

③

$$\pi(s, a; \theta) = \frac{e^{\phi(s, a)^T \theta}}{\sum_{b \in \mathcal{A}} e^{\phi(s, b)^T \theta}}$$

$$\log \pi(s, a; \theta) = \phi(s, a)^T \theta - \log \left[\sum_{b \in \mathcal{A}} e^{\phi(s, b)^T \theta} \right]$$

$$\nabla_{\theta} \log \pi(s, a; \theta) = \phi(s, a) - \frac{1}{\sum_{b \in \mathcal{A}} e^{\phi(s, b)^T \theta}} \cdot \sum_{b \in \mathcal{A}} \phi(s, b) e^{\phi(s, b)^T \theta}$$

$$= \phi(s, a) - \frac{\sum_{b \in \mathcal{A}} \phi(s, b) e^{\phi(s, b)^T \theta}}{\sum_{b \in \mathcal{A}} e^{\phi(s, b)^T \theta}}$$

$$= \phi(s, a) - \sum_{b \in \mathcal{A}} \phi(s, b) \pi(s, b; \theta)$$

$$\nabla_{\theta} \log \pi(s, a; \theta) = \phi(s, a) - \sum_{b \in \mathcal{A}} \phi(s, b) \pi(s, b; \theta)$$

$$\text{let } Q(s, a; \omega) = \phi(s, a)^T \omega - \left(\sum_{b \in \mathcal{X}} \phi(s, b) \pi(s, b; \theta) \right)^T \omega$$

$$\mathbb{E}_{\tilde{\pi}}[Q(s, a; \omega)] = \sum_{a \in \mathcal{A}} \tilde{\pi}(s, a; \theta) \cdot Q(s, a; \omega)$$

$$= \sum_{a \in \mathcal{A}} \tilde{\pi}(s, a; \theta) \left[\phi(s, a)^T \omega - \left(\sum_{b \in \mathcal{X}} \phi(s, b) \tilde{\pi}(s, b; \theta) \right)^T \omega \right]$$

$$= \sum_{a \in \mathcal{A}} \left[\frac{e^{\phi(s, a)^T \theta} \phi(s, a)^T \omega}{\sum_{b \in \mathcal{X}} e^{\phi(s, b)^T \theta}} - \frac{e^{\phi(s, a)^T \theta}}{\sum_{b \in \mathcal{X}} e^{\phi(s, b)^T \theta}} \cdot \sum_{b \in \mathcal{X}} \left[\frac{\phi(s, b) e^{\phi(s, b)^T \theta}}{\sum_{c \in \mathcal{X}} e^{\phi(s, c)^T \theta}} \right]^T \omega \right]$$

$$= \sum_{a \in \mathcal{A}} \left[\frac{e^{\phi(s, a)^T \theta} \phi(s, a)^T \omega}{\sum_{b \in \mathcal{X}} e^{\phi(s, b)^T \theta}} - e^{\phi(s, a)^T \theta} \cdot \frac{\sum_{b \in \mathcal{X}} \phi(s, b)^T \omega}{\sum_{c \in \mathcal{X}} e^{\phi(s, c)^T \theta}} \right]$$

$$= \sum_{a \in \mathcal{A}} \left[\frac{e^{\phi(s, a)^T \theta} \cdot \phi(s, a)^T \omega - e^{\phi(s, a)^T \theta} \cdot \sum_{b \in \mathcal{X}} \phi(s, b)^T \omega}{\sum_{b \in \mathcal{X}} e^{\phi(s, b)^T \theta}} \right]$$

$$= 0$$