Lecture 7

Translation Summary:

T : translation matrix

A : Data matrix

"M"

$$A' = (T @ A.T).T$$

Translating 3 variables by amounts t_x, t_y, t_z

$$T = \begin{bmatrix} 1 & 0 & 0 & t_x \\ 0 & 1 & 0 & t_y \\ 0 & 0 & 1 & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

4

4

Assignment to the plug-in

np.eye(4) ← 4x4 identity matrix

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

27A Column of 1s in A

$$T = \begin{bmatrix} 1 & 0 & 0 & -32 \\ 0 & 1 & 0 & -80 \\ 0 & 0 & 1 & 10 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad A.T = \begin{bmatrix} 32 & 6 \\ 105 & 85 \\ 50 & 40 \\ 1 & 1 \end{bmatrix}$$

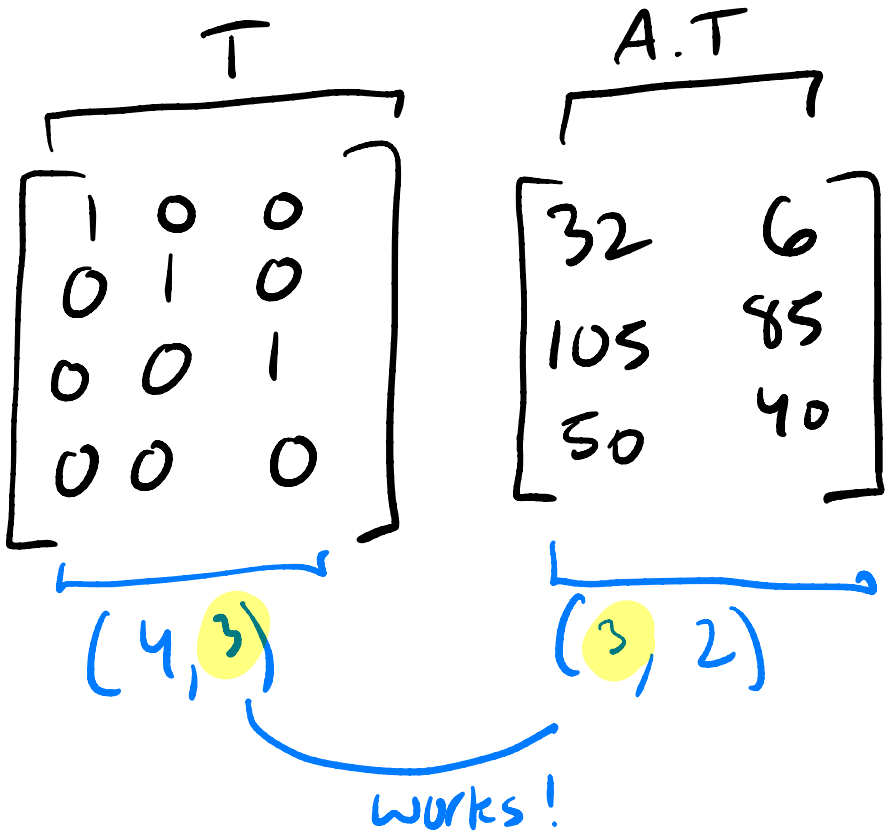
What if we got rid of the col of 1s in A?

$$T = \begin{bmatrix} 1 & 0 & 0 & -32 \\ 0 & 1 & 0 & -80 \\ 0 & 0 & 1 & 10 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad A.T = \begin{bmatrix} 32 & 6 \\ 105 & 85 \\ 50 & 40 \end{bmatrix}$$

$(4,4)$ $(3,2)$

Can't multiply them!

Try deleting col from T to make it work:



