Computer Vision I - *Basics of Image Processing – Part 1*

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28/10/2014

Link to lectures

- Slides of Lectures and Exercises will be online:

  http://www.inf.tu-dresden.de/index.php?node_id=2091&ln=de

  (on our webpage > teaching > Computer Vision 1)

- No lecture on 28.11.2014

Roadmap: Basics of Digital Image Processing

- What is an Image?
- Point operators (ch. 3.1)
- Filtering: (ch. 3.2, ch 3.3, ch. 3.4) – main focus
  - Linear filtering
  - Non-linear filtering
- Multi-scale image representation (ch. 3.5)
- Edges detection and linking (ch. 4.2)
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- Interest Point detection (ch. 4.1.1)

What is an Image

- We can think of the image as a function:

$$I(x,y), \qquad I:\ \mathbb{R}\times\mathbb{R}\to\mathbb{R}$$

  - For every 2D point (pixel) it tells us the amount of light it receives
  - The size and range of the sensor is limited:

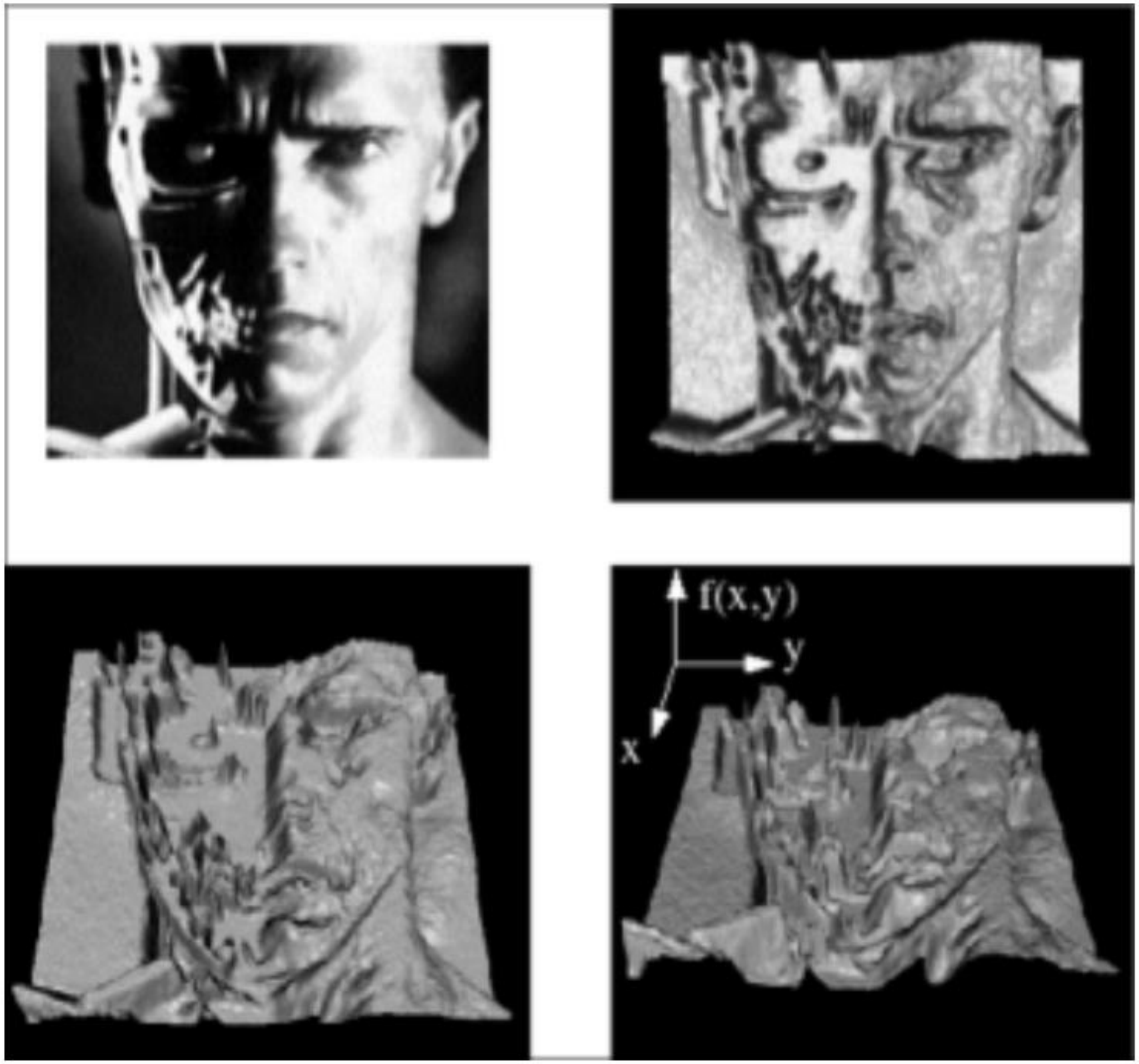
$$I(x,y), \qquad I:[a,b]\times[c,d]\to[0,m]$$

  - Colour image is then a vector-valued function:

$$I(x,y)=\begin{pmatrix} I_R(x,y) \\ I_G(x,y) \\ I_B(x,y) \end{pmatrix}, \qquad I:[a,b]\times[c,d]\to[0,m]^3$$

  - Comment, in most lectures we deal with grey-valued images and extension to colour is "obvious"

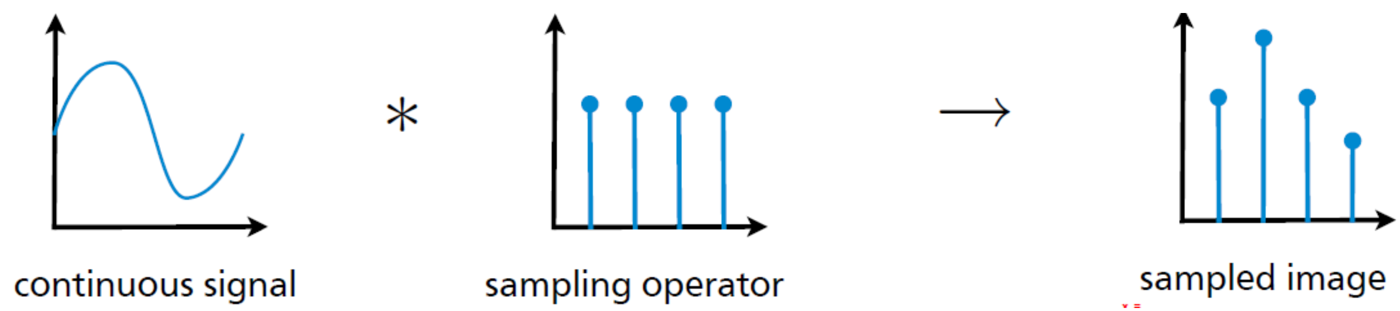
Images as functions



[from Steve Seitz]

Digital Images

- We usually do not work with spatially continuous functions, since our cameras do not sense in this way.
- Instead we use (spatially) discrete images
- Sample the 2D domain on a regular grid (1D version)



