Completely Fair Scheduler

Dr. Naser Al Madi

Learning objectives

- Some project feedback and software design tips
- Completely Fair Scheduler
- Red-black Trees

Project feedback

*I will continue to return graded work before the next project is due.

How to organize your notebook

Coupling, cohesion, and dependency injection

- SOLID Design: The Open-Close Principle

operating_system.kernel(scheduler.FCFS_scheduler)

vs.

operating_system.kernel(“FCFS”)

Pycodestyle

https://www.python.org/dev/peps/pep-0008/

Black

Pytest

Completely Fair Scheduling

Completely Fair Scheduling (CFS)

- Linux scheduler since 2.6.23 (2007)
- No heuristics.
- Elegant handling of I/O and CPU bound processes.
- By Ingo Molnár based on work from Con Kolivas

Ideal fairness

- Divide processor time equally among processes.

If there are N processes in the system, each process should get (100/N)% of the CPU time.

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process	Burst (ms)
A	8
B	4
C	16
D	4

A	1	2	3	4	6	8			
B	1	2	3	4					
C	1	2	3	4	6	8	12	16	
D	1	2	3	4					

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4 ms slice: each process got 4/4 = 1ms or CPU time

A	1	2	3	4	6	8			
B	1	2	3	4					
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4 ms slice: each process got 4/2 = 2ms or CPU time

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B	1	2	3	4					
C	1	2	3	4	6	8	12	16	
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Virtual Runtime

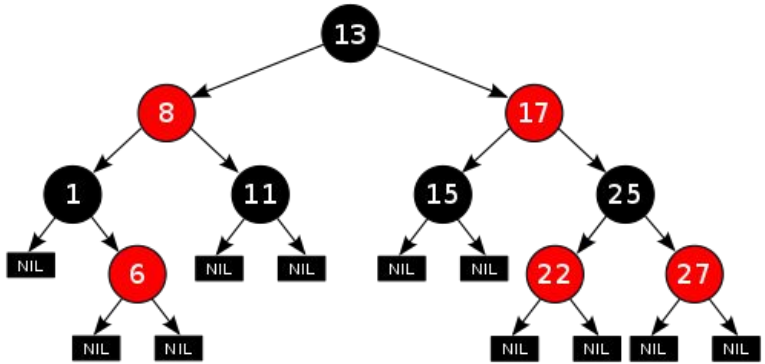
- In the PCB of each process a value called virtual runtime is added (vruntime).
- At each scheduling point, if process has run for t ms, then (vruntime += t).
- Vruntime for a process therefore monotonically increases.
- Implemented as unsigned 64 bit integer.

CFS main concept

- When timer interrupt occurs:
  - Choose the task with the lowest vruntime.
  - Compute its **dynamic timeslice**.
  - Program the high resolution time with this timeslice.
- The process begins to execute in the CPU.
- When interrupt occurs again, context switch if there is another task with a smaller runtime.

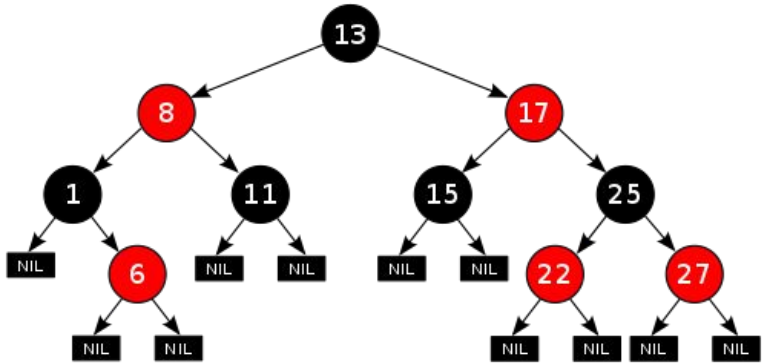
Picking the next process to run

- CFS uses a red-black tree.
  - Each node in the tree represents a runnable task.
  - Nodes ordered according to their vruntime
  - Nodes on the left have lower vruntime
  - The left most node is the task with the least vruntime



Picking the next process to run

- At a context switch:
  - Pick the left most node in tree in O(1)
  - If the finished process is runnable, it is inserted into the tree depending on the new vruntime in O(log(n))
  - Tasks move from left to right of the tree after execution. Starvation is avoided.



Why red black tree?

- Self balancing
- All operations are O(log n)

I/O and CPU bound processes

- What we need,
  - I/O bound should get higher priority and get a longer time to execute compared to CPU bound.
  - CFS achieves this efficiently:
    - I/O bound processes have small CPU bursts therefore will have a low vruntime.
    - They would appear towards the left of the tree, thus are given higher priorities.
    - I/O bound processes will typically have larger time slices because they have smaller vruntime.

Priority in CFS

Priority used to weigh the vruntime with nice value

- If process has run for t ms, then:

$$vruntime \mathrel{+}= t * (weight)$$

- A low priority means that time moves at a faster rate compared to high priority task.

New processes

- Gets added to the RB-tree
- Starts with an initial value of min_vruntime
- This ensures that it gets to execute quickly

Red-Black Trees

Let’s start by refreshing our memory!!

Motivating example:

You have a collection of data items, and you want to find a specific element in the data:

5	1	3	2	7	8	0	6

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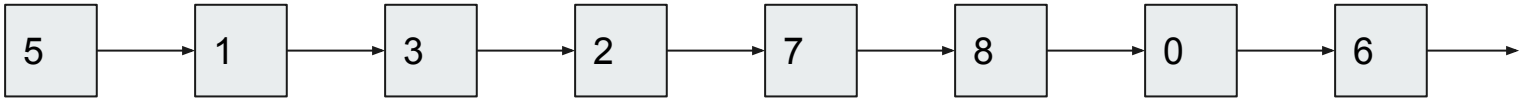

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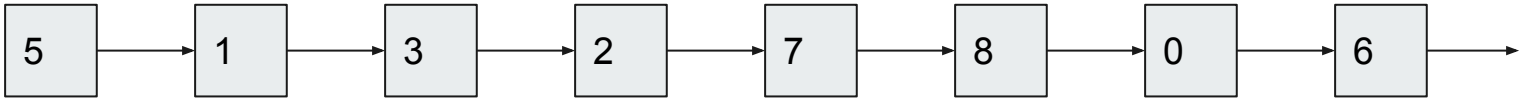

What about a linked list? What is the worst case time complexity for search? Also **O(n)** if the element we're looking for is the last element in the list.

Average time complexity

Data structure	search
Array	O(n)
Linkedlist	O(n)
Binary search tree	O(log(n))

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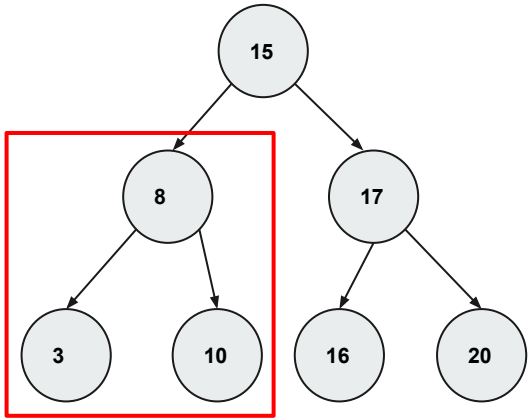

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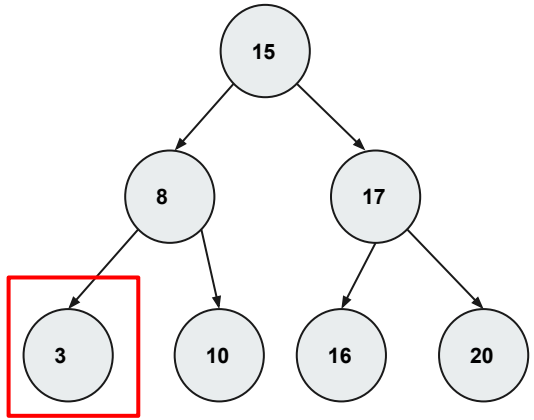

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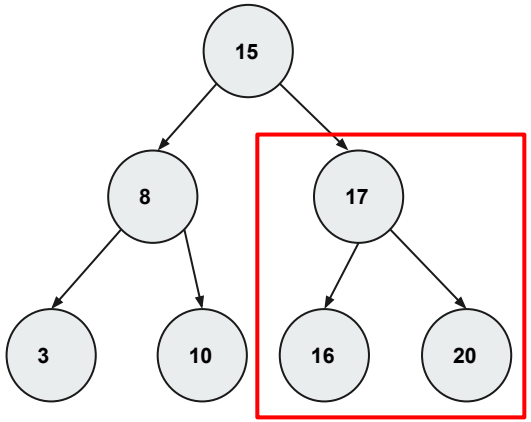

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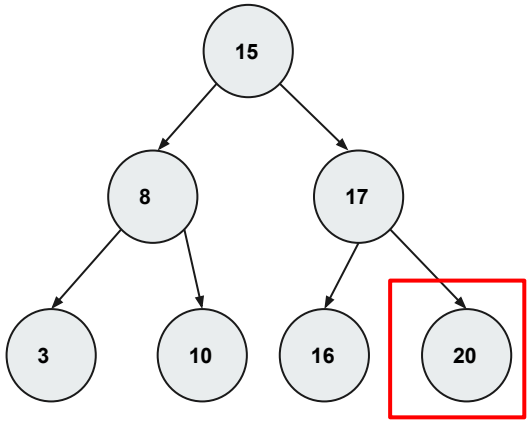

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- All values on the left subtree are less than the value of the node.
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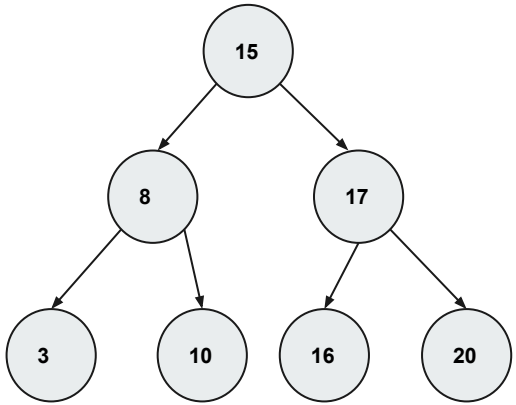
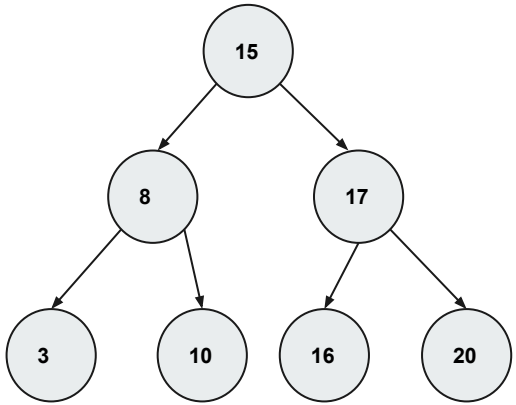
- The left and right subtree each must also be binary search trees.



BST search performance:

- Based on the tree structure we can search without visiting every single node.
- To search for any value we use the following logic:
  - Start at root.
  - Compare node value to search item, if search item is greater go right.
  - If search item is smaller then go left.



BST search performance:

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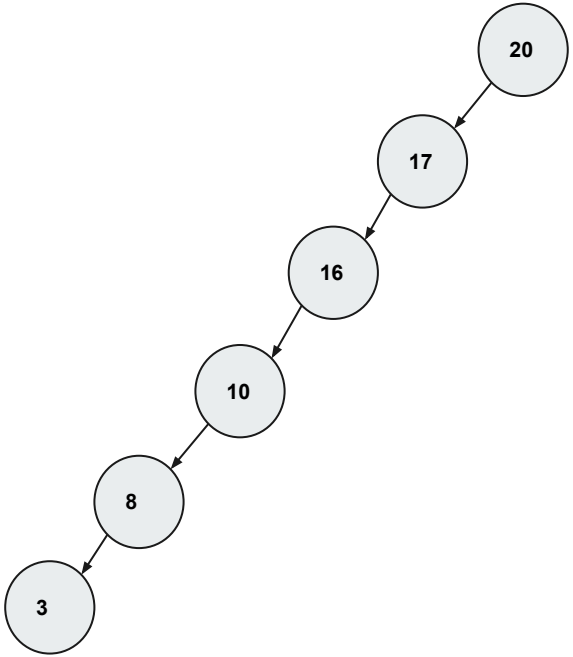
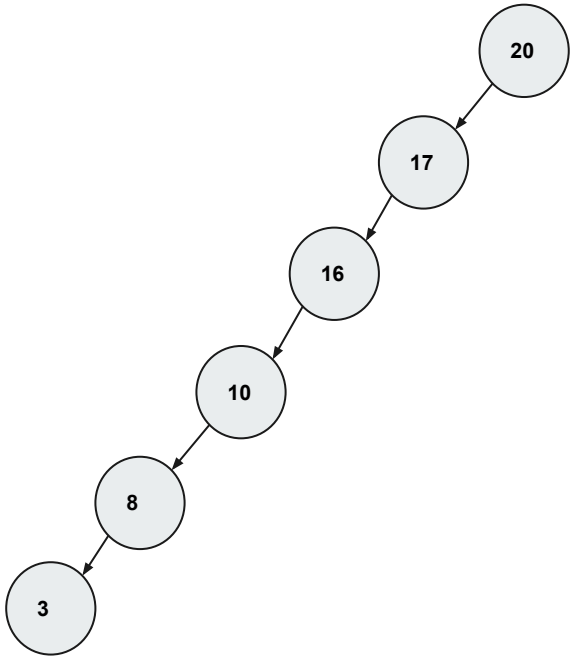


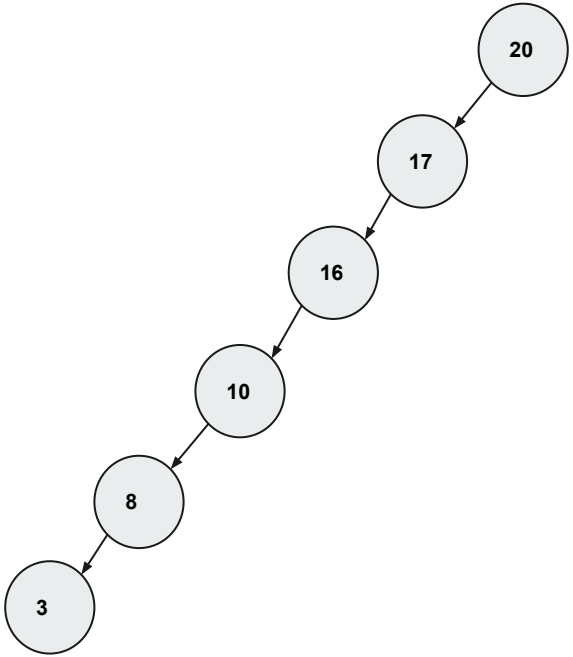
BST search performance:

- Is this a binary search tree?
- Yes! Since each left subtree is smaller than the value of the node.



BST search performance:

- What is the time complexity for searching this BST?



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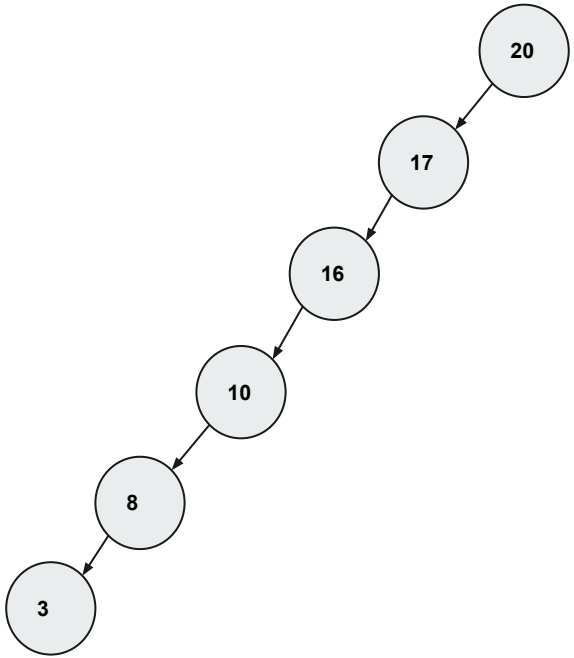

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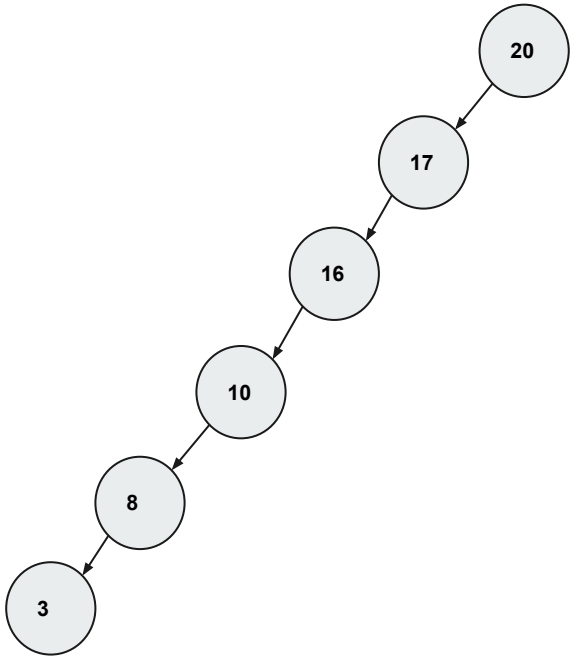
- In other words, the big O time complexity is equal to the height of the tree.



BST search performance:

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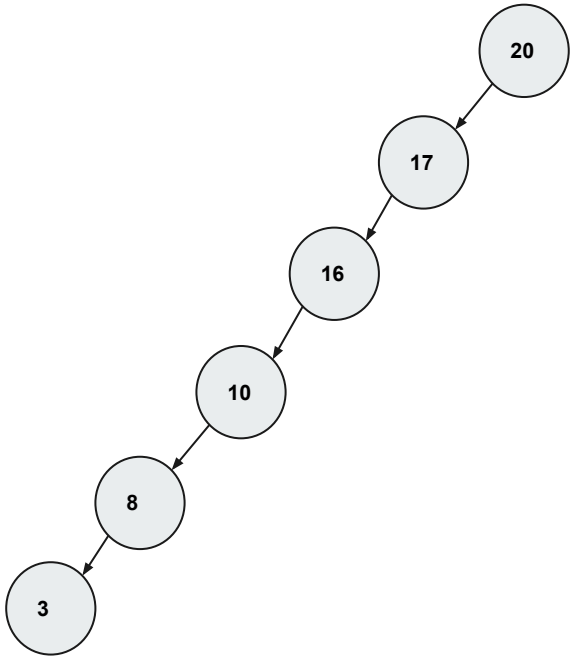
- That's why we prefer a balanced BST.



