MA274, Fall 2021 — Problem Set 1

These problems are mostly about sets, set membership, and subsets, reflecting the material in chapter 1 of *Numbers, Space, and the Structures of Mathematics*, which I'll denote by NSSM. You should read chapter 1 carefully, working through the many examples.

Most of these problems ask you to write a proof. The proof structures provided in NSSM are intended to help you do this. Please use them. Most of the proofs should be quite short.

This assignment does not need to be done with LaTeX, but if you are want to try to do that, go for it. I will place a (simplified) copy of the LaTeX file for this problem set on the course Moodle page.

Grading for problem sets will work like this:

a. I will randomly select four problems to be graded.

b. The grader will assign up to two points per question. So if you get all questions correct you will have eight points so far.

c. The grader will add up to two "style points." These will reflect how well you have explained your answers.

The resulting grade should be interpreted roughly as follows: 9 is a "perfect score"; 10 means you have done something exceptionally good; scores above 7 mean you are doing ok; anything less than that is an alarm bell.

1. In NSSM, the notation for an open interval is (a, b). The notation for a point in $\mathbb{R}^2$ is (x, y), which looks exactly the same. An alternative is to write $]a, b[$ for the open interval. Is that notation better or worse? (There is no absolute answer, of course, but you should state your reasons for preferring one or the other.)

2. Show that the interval $(-1, 1)$ is a subset of the interval $[-2, 4]$.

3. Show that the interval $(0, 1)$ is a subset of the set $X = \{x \in \mathbb{R} : x^2 \leqslant x\}$. Are the two sets the same?

4. NSSM, Section 1.7, Exercise 2.

5. NSSM, Section 1.7, Exercise 4.

6. NSSM, Section 1.7, Exercise 7.

7. NSSM, Section 1.7, Exercise 11.

8. NSSM, Section 1.7, Exercise 14.

9. NSSM, Section 1.7, Exercise 16.

10. Let X be a set. Explain why these two statements say exactly the same thing:

- $a \in X$
- $\{a\} \subset X$

(By "explain" I mean that you should write one or two sentences in English that unpack the meaning of both notations, thereby showing that they say the same thing.)