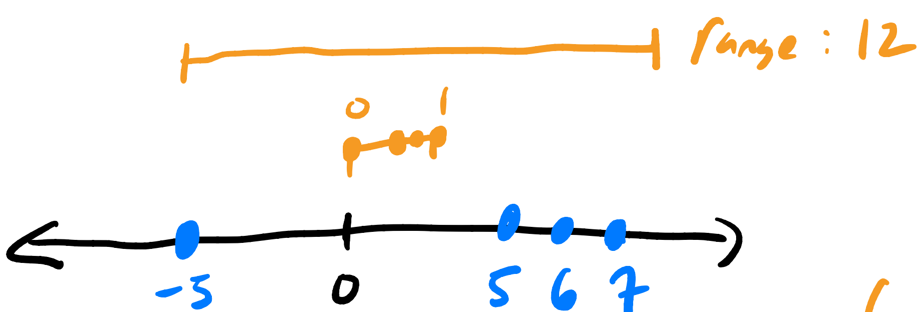
Lecture 8: Normalization

a Type of transformation of data involves <u>both</u> a T and S.

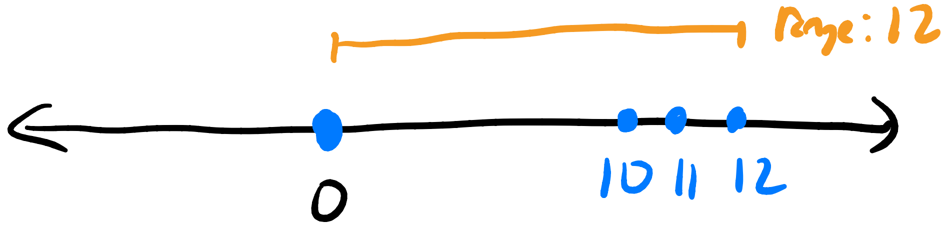
Normalization with one var x : [2, 1, 3, 5, ...]

<u>Basic goal</u>: Change data so that each value is between 0 and 1



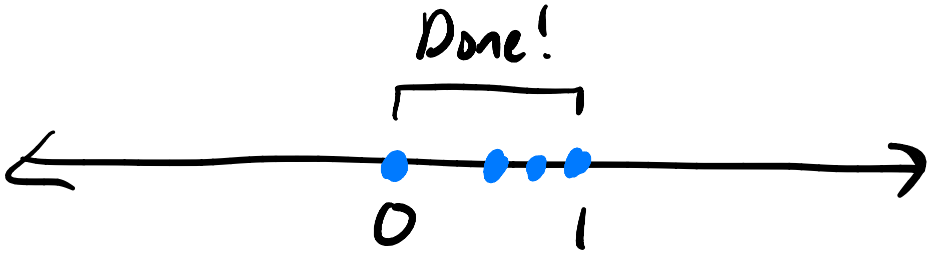
$-(-5) \Rightarrow +5$

1) Subtract -5 $\Rightarrow$ subtract min : $-\text{min}$



2) Divide by <u>12</u> : range : max − min] based on initial data w/o any transformations — A

$\Rightarrow$ $1/\text{range} = 1/(\text{max} - \text{min})$



data

Formula:

$$\frac{x_i - min}{max - min} = \overbrace{x_i'}^{\text{normalized data value.}}$$

Where is Translation? $\boxed{-min}$

Where is Scaling? $\boxed{1/range = 1/(max-min)}$

$$\overbrace{A'}^{\text{normalized data}} = (T @ S @ A.T).T \quad \underline{\text{or}}$$

$$\boxed{A' = (\underbrace{S @ \underbrace{T @ A.T}_{\text{translate 1st}}}_{\text{Scale 2nd } \frac{1}{\text{range}}}).T}$$

Option 1: Global normalization ("normalize together")

For T Translate each variable by $-\min$ over entire dataset — not count homogeneous coord.

