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Encryption & Decryption – 2

Sujeet Shenoi

Tandy School of Computer Science
University of Tulsa, Tulsa, OK 74104
sujeet@utulsa.edu



Secret & Public Key Encryption Algorithms

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Secret Key Algorithms (Symmetric)

- One key for encryption and decryption ($K_E = K_D = K$)
- $C = \{ P \}_K$ and $P = \{ C \}_K$
- One key per channel ($\#keys = n \cdot (n-1)/2$)

Public Key Algorithms (Asymmetric)

- Separate keys for encryption and decryption ($K_E \neq K_D$)
- $C = \{ P \}_{K_E}$ and $P = \{ C \}_{K_D}$
- $C = \{ P \}_{K_D}$ and $P = \{ C \}_{K_E}$
- Two keys per user ($\#keys = 2 \cdot n$)



Secret Key Algorithms

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- Data Encryption Standard (DES)
- Escrowed Encryption Standard (EES): Skipjack
- Advanced Encryption Standard (AES)

Secret Key Algorithms (Symmetric)

- Single Key for A-B Channel: (K_{AB})
- K_{AB} : Secret (known only to A and B)
- $A \rightarrow B$: $C = \{P\}_{K_{AB}}$ (and $P = \{C\}_{K_{AB}}$)
- $B \rightarrow A$: $C = \{P\}_{K_{AB}}$ (and $P = \{C\}_{K_{AB}}$)



Symmetric Key Systems

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Problems

- Revealed keys
- Key distribution
- Large number of keys ($n \cdot (n-1)/2$ keys)

Data Encryption Standard (DES)

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- NIST (1977)
- Developed for use by the general public
- Accepted as a cryptographic standard worldwide
- Hardware and software implementations
- Algorithm
 - Complex combination of substitution and transposition (Product Cipher)
 - 64-bit plaintext blocks; 56-bit keys
 - 16-round algorithm
 - Same algorithm for encryption and decryption



DES Algorithm (contd.)

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Algorithm Description

- Initial Permutation
- 16 Cycles (with Key Transformation)
- Inverse Initial Permutation
- Cycle Description
 - Split into Left and Right Halves: 32 bits each
 - Expansion Permutation: 32 bits → 48 bits (Right Half only)
 - XOR with Transformed Key: 48 bits (Right Half only)
 - S-Box (Substitution Choice): 48 bits → 32 bits (Right Half only)
 - P-Box (Permutation): 32 bits (Right Half only)
 - XOR with Original Left Half: 32 bits (Right Half only)
 - Concatenation of Original Right Half and Right Half



DES Schematic

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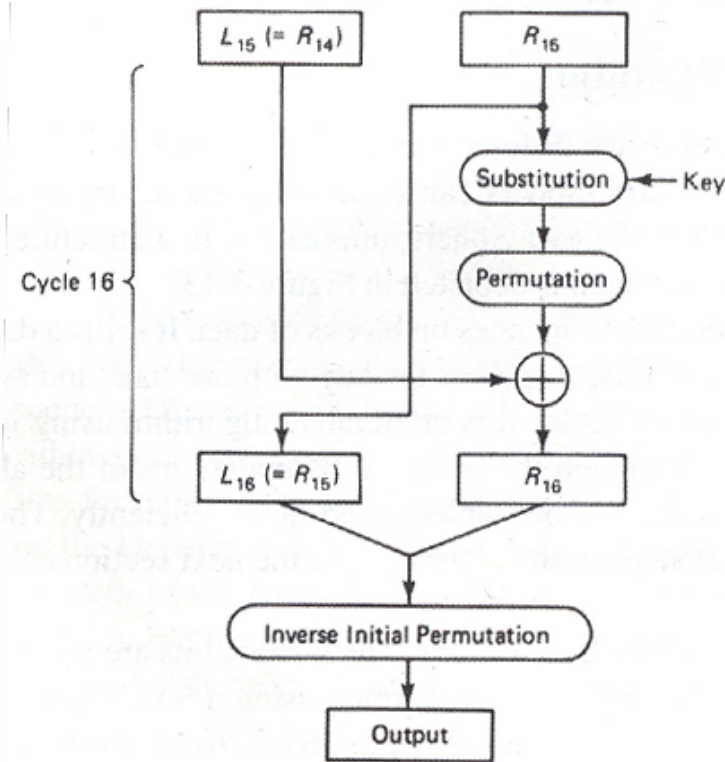
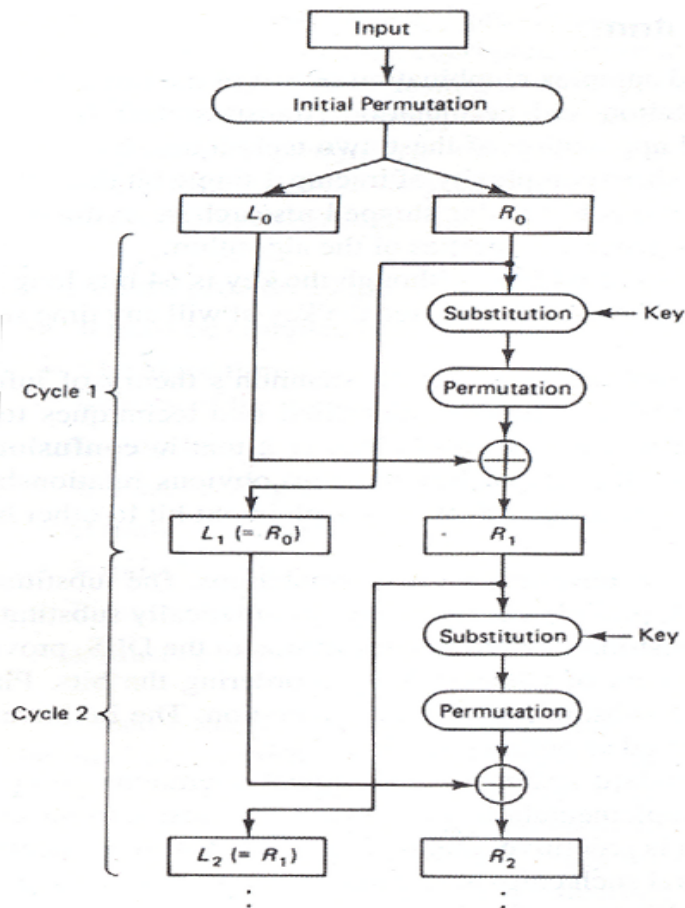


Figure 3-12 Cycles of Substitution and Permutation

Initial Permutation

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Table 3-8 Initial Permutation

Bit	Goes to Position							
1–8	40	8	48	16	56	24	64	32
9–16	39	7	47	15	55	23	63	31
17–24	38	6	46	14	54	22	62	30
25–32	37	5	45	13	53	21	61	29
33–40	36	4	44	12	52	20	60	28
41–48	35	3	43	11	51	19	59	27
49–56	34	2	42	10	50	18	58	26
57–64	33	1	41	9	49	17	57	25

Final Permutation

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Table 3-9 Final Permutation (Inverse Initial Permutation)

Bit	Goes to Position							
1–8	58	50	42	34	26	18	10	2
9–16	60	52	44	36	28	20	12	4
17–24	62	54	46	38	30	22	14	6
25–32	64	56	48	40	32	24	16	8
33–40	57	49	41	33	25	17	9	1
41–48	59	51	43	35	27	19	11	3
49–56	61	53	45	37	29	21	13	5
57–64	63	55	47	39	31	23	15	7