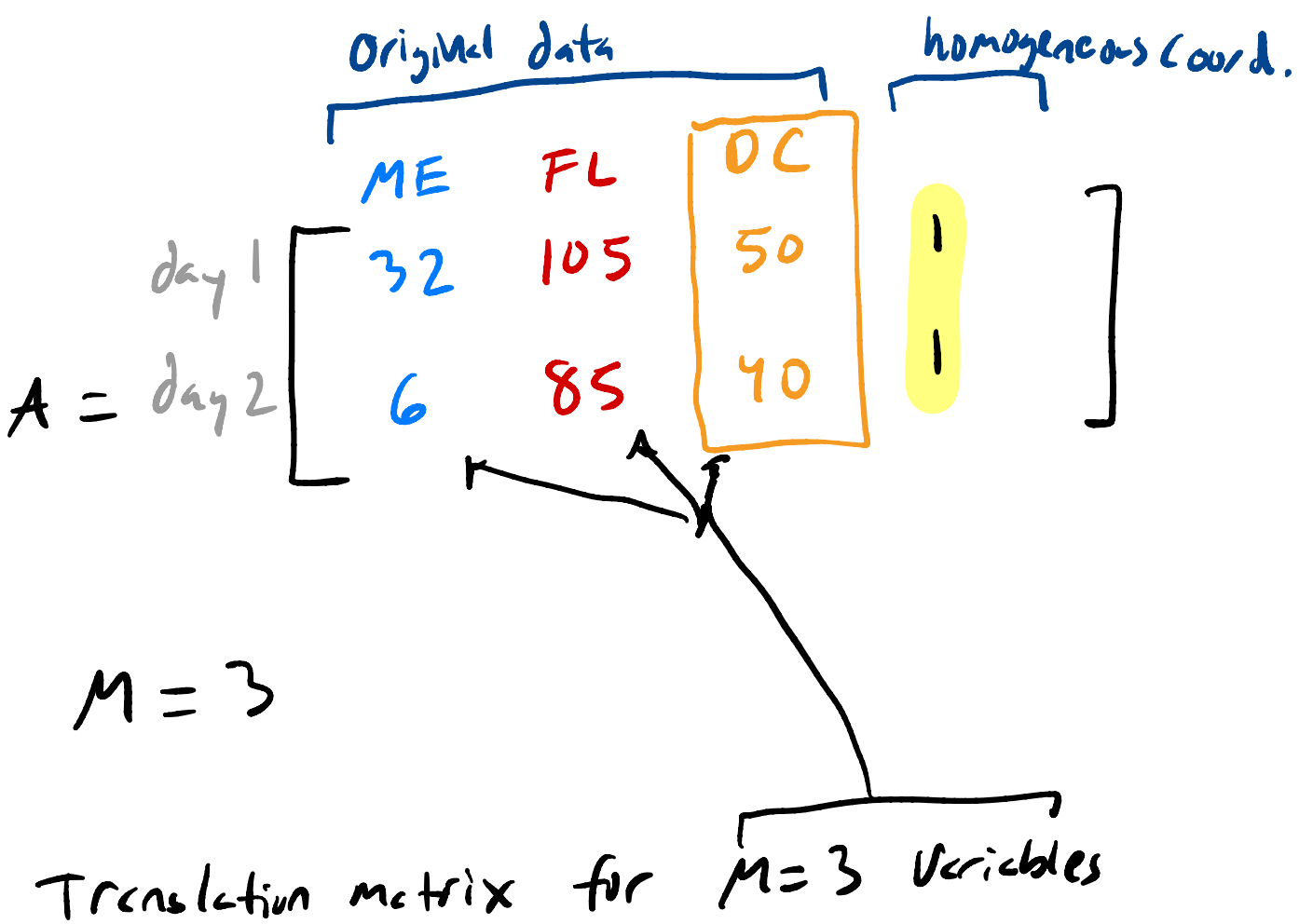
Where should shifts go!?!?

Will not work ⇒ we need that col of 1s

☆ Need col of 1s Appended to data

"fake" Ucrible

homogeneous Coordinate



$M = 3$

Translation matrix for $M = 3$ variables

4×4.

