Matrix Multiplication

We will need to multiply matrices storing data to perform operations on them next week.
The matrix multiplication will apply a transformation to the data. What these are will be the focus of project 2.

Matrix multiplication:

- We will review how to multiply matrices. Important to know, but in this class focus will be:

  1) How do we setup matrix multiplication so Numpy can do the math for us?

  2) How do we interpret results when we apply matrix multiplication to data?

- Useful to understand what Numpy is doing & debug simple examples.

☐ Computing AB:

Matrix multiplication between matrix A & matrix B.

1) Before doing AB:

we check to see if the shapes are valid.

☆ Need <u>inner dimensions</u> of shapes to match:

e.g. A.shape = (2, 2) B.shape = (2, 10)

$$AB = (2, \underline{2}) \times (\underline{2}, 10)$$

match! multiplication valid!

e.g. AC C.shape = (3, 2)

$$AC = (2, \underline{2}) \times (\underline{3}, 2)$$

<u>mismatch</u>! Cannot do it!

- If we have a mismatch but need to do the multiplication, we can <u>transpose</u> a matrix to try to get matching inner dimensions.

numpy notation to do transpose

$C : (3, 2)$ $\qquad\qquad$ $C.T : (2, 3)$

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix} \xrightarrow{\text{Transpose}} \begin{bmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \end{bmatrix}$$

$\longleftrightarrow$

flip row and cols

$A\,C.T : (2, \underline{2}) \times (\underline{2}, 3)$

works!

2) If the shapes are compatible, we figure out the final output shape

$\Rightarrow$ Output.shape = (# rows of left mat, # cols of right mat)

e.g. $AB = (2, 2) \times (2, 10)$

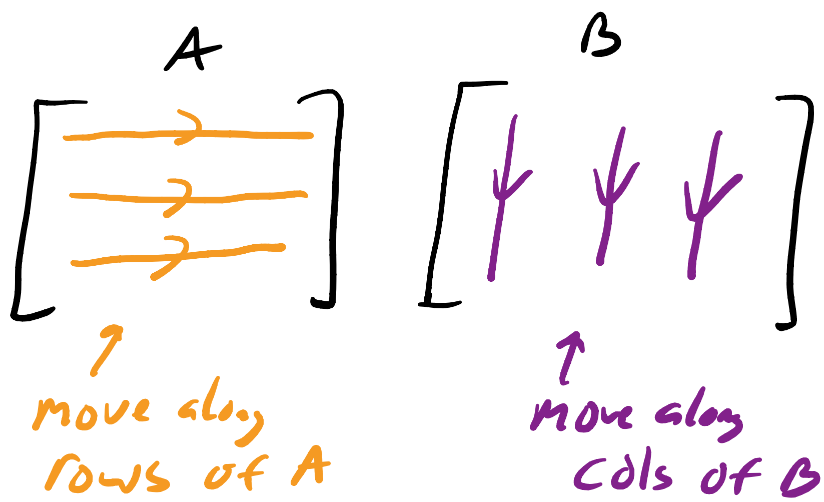
$$\Rightarrow \underbrace{(2, 10)}_{\text{output shape}}$$

3) To multiply matrices A and B we:

a) Go along the **rows** of the left matrix (A)