- Intensity/color values usually also discrete. Quantize the values per channel (e.g. 8 bit per channel)

y = \ x =	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72
41	210	209	204	202	197	247	143	71	64	80	84	54	54	57	58
42	206	196	203	197	195	210	207	56	63	58	53	53	61	62	51
43	201	207	192	201	198	213	156	69	65	57	55	52	53	60	50
44	216	206	211	193	202	207	208	57	69	60	55	77	49	62	61
45	221	206	211	194	196	197	220	56	63	60	55	46	97	58	106
46	209	214	224	199	194	193	204	173	64	60	59	51	62	56	48
47	204	212	213	208	191	190	191	214	60	62	66	76	51	49	55
48	214	215	215	207	208	180	172	188	69	72	55	49	56	52	56
49	209	205	214	205	204	196	187	196	86	62	66	87	57	60	48
50	208	209	205	203	202	186	174	185	149	71	63	55	55	45	56
51	207	210	211	199	217	194	183	177	209	90	62	64	52	93	52
52	208	205	209	209	197	194	183	187	187	239	58	68	61	51	56
53	204	206	203	209	195	203	188	185	183	221	75	61	58	60	60
54	200	203	199	236	188	197	183	190	183	196	122	63	58	64	66
55	205	210	202	203	199	197	196	181	173	186	105	62	57	64	63

Comment on Continuous Domain / Range

- There is a branch of computer vision research ("variational methods"), which operates on continuous domain for input images and output results

- Continuous domain methods are typically used for physics-based vision: segmentation, optical flow, etc. (we may consider this briefly in later lectures)

- Continues domain methods then use different optimization techniques, but still discretize in the end.

- In this lecture and other lectures we mainly operate in discrete domain and discrete or continuous range for output results

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Point operators

- Point operators work on every pixel independently:

$$J(x,y) = h\big(I(x,y)\big)$$

- Examples for h:
  - Control contrast and brightness; $h(z) = az^b + c$

original

Contrast enhanced

Example

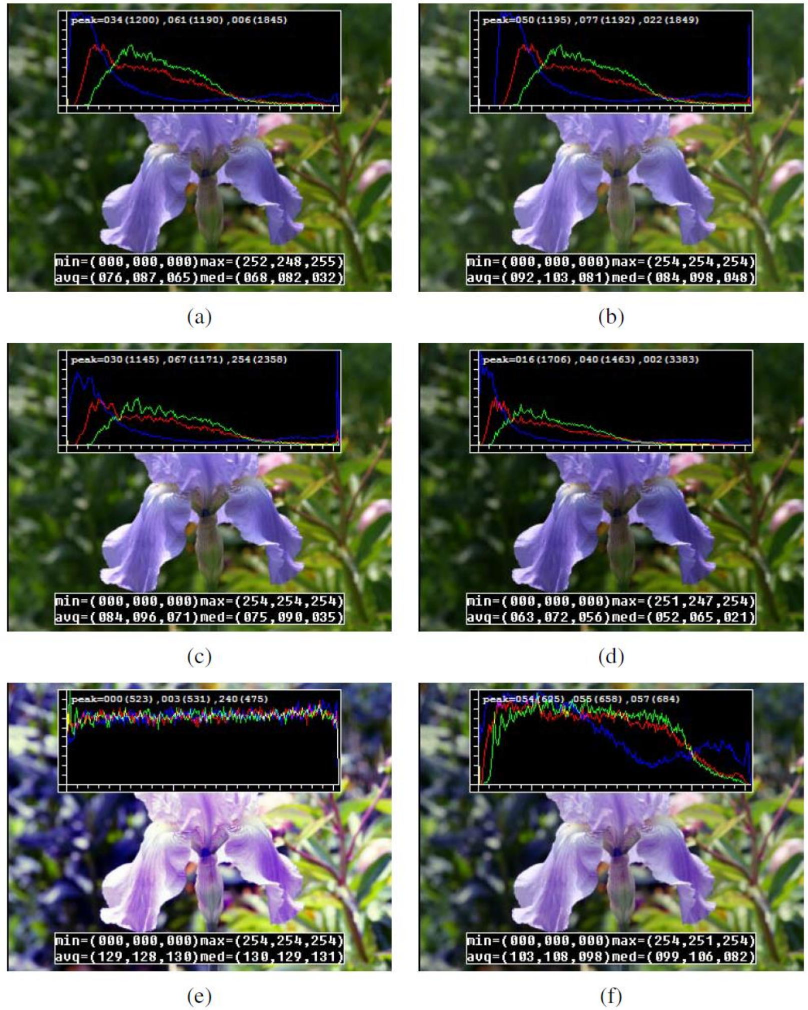
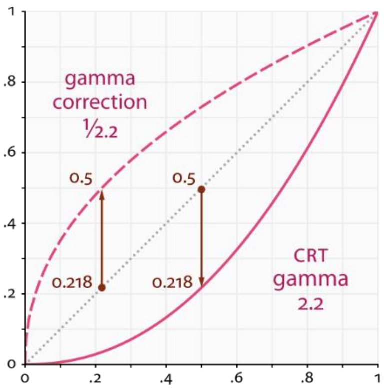


Figure 3.2 Some local image processing operations: (a) original image along with its three color (per-channel) histograms; (b) brightness increased (additive offset, $b = 16$); (c) contrast increased (multiplicative gain, $a = 1.1$); (d) gamma (partially) linearized ($\gamma = 1.2$); (e) full histogram equalization; (f) partial histogram equalization.

Example for Point operators: Gamma correction



Intensity range: [0,1]

<u>Inside cameras:</u>
$h(z) = z^{1/\gamma}$ where
often $\gamma = 2.2$ (called gamma correction)
"makes image brighter"

In (old) CRT monitors
An intensity z is perceived as:
$h(z) = z^{\gamma}$ ($\gamma = 2.2$ typically)
"perceive image as darker"

<u>Today:</u> even with "linear mapping" monitors, it is good to keep the gamma corrected image. Since human vision is more sensitive in dark areas.

<u>Important:</u> for many tasks in vision, e.g. estimation of a normal, it is good to run $h(z) = z^{\gamma}$ to get to a linear function

Example for Point Operators: Alpha Matting

Foreground F

Background B

Matte α
(amount of transparency)

Composite C

$$C(x,y) = \alpha(x,y)F(x,y) + \big(1 - \alpha(x,y)\big)B(x,y)$$

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Linear Filters / Operators

- Properties:
  - Homogeneity: $T[aX] = aT[X]$
  - Additivity: $T[X + Y] = T[X] + T[Y]$
  - Superposition: $T[aX + bY] = aT[X] + bT[Y]$

- Example:
  - Convolution
  - Matrix-Vector operations

Convolution

- Replace each pixel by a linear combination of its neighbours and itself.
- 2D convolution (discrete)

$$g = f * h$$

Centred at 0,0

45	60	98	127	132	133	137	133
46	65	98	123	126	128	131	133
47	65	96	115	119	123	135	137
47	63	91	107	113	122	138	134
50	59	80	97	110	123	133	134
49	53	68	83	97	113	128	133
50	50	58	70	84	102	116	126
50	50	52	58	69	86	101	120

image $f(x,y)$

*

0.1	0.1	0.1
0.1	0.2	0.1
0.1	0.1	0.1

filter (kernel) $h(x,y)$

=

69	95	116	125	129	132
68	92	110	120	126	132
66	86	104	114	124	132
62	78	94	108	120	129
57	69	83	98	112	124
53	60	71	85	100	114

filtered image $g(x,y)$

smaller output?

$$g(x,y) = \sum_{k,l} f(x-k, y-l)h(k,l)$$

"the image f is implicitly mirrored"