$(M+1, M+1)$

and Shape A w/ homogeneous coord: $(N, M+1)$

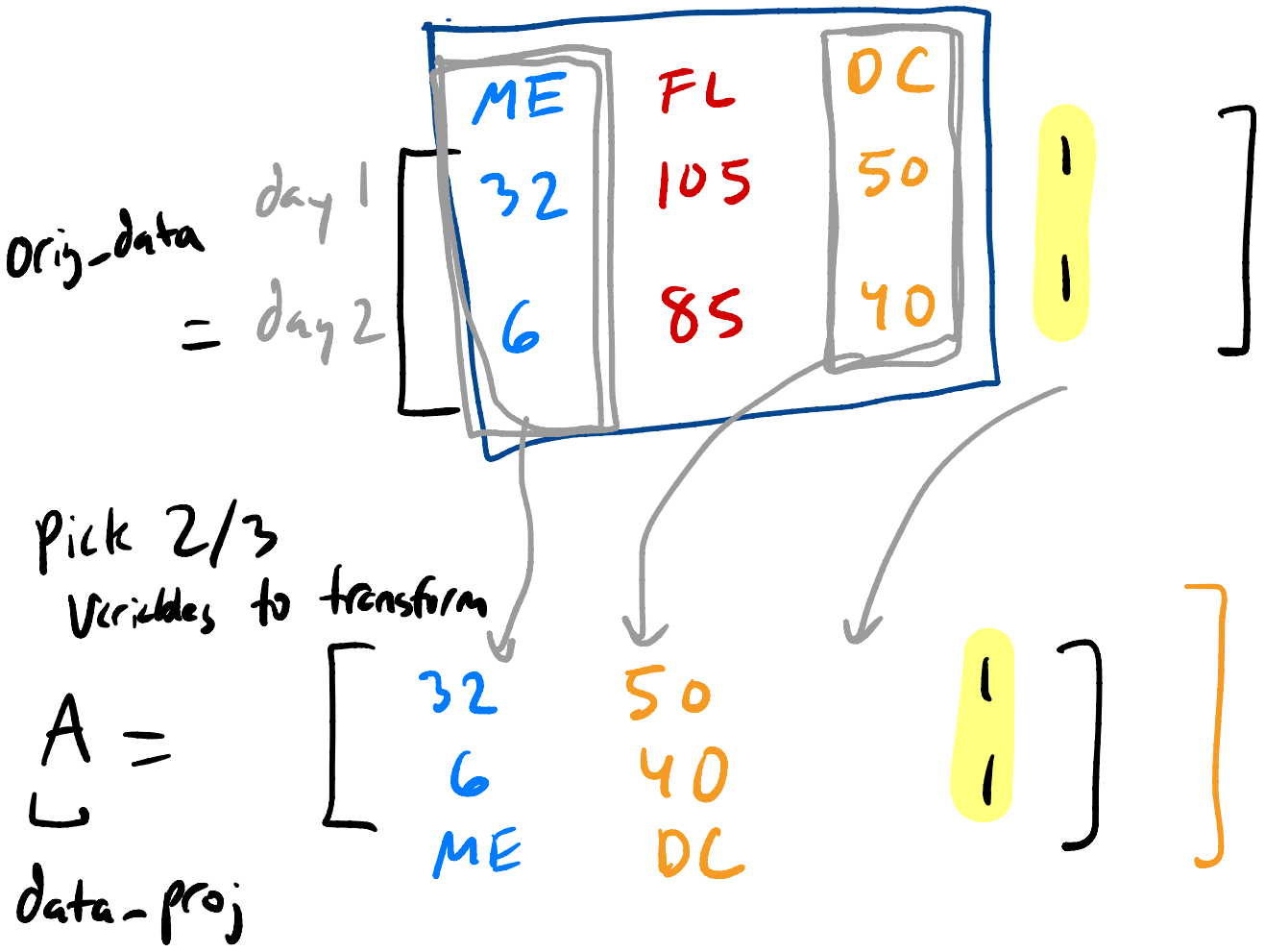
Scaling

Replace T matrix with an S matrix.

