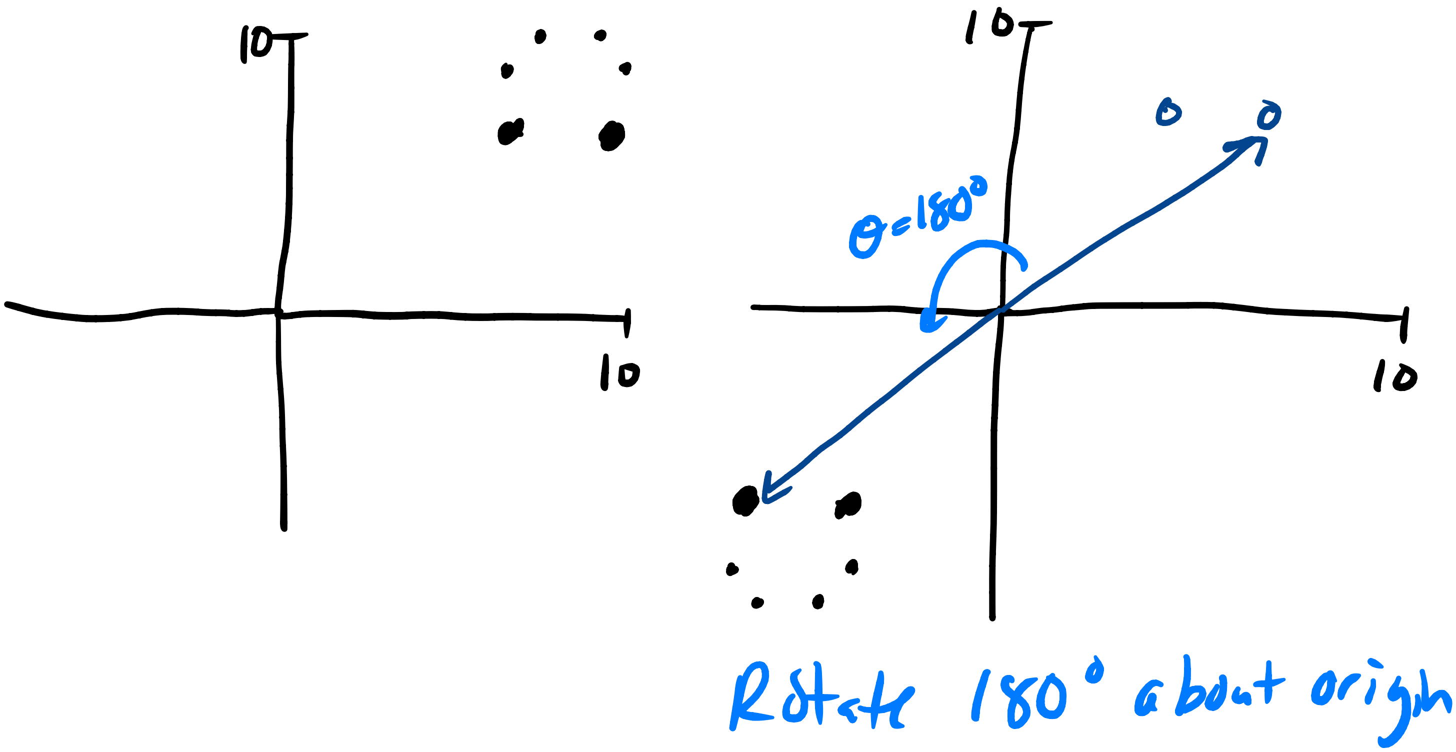
2D Rotation Matrix:

Another common operation on data that we can accomplish rotate data about a point, axis, or vector.

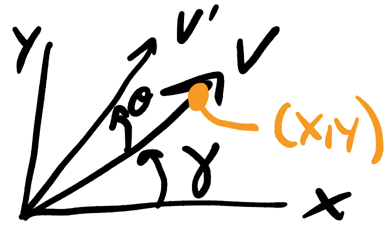
Example:



* Above, think of each data sample being end point of vector thru origin $\Rightarrow$ vectors each rotated by $180° = \theta$.

- We can determine actual mapping by converting to polar coordinates and considering a case where the data vector is not x-axis aligned:

$x = r\cos\gamma$
$y = r\sin\gamma$



$$x' = r\cos(\gamma+\theta) = r[\underbrace{\cos\gamma}_{x}\cos\theta - \underbrace{\sin\gamma}_{y}\sin\theta]$$

$$= x\cos\theta - y\sin\theta$$

$$y' = r\sin(\gamma+\theta) = r[\underbrace{\sin\gamma}_{y}\cos\theta + \underbrace{\cos\gamma}_{x}\sin\theta]$$

$$= y\cos\theta + x\sin\theta$$

$$x' = x\cos\theta - y\sin\theta$$
$$y' = x\sin\theta + y\cos\theta$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \overbrace{\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}}^{R(\theta)} \begin{bmatrix} x \\ y \end{bmatrix}$$

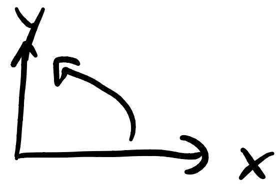
↑ rotated data sample

↑ Data Sample

☆ Makes sense $\cos\theta$ on main diagonal. If $\theta = 0$:

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} \Rightarrow \text{identity!} \quad \begin{matrix} x' = x \\ y' = y \end{matrix}$$

If $\theta = 90^\circ$



$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} \qquad \begin{matrix} x' = 0 \\ y' = x \end{matrix} \quad \checkmark$$

Makes Sense!

Add homogeneous coord to $R(\theta)$:

$$R(\theta) = \underbrace{\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{3 \times 3}$$

Apply to entire data matrix A :

$$\underset{\text{smile}}{A'} = (R(180) @ \underset{\text{frown}}{A}.T).T$$