Prove: $\underbrace{G_t - V(S_t)}$ $= \sum_{u=t}^{T-1} \gamma^{u-t}\left(R_{u+1} + \gamma \cdot V(S_{u+1}) - V(S_u)\right)$

$$= \sum_{u=t}^{T-1} \gamma^{u-t} R_{u+1} + \gamma^{u-t+1} V(S_{u+1}) - \gamma^{u-t} V(S_u)$$

$$G_t = \sum_{i=t+1}^{\infty} \gamma^{i-t-1} \cdot R_i = R_{t+1} + \gamma \cdot R_{t+2} + \gamma^2 R_{t+3} + \dots$$

$$V(S_t) = \mathbb{E}[G_t \mid S_t = s] \quad \forall s$$

$$\Rightarrow \sum_{i=t+1}^{\infty} \gamma^{i-t-1} R_i \quad - V(S_t)$$

$$\Rightarrow \sum_{i=t}^{\infty} \gamma^{i-t} R_{i+1} \quad - V(S_t)$$

$R_i = 0$ for all $i > T$

$$\Rightarrow \sum_{i=t}^{T-1} \gamma^{i-t} R_{i+1} \quad - V(S_t)$$

Subtract term from RHS

$$\Rightarrow -V(S_t) = \sum_{u=t}^{T-1} \gamma^{u-t+1} V(S_{u+1}) - \gamma^{u-t} V(S_u)$$

$$\Rightarrow -\mathbb{E}[G_t \mid S_t = S_t] = \sum_{u=t}^{T-1} \gamma^{u-t+1} \mathbb{E}[G_t \mid S_t = S_{u+1}] - \gamma^{u-t} \mathbb{E}[G_t \mid S_t = S_u]$$

$$\Rightarrow \quad -\mathbb{E}\left[\sum_{i=t+1}^{T} \gamma^{i-t+1} R_i \,\middle|\, S_t = S_t\right] = \sum_{u=t}^{T-1} \gamma^{u-t+1}\, \mathbb{E}\left[\sum_{i=t+1}^{T} \gamma^{i-t-1} R_i \,\middle|\, S_t = S_{u+1}\right] - \sum_{u=t}^{T-1} \gamma^{u-t}\, \mathbb{E}\left[\sum_{i=t+1}^{T} \gamma^{i-t-1} R_i \,\middle|\, S_t = S_u\right]$$

$$= \sum_{u=t}^{T-1} \gamma^{u-t+1}\, \mathbb{E}\left[\sum_{i=t+1}^{T} \gamma^{i-t-1} R_i \,\middle|\, S_t = S_{u+1}\right] - \sum_{u=t+1}^{T-1} \gamma^{u-t}\, \mathbb{E}\left[\sum_{i=t+1}^{T} \gamma^{i-t-1} R_i \,\middle|\, S_t = S_u\right] - \mathbb{E}\left[\sum_{i=t+1}^{T} \gamma^{i-t-1} R_i \,\middle|\, S_t = S_t\right]$$

Take out first element

$$= \cancel{\sum_{u=t}^{T-1} \gamma^{u-t+1}\, \mathbb{E}\left[\sum_{i=t+1}^{T} \gamma^{i-t-1} R_i \,\middle|\, S_t = S_{u+1}\right]} - \cancel{\sum_{u=t}^{T-1} \gamma^{u-t+1}\, \mathbb{E}\left[\sum_{i=t+1}^{T} \gamma^{i-t-1} R_i \,\middle|\, S_t = S_{u+1}\right]} - \mathbb{E}\left[\sum_{i=t+1}^{T} \gamma^{i-t-1} R_i \,\middle|\, S_t = S_t\right]$$

$$-\mathbb{E}\left[\sum_{i=t+1}^{T} \gamma^{i-t+1} R_i \,\middle|\, S_t = S_t\right] = -\mathbb{E}\left[\sum_{i=t+1}^{T} \gamma^{i-t-1} R_i \,\middle|\, S_t = S_t\right]$$