Contents

Preface xiii

I Foundations

_	Introduction 3
1	The Role of Algorithms in Computing 5 1.1 Algorithms 5 1.2 Algorithms as a technology 11
2	Getting Started 16 2.1 Insertion sort 16 2.2 Analyzing algorithms 23 2.3 Designing algorithms 29
3	Growth of Functions 43 3.1 Asymptotic notation 43 3.2 Standard notations and common functions 53
*	 Divide-and-Conquer 65 4.1 The maximum-subarray problem 68 4.2 Strassen's algorithm for matrix multiplication 75 4.3 The substitution method for solving recurrences 83 4.4 The recursion-tree method for solving recurrences 88 4.5 The master method for solving recurrences 93 4.6 Proof of the master theorem 97
<i>5</i> ★	 Probabilistic Analysis and Randomized Algorithms 114 5.1 The hiring problem 114 5.2 Indicator random variables 118 5.3 Randomized algorithms 122 5.4 Probabilistic analysis and further uses of indicator random variables 130

II Sorting and Order Statistics

Introduction	147

6 Heapsort 151

- 6.1 Heaps *151*
- 6.2 Maintaining the heap property 154
- 6.3 Building a heap 156
- 6.4 The heapsort algorithm 159
- 6.5 Priority queues 162

7 Quicksort 170

- 7.1 Description of quicksort 170
- 7.2 Performance of quicksort 174
- 7.3 A randomized version of quicksort 179
- 7.4 Analysis of quicksort 180

8 Sorting in Linear Time 191

- 8.1 Lower bounds for sorting 191
- 8.2 Counting sort 194
- 8.3 Radix sort 197
- 8.4 Bucket sort 200

9 Medians and Order Statistics 213

- 9.1 Minimum and maximum 214
- 9.2 Selection in expected linear time 215
- 9.3 Selection in worst-case linear time 220

III Data Structures

Introduction 229

10 Elementary Data Structures 232

- 10.1 Stacks and queues 232
- 10.2 Linked lists 236
- 10.3 Implementing pointers and objects 241
- 10.4 Representing rooted trees 246

11 Hash Tables 253

- 11.1 Direct-address tables 254
- 11.2 Hash tables 256
- 11.3 Hash functions 262
- 11.4 Open addressing 269
- ★ 11.5 Perfect hashing 277

Contents

12	Binary	Search	Trees	286
		~		

- 12.1 What is a binary search tree? 286
- 12.2 Querying a binary search tree 289
- 12.3 Insertion and deletion 294
- ★ 12.4 Randomly built binary search trees 299

13 Red-Black Trees 308

- 13.1 Properties of red-black trees 308
- 13.2 Rotations *312*
- 13.3 Insertion *315*
- 13.4 Deletion 323

14 Augmenting Data Structures 339

- 14.1 Dynamic order statistics 339
- 14.2 How to augment a data structure 345
- 14.3 Interval trees 348

IV Advanced Design and Analysis Techniques

Introduction 357

15 Dynamic Programming 359

- 15.1 Rod cutting 360
- 15.2 Matrix-chain multiplication 370
- 15.3 Elements of dynamic programming 378
- 15.4 Longest common subsequence 390
- 15.5 Optimal binary search trees 397

16 Greedy Algorithms 414

- 16.1 An activity-selection problem 415
- 16.2 Elements of the greedy strategy 423
- 16.3 Huffman codes 428
- ★ 16.4 Matroids and greedy methods 437
- ★ 16.5 A task-scheduling problem as a matroid 443

17 Amortized Analysis 451

- 17.1 Aggregate analysis 452
- 17.2 The accounting method 456
- 17.3 The potential method 459
- 17.4 Dynamic tables 463

V Advanced Data Structures

		40
ntrod	luction	48

18 B-Trees 484

- 18.1 Definition of B-trees 488
- 18.2 Basic operations on B-trees 491
- 18.3 Deleting a key from a B-tree 499

19 Fibonacci Heaps 505

- 19.1 Structure of Fibonacci heaps 507
- 19.2 Mergeable-heap operations 510
- 19.3 Decreasing a key and deleting a node 518
- 19.4 Bounding the maximum degree 523

20 van Emde Boas Trees 531

- 20.1 Preliminary approaches 532
- 20.2 A recursive structure 536
- 20.3 The van Emde Boas tree 545

21 Data Structures for Disjoint Sets 561

- 21.1 Disjoint-set operations 561
- 21.2 Linked-list representation of disjoint sets 564
- 21.3 Disjoint-set forests 568
- ★ 21.4 Analysis of union by rank with path compression 573