Convolution

- Linear $h * (f_0 + f_1) = h * f_0 + h * f_1$
- Associative $(f * g) * h = f * (g * h)$
- Commutative $f * h = h * f$
- Shift-Invariant $g(x, y) = f(x + k, y + l)$ (for a neighborhood k, l)

$$\leftrightarrow (h * g)(x, y) = (h * f)(x + k, y + l)$$

(it means "behaves everywhere the same, i.e. it does not depend on the position in the image.")

$$\boxed{72}\boxed{88}\boxed{62}\boxed{52}\boxed{37} * \boxed{1/4}\boxed{1/2}\boxed{1/4} \Leftrightarrow$$

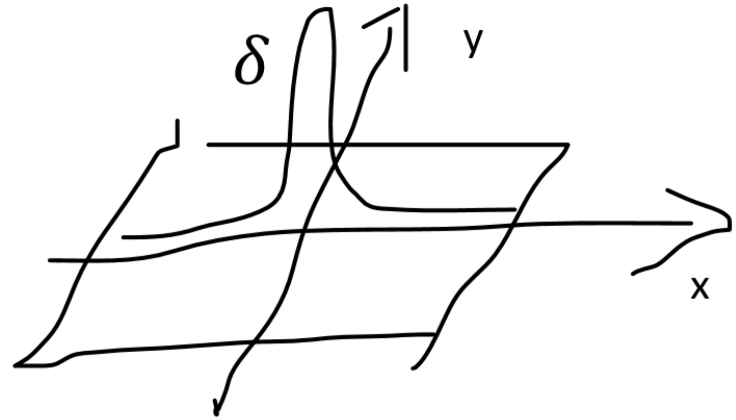
$$\frac{1}{4}\begin{bmatrix} 2 & 1 & . & . & . \\ 1 & 2 & 1 & . & . \\ . & 1 & 2 & 1 & . \\ . & . & 1 & 2 & 1 \\ . & . & . & 1 & 2 \end{bmatrix}\begin{bmatrix} 72 \\ 88 \\ 62 \\ 52 \\ 37 \end{bmatrix}$$

- Can be written in Matrix form: $g = H f$
- Correlation (not mirrored filter):

$$g(x, y) = \sum_{k,l} f(x + k, y + l) h(k, l)$$

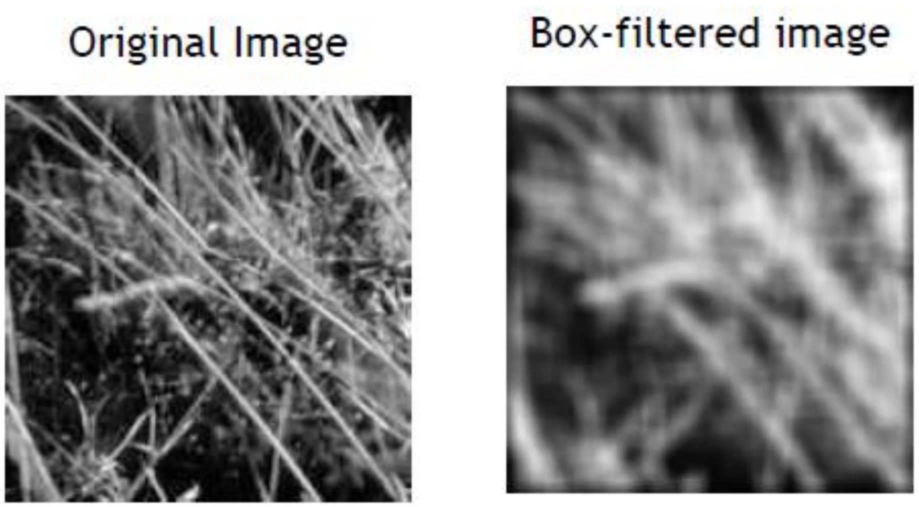
Examples

- Impulse function: $f = f * \delta$



- Box Filter:

$$\frac{1}{9} \cdot \begin{array}{|c|c|c|} \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline \end{array}$$



Application: Noise removal

- Noise is what we are not interested in:
  sensor noise (Gaussian, shot noise), quantisation artefacts, light fluctuation, etc.

- Typical assumption is that the noise is not correlated between pixels

- Basic Idea: neighbouring pixel contain information about intensity

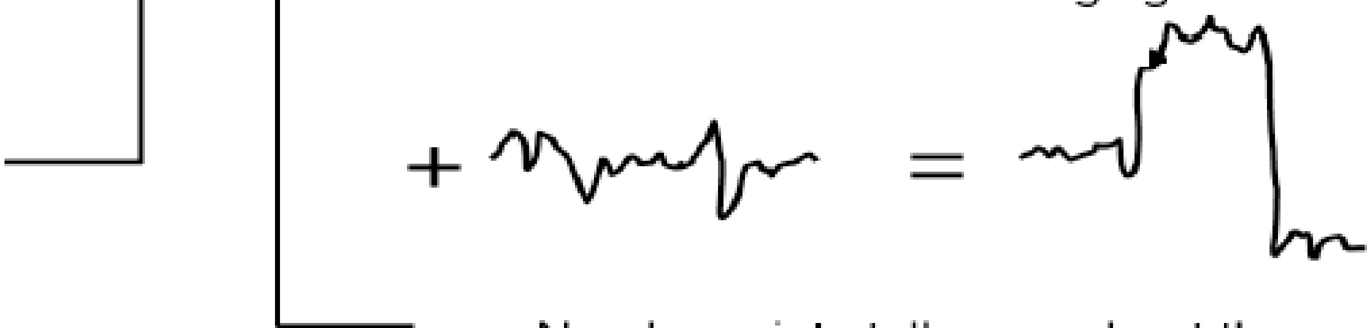
$$\begin{array}{|c|c|c|}\hline 2 & 3 & 3 \\ \hline 3 & 20 & 2 \\ \hline 3 & 2 & 3 \\ \hline \end{array} \rightarrow \begin{array}{|c|c|c|}\hline 2 & 3 & 3 \\ \hline 3 & 3 & 2 \\ \hline 3 & 2 & 3 \\ \hline \end{array}$$

Noise removal

Signal + Noise = Image

$$S + N = I$$

Neighborhood for averaging.



Nearby points tell more about the signal than distant ones.