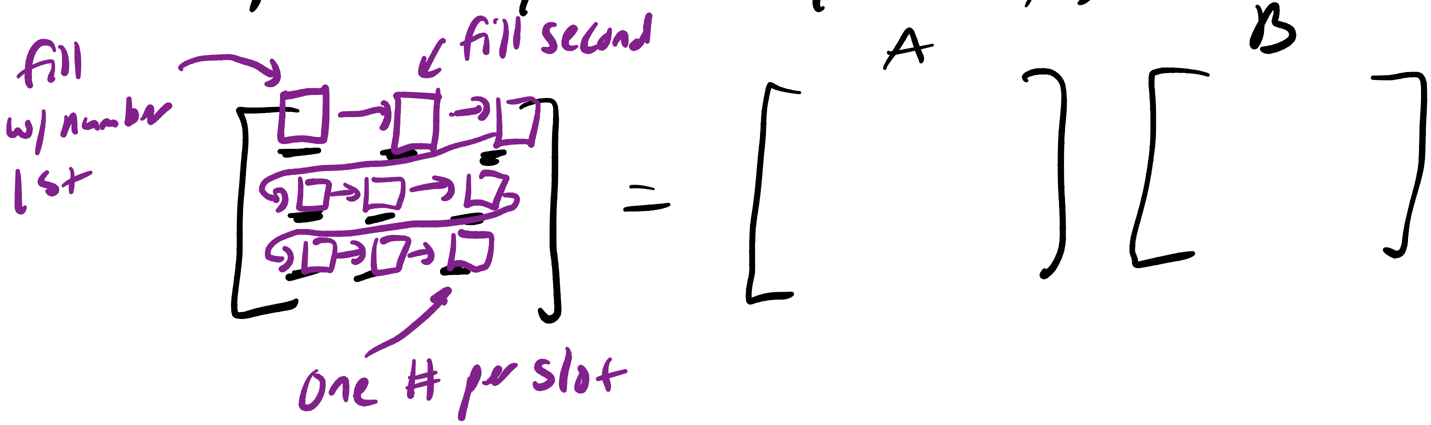
b) Go along the cols of the right matrix (B)

e.g. $AB =$



c) we fill in entries in "blank" output matrix one-by-one starting top-left, working rightward going row-by-row.

e.g. A.shape = B.shape = (3, 3)



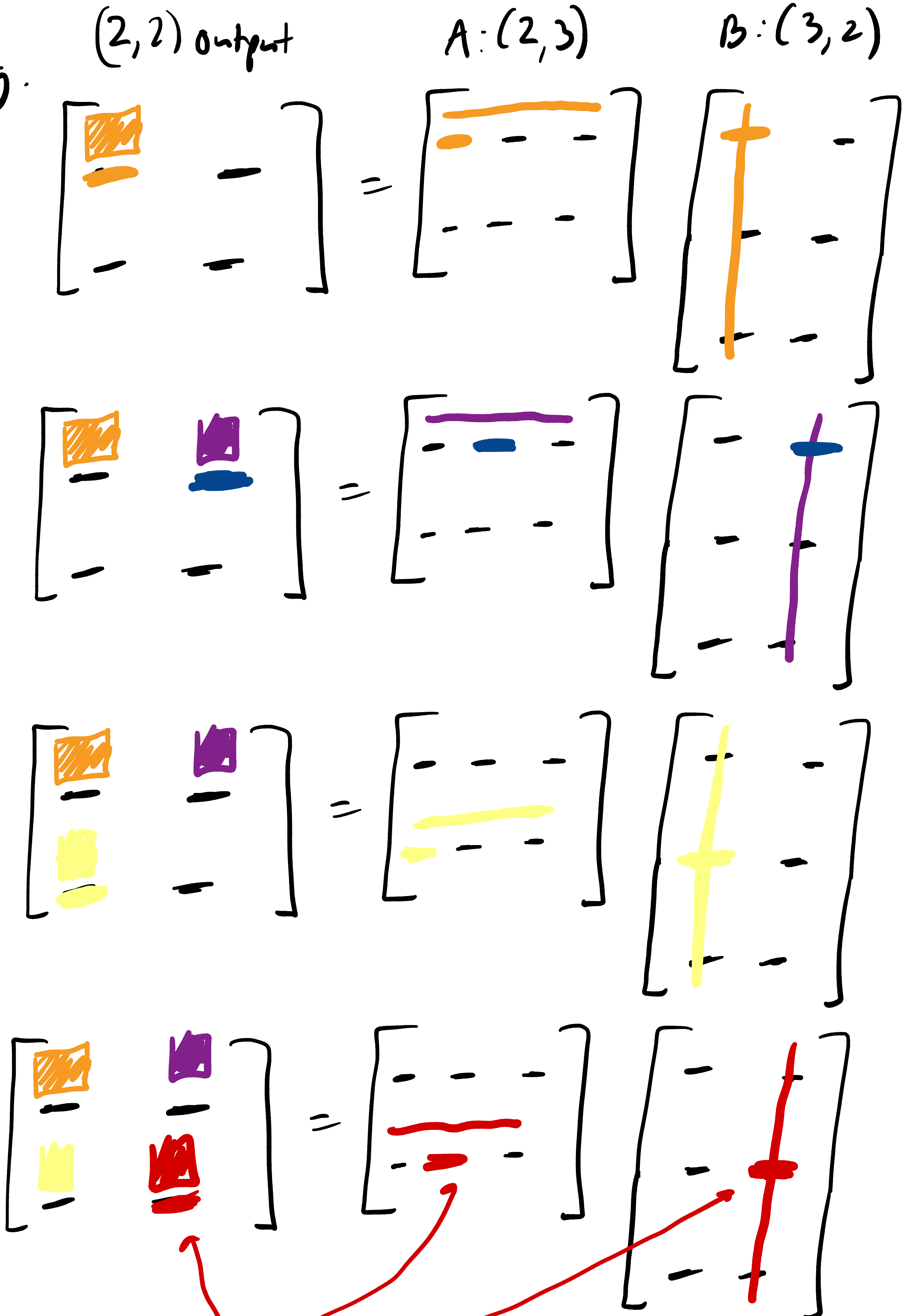
d) Each output matrix value determined by multiplying paired items in