Balanced Binary Search Tree

In addition to all the properties of a BST.

A binary search tree is balanced if and only if the depth of the two subtrees of every node never differ by more than 1.

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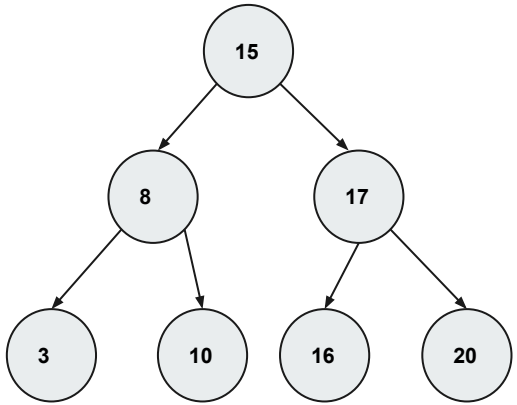

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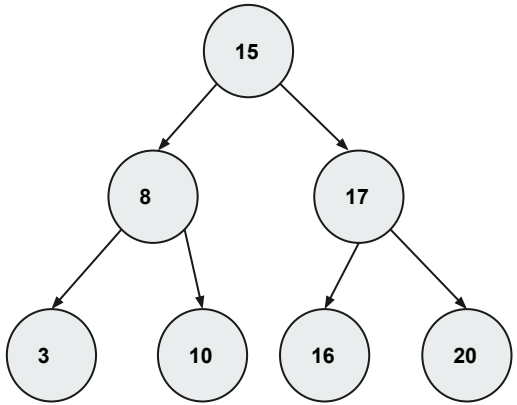

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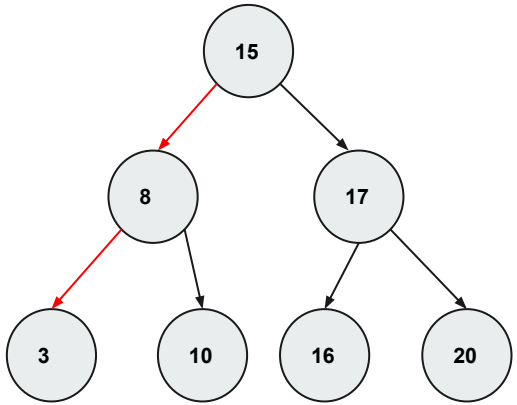

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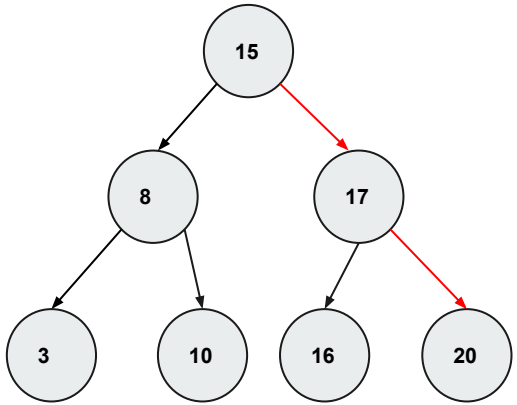

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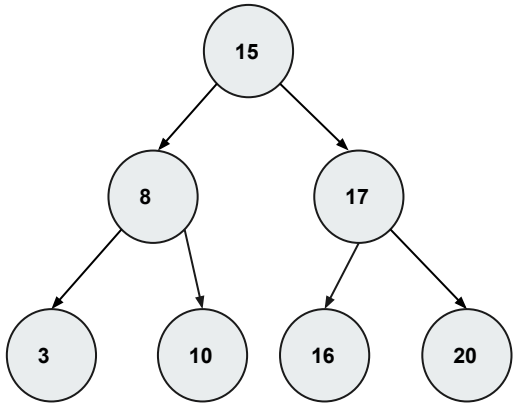

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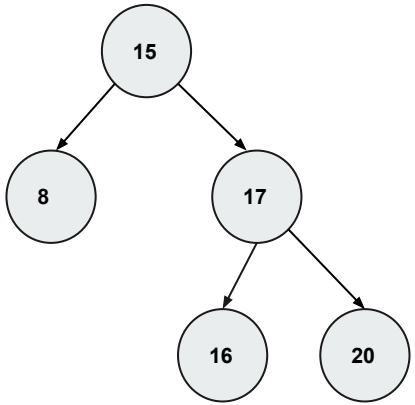

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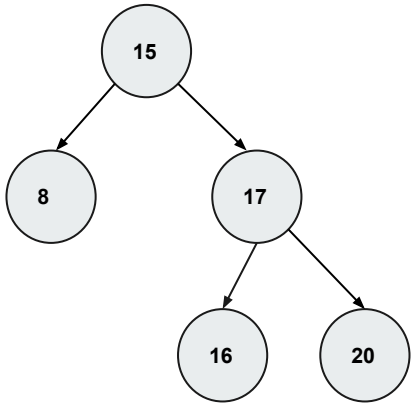

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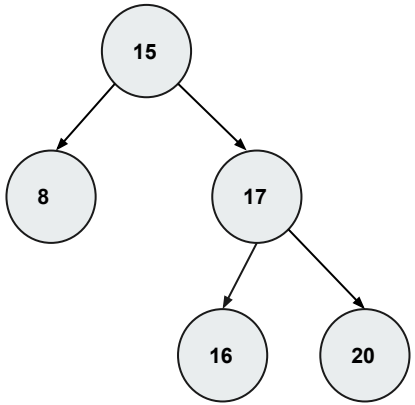

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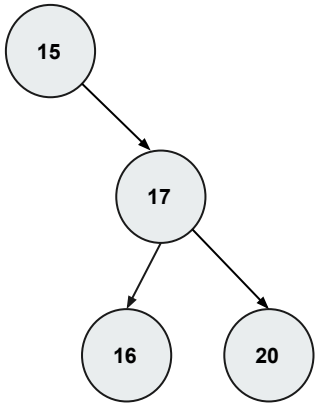

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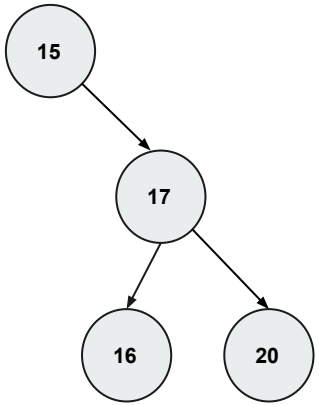

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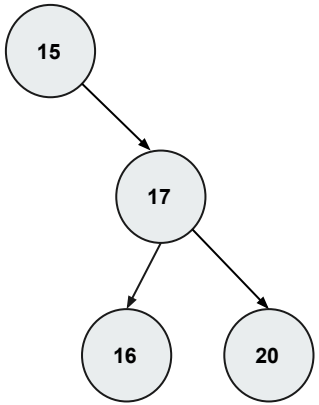
Not balanced



Red-black tree

- Special type of binary search trees
- **Approximately** balanced
- All operations in O(log(n))

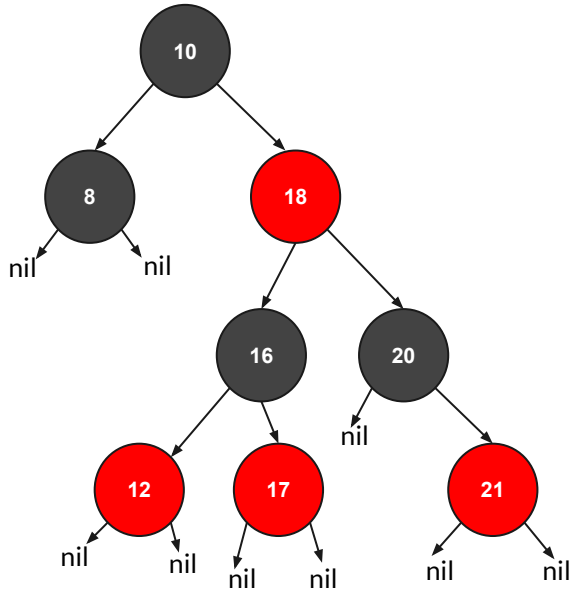
Better than heaps in:

- Delete any node in O(log(n))
- Not based on array implementation (no need for sequential memory)
- We can easily modify it to delete node with *min_vruntime* in O(1)

Red-black tree properties

*in addition to the properties of binary search trees

1. Every node is either black or red
2. Root and leaves (nil) are black
3. If a node is red, its children are black
4. All paths from a node to its nil descendants contain the same number of black nodes



Operations

- Search
- Insert
- Remove

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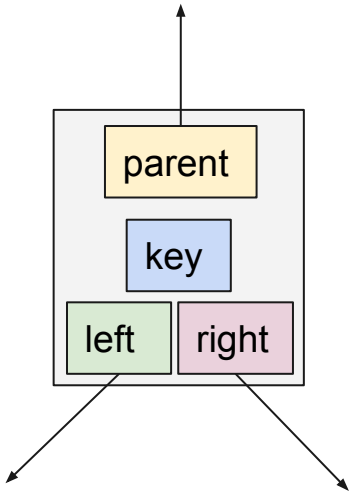
- Search
- Insert
- Remove

The two operations mess up the properties of a red-black tree, we solve this by rotation and recoloring.

RBTree node

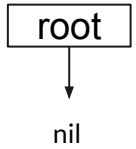
Each RBTree node keeps track of:

- Its value or key
- Left subtree
- Right subtree
- Its parent



Insert operation pseudocode

```
RB-Insert(T,z)
  y = nil[T]
  x = root[T]
  while x != nil[T]
     y = x
     if key[z] < key[x] then
        x = left[x]
     else
        x = right[x]
  p[z] = y
  if y = nil[T]
     root[T] = z
  else
    if key[z] < key[y] then
      left[y] = z
    else
      right[y] = z
  left[z] = nil[T]
  right[z] = nil[T]
  color[z] = RED
  RB-Insert-fixup(T,z)
```



Assuming:

- Z is a node
- T is the tree

Insert operation pseudocode

```
RB-Insert(T,z)
  y = nil[T]
  x = root[T]
  while x != nil[T]
     y = x
     if key[z] < key[x] then
        x = left[x]
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        x = right[x]
  p[z] = y
  if y = nil[T]
     root[T] = z
  else
    if key[z] < key[y] then
      left[y] = z
    else
      right[y] = z
  left[z] = nil[T]
  right[z] = nil[T]
  color[z] = RED
  RB-Insert-fixup(T,z)
```

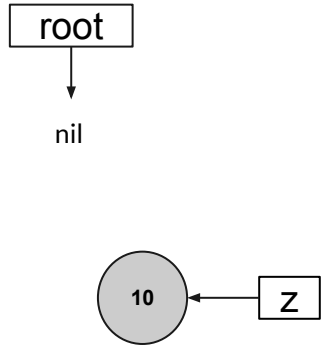

Assuming:

- Z is a node
- T is the tree

Insert operation pseudocode

```
RB-Insert(T,z)
  y = nil[T]
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  while x != nil[T]
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        x = right[x]
  p[z] = y
  if y = nil[T]
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  else
    if key[z] < key[y] then
      left[y] = z
    else
      right[y] = z
  left[z] = nil[T]
  right[z] = nil[T]
  color[z] = RED
  RB-Insert-fixup(T,z)
```

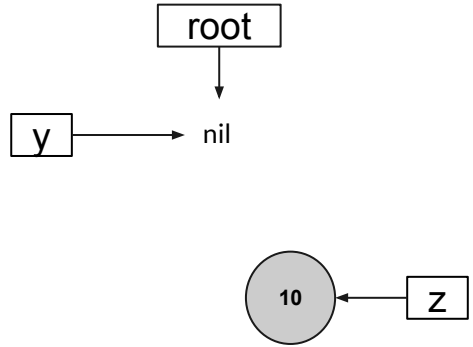

Insert operation pseudocode

```
RB-Insert(T,z)
  y = nil[T]
  x = root[T]
  while x != nil[T]
     y = x
     if key[z] < key[x] then
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        x = right[x]
  p[z] = y
  if y = nil[T]
     root[T] = z
  else
    if key[z] < key[y] then
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    else
      right[y] = z
  left[z] = nil[T]
  right[z] = nil[T]
  color[z] = RED
  RB-Insert-fixup(T,z)
```

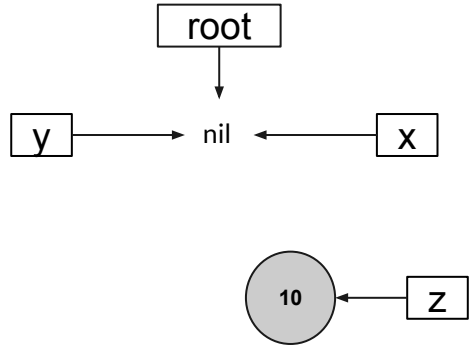

Insert operation pseudocode

```
RB-Insert(T,z)
  y = nil[T]
  x = root[T]
  while x != nil[T]
      y = x
      if key[z] < key[x] then
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        x = right[x]
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      left[y] = z
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  left[z] = nil[T]
  right[z] = nil[T]
  color[z] = RED
  RB-Insert-fixup(T,z)
```

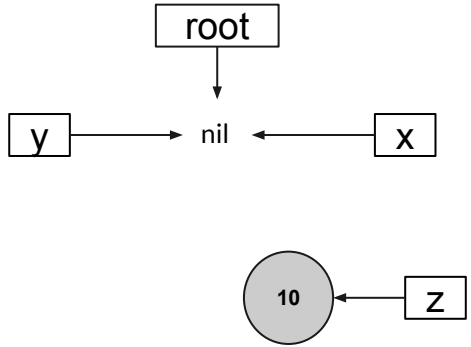

Insert operation pseudocode

```
RB-Insert(T,z)
  y = nil[T]
  x = root[T]
  while x != nil[T]
      y = x
      if key[z] < key[x] then
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        x = right[x]
  p[z] = y
  if y = nil[T]
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    else
      right[y] = z
  left[z] = nil[T]
  right[z] = nil[T]
  color[z] = RED
  RB-Insert-fixup(T,z)
```

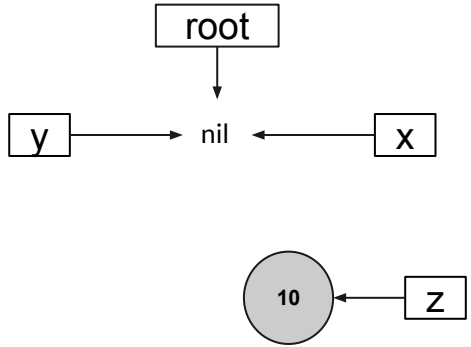

Insert operation pseudocode

```
RB-Insert(T,z)
  y = nil[T]
  x = root[T]
  while x != nil[T]
      y = x
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        x = right[x]
  p[z] = y
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     root[T] = z
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    else
      right[y] = z
  left[z] = nil[T]
  right[z] = nil[T]
  color[z] = RED
  RB-Insert-fixup(T,z)
```

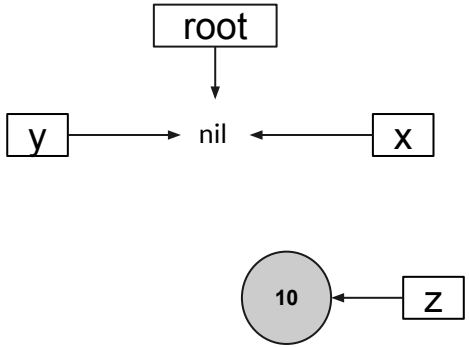

Insert operation pseudocode

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RB-Insert(T,z)
   y = nil[T]
   x = root[T]
   while x != nil[T]
      y = x
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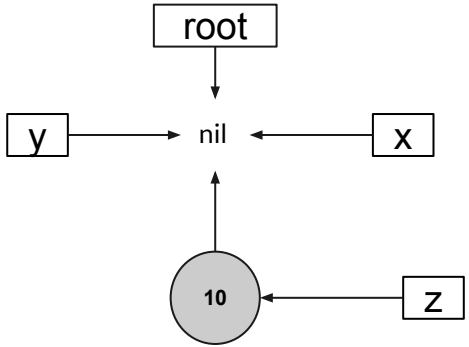
         x = right[x]
   p[z] = y
   if y = nil[T]
      root[T] = z
   else
     if key[z] < key[y] then
       left[y] = z
     else
       right[y] = z
   left[z] = nil[T]
   right[z] = nil[T]
   color[z] = RED
   RB-Insert-fixup(T,z)
```



Insert operation pseudocode

```
RB-Insert(T,z)
  y = nil[T]
  x = root[T]
  while x != nil[T]
     y = x
     if key[z] < key[x] then
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     else
        x = right[x]
  p[z] = y
  if y = nil[T]
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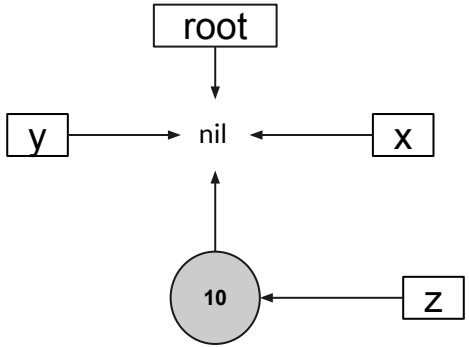
    if key[z] < key[y] then
      left[y] = z
    else
      right[y] = z
  left[z] = nil[T]
  right[z] = nil[T]
  color[z] = RED
  RB-Insert-fixup(T,z)
```



Insert operation pseudocode

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RB-Insert(T,z)
  y = nil[T]
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  while x != nil[T]
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        x = right[x]
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     root[T] = z
  else
    if key[z] < key[y] then
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    else
      right[y] = z
  left[z] = nil[T]
  right[z] = nil[T]
  color[z] = RED
  RB-Insert-fixup(T,z)
```

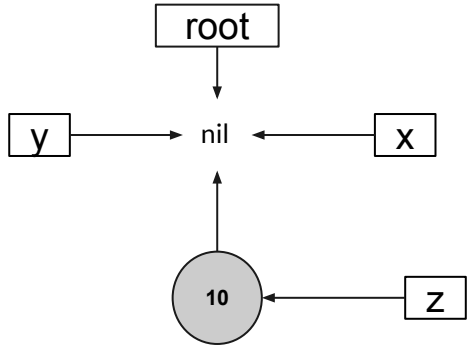

Insert operation pseudocode

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RB-Insert(T,z)
  y = nil[T]
  x = root[T]
  while x != nil[T]
     y = x
     if key[z] < key[x] then
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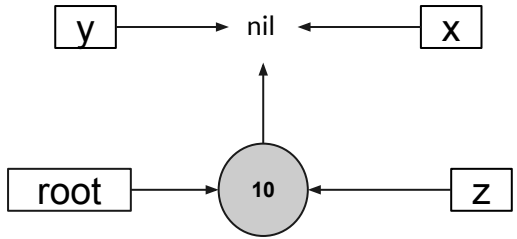
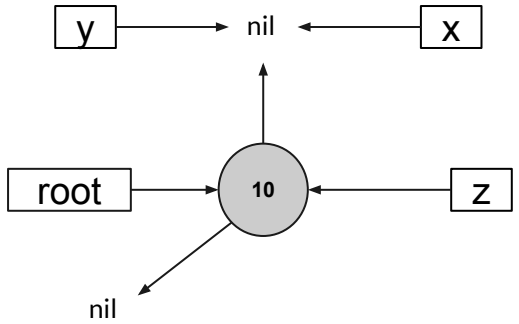
```



Insert operation pseudocode