The box filter does noise removal

- Box filter takes the mean in a neighbourhood

Image

Noise

Pixel-independent Gaussian noise added

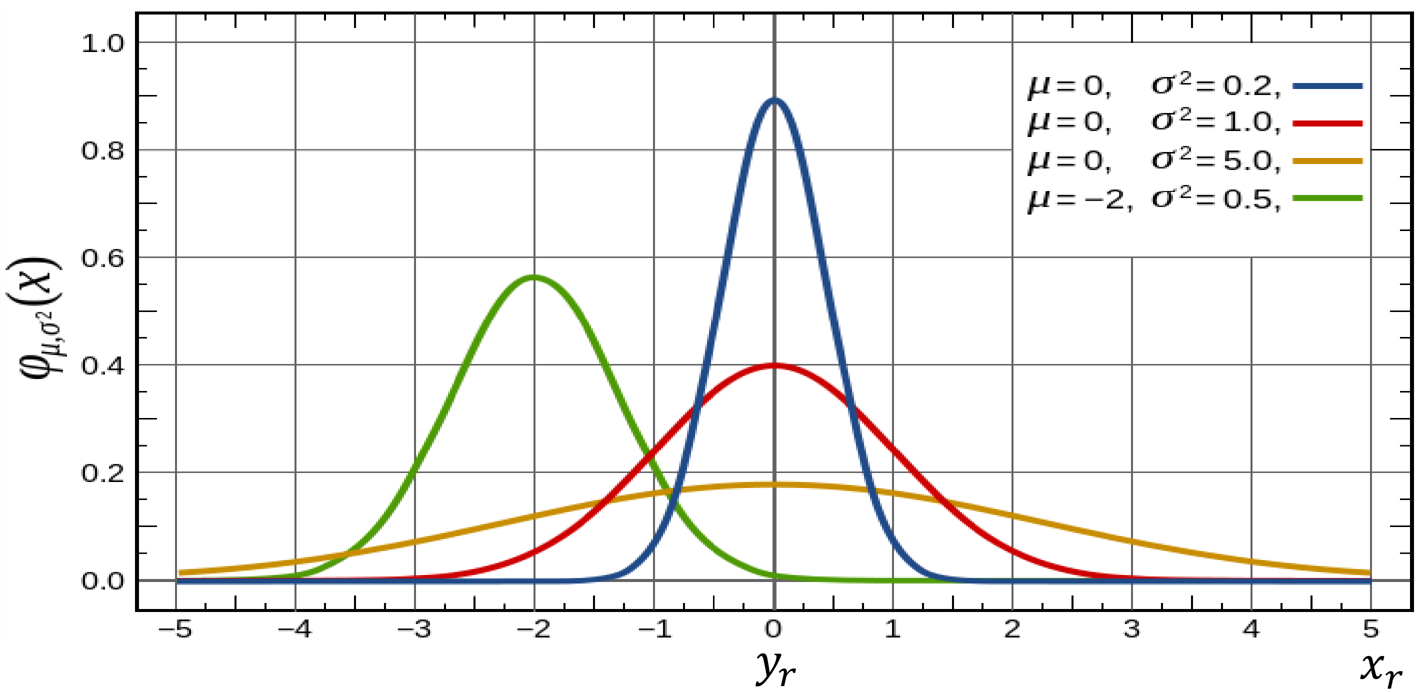
$$\frac{1}{9} \cdot \begin{array}{|c|c|c|}\hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline \end{array}$$

Filtered Image

Derivation of the Box Filter

- y_r is true gray value (color)
- x_r observed gray value (color)
- Noise model: Gaussian noise:

$$p(x_r|y_r) = N\,(x_r; y_r, \sigma) \sim \exp[-\frac{||x_r - y_r||^2}{2\sigma^2}]$$



Derivation of Box Filter

Further assumption: independent noise

$$p(x|y) \sim \prod_r \exp[-\frac{||x_r - y_r||^2}{2\sigma^2}]$$

Find the most likely solution for the true signal y

Maximum-Likelihood principle (probability maximization):

$$y^* = argmax_y \underbrace{p(y|x)}_{posterior} = argmax_y \frac{\overbrace{p(y)}^{prior}\overbrace{p(x|y)}^{likelihood}}{p(x)}$$

$p(x)$ is a constant (drop it out), assume (for now) uniform prior $p(y)$.

So we get:

$$p(y|x) = p(x|y) \sim \prod_r \exp[-\frac{||x_r - y_r||^2}{2\sigma^2}]$$

➡ the solution is trivial: $y_r = x_r$ for all r ☹

➡ additional assumptions about the **signal y** are necessary !!!

Derivation of Box Filter

Assumption: not uniform prior $p(y)$ but …
in a small vicinity $W(r) \subset D$ the "true" signal y_r is nearly constant

Only one y_r in a window $W(r)$

Maximum-a-posteriori:

$$p(y|x) \sim \prod_{r} \prod_{r' \in W(r)} \exp[-\frac{||x_{r'} - y_r||^2}{2\sigma^2}]$$

For one pixel r :

$$y_r^* = argmax_{y_r} \prod_{r' \in W(r)} \exp[-\frac{||x_{r'} - y_r||^2}{2\sigma^2}]$$

take neg. logarithm:

$$y_r^* = argmin_{y_r} \sum_{r' \in W(r)} \frac{||x_{r'} - y_r||^2}{2\sigma^2}$$

Derivation of Box Filter

How to do the minimization (factor $1/2\sigma^2$ is irrelevant):

$$y_r^* = argmin_{y_r} \sum_{r' \in W(r)} ||x_{r'} - y_r||^2$$

Take derivative and set to 0:

$$F(y_r) = \sum_{r' \in W(r)} ||x_{r'} - y_r||^2$$

$$\frac{\partial F}{\partial y_r} = 2\sum_{r'} (x_{r'} - y_r) = 2\sum_{r'} x_{r'} - |W| \cdot y_r \overset{!}{=} 0$$

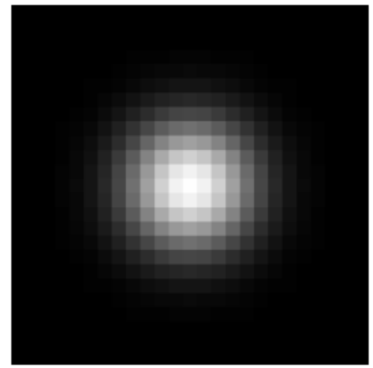
$$\Rightarrow \quad y_r^* = \frac{1}{|W|} \sum_{r'} x_{r'} \quad \text{(the average)}$$

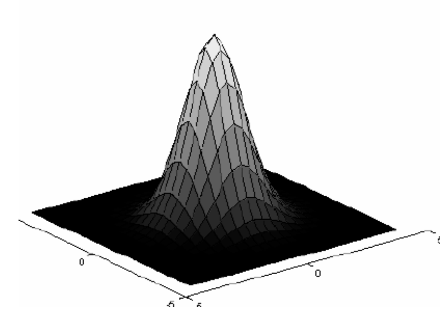
Box filter is optimal under pixel-independent, Gaussian Noise and constant signal in window

Gaussian (Smoothing) Filters

- Nearby pixels are weighted more than distant pixels
- Isotropic Gaussian (rotational symmetric)

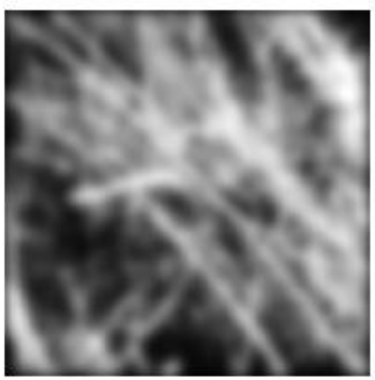
$$G_\sigma(x,y) = \frac{1}{2\pi\sigma^2} \exp\left(-\frac{x^2+y^2}{2\sigma^2}\right)$$