VI Graph Algorithms

Introduction 587

22 Elementary Graph Algorithms 589

- 22.1 Representations of graphs 589
- 22.2 Breadth-first search 594
- 22.3 Depth-first search 603
- 22.4 Topological sort 612
- 22.5 Strongly connected components 615

23 Minimum Spanning Trees 624

- 23.1 Growing a minimum spanning tree 625
- 23.2 The algorithms of Kruskal and Prim 631

Contents ix

24 Single-Source Shortest Paths 643

- 24.1 The Bellman-Ford algorithm 651
- 24.2 Single-source shortest paths in directed acyclic graphs 655
- 24.3 Dijkstra's algorithm 658
- 24.4 Difference constraints and shortest paths 664
- 24.5 Proofs of shortest-paths properties 671

25 All-Pairs Shortest Paths 684

- 25.1 Shortest paths and matrix multiplication 686
- 25.2 The Floyd-Warshall algorithm 693
- 25.3 Johnson's algorithm for sparse graphs 700

26 Maximum Flow 708

- 26.1 Flow networks 709
- 26.2 The Ford-Fulkerson method 714
- 26.3 Maximum bipartite matching 732
- ★ 26.4 Push-relabel algorithms 736
- ★ 26.5 The relabel-to-front algorithm 748

VII Selected Topics

Introduction 769

27 Multithreaded Algorithms 772

- 27.1 The basics of dynamic multithreading 774
- 27.2 Multithreaded matrix multiplication 792
- 27.3 Multithreaded merge sort 797

28 Matrix Operations 813

- 28.1 Solving systems of linear equations 813
- 28.2 Inverting matrices 827
- 28.3 Symmetric positive-definite matrices and least-squares approximation 832

29 Linear Programming 843

- 29.1 Standard and slack forms 850
- 29.2 Formulating problems as linear programs 859
- 29.3 The simplex algorithm 864
- 29.4 Duality 879
- 29.5 The initial basic feasible solution 886

30	Polynomials and the FFT 898	
	30.1 Representing polynomials 900	
	30.2 The DFT and FFT 906	
	30.3 Efficient FFT implementations 915	
31	Number-Theoretic Algorithms 926	
	31.1 Elementary number-theoretic notions 927	
	31.2 Greatest common divisor 933	
	31.3 Modular arithmetic 939	
	31.4 Solving modular linear equations 946	
	31.5 The Chinese remainder theorem 950	
	31.6 Powers of an element 954	
	31.7 The RSA public-key cryptosystem 958	
*	31.8 Primality testing 965	
*	31.9 Integer factorization 975	
32	String Matching 985	
	32.1 The naive string-matching algorithm 988	
	32.2 The Rabin-Karp algorithm 990	
	32.3 String matching with finite automata 995	
*	32.4 The Knuth-Morris-Pratt algorithm 1002	
33	Computational Geometry 1014	
	33.1 Line-segment properties 1015	
	2 71	<i>921</i>
	33.3 Finding the convex hull 1029	
	33.4 Finding the closest pair of points 1039	
34	NP-Completeness 1048	
	34.1 Polynomial time <i>1053</i>	
	34.2 Polynomial-time verification 1061	
	34.3 NP-completeness and reducibility 1067	
	34.4 NP-completeness proofs 1078	
	34.5 NP-complete problems 1086	
35	Approximation Algorithms 1106	
	35.1 The vertex-cover problem 1108	
	35.2 The traveling-salesman problem 1111	
	35.3 The set-covering problem 1117	
	35.4 Randomization and linear programming 1123	
	35.5 The subset-sum problem 1128	

VIII Appendix: Mathematical Background

	Introduction 1143
A	Summations 1145
	A.1 Summation formulas and properties 1145
	A.2 Bounding summations 1149
В	Sets, Etc. 1158
	B.1 Sets 1158
	B.2 Relations 1163
	B.3 Functions 1166
	B.4 Graphs 1168
	B.5 Trees 1173
C	Counting and Probability 1183
	C.1 Counting <i>1183</i>
	C.2 Probability 1189
	C.3 Discrete random variables 1196
	C.4 The geometric and binomial distributions 1201
*	C.5 The tails of the binomial distribution 1208
D	Matrices 1217
	D.1 Matrices and matrix operations 1217
	D.2 Basic matrix properties 1222

Bibliography 1231

Index 1251