```
RB-Insert(T,z)
   y = nil[T]
   x = root[T]
   while x != nil[T]
      y = x
      if key[z] < key[x] then
         x = left[x]
      else
         x = right[x]
   p[z] = y
   if y = nil[T]
      root[T] = z
   else
     if key[z] < key[y] then
       left[y] = z
     else
       right[y] = z
   left[z] = nil[T]
   right[z] = nil[T]
   color[z] = RED
   RB-Insert-fixup(T,z)
```



Insert operation pseudocode

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RB-Insert(T,z)
   y = nil[T]
   x = root[T]
   while x != nil[T]
      y = x
      if key[z] < key[x] then
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      else
         x = right[x]
   p[z] = y
   if y = nil[T]
      root[T] = z
   else
     if key[z] < key[y] then
       left[y] = z
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       right[y] = z
   left[z] = nil[T]
   right[z] = nil[T]
   color[z] = RED
   RB-Insert-fixup(T,z)
```

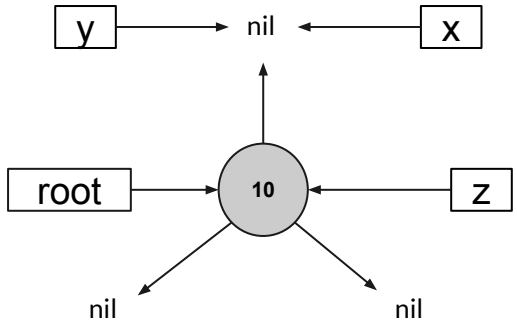

Insert operation pseudocode

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RB-Insert(T,z)
  y = nil[T]
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  while x != nil[T]
     y = x
     if key[z] < key[x] then
        x = left[x]
     else
        x = right[x]
  p[z] = y
  if y = nil[T]
     root[T] = z
  else
    if key[z] < key[y] then
      left[y] = z
    else
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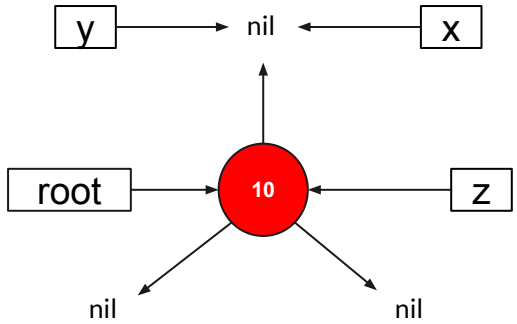
  left[z] = nil[T]
  right[z] = nil[T]
  color[z] = RED
  RB-Insert-fixup(T,z)
```



Insert operation pseudocode

```
RB-Insert(T,z)
  y = nil[T]
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        x = right[x]
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    if key[z] < key[y] then
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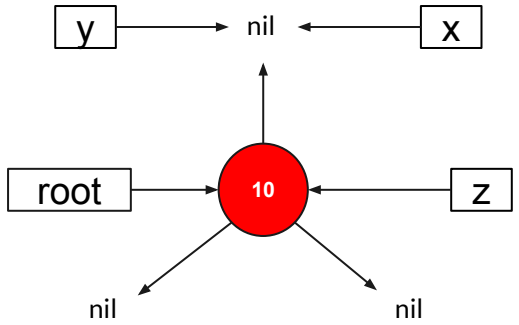
  left[z] = nil[T]
  right[z] = nil[T]
  color[z] = RED
  RB-Insert-fixup(T,z)
```



RB-Insert-fixup() is the function that restores the RBTree properties after insert. We will talk about it in a bit.

Insert operation pseudocode

```
RB-Insert(T,z)
  y = nil[T]
  x = root[T]
  while x != nil[T]
     y = x
     if key[z] < key[x] then
        x = left[x]
     else
        x = right[x]
  p[z] = y
  if y = nil[T]
     root[T] = z
  else
    if key[z] < key[y] then
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    else
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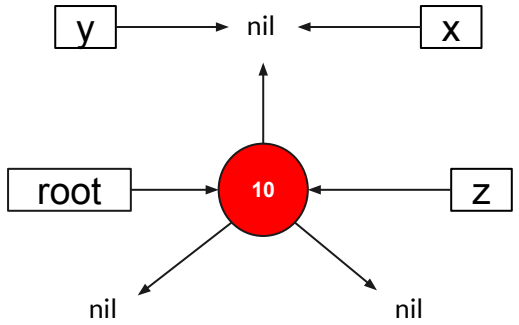
  left[z] = nil[T]
  right[z] = nil[T]
  color[z] = RED
  RB-Insert-fixup(T,z)
```



Is this a valid RB-tree?

Insert operation pseudocode

```
RB-Insert(T,z)
  y = nil[T]
  x = root[T]
  while x != nil[T]
     y = x
     if key[z] < key[x] then
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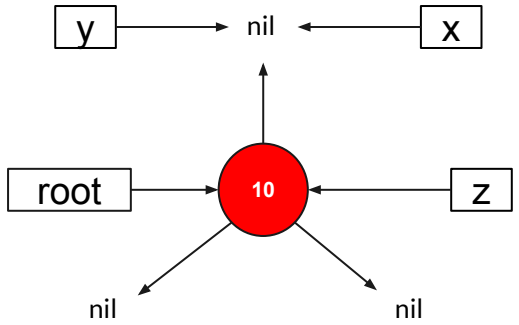
     else
        x = right[x]
  p[z] = y
  if y = nil[T]
     root[T] = z
  else
    if key[z] < key[y] then
      left[y] = z
    else
      right[y] = z
  left[z] = nil[T]
  right[z] = nil[T]
  color[z] = RED
  RB-Insert-fixup(T,z)
```



Is this a valid RB-tree? Nope (root must be black)

Insert operation pseudocode

```
RB-Insert(T,z)
  y = nil[T]
  x = root[T]
  while x != nil[T]
     y = x
     if key[z] < key[x] then
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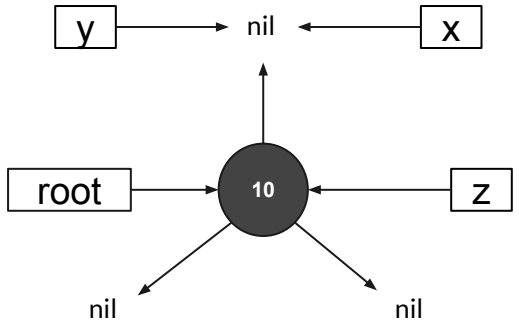
        x = right[x]
  p[z] = y
  if y = nil[T]
     root[T] = z
  else
    if key[z] < key[y] then
      left[y] = z
    else
      right[y] = z
  left[z] = nil[T]
  right[z] = nil[T]
  color[z] = RED
  RB-Insert-fixup(T,z)
```

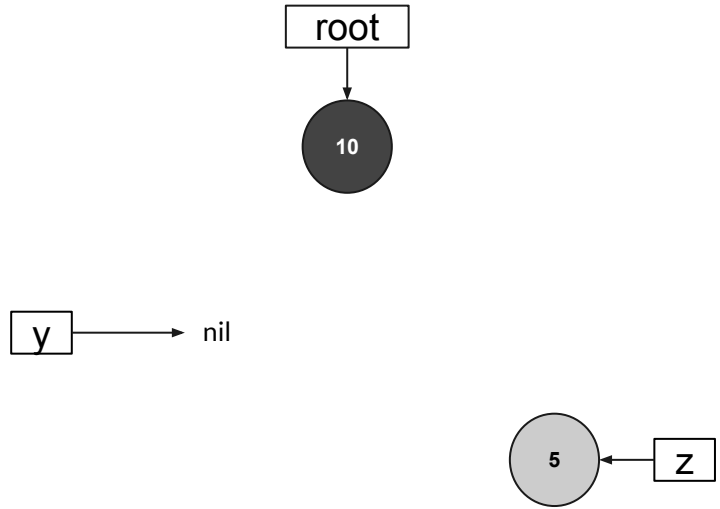


Is this a valid RB-tree? Nope (root must be black)

More interesting example

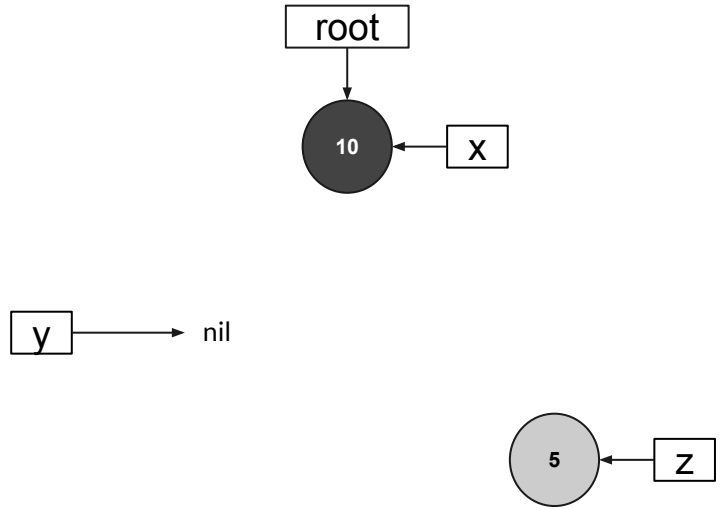
Insert operation pseudocode

```
RB-Insert(T,z)
  y = nil[T]
  x = root[T]
  while x != nil[T]
     y = x
     if key[z] < key[x] then
        x = left[x]
     else
        x = right[x]
  p[z] = y
  if y = nil[T]
     root[T] = z
  else
    if key[z] < key[y] then
      left[y] = z
    else
      right[y] = z
  left[z] = nil[T]
  right[z] = nil[T]
  color[z] = RED
  RB-Insert-fixup(T,z)
```



Insert operation pseudocode

```
RB-Insert(T,z)
   y = nil[T]
   x = root[T]
   while x != nil[T]
      y = x
      if key[z] < key[x] then
         x = left[x]
      else
         x = right[x]
   p[z] = y
   if y = nil[T]
      root[T] = z
   else
     if key[z] < key[y] then
       left[y] = z
     else
       right[y] = z
   left[z] = nil[T]
   right[z] = nil[T]
   color[z] = RED
   RB-Insert-fixup(T,z)
```



Insert operation pseudocode