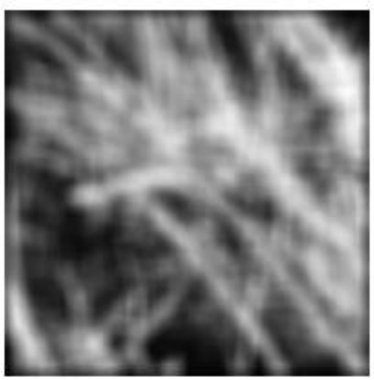


Original Image

Gaussian-filtered image



Box-filtered image

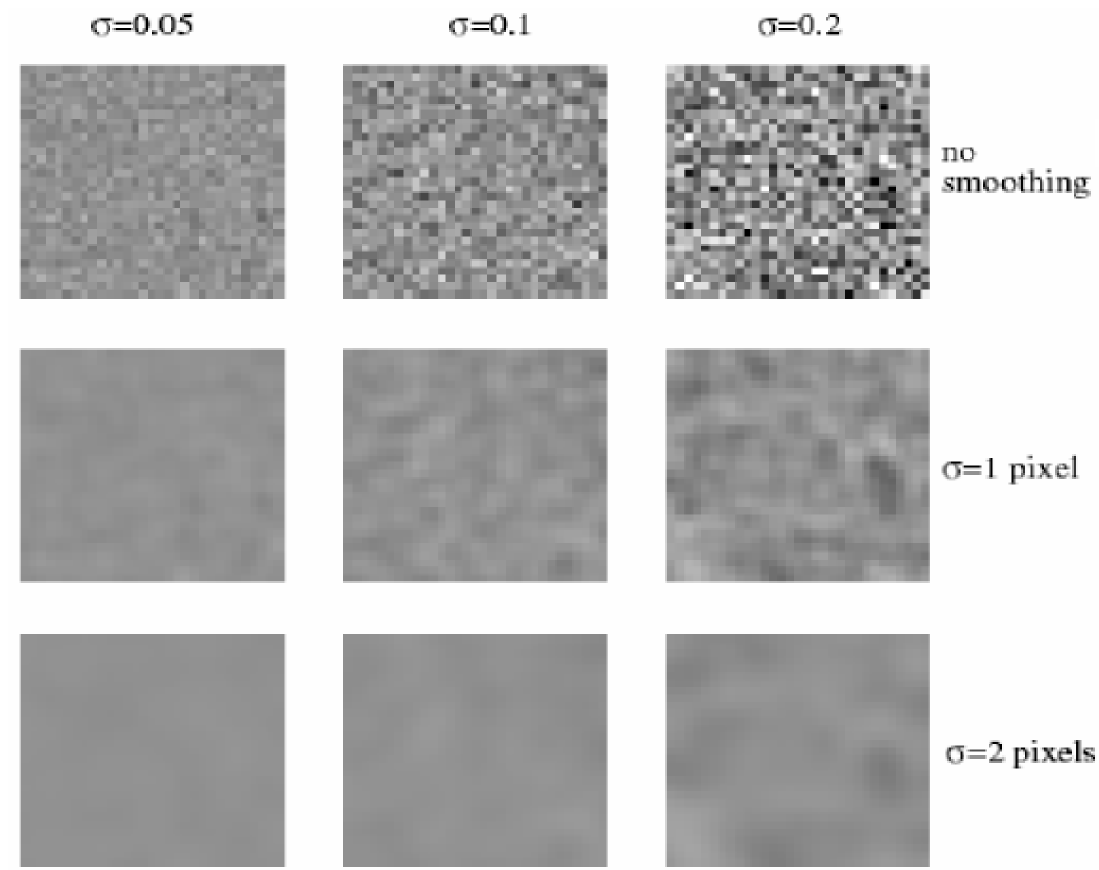


Gaussian Filter

Input: constant grey-value image

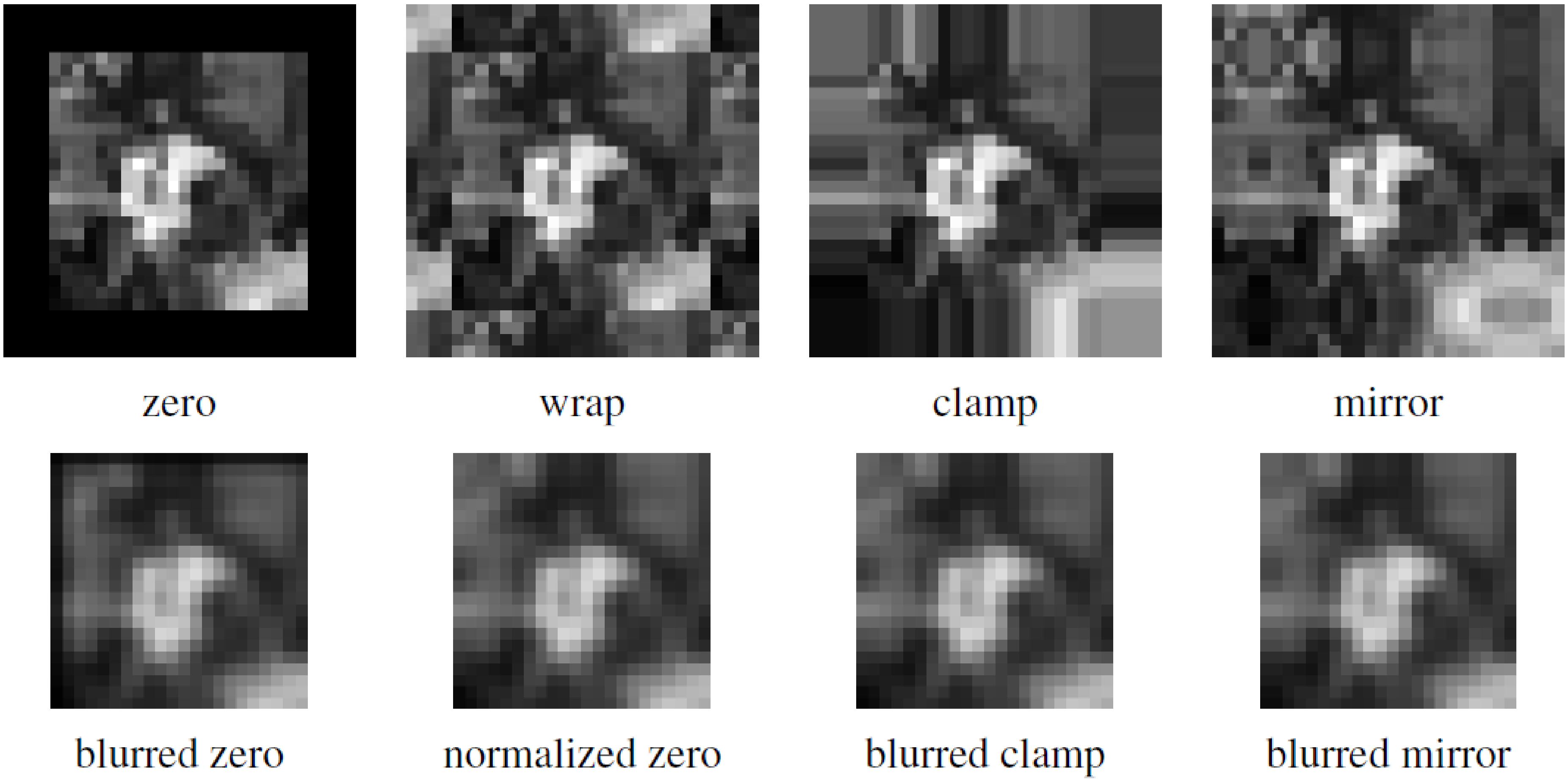
noise increases

smoothing increases



More noise needs larger sigma

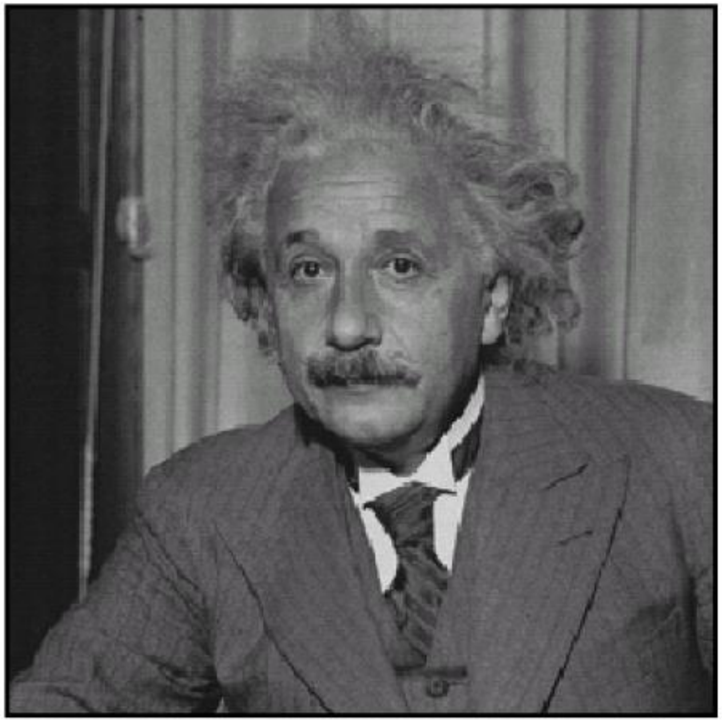
Handling the Boundary (Padding)