DES Cycle

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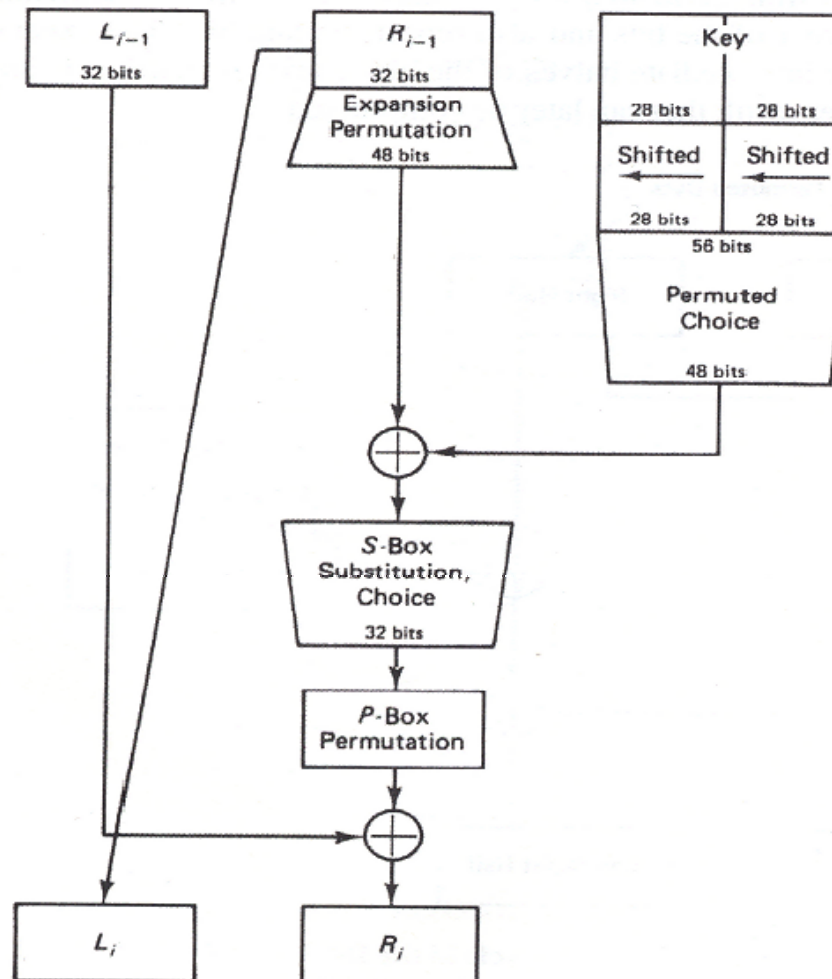


Figure 3-16

Expansion Permutation

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Table 3-3 Expansion Permutation

Bit	1	2	3	4	5	6	7	8
Moves to Position	2,48	3	4	5,7	6,8	9	10	11,13
Bit	9	10	11	12	13	14	15	16
Moves to Position	12,14	15	16	17,19	18,20	21	22	23,25
Bit	17	18	19	20	21	22	23	24
Moves to Position	24,26	27	28	29,31	30,32	33	34	35,37
Bit	25	26	27	28	29	30	31	32
Moves to Position	36,38	39	40	41,43	42,44	45	46	47,1

S-Boxes

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Expanded $R_{i-1} \oplus$ Key

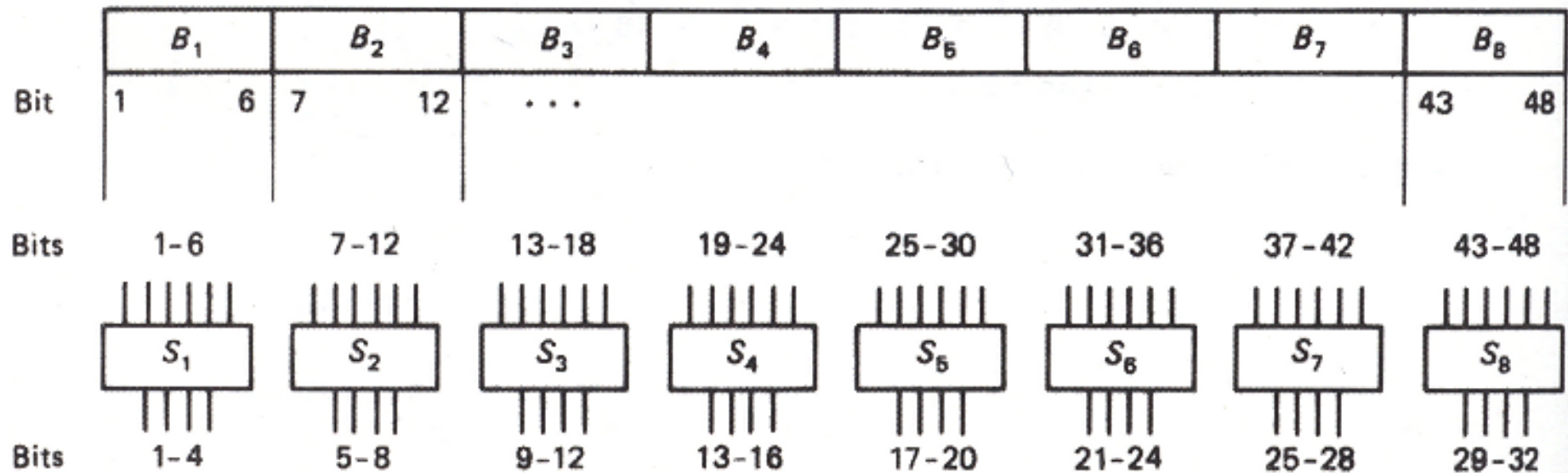


Figure 3-18 S-Boxes Operating on Eight 6-bit Blocks

S-Boxes

Sec. 3.9 The Data Encryption Standard (DES)

