which the use of the ESP8266 module is taught, and so more detailed plans will be realised once the course is completed.

4.6 Power

The board will be battery powered, so research will need to be performed into the size and type of battery required to sustain the board for a reasonable amount of usage. What 'reasonable' constitutes will be determined by future conversations with teachers. Thought will have to go into how a charging interface may be designed; it could be powered from the mains, but an adapter will need to be incorporated to bring the voltage to an appropriate level, or it could be powered via a USB port, which could reduce complexity but may not be as convenient as the wall socket.

4.7 Additional Interfaces

It may be of use to include additional interfaces such as buttons, switches and an LCD screen or 7-segment display. These may not serve any purpose when the product is first put into production, but would serve as an available resource to be used when software is updated in the future. It would also give the flexibility of allowing us to add functionality nearing the end of this design cycle through the relatively easy task of writing extra code. rather than the less-easily accomplished task of redesigning and rebuilding hardware. This would, however, increase material costs, which will be taken into account when making the final decision.

Chapter 5

Evaluation Plan

The majority of the evaluation for this project is qualitative and subjective; the true judge of the success of the product will be the reactions of the teachers and students who will use the product. We will want to know if our design met our aim of helping teachers to keep a record of students' work, and to more easily track and monitor student progress, and also if it provides a friendly user experience for both teachers and students.

This should be achieved through communication with primary maths teachers, and visits to nearby schools where we can see how a classroom might make use of the product. We should observe the way in which the product is used, making note of unexpected ways in which the product is used or handled. Not being part of the target demographic for the product, it is likely there are some use-cases which we have not anticipated, and such instances would be taken into account if design for the product were to be continued further. The students should be monitored to see what, if any, frustrations they have with operating the product, and if they enjoyed using it. The teachers should be surveyed about the ease of set-up of the product (including charging and distribution among children) and about the merit the product has as an educational tool compared to other methods they have used.

The ideal evaluation would be a long-term study of a class using the product over the course of a year, along with a similar class using traditional rods or other methods of teaching, and comparing each class' performance and satisfaction over that year. This is, unfortunately, out of the scope of this project, so we have to settle with a much shorter-term survey.

Since this report focusses only on the hardware aspect of the product, evaluation considerations must focus on that aspect too. While it can be difficult to separate the concepts of software and hardware when trying to evaluate the product, one feature that is exclusive to the hardware is the reliability of rod placement detection. The accurate detection of rods is vital in ensuring children do not get confused when using the product, not only because it may cause them to misunderstand the lesson they are supposed to learn, but if they suspect the product does not function properly, they may be less inclined to engage with it in future.

To evaluate reliability, multiple possible configurations of rods should be tested and the response of the board should be recorded. The test should include extreme and edge cases, such as filling the entire board with rods, quickly placing and removing a rod, and adding many rods in quick succession. There is no margin of error, so every test needs to pass.

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