```
RB-Insert(T,z)
  y = nil[T]
  x = root[T]
  while x != nil[T]
     y = x
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        x = left[x]
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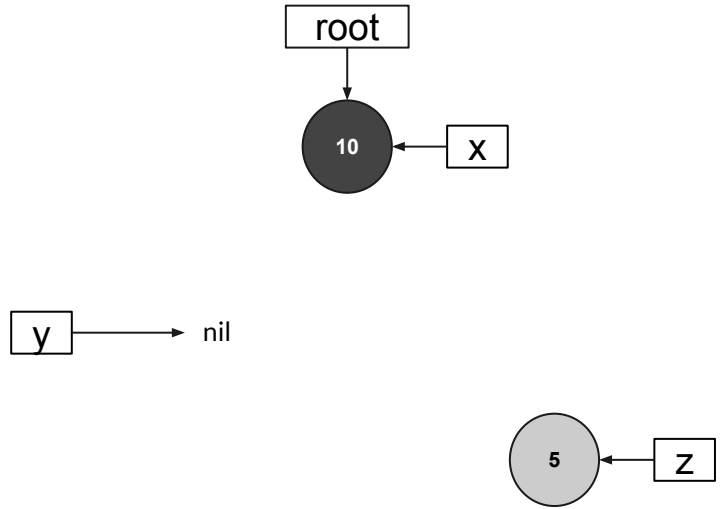
        x = right[x]
  p[z] = y
  if y = nil[T]
     root[T] = z
  else
    if key[z] < key[y] then
      left[y] = z
    else
      right[y] = z
  left[z] = nil[T]
  right[z] = nil[T]
  color[z] = RED
  RB-Insert-fixup(T,z)
```



Insert operation pseudocode

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  y = nil[T]
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        x = right[x]
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  if y = nil[T]
     root[T] = z
  else
    if key[z] < key[y] then
      left[y] = z
    else
      right[y] = z
  left[z] = nil[T]
  right[z] = nil[T]
  color[z] = RED
  RB-Insert-fixup(T,z)
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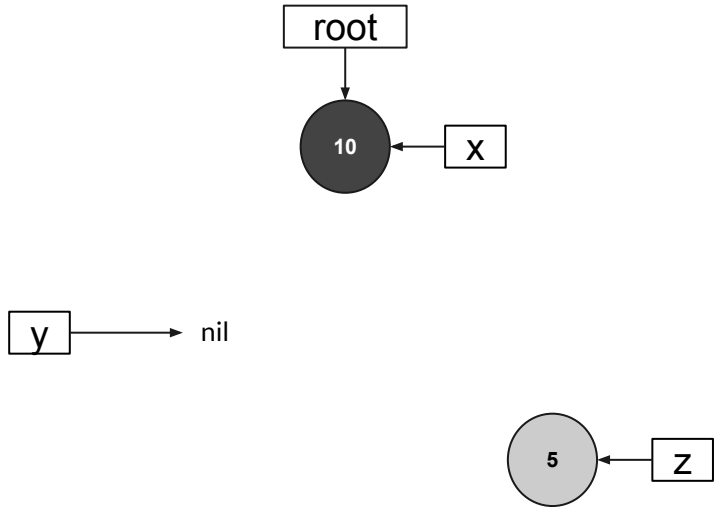

Insert operation pseudocode

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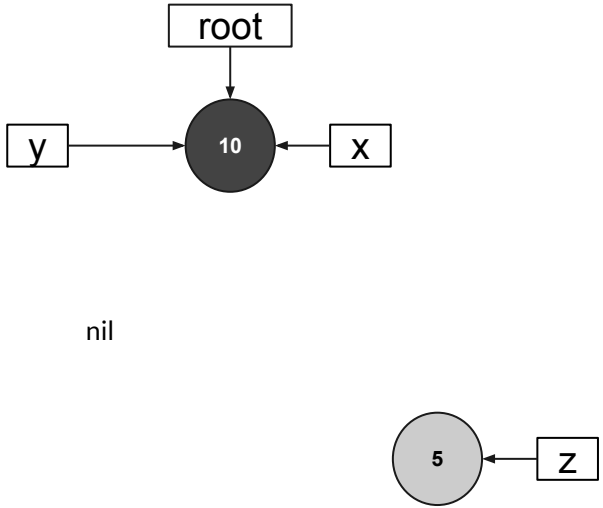

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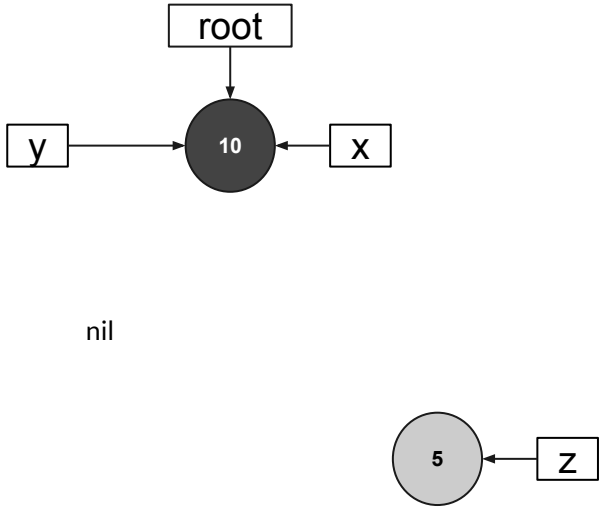

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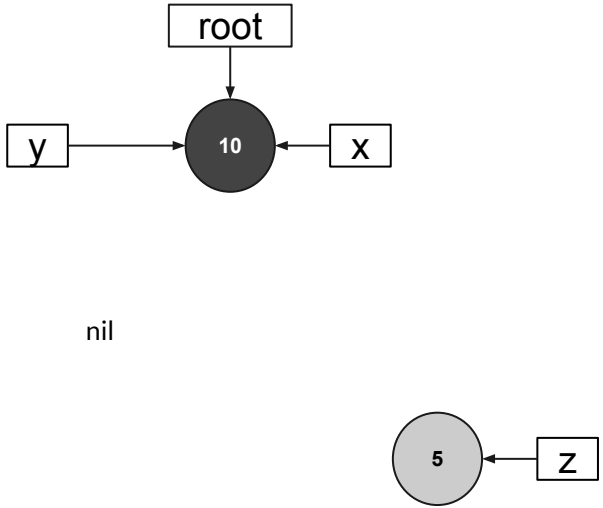

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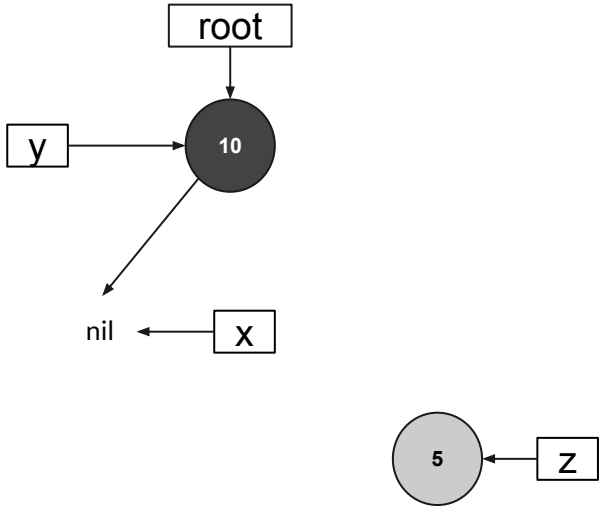

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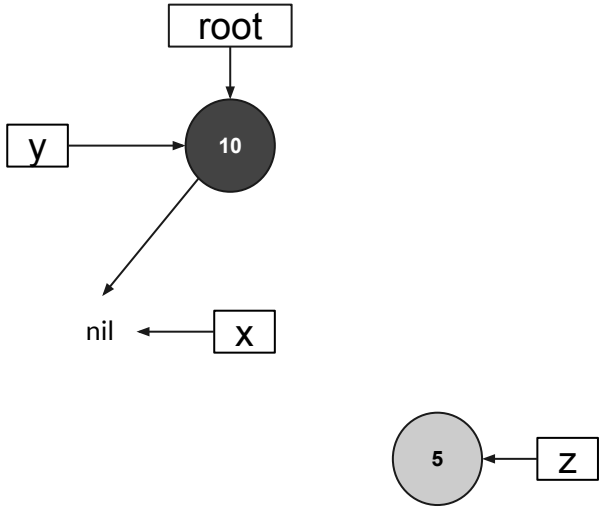

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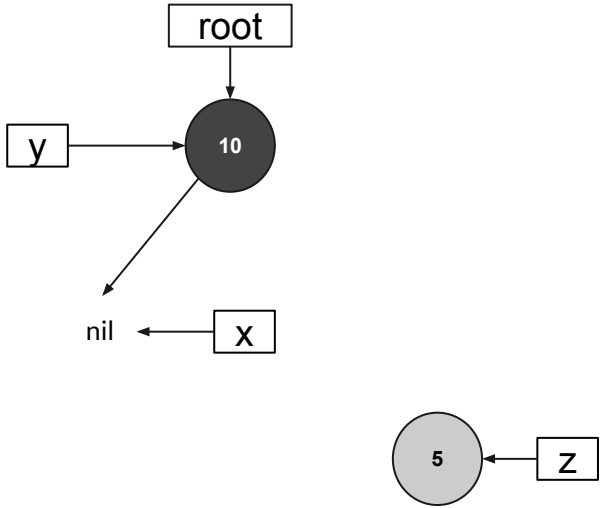

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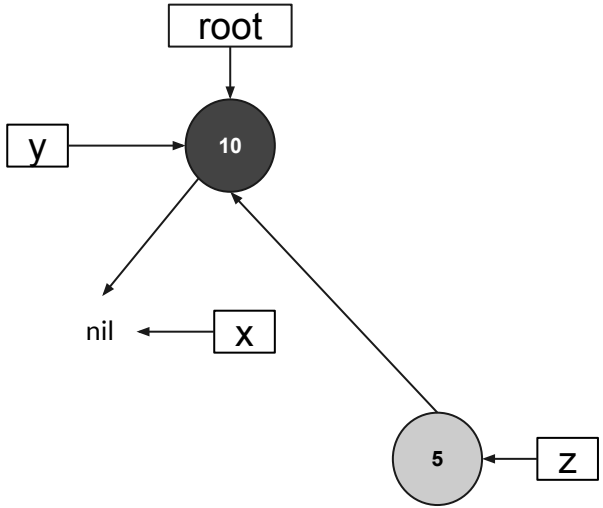

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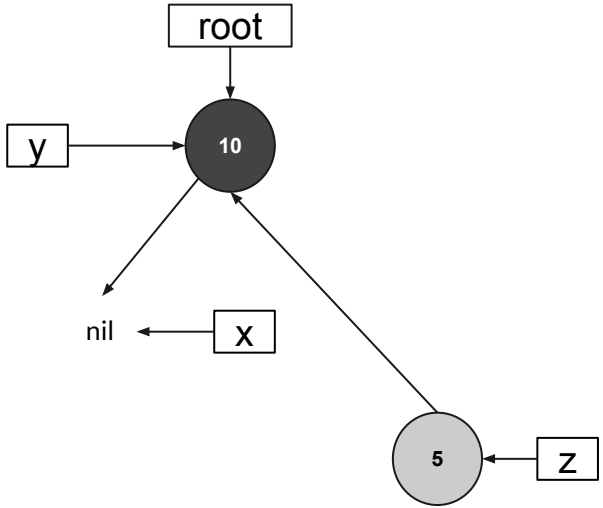

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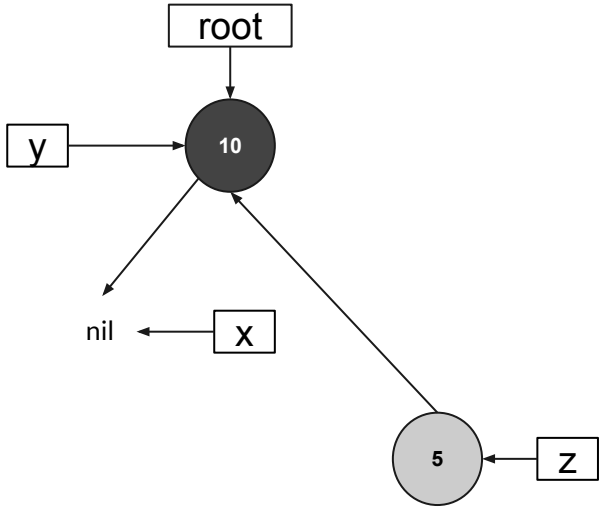

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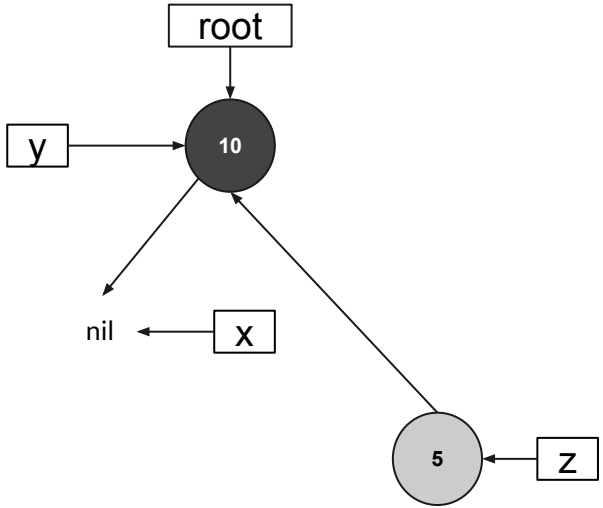

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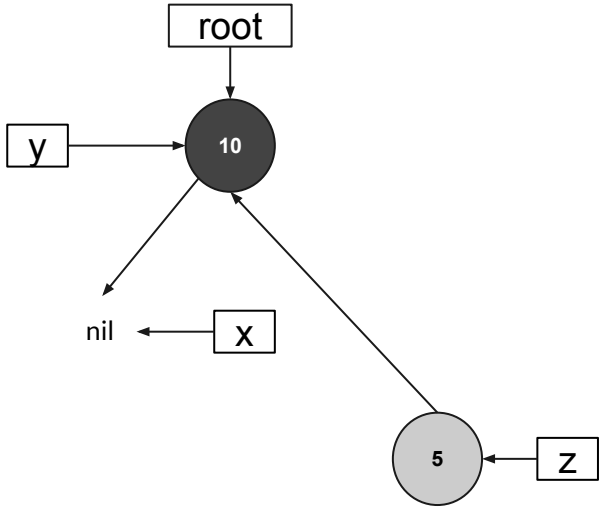

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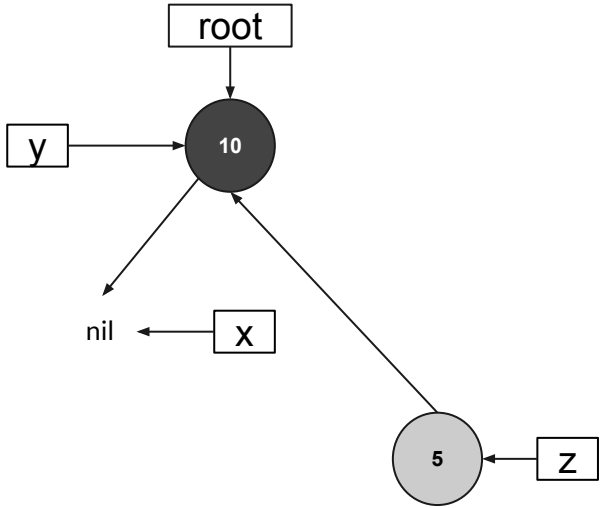

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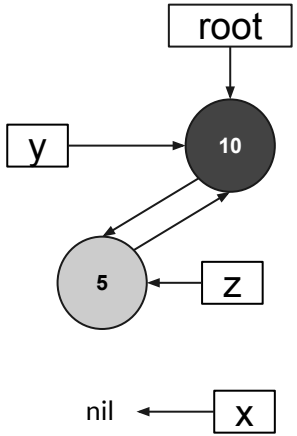

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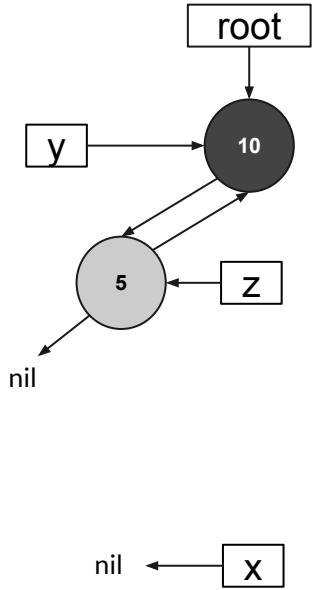

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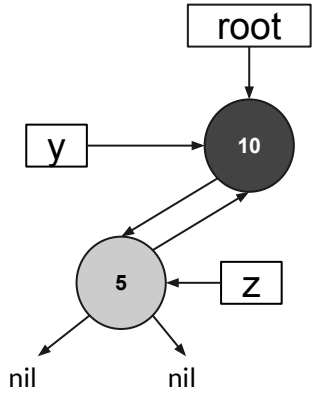

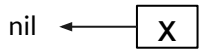

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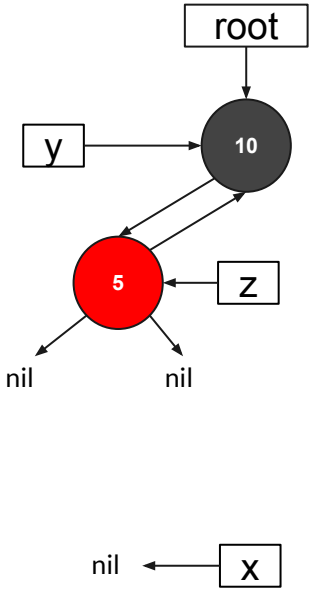

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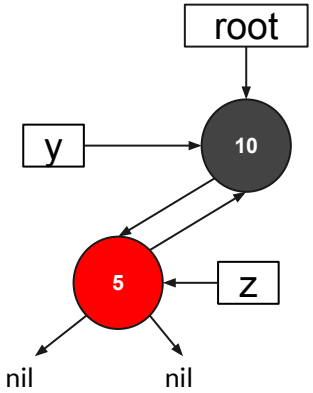

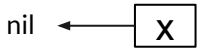

Nothing to fixup, this is a valid RB-tree

Let’s complicate this a bit

Insert operation pseudocode

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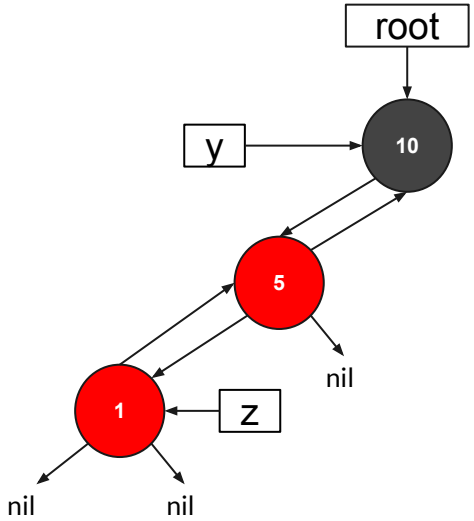

Let's say we added the 1, is this a valid RB-tree?

Insert operation pseudocode

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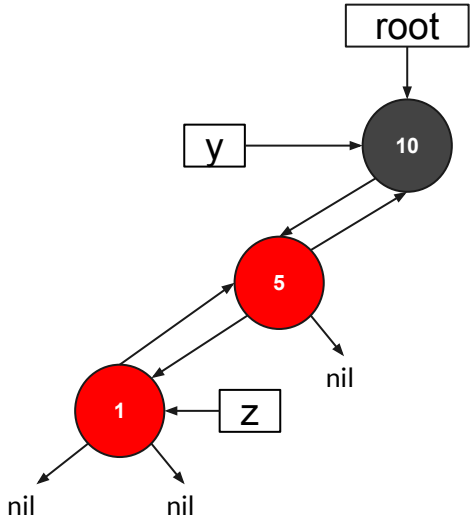

Let's say we added the 1, is this a valid RB-tree? Nope.

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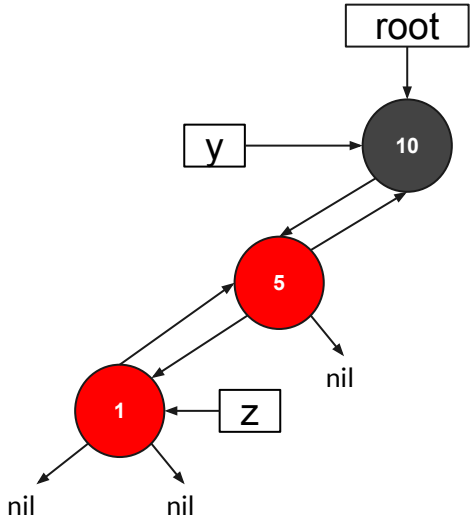


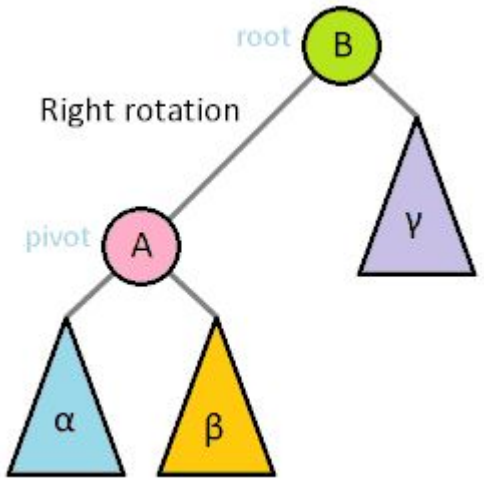
Let's say we added the 1, is this a valid RB-tree? Nope, we need to fix this tree.

Insert-fixup

Insert-fixup

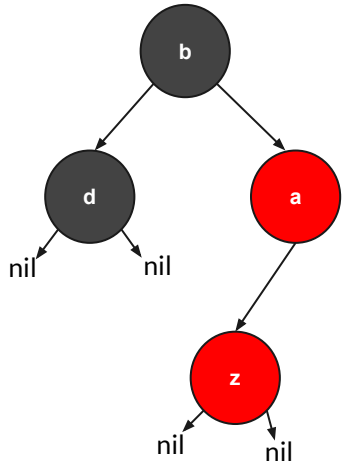
We can restore the properties of a red-black tree by:

- Recoloring
- Rotation



Fixup

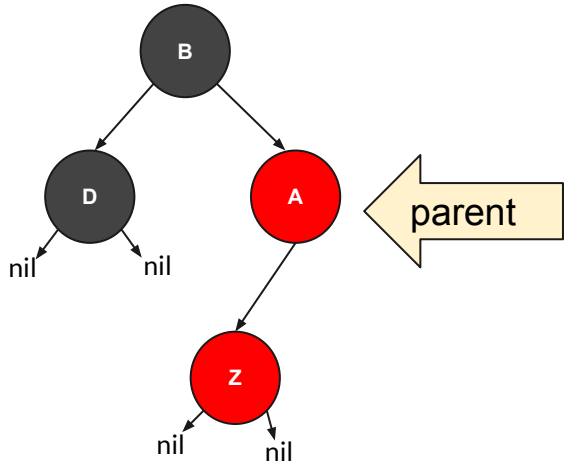
We have 4 scenarios, to learn them we must agree on some terminology:



Fixup

We have 4 scenarios, to learn them we must agree on some terminology. Z's relations are:

A: Parent
B: Grandparent
D: uncle



Fixup

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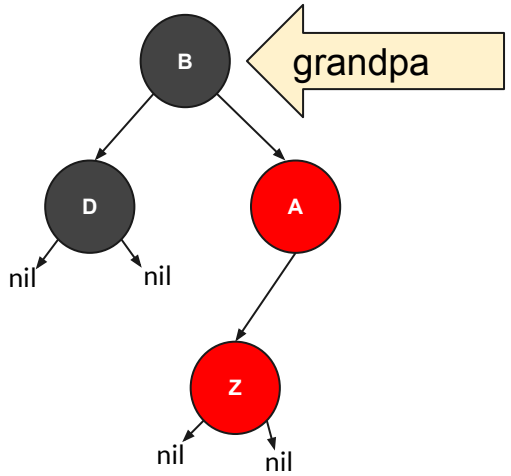

Fixup

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A: Parent
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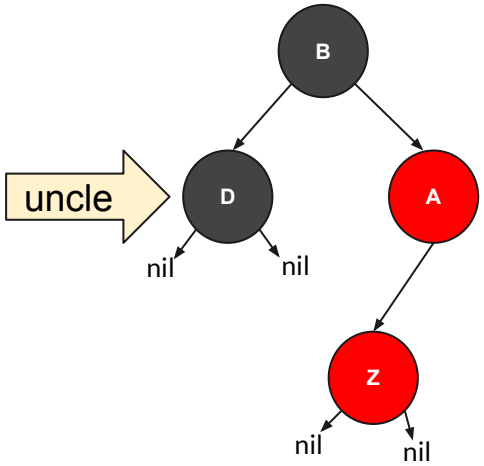


Fixup scenarios

Case 0: Z is the root

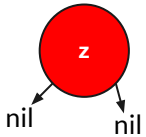
Case 1: Z's uncle is red

Case 2: Z's uncle is black (triangle)

Case 3: Z's uncle is black (line)

Case 0: Z is the root

1. Color Z black
2. Done!



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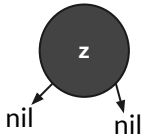
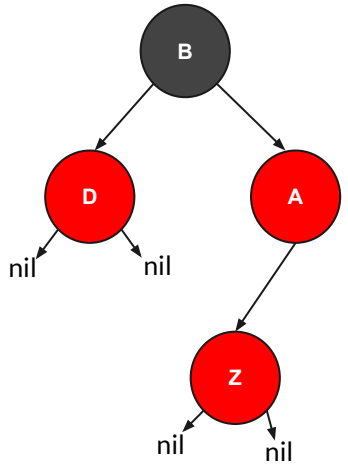


Case 1: Z's uncle is red

1. Recolor parent, uncle, and grandparent.
2. Done!



Case 1: Z's uncle is red

1. Recolor parent, uncle, and grandparent.
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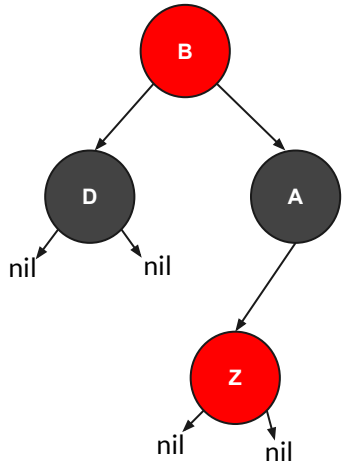
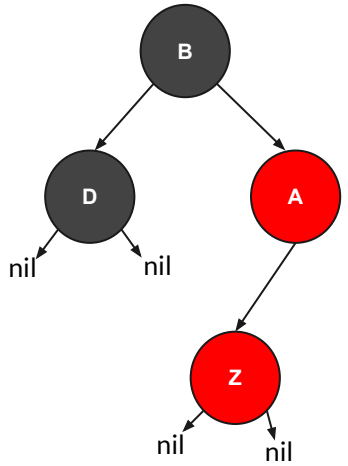


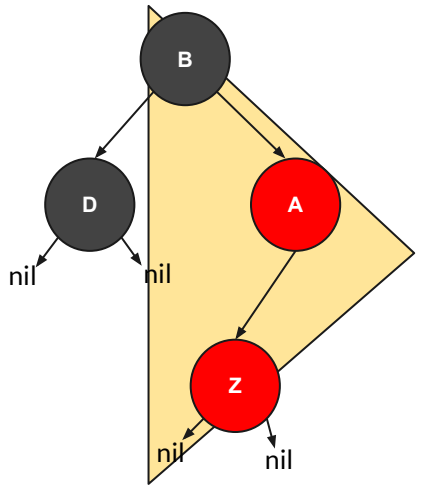
Case 2: Z's uncle is black (triangle)

Triangle: Z, its parent, and grandpa form a triangle



Case 2: Z's uncle is black (triangle)

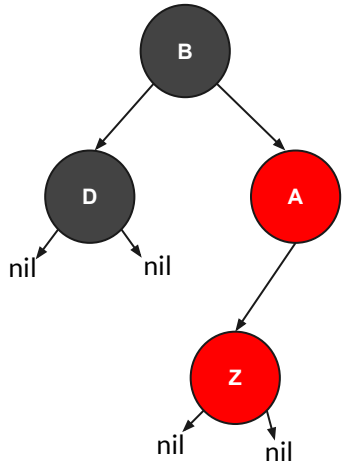
Triangle: Z, its parent, and grandpa form a triangle



Case 2: Z's uncle is black (triangle)

Triangle: Z, its parent, and grandpa form a triangle.

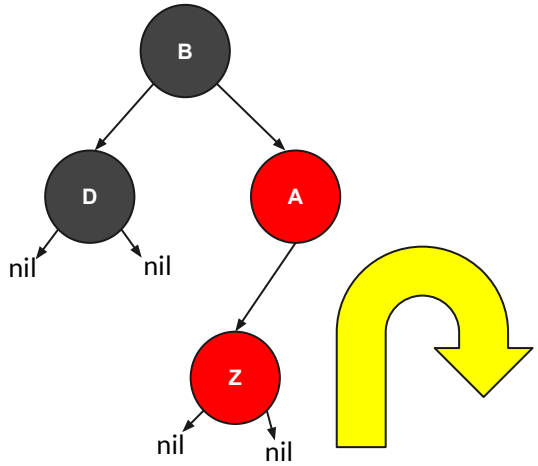
1. Rotate Z's parent in the opposite direction to Z.



Case 2: Z's uncle is black (triangle)

Triangle: Z, its parent, and grandpa form a triangle.

1. Rotate Z's parent in the opposite direction to Z.

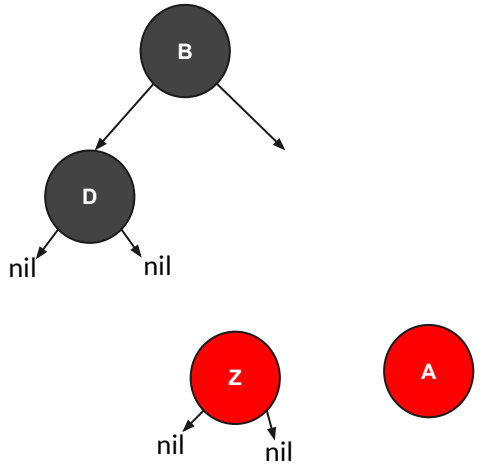


In this example, Z is on the left, so we rotate A to the right.

Case 2: Z's uncle is black (triangle)

Triangle: Z, its parent, and grandpa form a triangle.

1. Rotate Z's parent in the opposite direction to Z.

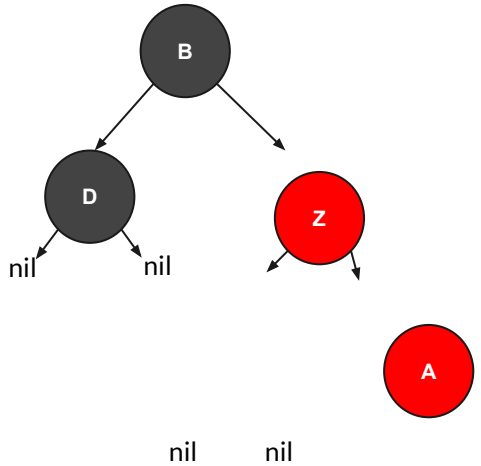


In this example, Z is on the left, so we rotate A to the right.

Case 2: Z's uncle is black (triangle)

Triangle: Z, its parent, and grandpa form a triangle.

1. Rotate Z's parent in the opposite direction to Z.



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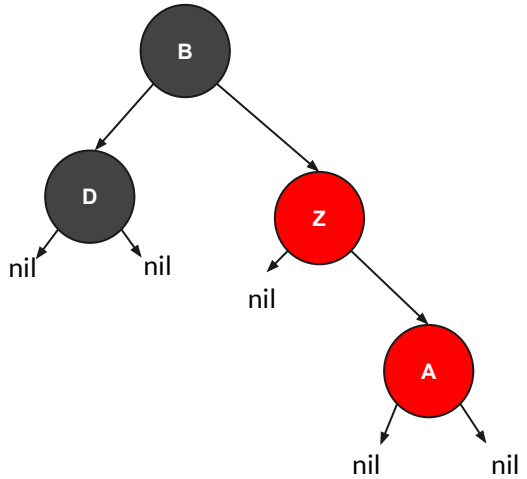
Case 2: Z's uncle is black (triangle)

Triangle: Z, its parent, and grandpa form a triangle.

1. Rotate Z's parent in the opposite direction to Z.

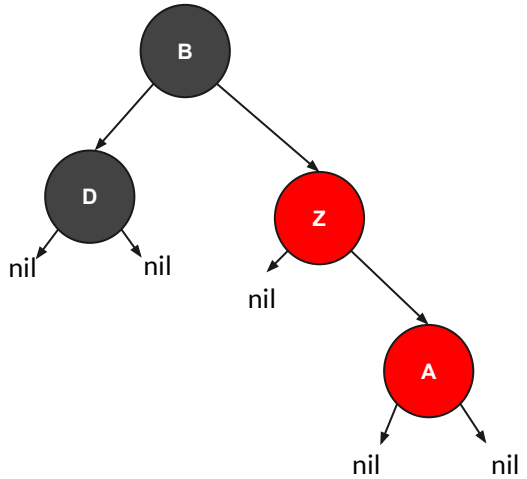


In this example, Z is on the left, so we rotate A to the right.

Case 2: Z's uncle is black (triangle)

Triangle: Z, its parent, and grandpa form a triangle.

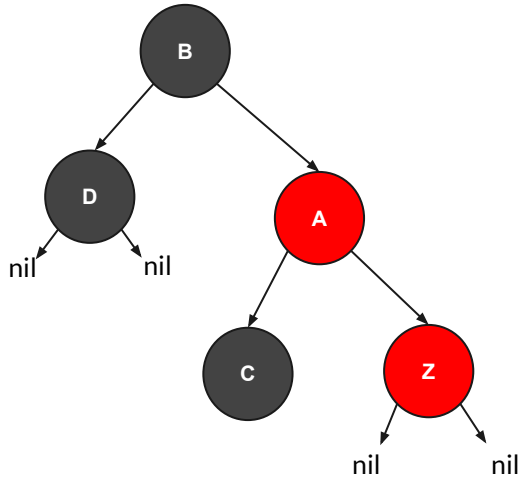
1. Rotate Z's parent in the opposite direction to Z.



I realize that this oversimplified example does not seem to solve the problem yet. We will see this in a realistic example in a bit.

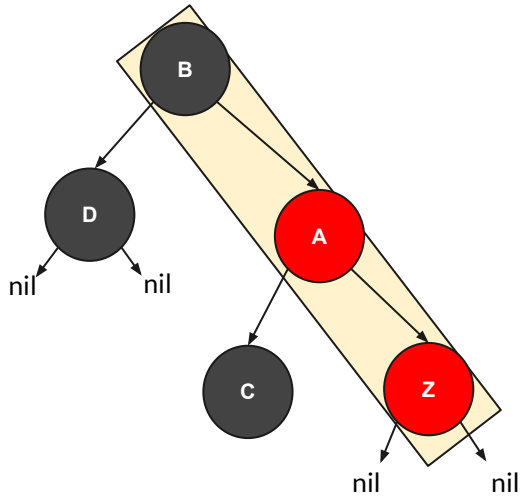
Case 3: Z's uncle is black (line)

Line: Z, its parent, and grandpa form a line.



Case 3: Z's uncle is black (line)

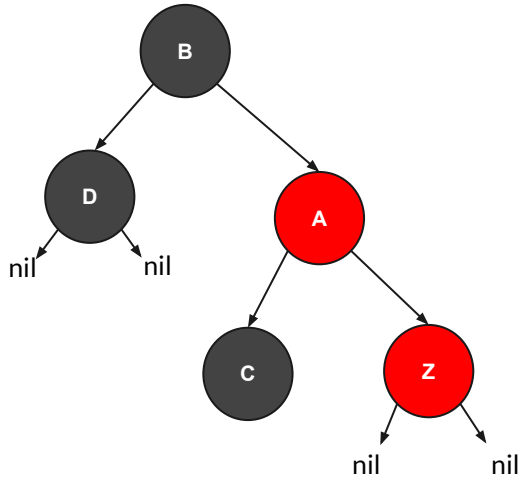
Line: Z, its parent, and grandpa form a line.



Case 3: Z's uncle is black (line)

Line: Z, its parent, and grandpa form a line.

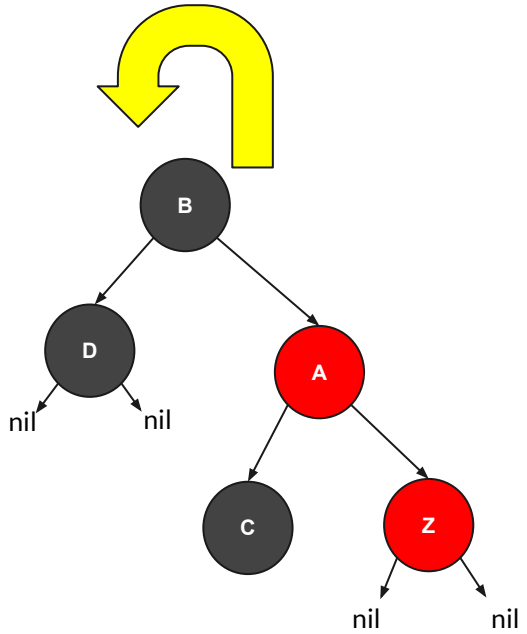
1. Rotate Z's **grandparent** in the opposite direction to Z.



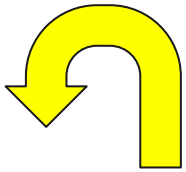
Case 3: Z's uncle is black (line)

Line: Z, its parent, and grandpa form a line.

1. Rotate Z's **grandparent** in the opposite direction to Z.

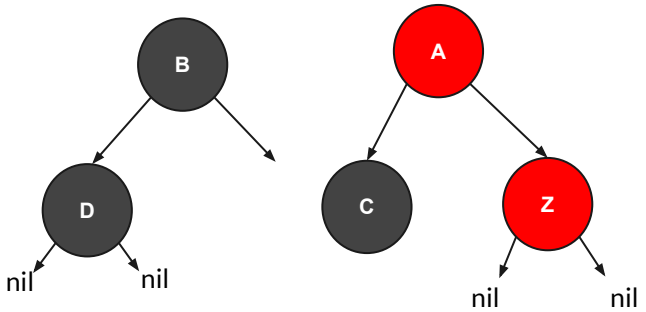


Case 3: Z's uncle is black (line)



Line: Z, its parent, and grandpa form a line.

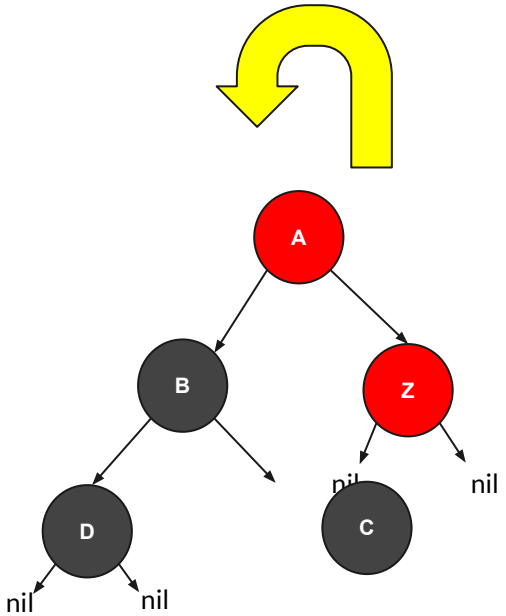
1. Rotate Z's **grandparent** in the opposite direction to Z.



Case 3: Z's uncle is black (line)

Line: Z, its parent, and grandpa form a line.

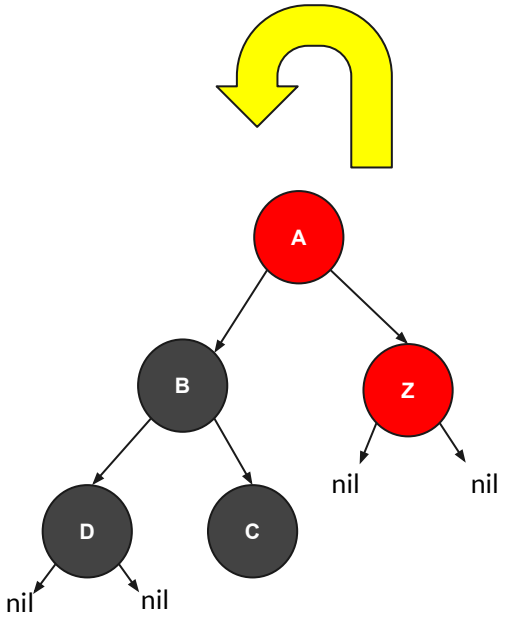
1. Rotate Z's **grandparent** in the opposite direction to Z.



Case 3: Z's uncle is black (line)

Line: Z, its parent, and grandpa form a line.

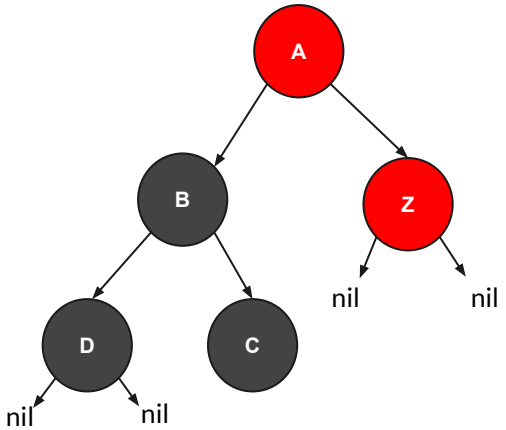
1. Rotate Z's **grandparent** in the opposite direction to Z.



Case 3: Z's uncle is black (line)

Line: Z, its parent, and grandpa form a line.

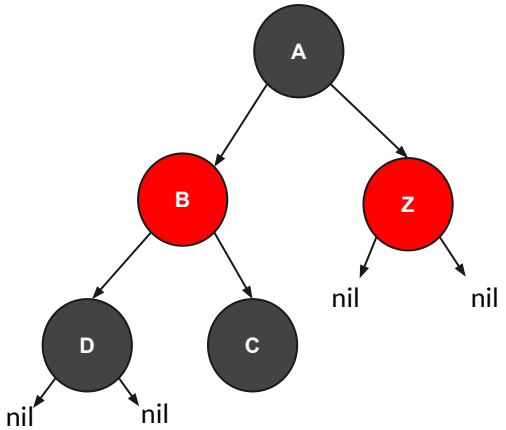
1. Rotate Z's **grandparent** in the opposite direction to Z.
2. Recolor parent and grandparent.



Case 3: Z's uncle is black (line)

Line: Z, its parent, and grandpa form a line.

1. Rotate Z's **grandparent** in the opposite direction to Z.
2. Recolor parent and grandparent.



Fixup scenarios summary

Case 0: Z is the root => color Z black

Case 1: Z's uncle is red => recolor parent, grandparent, and uncle

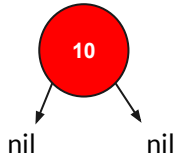
Case 2: Z's uncle is black (triangle) => rotate parent

Case 3: Z's uncle is black (line) => rotate grandparent + recolor parent and grandparent

Fixup pseudocode: case 0

Fixup pseudocode: Case 0

