

## Lecture 7

### Translation Summary:

$T$ : translation matrix

$A$ : Data matrix  
"M"

$$A' = (T @ A \cdot T) \cdot T$$

Translating 3 Variables by amounts  $t_x, t_y, t_z$

$$T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$y$

$t_x$

$t_y$

$t_z$

$y$

Assignment to  $y$  for plug in

$np.eye(4)$  4x4 identity matrix

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

## A Column of 1s in A

$$\begin{array}{c} T \\ \overbrace{\quad\quad\quad}^T \\ \left[ \begin{array}{cccc} 1 & 0 & 0 & -32 \\ 0 & 1 & 0 & -80 \\ 0 & 0 & 1 & 10 \\ 0 & 0 & 0 & 1 \end{array} \right] \end{array} \qquad \begin{array}{c} A.T \\ \overbrace{\quad\quad\quad}^{A.T} \\ \left[ \begin{array}{cc} 32 & 6 \\ 105 & 85 \\ 50 & 40 \\ 1 & 1 \end{array} \right] \end{array}$$

What if we got rid of the col of 1s in A?

$$\begin{array}{c} T \\ \overbrace{\quad\quad\quad}^T \\ \left[ \begin{array}{cccc} 1 & 0 & 0 & -32 \\ 0 & 1 & 0 & -80 \\ 0 & 0 & 1 & 10 \\ 0 & 0 & 0 & 1 \end{array} \right] \end{array} \qquad \begin{array}{c} A.T \\ \overbrace{\quad\quad\quad}^{A.T} \\ \left[ \begin{array}{cc} 32 & 6 \\ 105 & 85 \\ 50 & 40 \end{array} \right] \end{array}$$

(4,4)      (3,2)

Can't multiply them!

Try deleting col from T to make it work:

$$\begin{array}{c} T \\ \left[ \begin{array}{ccc} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{array} \right] \\ \xrightarrow{(4, 3)} \end{array} \quad \begin{array}{c} A \cdot T \\ \left[ \begin{array}{cc} 32 & 6 \\ 105 & 85 \\ 50 & 40 \end{array} \right] \\ \xrightarrow{(3, 2)} \\ \text{works!} \end{array}$$

Where should shifts go ?? ??

Will not work  $\Rightarrow$  we need that col of 1s.

\* Need col of 1s Appended to data  
"fake" variable  
homogeneous coordinate

Original data

	ME	FL	DC	homogeneous coord.
day 1	32	105	50	1
A = day 2	6	85	40	-

$$M = 3$$

Translation matrix for  $M=3$  variables

$$\begin{matrix} 4 \times 4 \\ (M+1, M+1) \end{matrix}$$

and Shape A w/ homogeneous (coord):  $(N, M+1)$

## Scaling

Replace T matrix with an S matrix.

Scaled/manipulated data

$$A' = (S \odot A \cdot T) \cdot T$$

New transformation matrix.

orig-data

$$= \begin{bmatrix} \text{day 1} & \text{ME} & \text{FL} & \text{DC} \\ & 32 & 105 & 50 \\ \text{day 2} & 6 & 85 & 40 \end{bmatrix}$$