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Table 3-6 S-Boxes of DES

Box	Row	Column															
		0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
S ₁	0	14	4	13	1	2	15	11	8	3	10	6	12	5	9	0	7
	1	0	15	7	4	14	2	13	1	10	6	12	11	9	5	3	8
	2	4	1	14	8	13	6	2	11	15	12	9	7	3	10	5	0
	3	15	12	8	2	4	9	1	7	5	11	3	14	10	0	6	13
S ₂	0	15	1	8	14	6	11	3	4	9	7	2	13	12	0	5	10
	1	3	13	4	7	15	2	8	14	12	0	1	10	6	9	11	5
	2	0	14	7	11	10	4	13	1	5	8	12	6	9	3	2	15
	3	13	8	10	1	3	15	4	2	11	6	7	12	0	5	14	9
S ₃	0	10	0	9	14	6	3	15	5	1	13	12	7	11	4	2	8
	1	13	7	0	9	3	4	6	10	2	8	5	14	12	11	15	1
	2	13	6	4	9	8	15	3	0	11	1	2	12	5	10	14	7
	3	1	10	13	0	6	9	8	7	4	15	14	3	11	5	2	12
S ₄	0	7	13	14	3	0	6	9	10	1	2	8	5	11	12	4	15
	1	13	8	11	5	6	15	0	3	4	7	2	12	1	10	14	9
	2	10	6	9	0	12	11	7	13	15	1	3	14	5	2	8	4
	3	3	15	0	6	10	1	13	8	9	4	5	11	12	7	2	14
S ₅	0	2	12	4	1	7	10	11	6	8	5	3	15	13	0	14	9
	1	14	11	2	12	4	7	13	1	5	0	15	10	3	9	8	6
	2	4	2	1	11	10	13	7	8	15	9	12	5	6	3	0	14
	3	11	8	12	7	1	14	2	13	6	15	0	9	10	4	5	3
S ₆	0	12	1	10	15	9	2	6	8	0	13	3	4	14	7	5	11
	1	10	15	4	2	7	12	9	5	6	1	13	14	0	11	3	8
	2	9	14	15	5	2	8	12	3	7	0	4	10	1	13	11	6
	3	4	3	2	12	9	5	15	10	11	14	1	7	6	0	8	13
S ₇	0	4	11	2	14	15	0	8	13	3	12	9	7	5	10	6	1
	1	13	0	11	7	4	9	1	10	14	3	5	12	2	15	8	6
	2	1	4	11	13	12	3	7	14	10	15	6	8	0	5	9	2
	3	6	11	13	8	1	4	10	7	9	5	0	15	14	2	3	12
S ₈	0	13	2	8	4	6	15	11	1	10	9	3	14	5	0	12	7
	1	1	15	13	8	10	3	7	4	12	5	6	11	0	14	9	2
	2	7	11	4	1	9	12	14	2	0	6	10	13	15	3	5	8
	3	2	1	14	7	4	10	8	13	15	12	9	0	3	5	6	11



P-Box

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Table 3-7 Permutation Box *P*

<i>Bit</i>	<i>Goes to Position</i>							
1–8	9	17	23	31	13	28	2	18
9–16	24	16	30	6	26	20	10	1
17–24	8	14	25	3	4	29	11	19
25–32	32	12	22	7	5	27	15	21

DES Cycle

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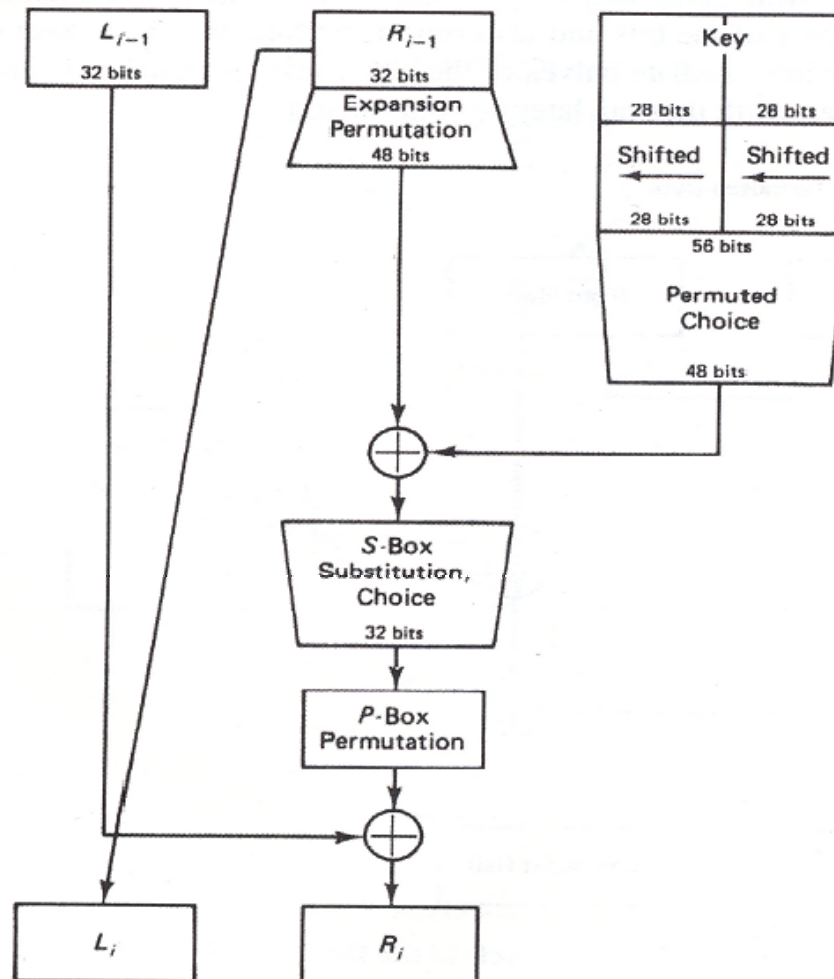


Figure 3-16

Key Shift

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Table 3-4 Bits Shifted by Cycle Number

Cycle Number	Bits Shifted
1	1
2	1
3	2
4	2
5	2
6	2
7	2
8	2
9	2
10	2
11	2
12	2
13	2
14	2
15	2
16	1

Choice Permutation

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Table 3-5 Choice Permutation to Select 48 Key Bits

Key Bit	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Selected for Position	5	24	7	16	6	10	20	18	—	12	3	15	23	1
Key Bit	15	16	17	18	19	20	21	22	23	24	25	26	27	28
Selected for Position	9	19	2	—	14	22	11	—	13	4	—	17	21	8
Key Bit	29	30	31	32	33	34	35	36	37	38	39	40	41	42
Selected for Position	47	31	27	48	35	41	—	46	28	—	39	32	25	44
Key Bit	43	44	45	46	47	48	49	50	51	52	53	54	55	56
Selected for Position	—	37	34	43	29	36	38	45	33	26	42	—	30	40

DES Cycle

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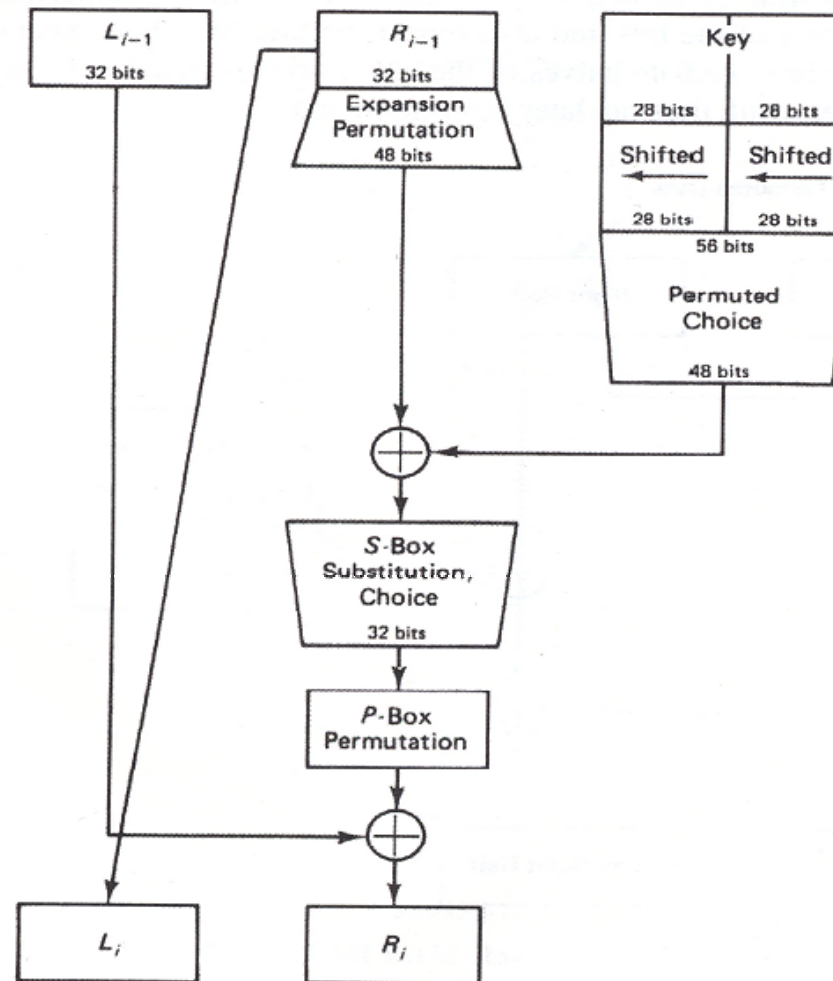


Figure 3-16

DES Algorithm (contd.)