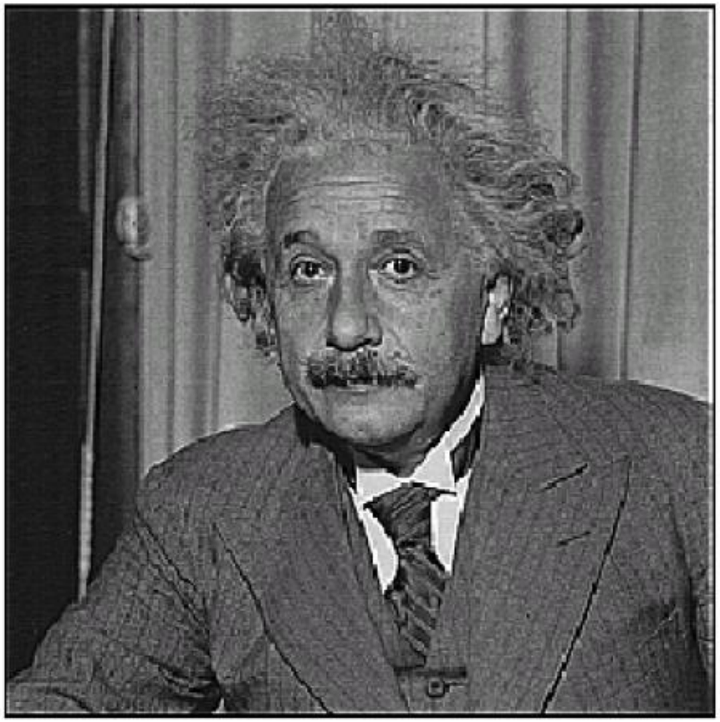
Gaussian for Sharpening

Sharpen an image by amplifying what “smoothing removes”:

$$g = f + \gamma\,(f - h_{blur} * f)$$



original



sharpened

How to compute convolution efficiently?

- Separable filters (next)
- Fourier transformation (wait 2 lectures)
- Integral Image trick (see exercise)

Important for later (integral Image trick):

- Naive implementation would be $O(Nw)$ where w is the number of elements in box filter
- The Box filter (mean filter) can be computed in $O(N)$.

$$\frac{1}{9} \cdot \begin{array}{|c|c|c|}\hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline \end{array}$$

45	60	98	127	132	133	137	133
46	65	98	123	126	128	131	133
47	65	96	115	119	123	135	137
47	63	91	107	113	122	138	134
50	59	80	97	110	123	133	134
49	53	68	83	97	113	128	133
50	50	58	70	84	102	116	126
50	50	52	58	69	86	101	120

image

*

0.1	0.1	0.1
0.1	0.2	0.1
0.1	0.1	0.1

filter (kernel)

=

69	95	116	125	129	132
68	92	110	120	126	132
66	86	104	114	124	132
62	78	94	108	120	129
57	69	83	98	112	124
53	60	71	85	100	114

filtered image

$$g(x,y) = \sum_{k,l} f(x-k, y-l)h(k,l)$$

Separable filters

For some filters we have: $f * h = f \; * (h_x * h_y)$

Where h_x, h_y are 1D filters.

Example Box filter:

$$\underbrace{\frac{1}{9} \cdot \begin{array}{|c|c|c|} \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline \end{array}}_{h_x * h_y} = \frac{1}{3} \cdot \underbrace{\begin{array}{|c|c|c|} \hline 1 & 1 & 1 \\ \hline \end{array}}_{h_x} * \frac{1}{3} \cdot \underbrace{\begin{array}{|c|} \hline 1 \\ \hline 1 \\ \hline 1 \\ \hline \end{array}}_{h_y}$$

Now we can do two 1D convolutions:

$$f * h = f \; * \left(h_x * h_y\right) = (f * h_x) \; * h_y$$

Naïve implementation for 3x3 filter: 9N operations versus 3N+3N operations

Can any filter be made separable?

Note:
$$\underbrace{\frac{1}{9}\cdot\begin{array}{|c|c|c|}\hline 1&1&1\\\hline 1&1&1\\\hline 1&1&1\\\hline\end{array}}_{h_x * h_y} = \underbrace{\frac{1}{3}\cdot\begin{array}{|c|c|c|}\hline 1&1&1\\\hline\end{array}}_{h_x} * \underbrace{\frac{1}{3}\cdot\begin{array}{|c|}\hline 1\\\hline 1\\\hline 1\\\hline\end{array}}_{h_y} = \underbrace{\frac{1}{3}\cdot\begin{array}{|c|}\hline 1\\\hline 1\\\hline 1\\\hline\end{array}}_{h_x} \cdot \underbrace{\frac{1}{3}\cdot\begin{array}{|c|c|c|}\hline 1&1&1\\\hline\end{array}}_{h_y}$$

Apply SVD to the kernel matrix:

$$\boldsymbol{A} = \left[\begin{array}{c|c|c} u_0 & \cdots & u_{p-1} \end{array}\right]\begin{bmatrix}\sigma_0 & & \\ & \ddots & \\ & & \sigma_{p-1}\end{bmatrix}\left[\begin{array}{c} v_0^T \\ \hline \cdots \\ \hline v_{p-1}^T \end{array}\right]$$

$$= \sum_{j=0}^{p-1} \sigma_j u_j v_j^T,$$

If all σ_i are 0 (apart from σ_0) then it is separable.