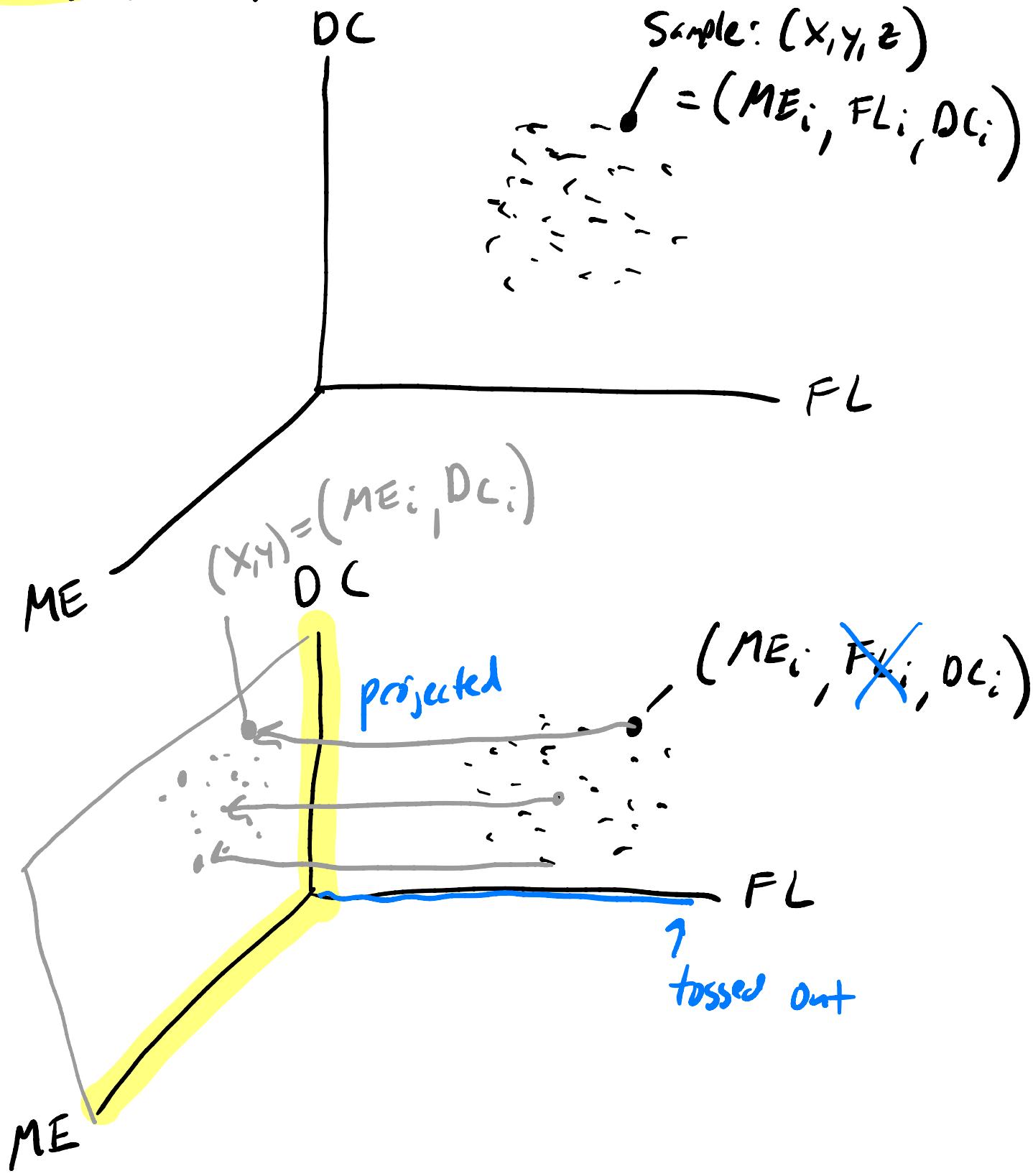
Pick 2/3 variables to transform

$$A = \begin{bmatrix} 32 & 50 \\ 6 & 40 \\ \text{ME} & \text{DC} \end{bmatrix}$$

M = 2

data-proj

## Orthographic projection



$$A = \begin{bmatrix} 32 & 50 \\ 6 & 40 \\ ME & DC \end{bmatrix} \quad \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$A' = (S @ A.T) . T$$

$$A.T = \begin{bmatrix} 32 & 6 \\ 50 & 40 \\ 1 & 1 \end{bmatrix}$$

$$S = \begin{bmatrix} 0.5 & 0 & 0 \\ 0 & 0.1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$S @ A.T = \begin{bmatrix} 0.5 & 0 & 0 \\ 0 & 0.1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 32 & 6 \\ 50 & 40 \\ 1 & 1 \end{bmatrix}$$

Shapes work? ✓  $(3, 3)$   $\times (3, 2)$  good!

Output shape?  $(3, 2)$

$$\begin{bmatrix} *1 & *2 \\ *3 & *4 \\ *5 & *6 \end{bmatrix} = \begin{bmatrix} 6.5 & 0 & 0 \\ 0 & 0.1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 32 & 6 \\ 50 & 40 \\ 1 & 1 \end{bmatrix}$$

$$*1 = 16$$

$$*2 = 3$$

$$*3 = 5$$

$$*4 = 4$$

$$*5 = 1$$

$$*6 = 1 \quad (3, 2)$$

$$\begin{bmatrix} 16 & 3 \\ 5 & 4 \\ 1 & 1 \end{bmatrix} \xrightarrow{\text{transpose}} \text{transpose}$$

$$A' = \begin{bmatrix} 16 & 5 & 1 \\ 3 & 4 & 1 \end{bmatrix} \quad \text{Compare with } A:$$

$$A = \begin{bmatrix} 32 & 50 & 1 \\ 6 & 40 & 1 \\ ME & DC & \end{bmatrix}$$

ME: What happened to these values? 1/2 as big

DC: What happened? 1/10 as big

Scale factor/multiplier on data in 1st Var

$$S = \begin{bmatrix} 6.5 & 0 & 0 \\ 0 & 0.1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Scale factor for 2nd Var  
for homogeneous Coord.

$$S_2(S_x, S_y) = \begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Scaling  
matrix for  
2 Vcr

$$S_3(S_x, S_y, S_z) = \begin{bmatrix} S_x & 0 & 0 & 0 \\ 0 & S_y & 0 & 0 \\ 0 & 0 & S_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$\underbrace{\qquad\qquad\qquad}_{(4,4)}$

In general:  $M$  Variables.

list of Scale factors for each variable — len  $M$

$\overbrace{\qquad\qquad\qquad}^M$

$$S(S_1, S_2, \dots, S_M) = \begin{bmatrix} S_1 & 0 & 0 & \dots & 0 \\ 0 & S_2 & \dots & & \vdots \\ \vdots & & & & \\ 0 & 0 & \dots & 0 & 1 \end{bmatrix}$$

Shape:  $(M+1, M+1)$

Translation in general: T for M Variables

Shift for Var 1      Shift for Var 2

$$T(t_1, t_2, \dots, t_m) = \begin{bmatrix} 1 & 0 & 0 & \dots & & f_1 \\ 0 & 1 & 0 & \dots & & f_2 \\ \vdots & & & \ddots & & \vdots \\ 0 & \dots & & & 0 & t_m \\ & & & & 1 & \\ & & & & 0 & 1 \end{bmatrix}$$

Shape:  $(M+1, M+1)$