```
RB-Insert-fixup(T,z) {
 while(z's parent is Red) {
    set y to be z's uncle
    if uncle y is Red {
          color parent and uncle black
          color grandparent red
          set z to grandparent
    }
    else {  // the uncle is black
         if (triangle) {
                   set z to parent
                   rotate to parent
                  }
         // rotate the grandparent and finish
         color parent of z black
         color grandparent of z red
         rotate grand parent of z
       }
  } // end while
 color root black
}
```



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                   set z to parent
                   rotate to parent
                  }
         // rotate the grandparent and finish
         color parent of z black
         color grandparent of z red
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       }
  } // end while
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}
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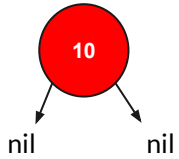

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         rotate grand parent of z
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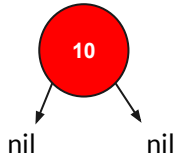

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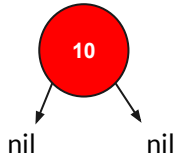

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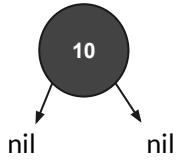

Fixup pseudocode: case 1

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         rotate grand parent of z
        }
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```

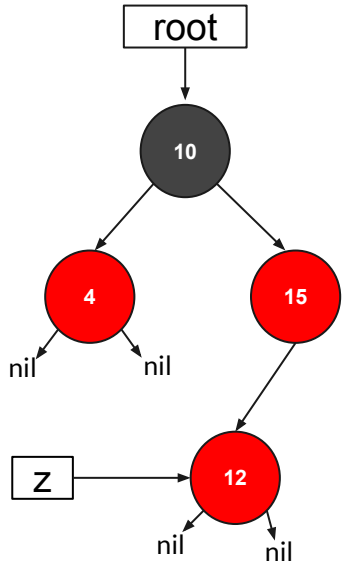

Fixup pseudocode: Case 1

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RB-Insert-fixup(T,z) {
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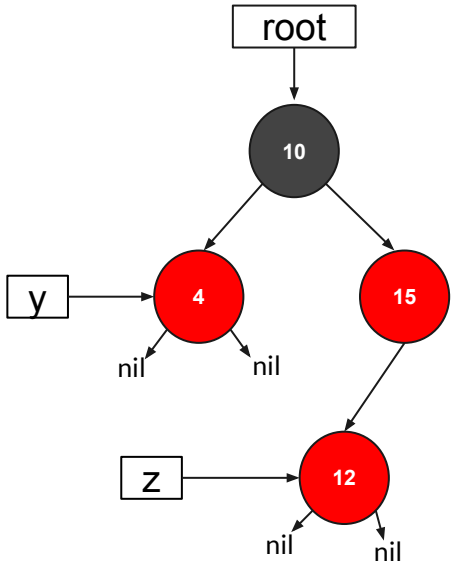
```