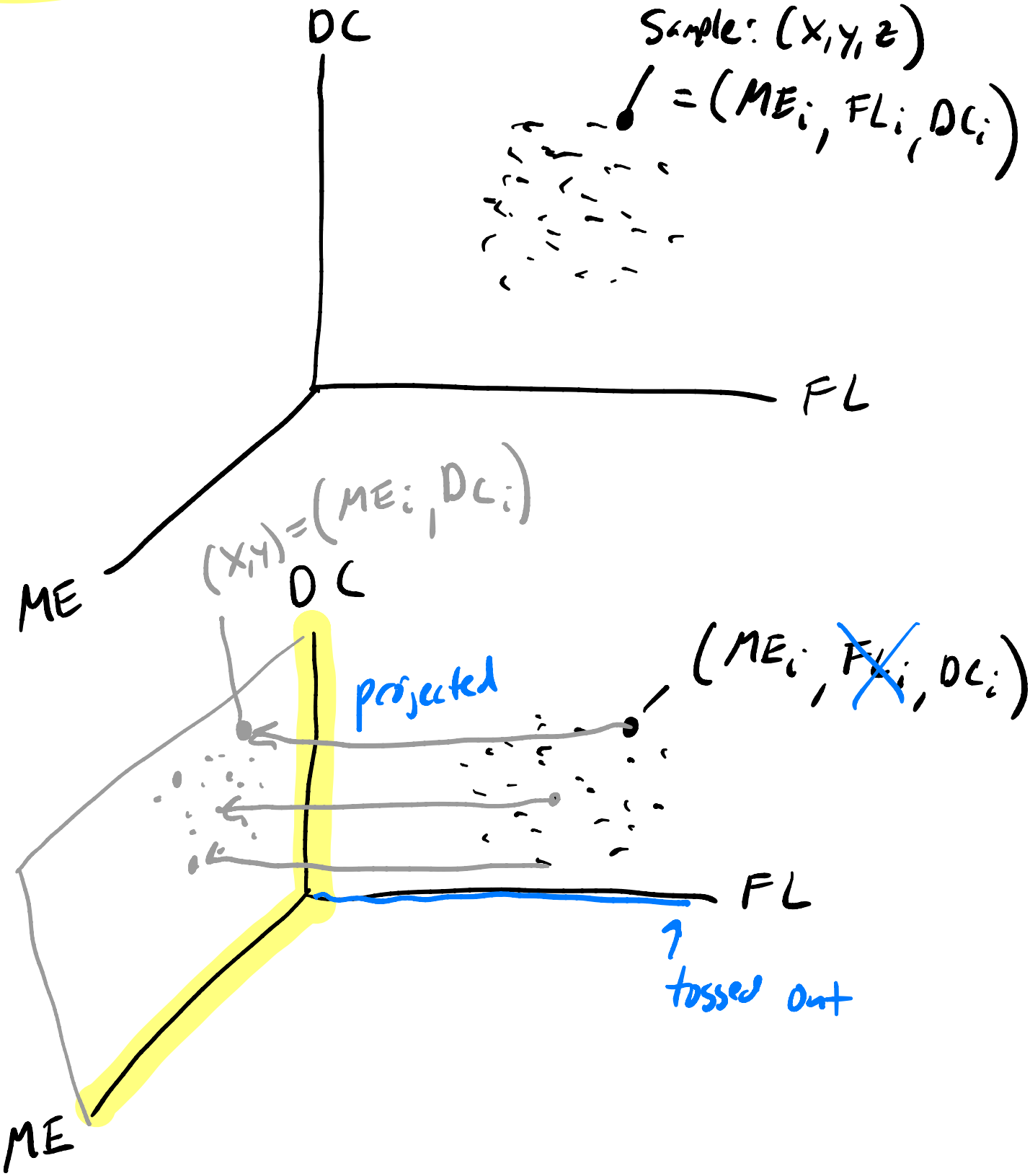
Scaled/manipulated data

$$A' = (S @ A.T).T$$

S: new transformation matrix



Orthographic projection



$$A = \begin{bmatrix} 32 & 50 & 1 \\ 6 & 40 & 1 \\ ME & DC & \end{bmatrix}$$

$$A' = (S @ A.T).T$$

$$A.T = \begin{bmatrix} 32 & 6 \\ 50 & 40 \\ 1 & 1 \end{bmatrix}$$

$$S = \begin{bmatrix} 0.5 & 0 & 0 \\ 0 & 0.1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$S @ A.T = \begin{bmatrix} 0.5 & 0 & 0 \\ 0 & 0.1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 32 & 6 \\ 50 & 40 \\ 1 & 1 \end{bmatrix}$$

Shapes work? ✓ $(3, 3) \times (3, 2)$ good!