For S Scale each variable by $1/\text{range}$, where

$$\text{range} = \underbrace{\max - \min}_{\text{determined over entire matrix } A}$$

Example:

global max

$$A = \begin{array}{ccc} \text{ME} & \text{FL} & \\ \left[\begin{matrix} 32 \\ 5 \\ 10 \end{matrix}\right. & \begin{matrix} 100 \\ 90 \\ 80 \end{matrix} & \left.\begin{matrix} 1 \\ 1 \\ 1 \end{matrix}\right] \end{array}$$

$$\boxed{A' = (S @ \underline{T @ A.T}).T}$$

$$\overbrace{\begin{bmatrix} 32 & 5 & 10 \\ 100 & 90 & 80 \\ 1 & 1 & 1 \end{bmatrix}}^{A.T}$$

$$T = \begin{bmatrix} 1 & 0 & -5 \\ 0 & 1 & -5 \\ 0 & 0 & 1 \end{bmatrix}$$

$$T @ A.T = \begin{bmatrix} 1 & 0 & -5 \\ 0 & 1 & -5 \\ 0 & 0 & 1 \end{bmatrix} \overbrace{\begin{array}{c} \begin{bmatrix} 32 & 5 & 10 \\ 100 & 90 & 80 \\ 1 & 1 & 1 \end{bmatrix} \end{array}}^{A.T} \begin{array}{l} ME \\ FL \\ \end{array}$$

$$= \begin{bmatrix} 27 & 0 & 5 \\ 95 & 85 & 75 \\ 1 & 1 & 1 \end{bmatrix}$$

$$S @ (T @ A.T)$$

$$\frac{1}{\text{range}} = \frac{1}{\text{max} - \text{min}}$$

overall

$$= \frac{1}{100 - 5} = \frac{1}{95}$$

$$S = \begin{bmatrix} \boxed{1/95} & 0 & 0 \\ 0 & \boxed{1/95} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \boxed{1/95} & 0 & 0 \\ 0 & \boxed{1/95} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 27 & 0 & 5 \\ 95 & 85 & 75 \\ 1 & 1 & 1 \end{bmatrix}$$

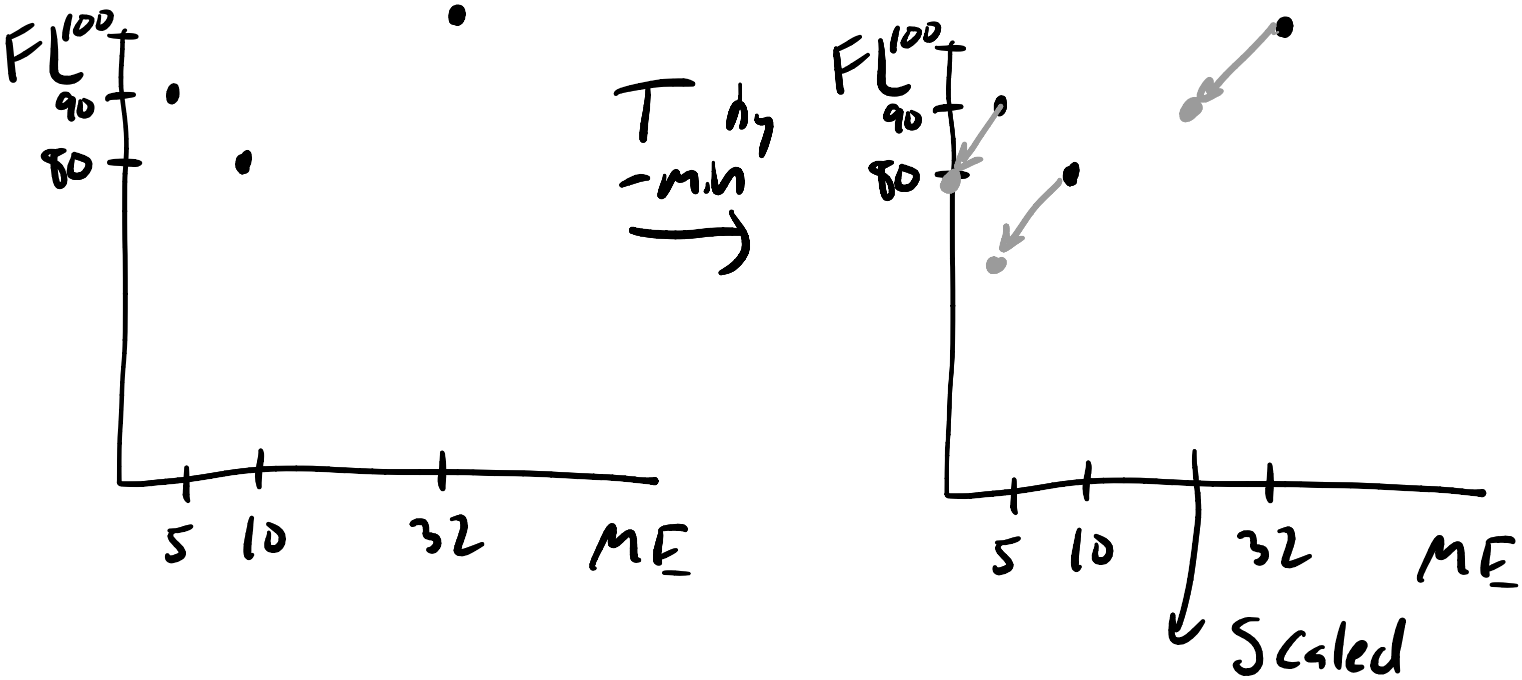
$$= \begin{bmatrix} 27/95 & 0 & 5/95 \\ 95/95 & 85/95 & 75/95 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 27/95 & 0 & 5/95 \\ 1 & 85/95 & 75/95 \\ 1 & 1 & 1 \end{bmatrix}$$