- A rows
- B cols

Then adding them up.

A Result from each computation = <u>Scalar always</u>

e.g.



row/col of output entry matches intersection of current row in A, col in B.

example with numbers:

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \qquad B = \begin{bmatrix} 0 & 2 \\ 4 & 6 \\ 1 & 1 \end{bmatrix}$$

shape = (2, 2) shape = (3, 2)

$$AB \Rightarrow (2, 2) \times (3, 2)$$

mismatch!

Assume we actually still want to multiply A + B.

Try: $B.T : (2, 3) = \begin{bmatrix} 0 & 4 & 1 \\ 2 & 6 & 1 \end{bmatrix}$

$$A\,B.T = (2, 2) \times (2, 3)$$

Output shape = (2, 3)

$$\begin{bmatrix} \underline{*1} & \underline{*2} & \underline{*3} \\ \underline{*4} & \underline{*5} & \underline{*6} \end{bmatrix} = \begin{array}{cc} A & B.T \\ \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} & \begin{bmatrix} 0 & 4 & 1 \\ 2 & 6 & 1 \end{bmatrix} \end{array}$$

$*1 = 1 \cdot 0 + 2 \cdot 2 = 4$

$*2 = 1 \cdot 4 + 2 \cdot 6 = 16$

$*3 = 1 \cdot 1 + 2 \cdot 1 = 3$

$*4 = 3 \cdot 0 + 4 \cdot 2 = 8$

$*5 = 3 \cdot 4 + 4 \cdot 6 = 12 + 24 = 36$

$*6 = 3 \cdot 1 + 4 \cdot 1 = 7$

$$AB.T = \begin{bmatrix} 4 & 16 & 3 \\ 8 & 36 & 7 \end{bmatrix} \Rightarrow \text{Done!}$$

e) Order matters with matrix multiplication, unlike Scalar multiplication:

$$\begin{array}{l} a = 1 \\ b = 2 \end{array} \Rightarrow \overset{1}{a} * \overset{2}{b} = \overset{2}{b} * \overset{1}{a} = 2 \checkmark$$

Not true for matrices: $\boxed{AB \mathrel{!}= BA}$

e.g. $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ $B = \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}$

$$AB = \begin{bmatrix} 1\cdot 5 + 2\cdot 7 & 1\cdot 6 + 2\cdot 8 \\ 3\cdot 5 + 4\cdot 7 & 3\cdot 6 + 4\cdot 8 \end{bmatrix} = \begin{bmatrix} 19 & 22 \\ 43 & 50 \end{bmatrix}$$

$\updownarrow$!=

$$BA = \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 5\cdot 1 + 6\cdot 3 & 5\cdot 2 + 6\cdot 4 \\ 7\cdot 1 + 8\cdot 3 & 7\cdot 2 + 8\cdot 4 \end{bmatrix} = \begin{bmatrix} 23 & 34 \\ 31 & 46 \end{bmatrix}$$

Now let's see how to do Matrix multiplication with Numpy