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         if (triangle) {
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}
```

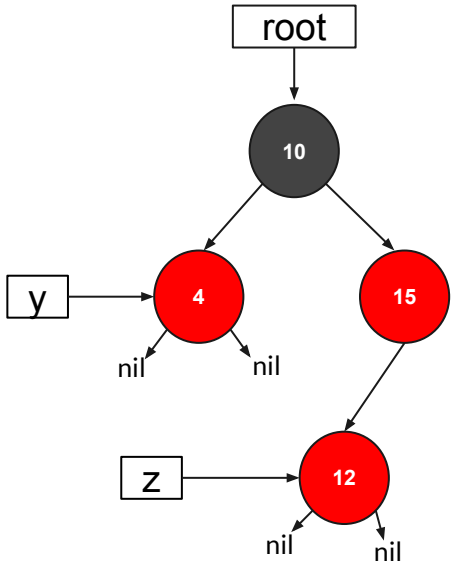

Fixup pseudocode: Case 1

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RB-Insert-fixup(T,z) {
 while(z's parent is Red) {
    set y to be z's uncle
    if uncle y is Red {
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          set z to grandparent
    }
    else {  // the uncle is black
         if (triangle) {
                  set z to parent
                  rotate to parent
                }
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         color parent of z black
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         rotate grand parent of z
        }
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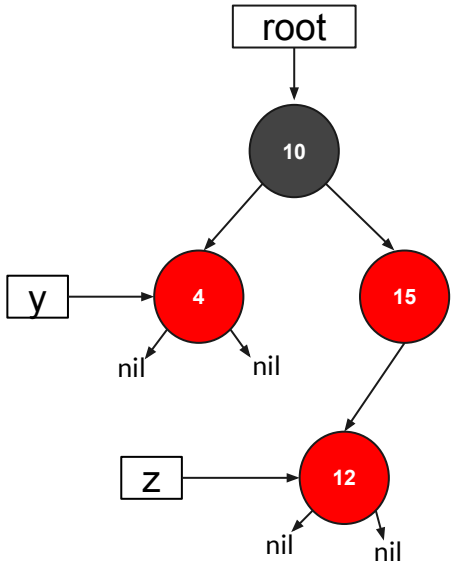
 color root black
}
```



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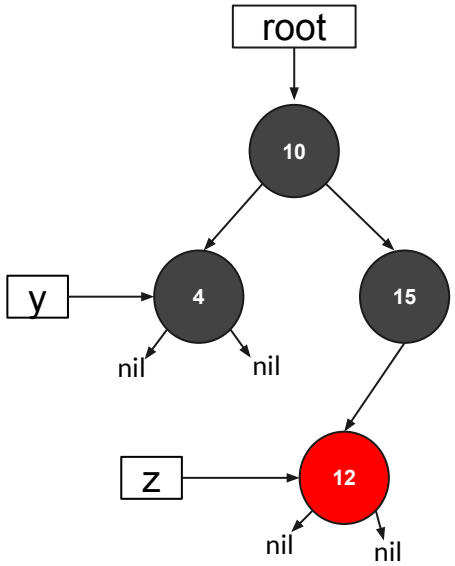

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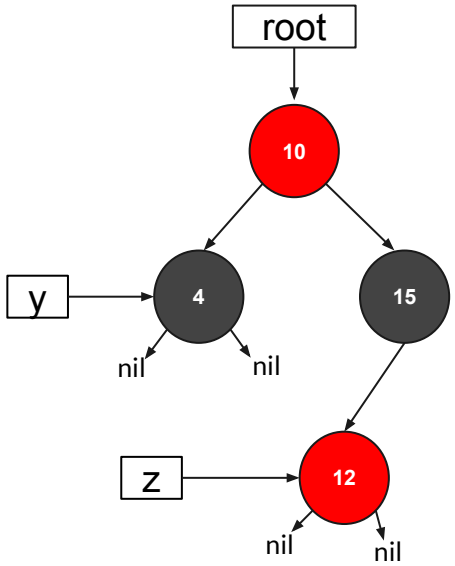

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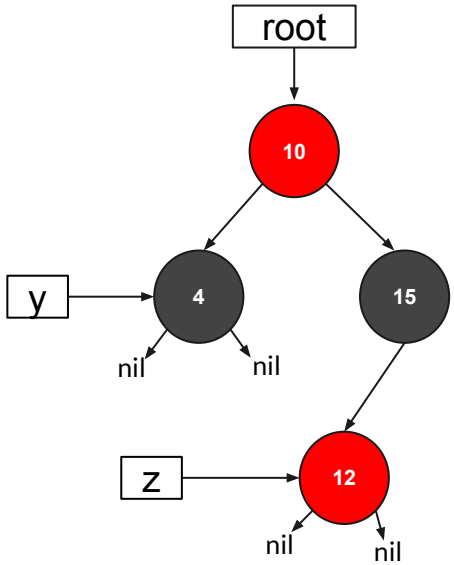

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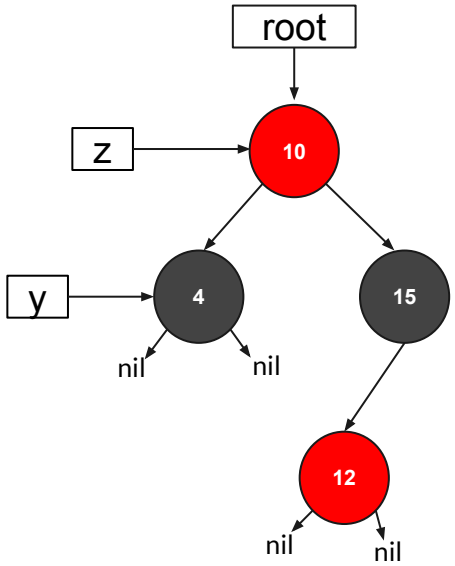

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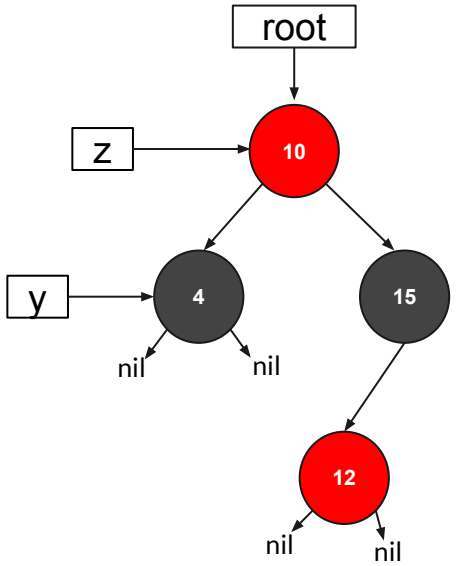

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}
```

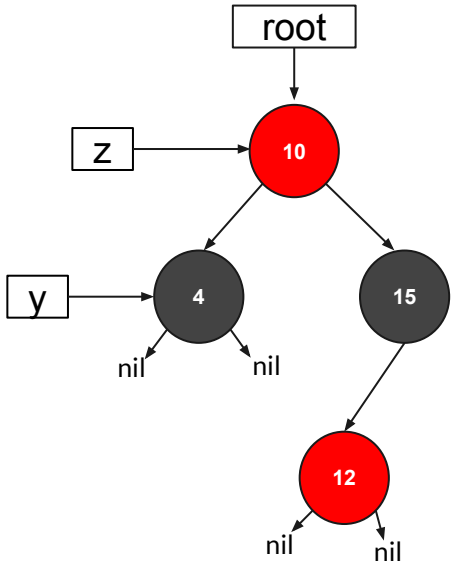

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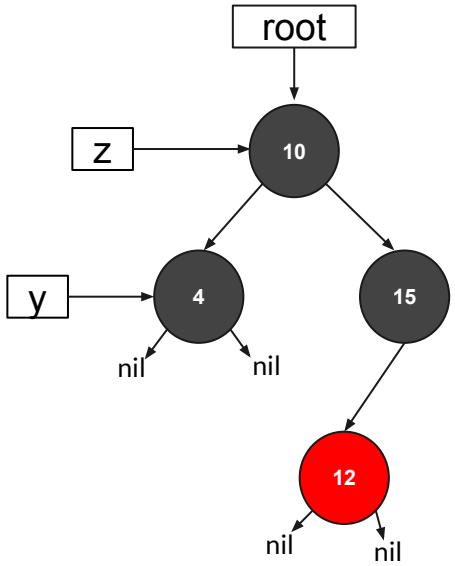

Fixup pseudocode: case 2

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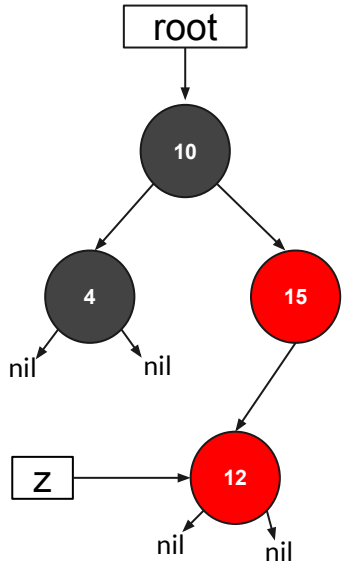
```



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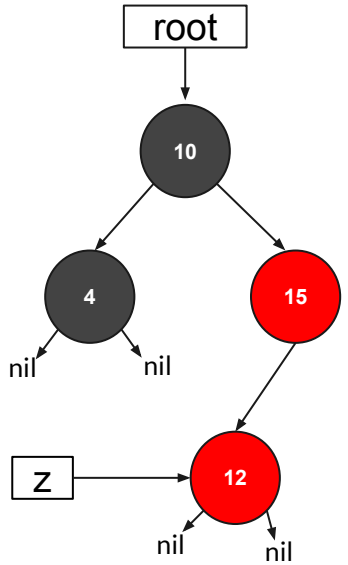

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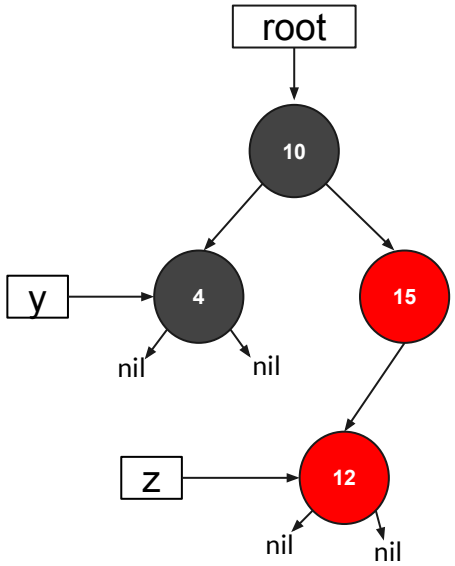

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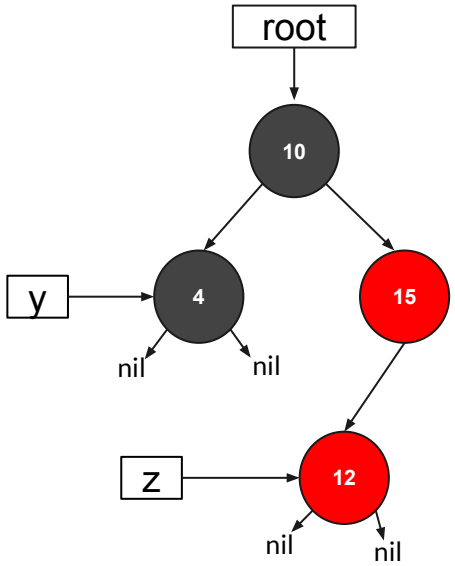

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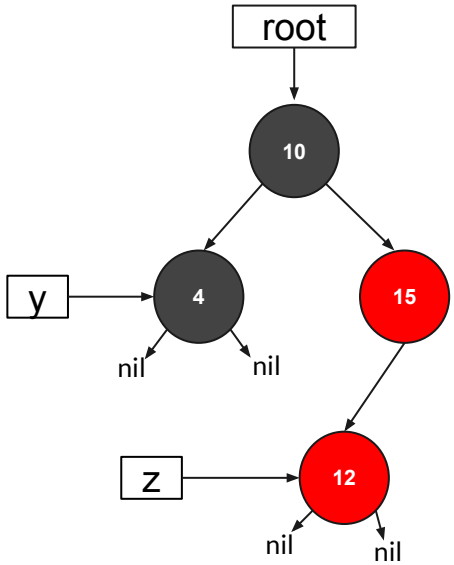

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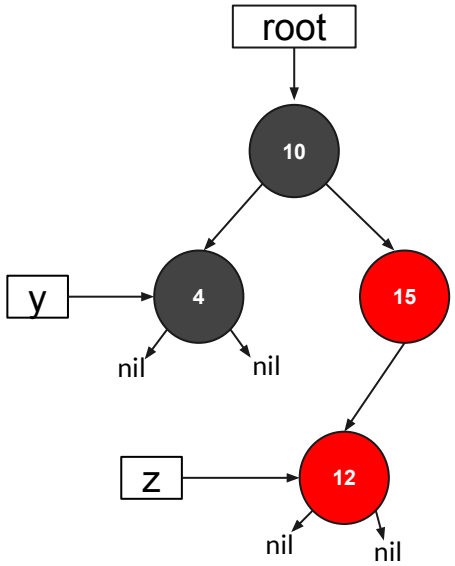

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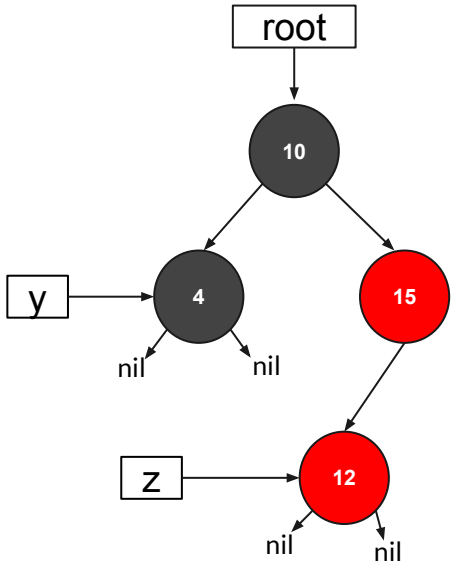

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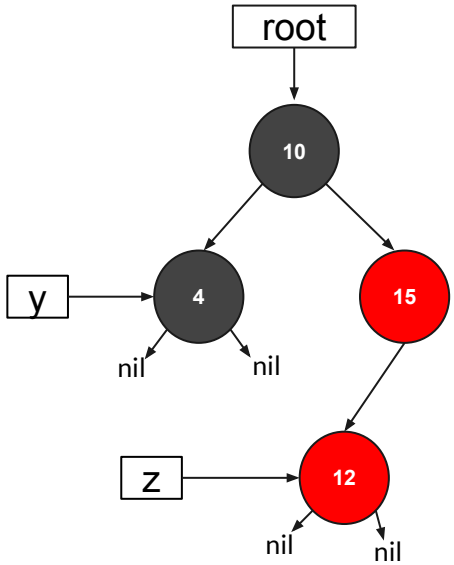

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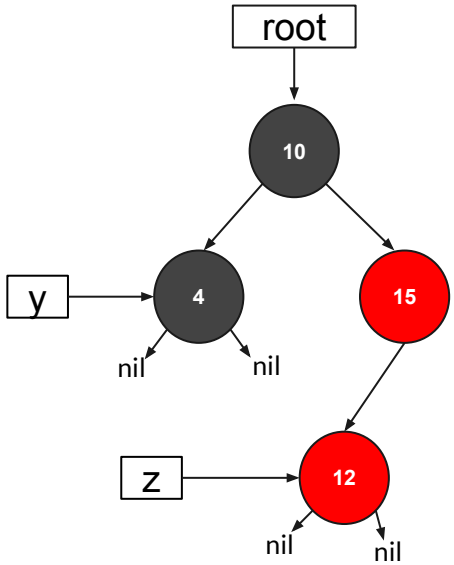

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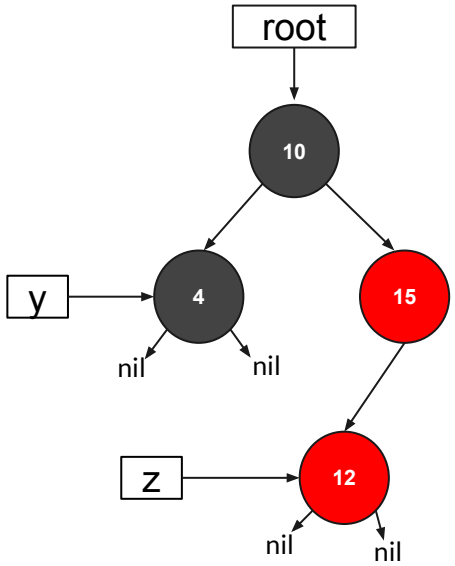

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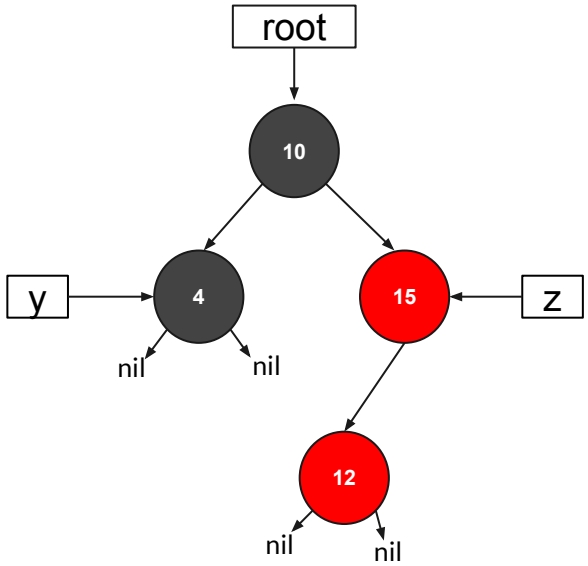