Example of separable filters

$$\frac{1}{K^2}\begin{array}{|c|c|c|c|}\hline 1 & 1 & \cdots & 1 \\ \hline 1 & 1 & \cdots & 1 \\ \hline \vdots & \vdots & 1 & \vdots \\ \hline 1 & 1 & \cdots & 1 \\ \hline \end{array}$$

$$\frac{1}{K}\begin{array}{|c|c|c|c|}\hline 1 & 1 & \cdots & 1 \\ \hline \end{array}$$

(a) box, $K = 5$

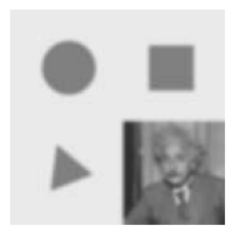
$$\frac{1}{16}\begin{array}{|c|c|c|}\hline 1 & 2 & 1 \\ \hline 2 & 4 & 2 \\ \hline 1 & 2 & 1 \\ \hline \end{array}$$

$$\frac{1}{4}\begin{array}{|c|c|c|}\hline 1 & 2 & 1 \\ \hline \end{array}$$

(b) bilinear

$$\frac{1}{256}\begin{array}{|c|c|c|c|c|}\hline 1 & 4 & 6 & 4 & 1 \\ \hline 4 & 16 & 24 & 16 & 4 \\ \hline 6 & 24 & 36 & 24 & 6 \\ \hline 4 & 16 & 24 & 16 & 4 \\ \hline 1 & 4 & 6 & 4 & 1 \\ \hline \end{array}$$

$$\frac{1}{16}\begin{array}{|c|c|c|c|c|}\hline 1 & 4 & 6 & 4 & 1 \\ \hline \end{array}$$



(c) “Gaussian”

$$\frac{1}{8}\begin{array}{|c|c|c|}\hline -1 & 0 & 1 \\ \hline -2 & 0 & 2 \\ \hline -1 & 0 & 1 \\ \hline \end{array}$$

$$\frac{1}{2}\begin{array}{|c|c|c|}\hline -1 & 0 & 1 \\ \hline \end{array}$$

$$\frac{1}{4}\begin{array}{|c|}\hline 1 \\ \hline 2 \\ \hline 1 \\ \hline \end{array}$$

(d) Sobel

$$\frac{1}{4}\begin{array}{|c|c|c|}\hline 1 & -2 & 1 \\ \hline -2 & 4 & -2 \\ \hline 1 & -2 & 1 \\ \hline \end{array}$$

$$\frac{1}{2}\begin{array}{|c|c|c|}\hline 1 & -2 & 1 \\ \hline \end{array}$$

(e) corner

Half-way break

3 minutes break

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Non-linear filters

- There are many different non-linear filters.
  We look at the following selection:
  - Median filter
  - Bilateral filter and Guided Filter
  - Morphological operations

Shot noise (Salt and Pepper Noise) - motivation

Original + shot noise

Gaussian filtered

Median filtered

Another example


Original


Noised


Mean

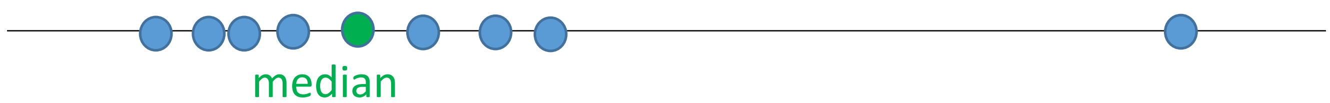

Median

Median Filter

Replace each pixel with the median in a neighbourhood:

5	6	5
4	20	5
4	6	5

5	6	5
4	5	5
4	6	5



Median filter: order the values and take the **middle** one

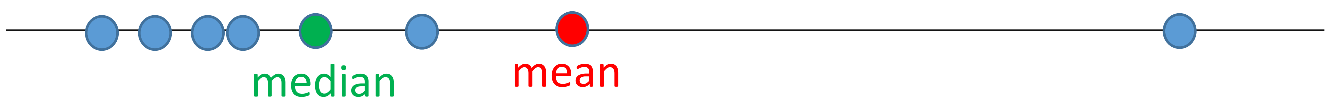
- No strong smoothing effect since values are not averaged
- Very good to remove outliers (shot noise)

Used a lot for post processing of outputs (e.g. optical flow)

Median Filter: Derivation

Reminder: for Gaussian noise we did solve the following ML problem

$$y_r^* = argmax_{y_r} \underbrace{\prod_{r' \in W(r)} \exp[-\frac{||x_{r'} - y_r||^2}{2\sigma^2}]}_{p(y|x)} = argmin_{y_r} \sum_{r' \in W(r)} ||x_{r'} - y_r||^2 = 1/|W| \sum_{r' \in W(r)} x_r$$



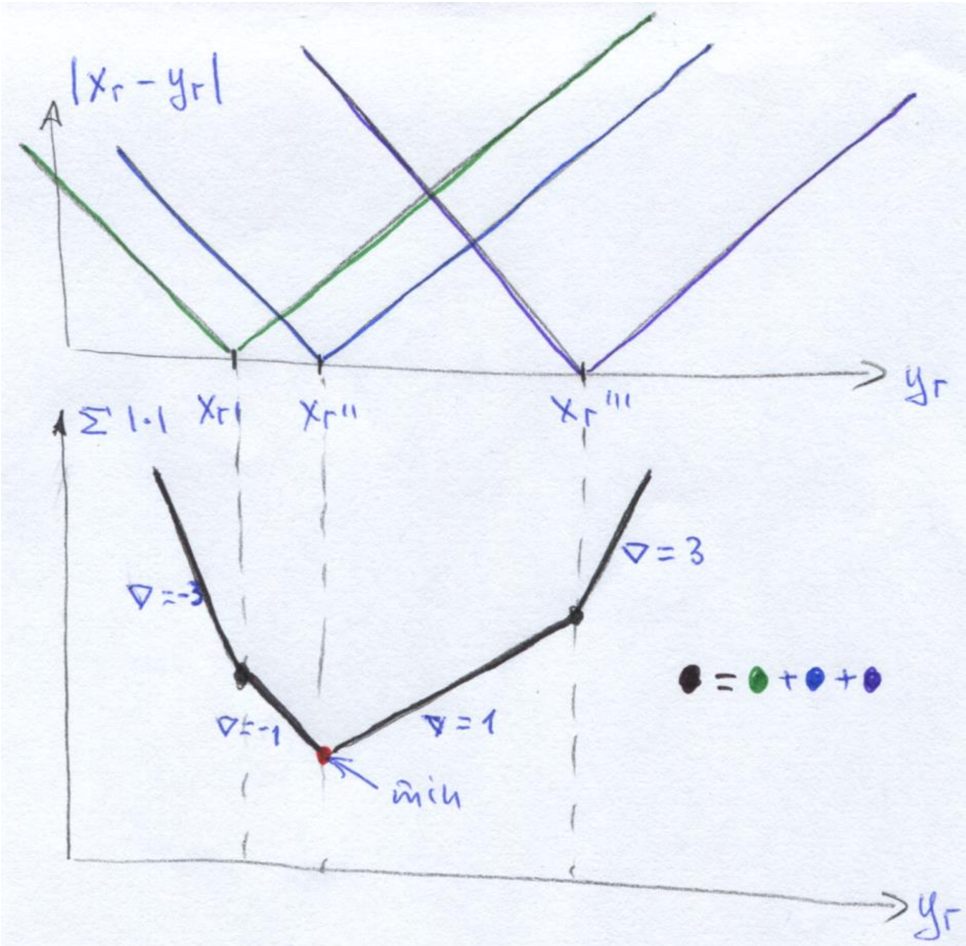
Does not look like a Gaussian distribution

For Median we solve the following problem:

$$y_r^* = argmax_{y_r} \prod_{r' \in W(r)} \exp[-\frac{|x_{r'} - y_r|}{2\sigma^2}] = argmin_{y_r} \sum_{r' \in W(r)} |x_{r'} - y_r| = Median\ (W(r))$$

Due to absolute norm it is more robust

Median Filter Derivation



Optimal solution is the
median of all values

minimize the following:

$$F(y_r) = \sum_{r' \in W(r)} |x_{r'} - y_r|$$

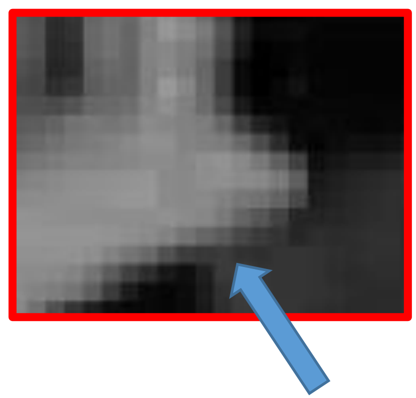
Problem: not differentiable ☹,
good news: it is convex ☺