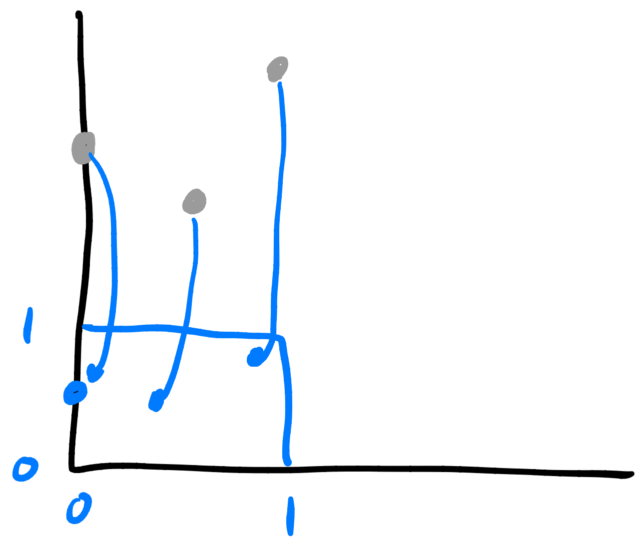
↙ transpose

$$= \begin{bmatrix} 27/95 & 1 & 1 \\ 0 & 85/95 & 1 \\ 5/95 & 75/95 & 1 \end{bmatrix} = A'$$

globally normalized data

<u>Sanity check</u>: There should be one 0 and one 1 in data matrix (normalized)


FL
100
90
80
5
10
32
ME
T by
-min
FL
100
90
80
5
10
32
ME
Scaled


1
0
0
1

Option 2: normalize seperately (per variable):

$$\overbrace{A'}^{\text{normalized data}} = (S @ T @ A.T).T$$

For T — Translate each var by - its own min

$\boxed{-\text{min}}$

For S — Scale each var by $\boxed{//\text{range}}$ — but max/min computed for each var seperately.

$$A = \begin{array}{ccc} \text{ME} & \text{FL} & \\ \end{array}$$
$$A = \begin{bmatrix} 32 & 100 & 1 \\ 5 & 90 & 1 \\ 10 & 80 & 1 \end{bmatrix}$$

$$T @ A.T = \begin{bmatrix} 1 & 0 & -5 \\ 0 & 1 & -80 \\ 0 & 0 & 1 \end{bmatrix} \overbrace{\begin{bmatrix} 32 & 5 & 10 \\ 100 & 90 & 80 \\ 1 & 1 & 1 \end{bmatrix}}^{A.T}$$

$$= \begin{bmatrix} 27 & 0 & 5 \\ 20 & 10 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

$$\underbrace{S}@T@A.T$$

$\frac{1}{\text{range}}$

Me: max - min = 32 - 5 = 27

FL: max - min = 100 - 80 = 20

$$\begin{bmatrix} \boxed{1/27} & 0 & 0 \\ 0 & \boxed{1/20} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \boxed{1/27} & 0 & 0 \\ 0 & \boxed{1/20} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 27 & 0 & 5 \\ 20 & 10 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