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Encryption

- $L_j = R_{j-1}$ (1)
- $R_j = L_{j-1} \oplus f(R_{j-1}, k_j)$ (2)

Decryption

- $R_{j-1} = L_j$ (3)
- $L_{j-1} = R_j \oplus f(R_{j-1}, k_j)$ (4)
- $L_{j-1} = R_j \oplus f(L_j, k_j)$ (5) (sub 3 in 4)
- Same hardware can be used for decryption
- Keys are submitted in reverse order $k_{16}, k_{15} \dots k_1$



Security Issues

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- **Design of Algorithm**

- Secrecy: Rationale for S-boxes, P-boxes and key transformations was not released. Congressional inquiry exonerated NSA, but details are still secret
- Possible Design Flaws: NSA released information about S-boxes. No S-box is a linear function of its input. Diffusion: Changing one S-box input changes at least two output bits. S-boxes were chosen to minimize differences between the number of 1s and 0s when any single input bit is held constant

- **Number of Iterations**



- Are 16 cycles sufficient? Experiments indicate 8 cycles are sufficient to eliminate any observed dependence

Security Issues (contd.)

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- **Key Length (Brute Force Attacks)**
 - Lucifer (IBM) has 128 bit keys; DES keys have 56 bits
 - 10 keys/s \Rightarrow 228 million years; 1 keys/ μ s \Rightarrow 2,280 years
 - Parallel Attack (Diffie and Hellman, 1977): 10^6 chips, each testing 1 key/ μ s, would require 20 hours for a brute force attack; \$50 million machine would cost \$20,000 per solution
 - 1997: Parallel Attack: 3,500 machines took 120 days (linear approach; 35,000 machines would require 12 days)
 - 1998: \$130,000 machine cracked a DES key in 112 hours
 - January 1999: EFF Team broke DES in 22 hours and 15 minutes using the Deep Crack supercomputer and 100,000 PCs; Speed was 256 billion keys/s
 - NSA will not re-certify DES



DES Weaknesses

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- **Complements**
 - If $C = \text{DES}(P, k)$, then $\neg C = \text{DES}(\neg P, \neg k)$
 - Not a serious problem
- **Weak Keys (4)**
 - Keys for which $C = \text{DES}(P, k)$ and $P = \text{DES}(C, k)$
 - Same sub-key is generated for each round
 - Occurs when each key half consists only of 0s or 1s
- **Semi-Weak Keys**
 - Keys for which $C = \text{DES}(P, k_1) = \text{DES}(P, k_2) \dots$
 - Multiple keys can decrypt message



Weak DES Keys

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Table 3-10 Weak DES Keys

Left Half	Right Half	Weak Key Value			
zeros	zeros	0101	0101	0101	0101
ones	ones	FEFE	FEFE	FEFE	FEFE
zeros	ones	1F1F	1F1F	0E0E	0E0E
ones	zeros	E0E0	E0E0	F1F1	F1F1

Semi-Weak DES Keys

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Table 3-11 Semi-Weak DES Key Pairs

01FE	01FE	01FE	01FE	FE01	FE01	FE01	FE01
1FE0	1FE0	0EF1	0EF1	E01F	E01F	F10E	F10E
01E0	01E0	01F1	01F1	E001	E001	F101	F101
1FFE	1FFE	0EFE	0EFE	FE1F	FE1F	FE0E	FE0E
011F	011F	010E	010E	1F01	1F01	0E01	0E01
E0FE	E0FE	F1FE	F1FE	FEE0	FEE0	FEF1	FEF1

DES Weaknesses (contd.)

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- **Design Weaknesses**

- Expansion permutation repeats first and fourth bits of every 4-bit series, crossing bits from neighboring 4-bit series
- S-box S_4 derives the last three output bits the same way as the first by complementing some of the input bits
- Two different, but carefully chosen, inputs to S-boxes can produce the same output

- **Key Clustering**

- Two or more keys produce the same encryption
- In addition to semiweak keys, other key clusters exist



DES Weaknesses (contd.)

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- **Differential Cryptanalysis**
 - Powerful code breaking technique
 - Uses carefully selected pairs of plaintext with subtle differences and studies the effects of these differences on the resulting ciphertext pairs
 - 6 key rounds: 2^8 tests
 - 10 key rounds: 2^{35} tests
 - 15 key rounds: 2^{52} tests
 - 16 key rounds: 2^{58} tests (brute force requires 2^{56} tests)
- **NSA will not re-certify DES**



Double Encryption

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DES is “weak”

Can we use two 56-bit DES keys back to back?

- $\{ \{ \text{Message} \}_{K_1} \}_{K_2}$
- 56-bit key $\Rightarrow 2^{56}$ possibilities
- Two 56-bit keys $\Rightarrow 2^{112}$ possibilities?
- No!
- 2^{57} possibilities (Merkle, 1981)



Triple DES (TDES)

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Same hardware/software

Encryption (EDE)

- TDES: $C = \text{DES}_{\text{Encrypt}}(\text{DES}_{\text{Decrypt}}(\text{DES}_{\text{Encrypt}}(P, k_1), k_2), k_3)$

Decryption (DED)

- TDES: $P = \text{DES}_{\text{Decrypt}}(\text{DES}_{\text{Encrypt}}(\text{DES}_{\text{Decrypt}}(C, k_3), k_2), k_1)$

nTDES

- 3TDES ($k_1 \neq k_2 \neq k_3$): Key Size (168 bits) Effective Size (112 bits)
- 2TDES ($k_1 = k_3 \neq k_2$): Key Size (112 bits) Effective Size (80 bits)
(Because of “Meet-in-the-Middle Attacks”)
- 1TDES ($k_1 = k_2 = k_3$): Key Size (56 bits) (1TDES = DES)



Escrowed Encryption Standard (EES)

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- Developed by NSA (1980s) to allow “legal” wiretapping
- AT&T encrypted telephone devices (1993)
 - Analog → Digital → Encrypt → ... → Decrypt → Digital → Analog
 - Unique key was generated for each session and transmitted
- Unit keys would be split into halves and kept by different escrow agencies
- Law enforcement agents would need court orders to obtain key halves (using information in LEAF)
- Sealed encryption devices



Clipper Chip

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- Skipjack (algorithm)
- Clipper (chip implementing Skipjack and LEAF)
- MOSAIC (program)
- Capstone (cryptographic device with key exchange)
- Tessera (Capstone chip)
- Fortezza (Capstone chip)
- Escrowed Encryption Standard (EES)



Clipper (contd.)

