Since the scenario requires trajectory planning in order for the robot to avoid the obstacle. In this chapter trajectories that are polynomial functions are investigated. To simplify the problem a robot with a single rotary joint is used.

Firstly, trajectories without obstacles are considered.

As the robot has to be at zero velocity at the start and beginning a cubic polynomial is need to fulfil all requirements of the trajectory. The joint angle and velocity can be described as a function of time with the coefficients .

Furthermore, there are four boundary conditions, where the joint angles and the end time are known values.

With that a system of equations can be put into a matrix to deduce the coefficients.

When using a cubic polynomial the start and end acceleration does not equal zero, which is a problem in reality, because a sudden acceleration or deceleration is impossible or lead to mechanical damage. To meet this a quintic polynomial function is required.

Two more boundary conditions can be formulated. The other four are still valid.

This leads to another system of equations which can be solved for the coefficients.

When there are obstacles in the workspace of a robot it is often required to define via points for the trajectory in order to avoid collisions. Assume that the via point is crossed at then there is one more boundary.

With that the cubic polynomial has to be expanded to a quartic polynomial.

The coefficients can be derived from the following equation system.

The same logic applies for the quintic polynomial, which becomes a hexic polynomial.

Solving for the coefficients.

Another way of connecting the via point to the trajectory is by using two separate polynomials for the first and second part of the movement. It is required that the speed and acceleration at the via point are constant.

The following conditions apply for this case. Assume that .

This leads to the following system of equations.

The last method to find the trajectory is by connecting two quintic polynomials.

There are four more conditions than in the previous method. Notice that even jerk and snap are now constant at the via point.

The following system of equations can then be used to find the coefficients.

MATLAB

A MATLAB script *trajectory\_planning* is developed to find and visualize the trajectories and their first and second derivatives. In order to deduce the coefficients for the polynomial functions, MATLAB functions are written where the previously shown equations are implemented.

The functions for the cubic and quintic trajectories *coefficients\_cubic* and *coefficients\_quintic* take and as inputs and return an array of the coefficients. with the function *polyval* the trajectory can be deduced and by using *polyder* the derivatives of the trajectory can be found.

This is performed with the sample parameters and . In the following figure x.x are the MATLAB plots for angle position, velocity and acceleration for the cubic and quintic trajectory.

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There is not much difference for the angle position. But for the angle velocity it is obvious that quintic starts and ends tangential to the x-axis and the maximum is higher than for cubic. Most notable are the differences in the angle acceleration as cubic starts with a positive value and ends with a negative value while quintic starts and ends with 0.

For the trajectories with via points the functions *coefficients\_quartic, coefficients\_hexic, coefficients\_cubic\_via* and *coefficients\_quintic\_via* take and as inputs and returns the coefficients. For the first two functions there is one set of coefficients while the second has one set for the first and one for the second part of the trajectory.

The via point is at and and the other parameters stay the same. Figure x.x shows the results of all methods.

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Hexic overshoots the angle position the most after crossing the via point and also has the highest angle velocity and acceleration. Quartic and two quintic have similar overshoot of the angle position and also similar maximum angle velocity, but for the angle acceleration two quintic is better as the start and end acceleration are 0. Two cubic has the lowest overshoot and the lowest angle velocity, but there are the expected problems with the angle acceleration.

In conclusion two quintic performers best in this setup, because of its advantages in the angle acceleration compared to two cubic and quartic and the lower overshoot and angle velocity compared hexic.