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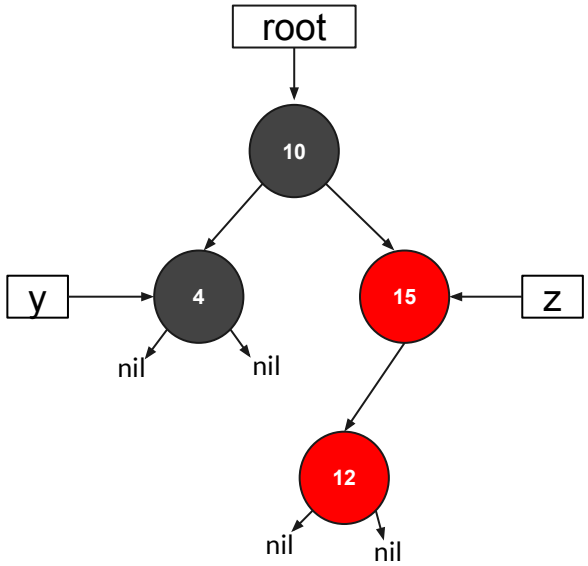

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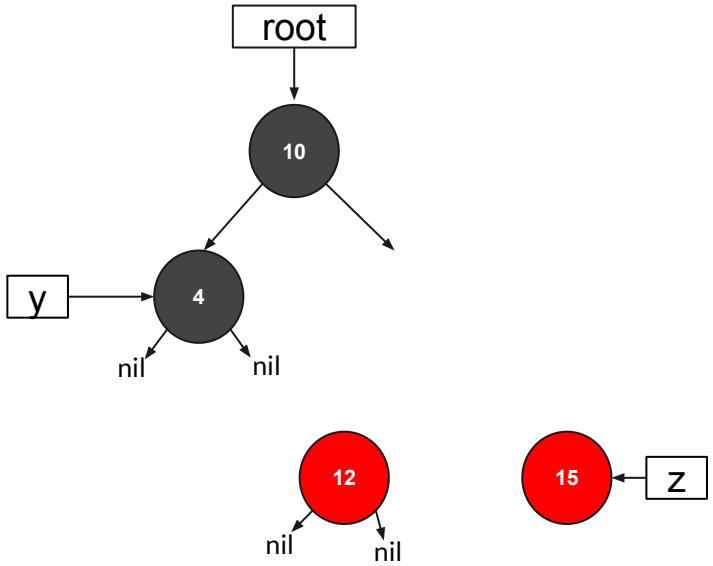

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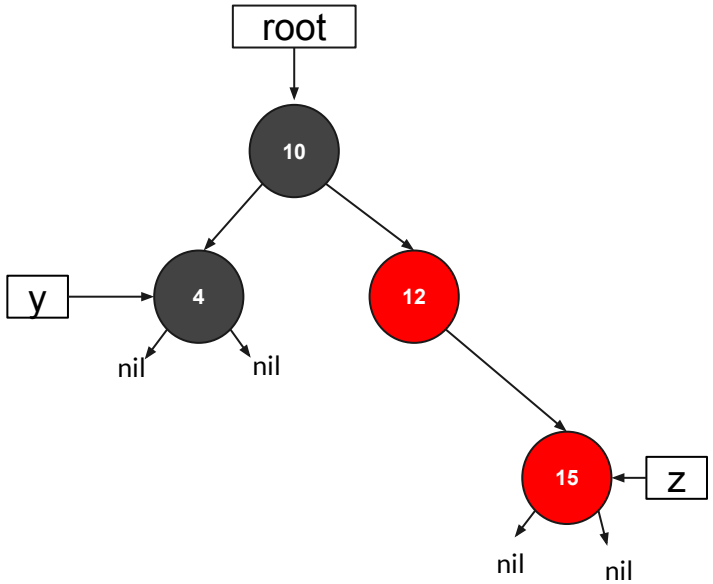

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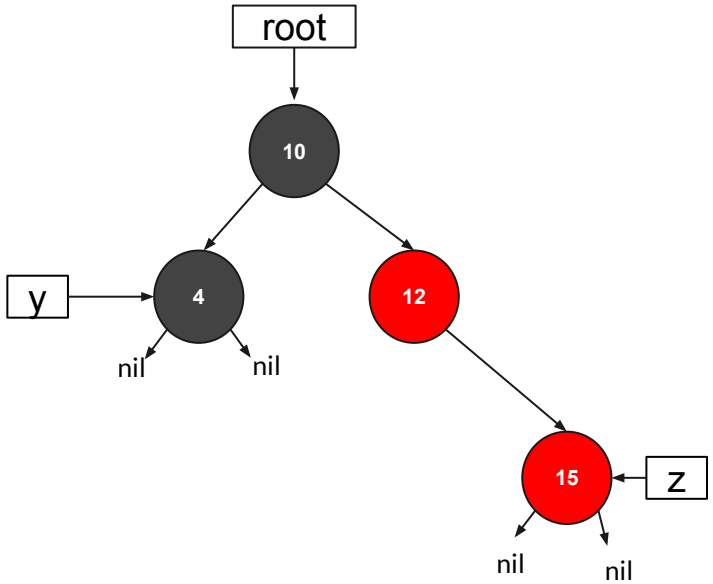

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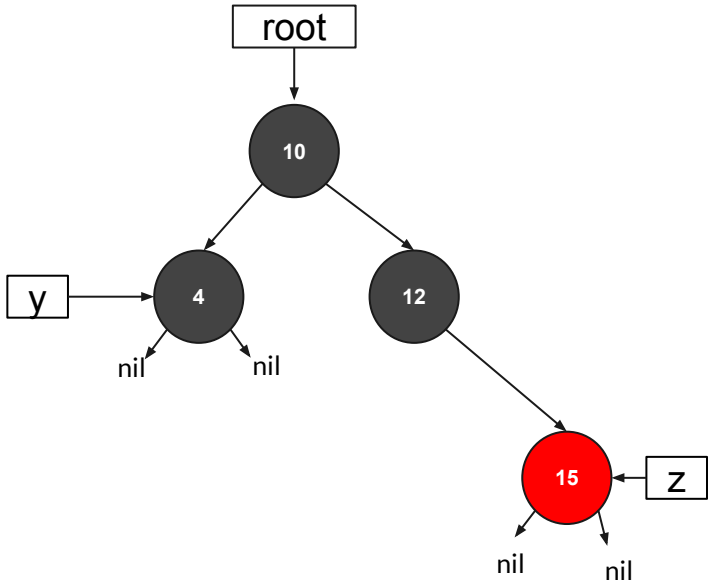

Fixup pseudocode: Case 2

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RB-Insert-fixup(T,z) {
 while(z's parent is Red) {
    set y to be z's uncle
    if uncle y is Red {
          color parent and uncle black
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          set z to grandparent
    }
    else {  // the uncle is black
         if (triangle) {
                   set z to parent
                   rotate to parent
                  }
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         color parent of z black
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         rotate grand parent of z
        }
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 color root black
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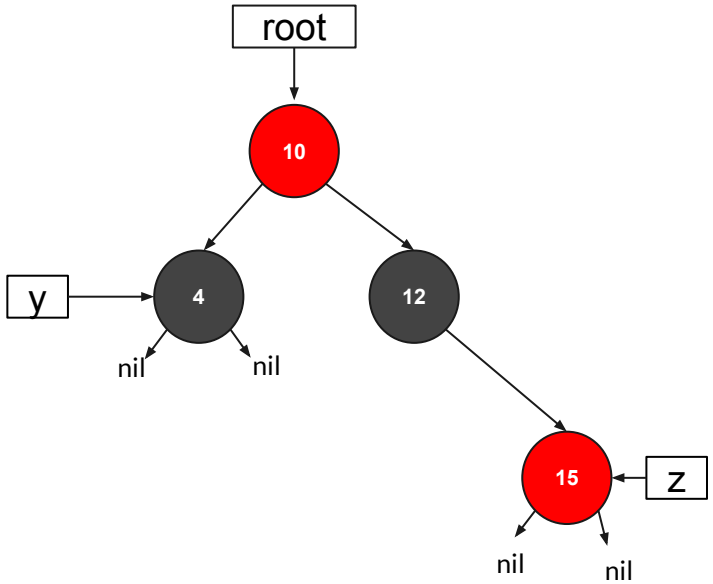

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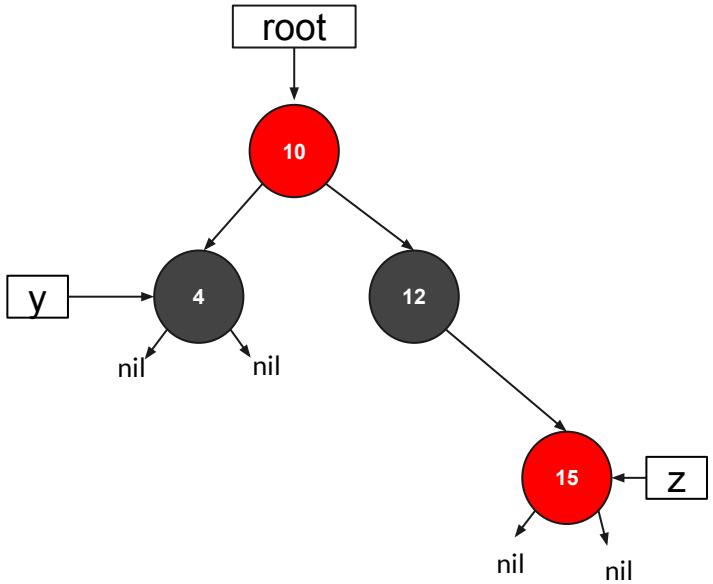

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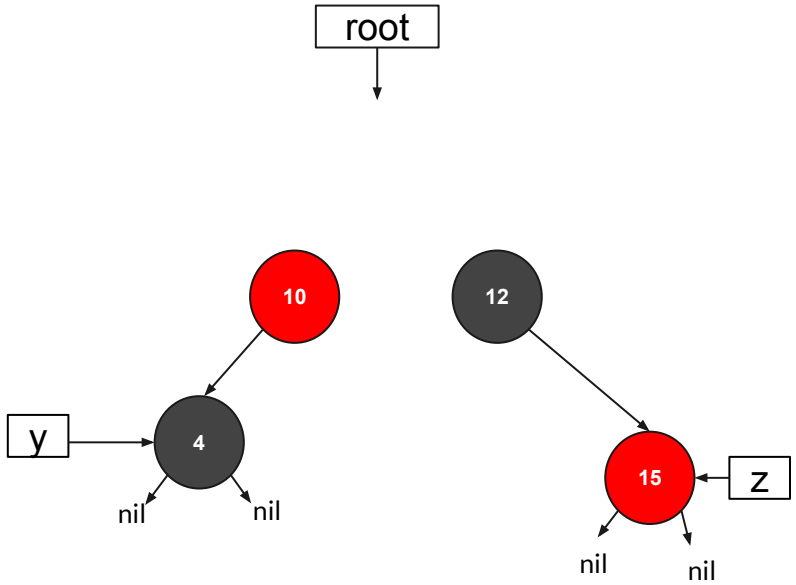

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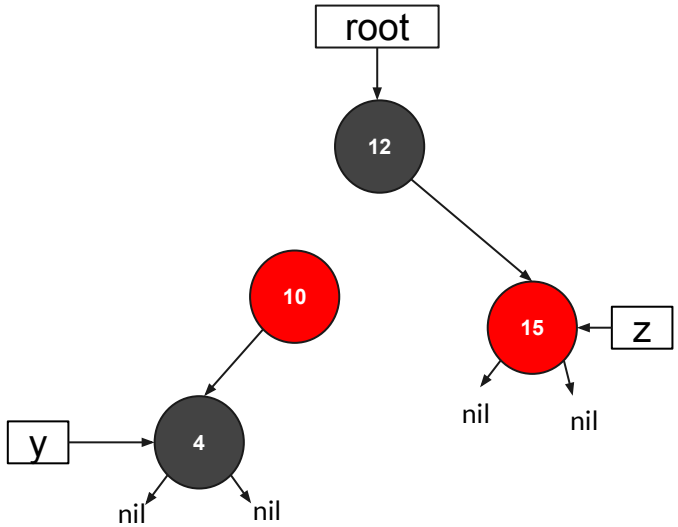

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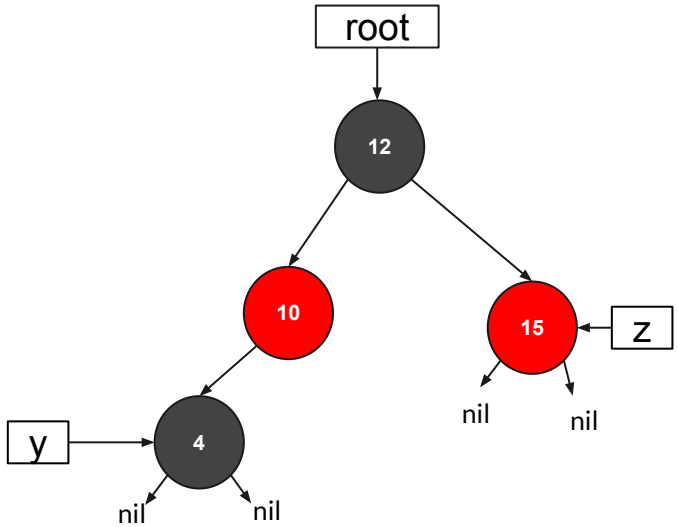

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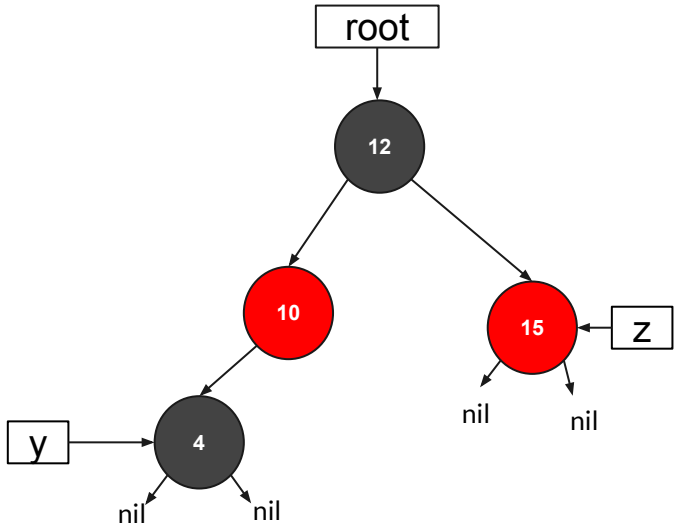

Fixup pseudocode: case 3

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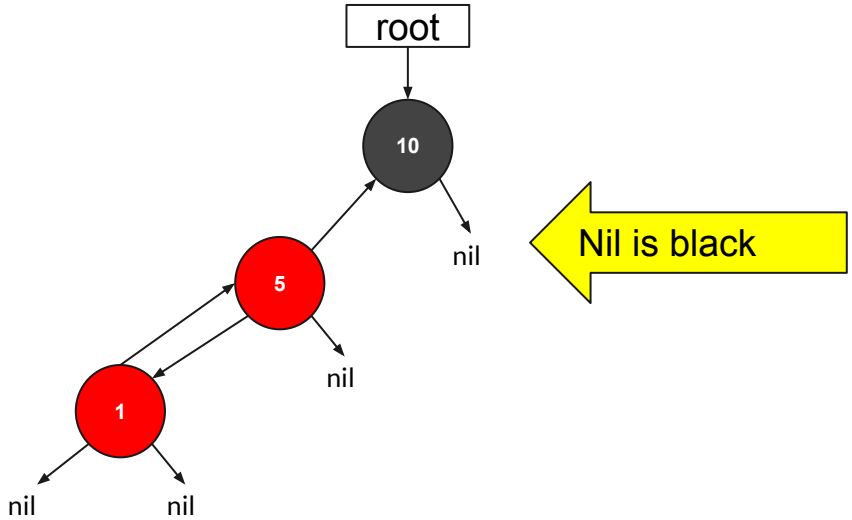

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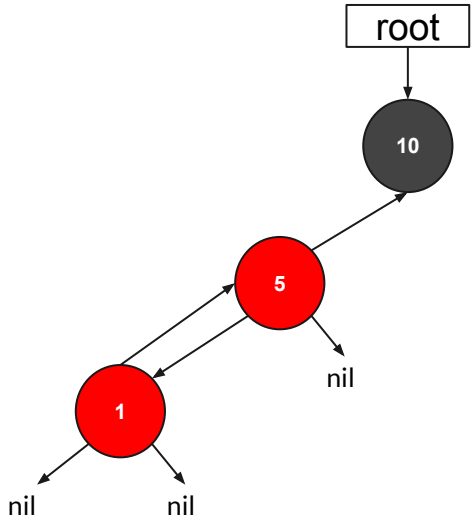

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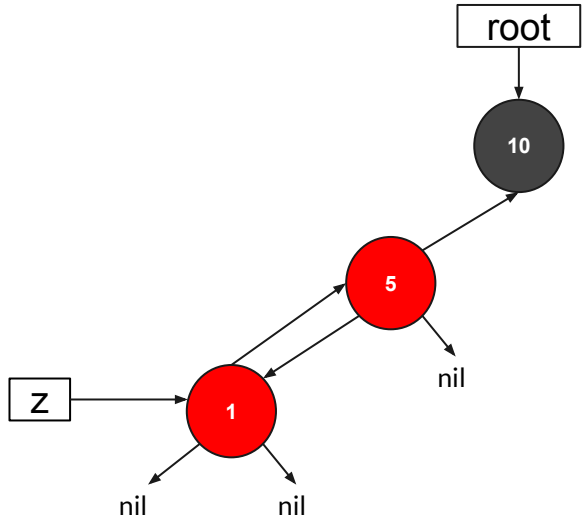

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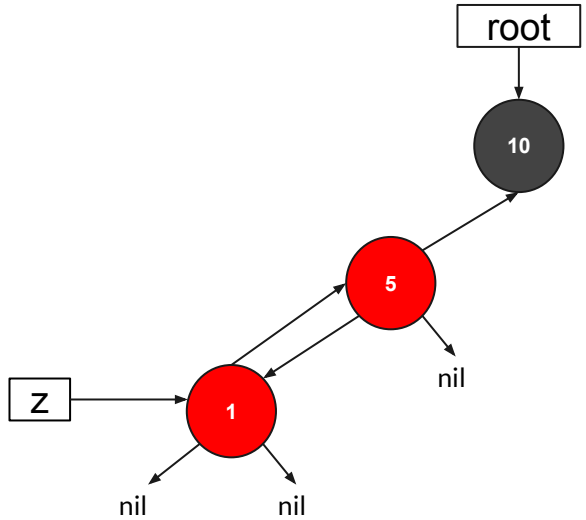