27/27

20/20

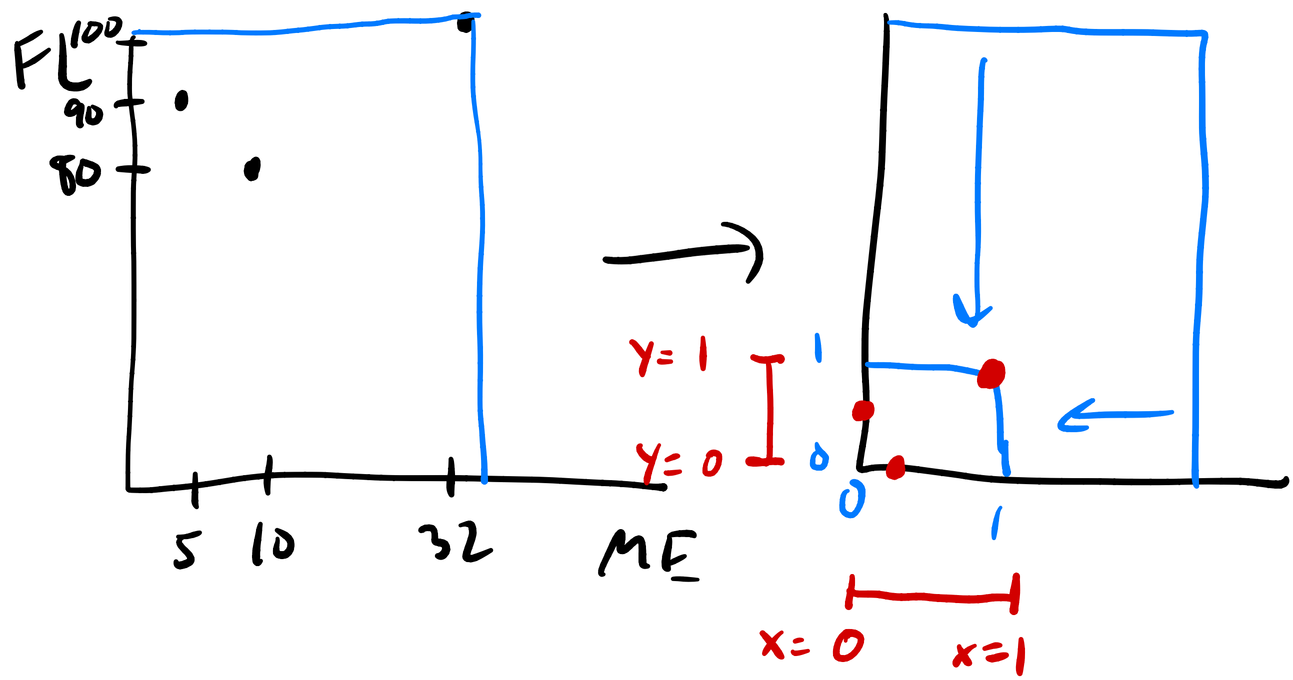
$$= \begin{bmatrix} 1 & 0 & 5/27 \\ 1 & 10/20 & 0 \\ 1 & 1 & 1 \end{bmatrix} \xrightarrow{\text{transpose}}$$

$$A' = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 10/10 & 1 \\ 5/27 & 0 & 1 \end{bmatrix}$$

Done!

data normalized separately.

Sanity check: one 1 and one 0 per data col.



$S: (m+1, m+1)$
$T: (m+1, m+1)$

Output shape: $(m+1, m+1)$

C : Compound matrix

$\underbrace{C}_{(m+1,m+1)} = S @ T$ 1st

apply only one transformation on data

$A' = ((C @ A.T).T$ 2nd

$C = \overbrace{S @ T @ S @ T @ S @ T}^{(m+1,m+1)}$ 1st

$A' = (C @ A.T).T$ 2nd

Same eq. to apply to data no matter how many transformations

☆ S@T != T@S