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Clipper Message Format

- $S \rightarrow R: \{M\}_k \cdot \{\{k\}_u \cdot \{n, a\}\}_f$
 - LEAF: $\{\{k\}_u \cdot \{n, a\}\}_f$
 - M: 64-bit block
 - k: 80-bit session key (randomly generated and transmitted)
 - u: 80-bit unit key (unique to Clipper unit; held in escrow)
 - n: 30-bit unit ID (unique to Clipper unit)
 - a: Escrow authenticator
 - f: 80-bit law enforcement key (common to Clipper family)



Skipjack Algorithm

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Algorithm Description

- 64-bit block (4 16-bit words: w_1, w_2, w_3, w_4)
- 32 Cycles (80-bit Key (10-bytes): cv_0, \dots, cv_9)
- Cycle Description
 - Rule A (8 Steps) {Decryption: Rule B^{-1} (8 Steps)}
 - Rule B (8 Steps) {Decryption: Rule A^{-1} (8 Steps)}
 - Rule A (8 Steps) {Decryption: Rule B^{-1} (8 Steps)}
 - Rule B (8 Steps) {Decryption: Rule A^{-1} (8 Steps)}
 - G^k Permutation {Decryption: $[G^k]^{-1}$ }(4-round Feistel structure)
- F Table (Fixed-byte substitution table)



Skipjack Algorithm (contd.)

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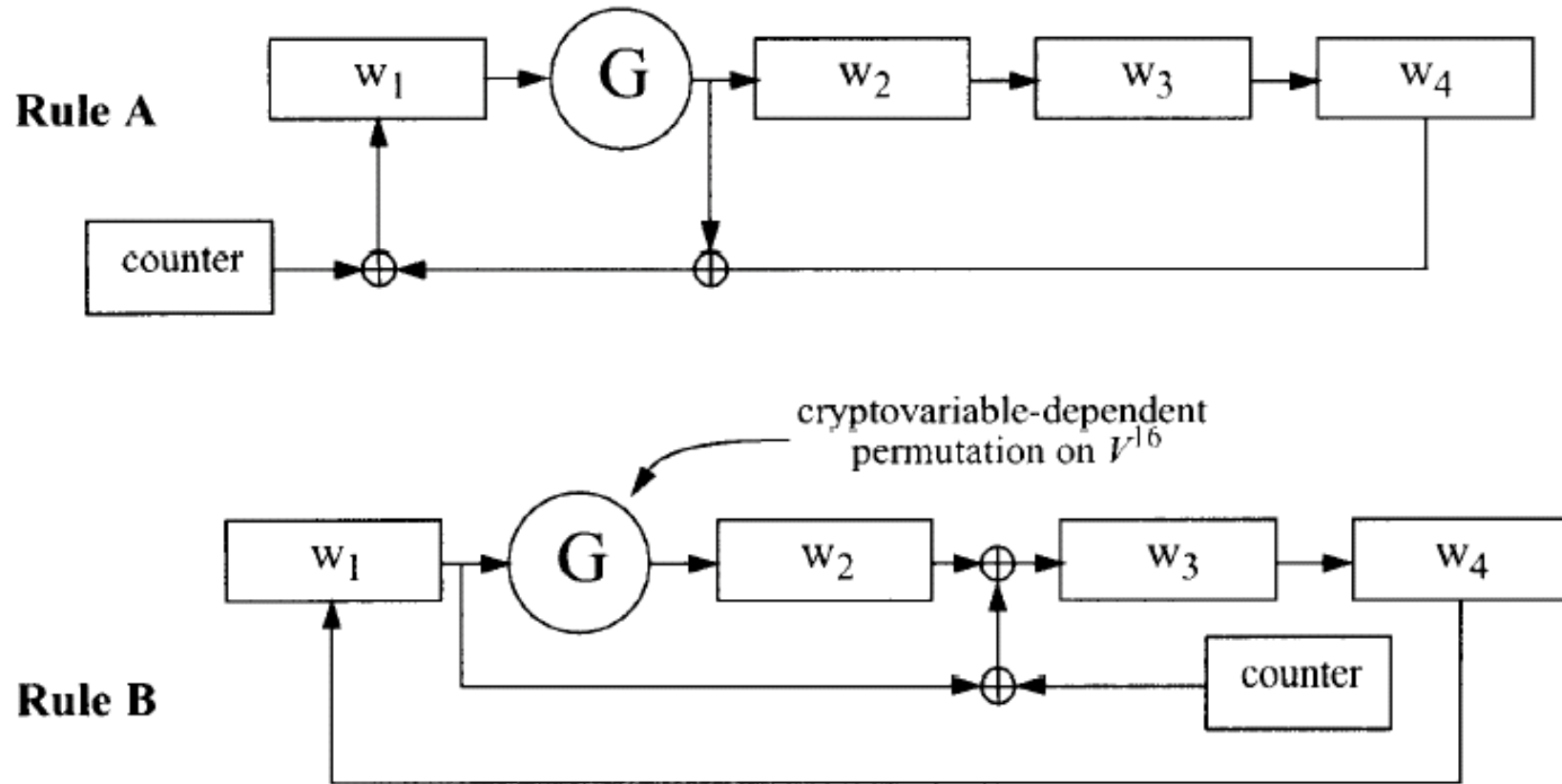


Figure 5. “SKIPJACK Stepping Rules”