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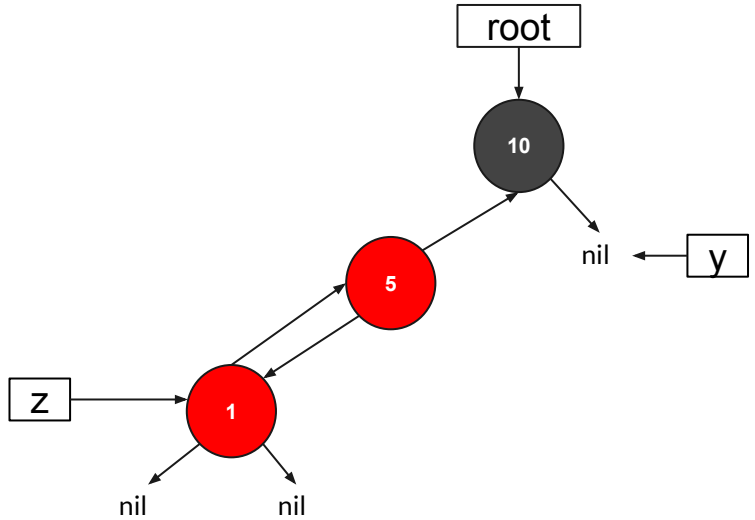

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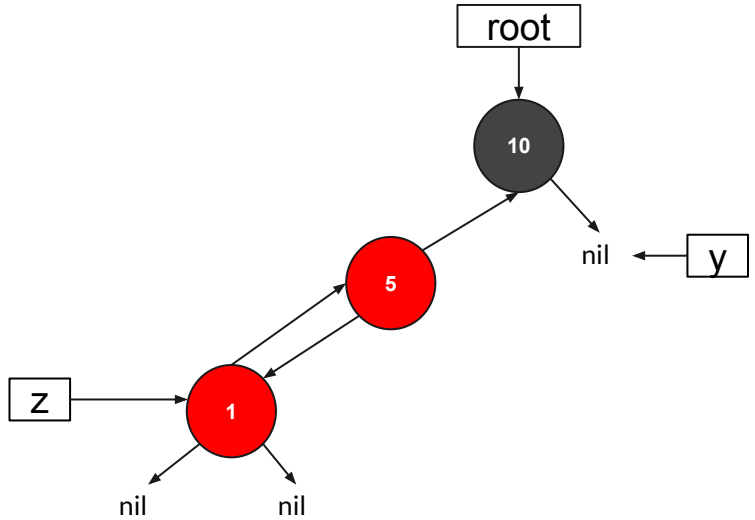

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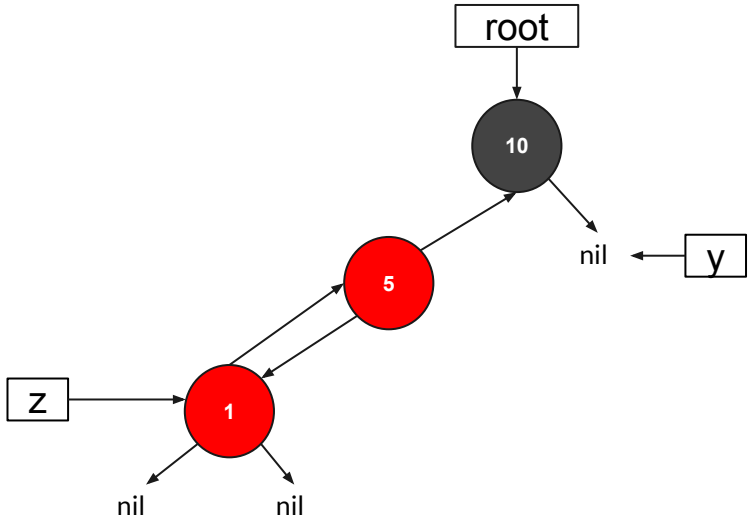

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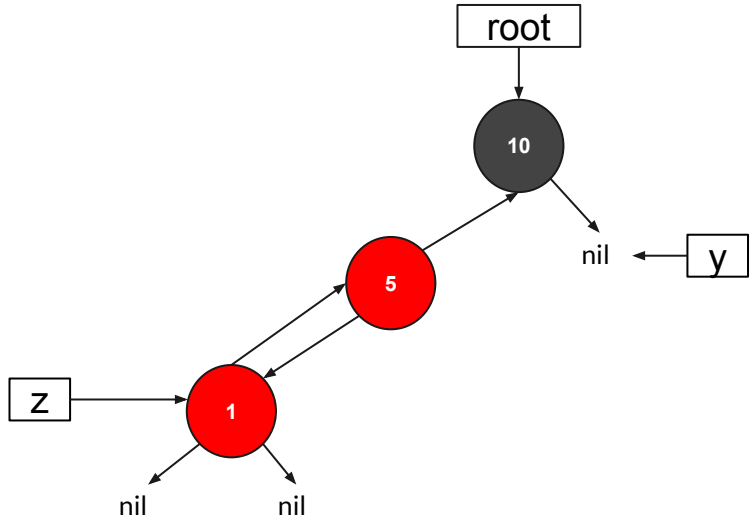

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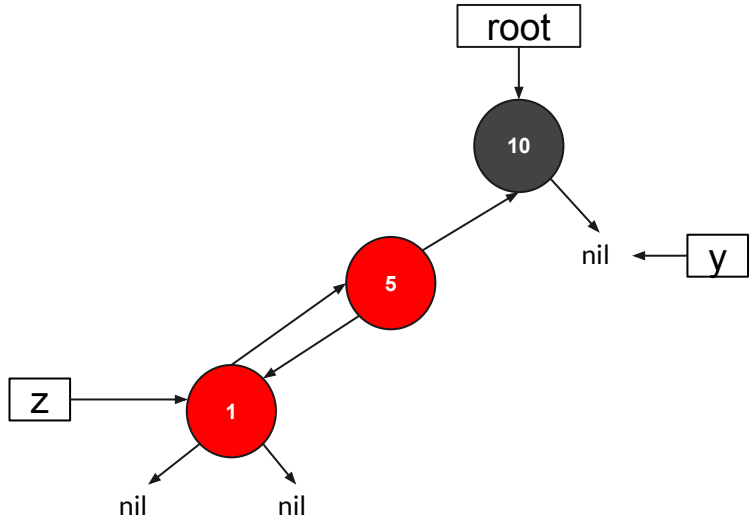

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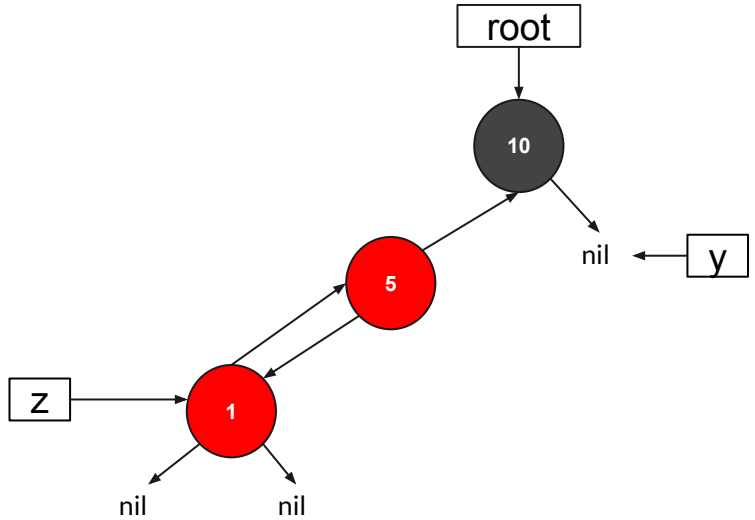

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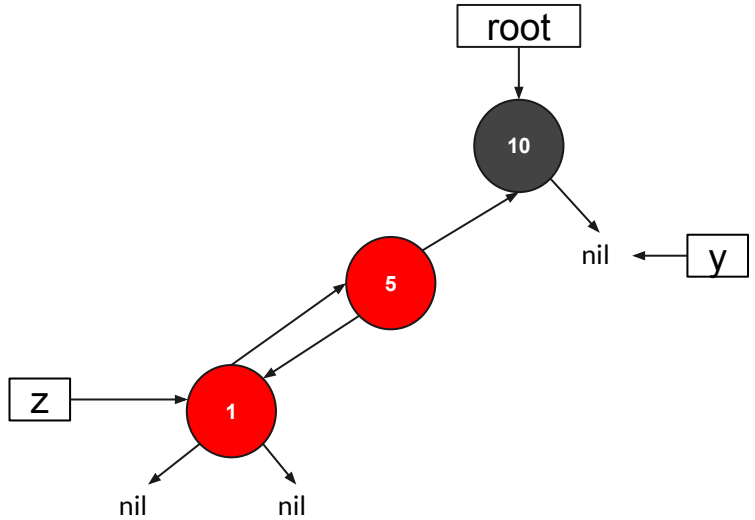

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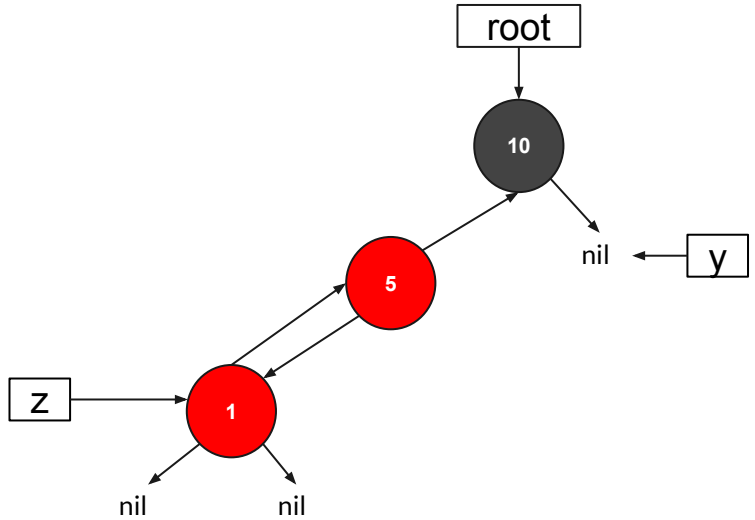

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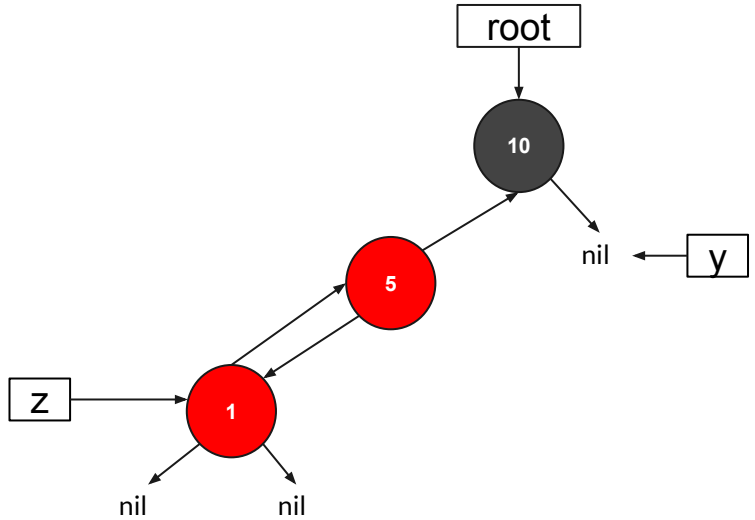

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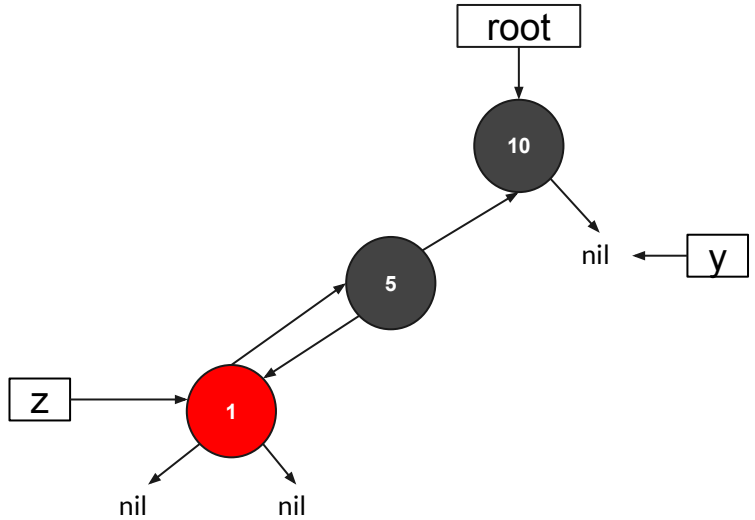

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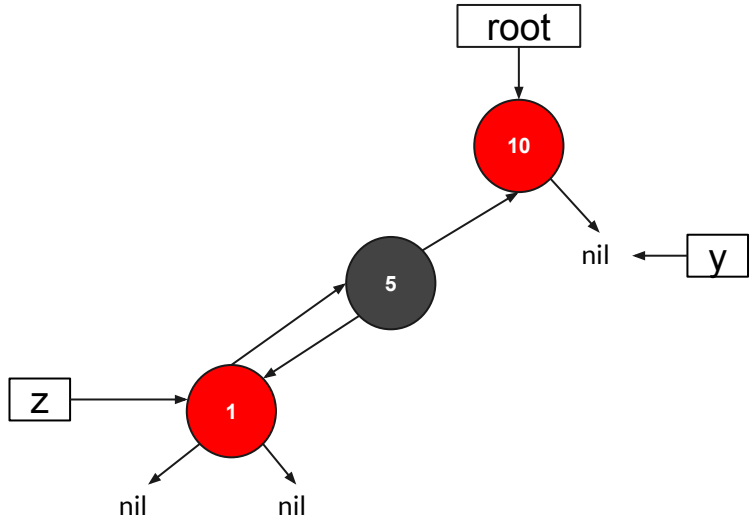

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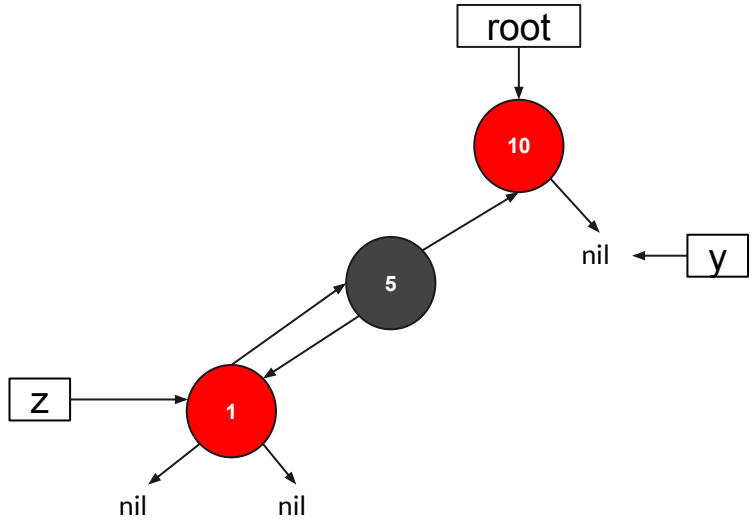

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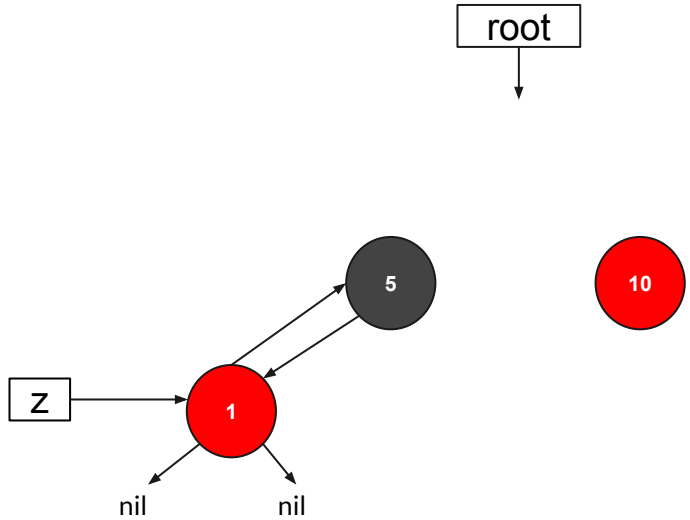

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}
```
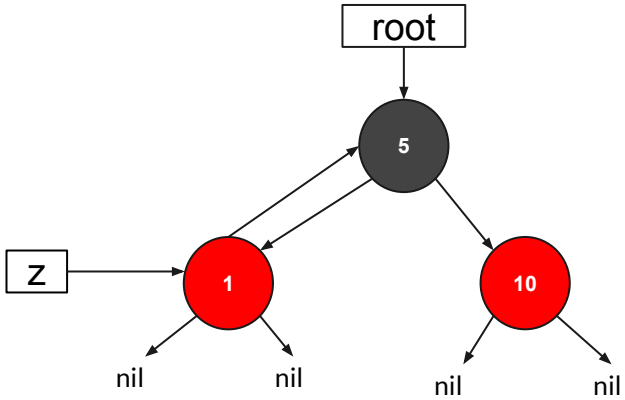
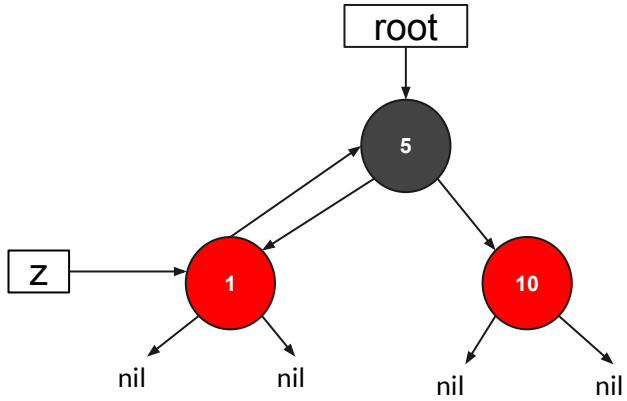



Fixup pseudocode: Case 3

```
RB-Insert-fixup(T,z) {
 while(z's parent is Red) {
    set y to be z's uncle
    if uncle y is Red {
          color parent and uncle black
          color grandparent red
          set z to grandparent
    }
    else {  // the uncle is black
         if (triangle) {
                  set z to parent
                  rotate to parent
                }
         // rotate the grandparent and finish
         color parent of z black
         color grandparent of z red
         rotate grand parent of z
       }
  } // end while
 color root black
}
```



Red-Black Tree Visualization

https://www.cs.usfca.edu/~galles/visualization/RedBlack.html

References

- Introduction to Operating Systems (Prof. Chester Rebeiro, IIT Madras)
- Understanding the Linux Kernel, 3rd Edition By Daniel P. Bovet, Marco Cesati
- Linux Kernel Development By Robert Love
- Red-black trees tutorial by Michael Sambol
- https://www.andrew.cmu.edu/user/mm6/95-771/examples/RedBlackTreeProject/dist/javadoc/redblacktreeproject/RedBlackTree.html