Motivation – Bilateral Filter

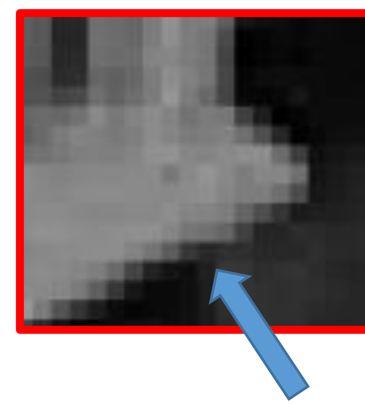
Original + Gaussian noise

Gaussian filtered

Bilateral filtered

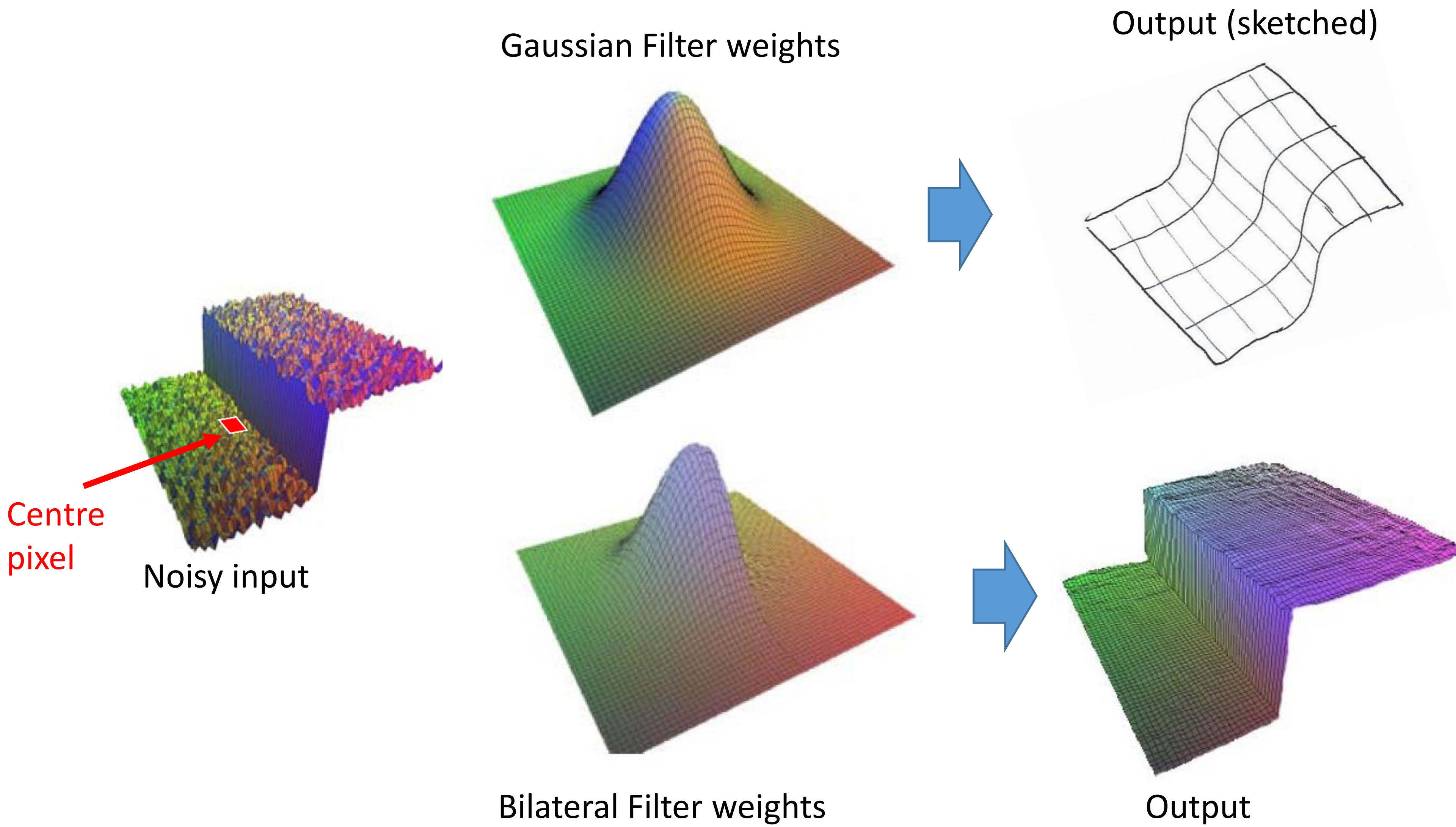


Edge over-smoothed



Edge not over-smoothed

Bilateral Filter – in pictures



Bilateral Filter – in equations

Filters looks at: a) distance to surrounding pixels (as Gaussian)
b) Intensity of surrounding pixels

$$g(i,j) = \frac{\sum_{k,l} f(k,l) w(i,j,k,l)}{\sum_{k,l} w(i,j,k,l)}$$

Linear combination

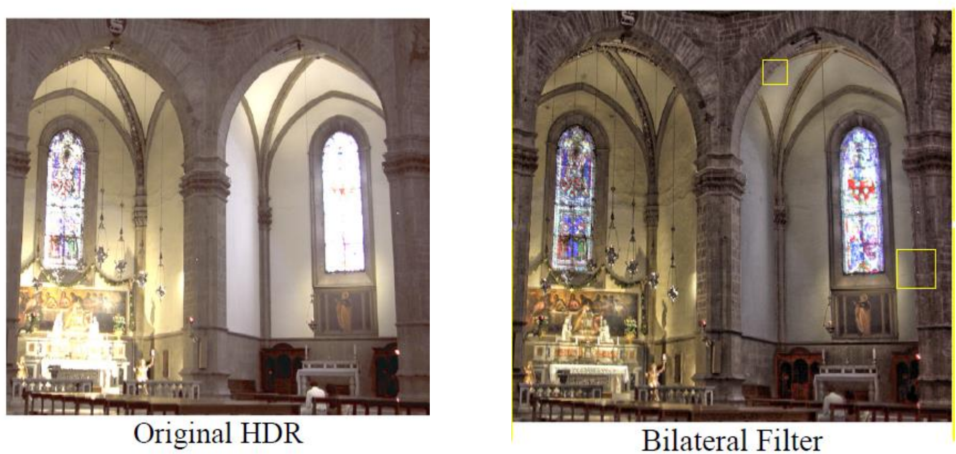
$$w(i,j,k,l) = \exp\left(-\underbrace{\frac{(i-k)^2 + (j-l)^2}{2\sigma_d^2}}_{\text{Same as Gaussian filter}} - \underbrace{\frac{\|f(i,j) - f(k,l)\|^2}{2\sigma_r^2}}_{\text{Consider intensity}} \right)$$

Problem: computation is slow $O(Nw)$; approximations can be done in $O(N)$
Comment: **Guided filter** (see later) is similar and can be computed exactly in $O(N)$

See a tutorial on: http://people.csail.mit.edu/sparis/bf_course/

Application: Bilateral Filter

Cartoonization



HDR compression
(Tone mapping)