Encryption Rules

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ENCRYPT

Rule A

$$w_1^{k+1} = G^k(w_1^k) \oplus w_4^k \oplus counter^k$$

$$w_2^{k+1} = G^k(w_1^k)$$

$$w_3^{k+1} = w_2^k$$

$$w_4^{k+1} = w_3^k$$

Rule B

$$w_1^{k+1} = w_4^k$$

$$w_2^{k+1} = G^k(w_1^k)$$

$$w_3^{k+1} = w_1^k \oplus w_2^k \oplus counter^k$$

$$w_4^{k+1} = w_3^k$$



Decryption Rules

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DECRYPT

Rule A⁻¹

$$w_1^{k-1} = [G^{k-1}]^{-1}(w_2^k)$$

$$w_2^{k-1} = w_3^k$$

$$w_3^{k-1} = w_4^k$$

$$w_4^{k-1} = w_1^k \oplus w_2^k \oplus counter^{k-1}$$

Rule B⁻¹

$$w_1^{k-1} = [G^{k-1}]^{-1}(w_2^k)$$

$$w_2^{k-1} = [G^{k-1}]^{-1}(w_2^k) \oplus w_3^k \oplus counter^{k-1}$$

$$w_3^{k-1} = w_4^k$$

$$w_4^{k-1} = w_1^k$$



G-Permutation

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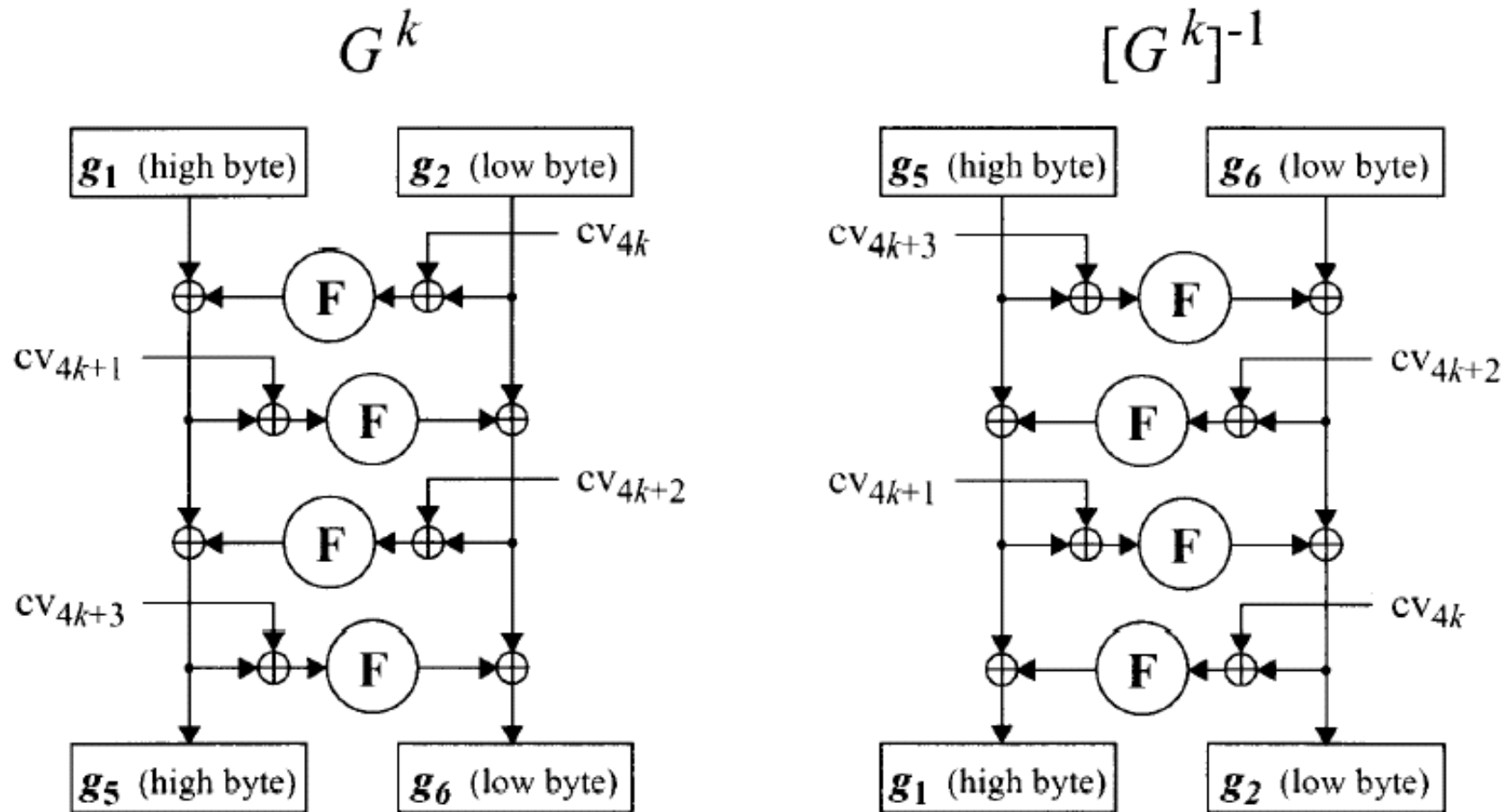


Figure 6. “G-permutation diagram”

F-Table

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	x0	x1	x2	x3	x4	x5	x6	x7	x8	x9	xA	xB	xC	xD	xE	xF
0x	a3	d7	09	83	f8	48	f6	f4	b3	21	15	78	99	b1	af	f9
1x	e7	2d	4d	8a	ce	4c	ca	2e	52	95	d9	1e	4e	38	44	28
2x	0a	df	02	a0	17	f1	60	68	12	b7	7a	c3	e9	fa	3d	53
3x	96	84	6b	ba	f2	63	9a	19	7c	ae	e5	f5	f7	16	6a	a2
4x	39	b6	7b	0f	c1	93	81	1b	ee	b4	1a	ea	d0	91	2f	b8
5x	55	b9	da	85	3f	41	bf	e0	5a	58	80	5f	66	0b	d8	90
6x	35	d5	c0	a7	33	06	65	69	45	00	94	56	6d	98	9b	76
7x	97	fc	b2	c2	b0	fe	db	20	e1	eb	d6	e4	dd	47	4a	1d
8x	42	ed	9e	6e	49	3c	cd	43	27	d2	07	d4	de	c7	67	18
9x	89	cb	30	1f	8d	c6	8f	aa	c8	74	dc	c9	5d	5c	31	a4
Ax	70	88	61	2c	9f	0d	2b	87	50	82	54	64	26	7d	03	40
Bx	34	4b	1c	73	d1	c4	fd	3b	cc	fb	7f	ab	e6	3e	5b	a5
Cx	ad	04	23	9c	14	51	22	f0	29	79	71	7e	ff	8c	0e	e2
Dx	0c	ef	bc	72	75	6f	37	a1	ec	d3	8e	62	8b	86	10	e8
Ex	08	77	11	be	92	4f	24	c5	32	36	9d	cf	f3	a6	bb	ac
Fx	5e	6c	a9	13	57	25	b5	e3	bd	a8	3a	01	05	59	2a	46



Skipjack Algorithm (contd.)

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Expected to be 36 years before the cost of breaking Skipjack is equal to the cost of breaking DES today

- Skipjack was classified until 1998
- Abruptly declassified
- Problems still exist
 - Once unit key (u) is known, all past, present and future transmissions are compromised
 - Knowing the unit key (u) makes it possible to fabricate messages



Advanced Encryption Standard (AES)

0100101010010101010101001100101010101000010101010101010100010100100—●

Rijndael Algorithm (Daeman & Rijmen, 2000)

- Federal standard (FIPS 197) in December 2001
- Features
 - A system breaking DES in 1 second would take 149 trillion years to break a 128-bit AES key (smallest key size)
 - Very good performance in hardware and software
 - Wide range of computing environments
 - Variable block and key lengths, and number of cycles
 - Simplicity, low memory requirements, sound design
 - Suitable for ATM, HDTV, B-ISDN, voice, satellite (> 1 Gbps requires dedicated hardware)



AES (contd.)

0100101010010101010101001100101010101000010101010101010100010100100—●

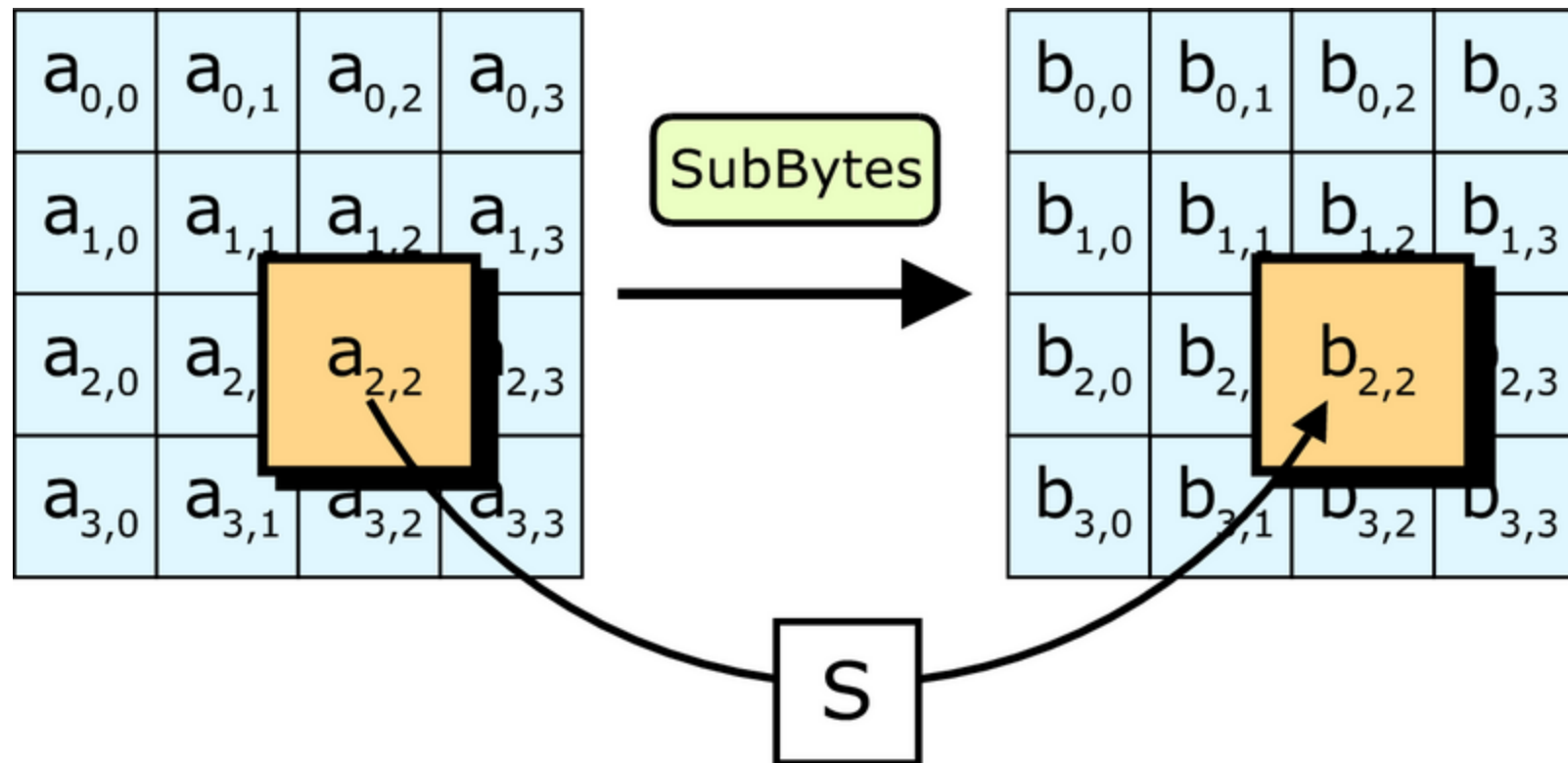
Details of AES Algorithm

- Substitution and permutation network
 - Most ciphers use a Feistel structure (some of the bits in intermediate states are simply transposed)
- AES uses three distinct invertible uniform transformations (layers)
 - Operates on 4x4 matrix (128-bit blocks) using a finite field
- AES Algorithm
 - SubBytes: Affine S-boxes (confusion)
 - ShiftRows: Mixes bytes of rows (diffusion)
 - MixColumns: Mixes, transforms bytes of columns (diffusion)
 - AddRoundKey: XOR of key to State (confusion)



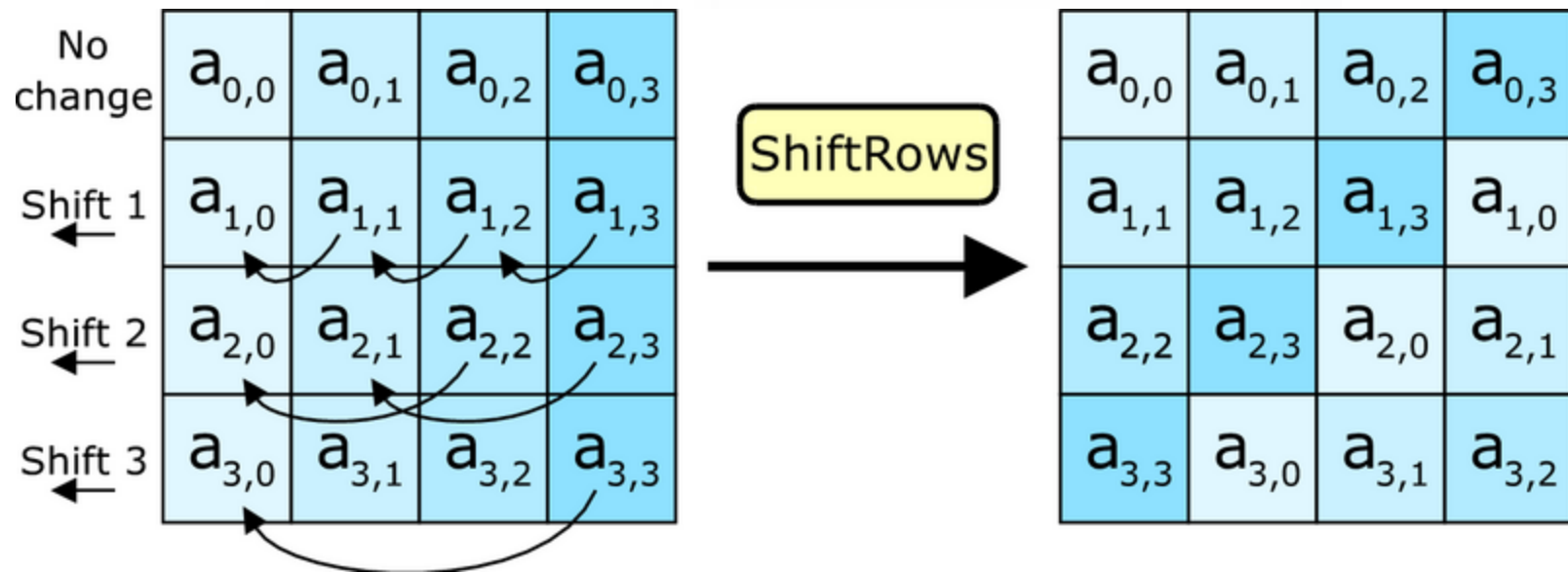
AES (contd.)

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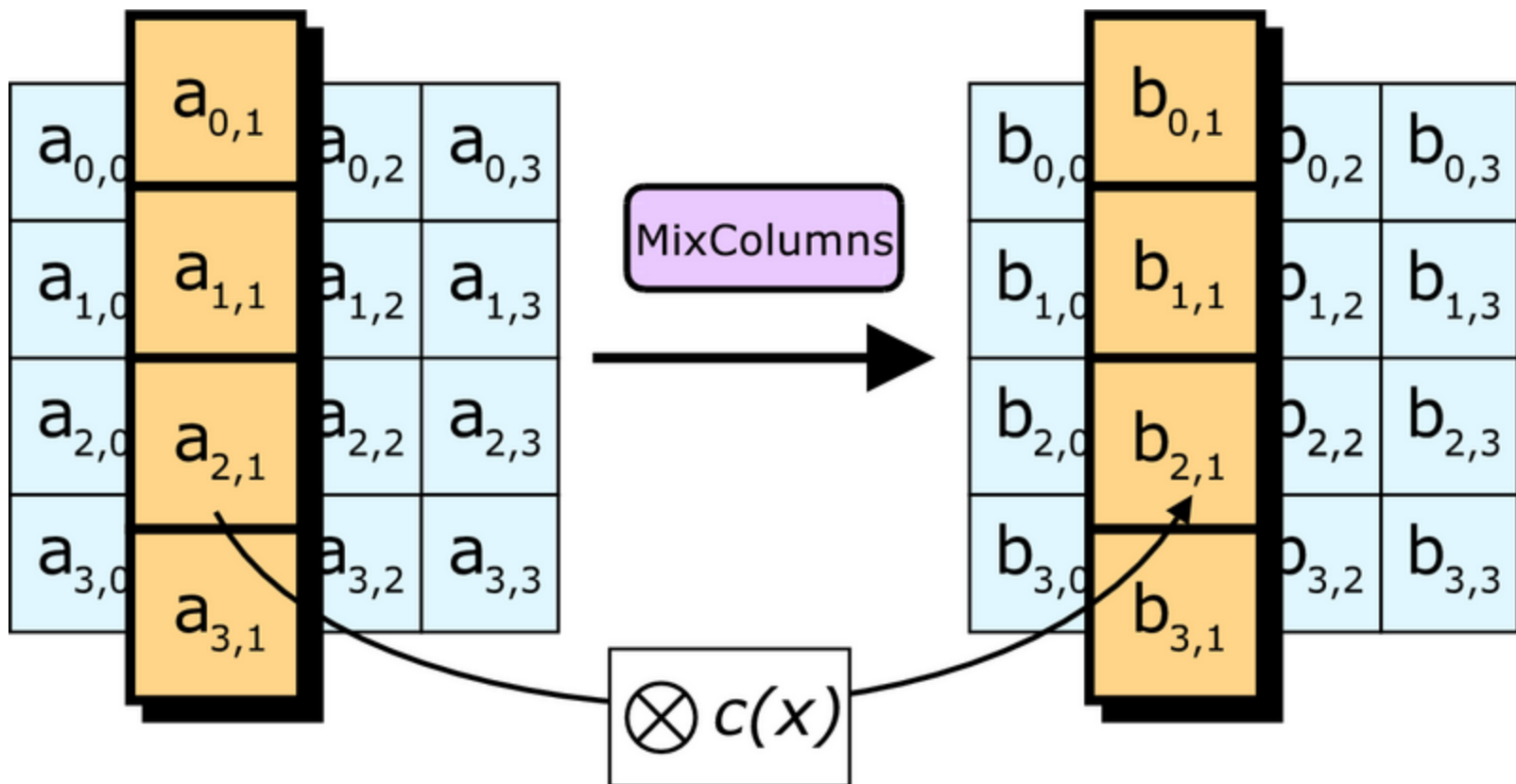
AES (contd.)

0100101010010101010101001100101010101000010101010101010100010100100—●



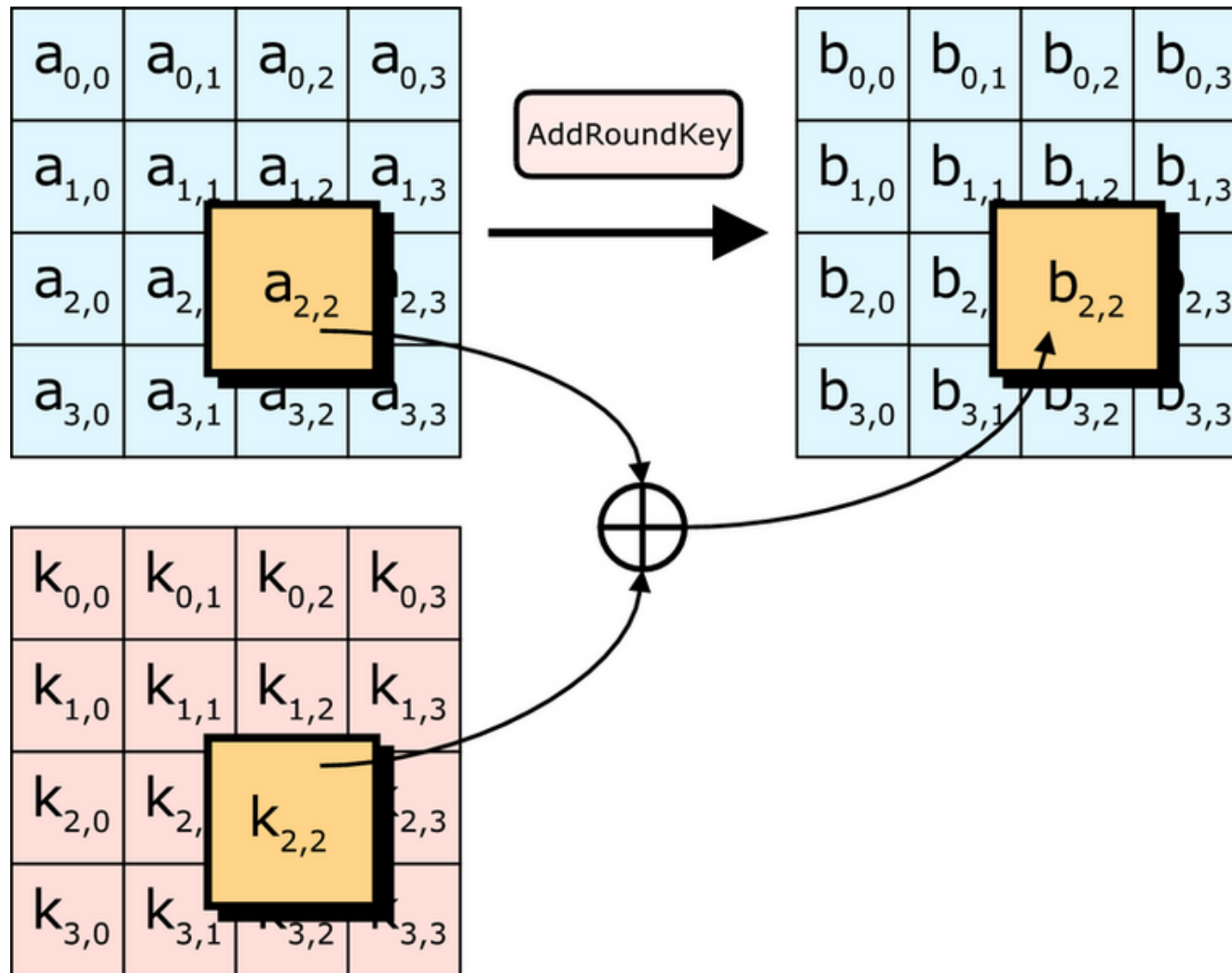
AES (contd.)

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AES (contd.)

0100101010010101010101001100101010101000010101010101010100010100100—●



AES (contd.)

01001010100101010101010011001010101010000101010101010101010100010100100—●

- Resistant to all known attacks
- Speed, code compactness, wide range of platforms (including smart card applications)
- Design simplicity; Strong math foundation
- Number of Rounds (N_r) (Text: defined as $N_r - 1$)
- Variable Block (N_b) and Key (N_k) sizes (4-byte words)

	$N_b = 4$	$N_b = 6$	$N_b = 8$
$N_k = 4:$	$N_r = 10$	$N_r = 12$	$N_r = 14$
$N_k = 6:$	$N_r = 12$	$N_r = 12$	$N_r = 14$
$N_k = 8:$	$N_r = 14$	$N_r = 14$	$N_r = 14$