Output shape? $(3, 2)$

$$\begin{bmatrix} *1 & *2 \\ *3 & *4 \\ *5 & *6 \end{bmatrix} = \begin{bmatrix} 0.5 & 0 & 0 \\ 0 & 0.1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 32 & 6 \\ 50 & 40 \\ 1 & 1 \end{bmatrix}$$

$*1 = 16$

$*2 = 3$

$*3 = 5$

$*4 = 4$

$*5 = 1$

$*6 = 1$

$$\begin{bmatrix} 16 & 3 \\ 5 & 4 \\ 1 & 1 \end{bmatrix} \quad (3,2)$$

$\longrightarrow$ transpose

$$A' = \begin{bmatrix} 16 & 5 & 1 \\ 3 & 4 & 1 \end{bmatrix}$$

Compare with A:

$$A = \begin{bmatrix} 32 & 50 & 1 \\ 6 & 40 & 1 \\ ME & DC & \end{bmatrix}$$

ME: What happened to these values? 1/2 as big

DC: What happened? 1/10 as big

$$S = \begin{bmatrix} 0.5 & 0 & 0 \\ 0 & 0.1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Scale factor/multiplier on data in 1st var

Scale factor for 2nd var

for homogeneous coord.

$$S_2(s_x, s_y) = \begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

S_2

Scaling matrix for 2 var

$$S_3(s_x, s_y, s_z) = \begin{bmatrix} s_x & 0 & 0 & 0 \\ 0 & s_y & 0 & 0 \\ 0 & 0 & s_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$(4, 4)$

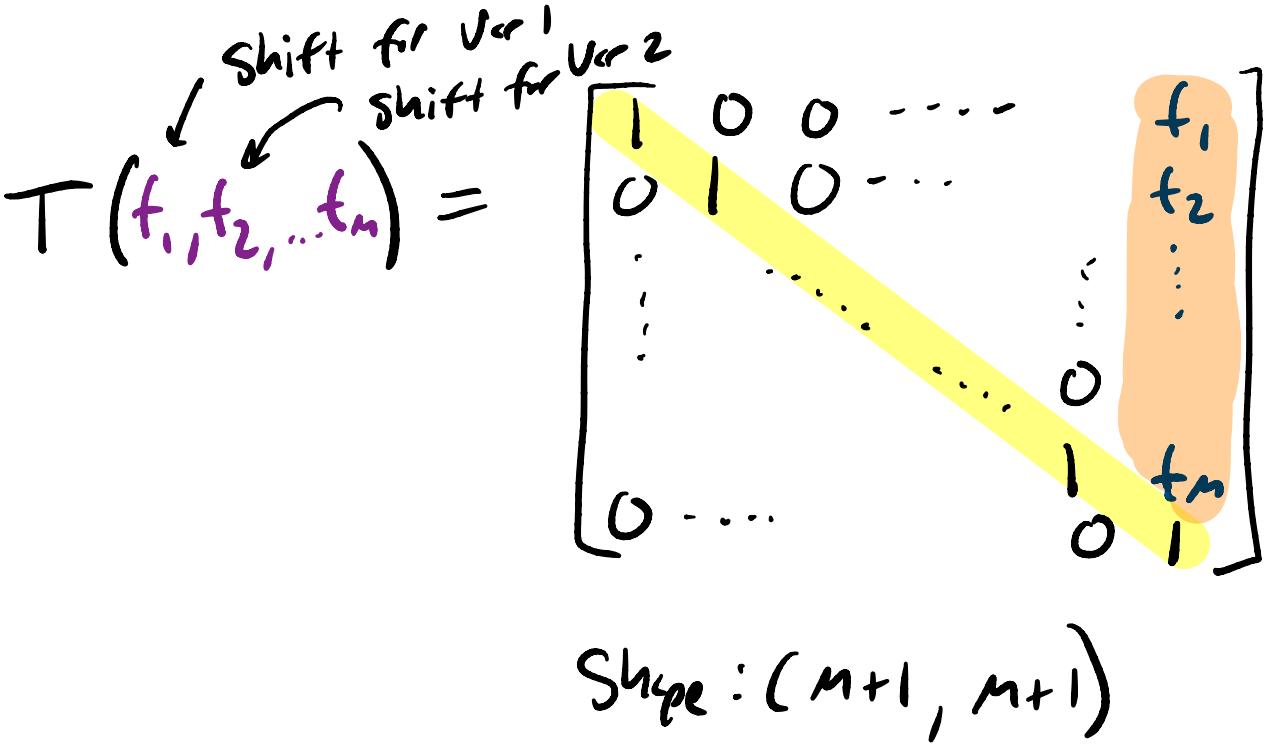
In general: M variables.

list of scale factors for each variable — len M

$$S(s_1, s_2, \ldots, s_M) = \begin{bmatrix} s_1 & 0 & 0 \cdots & 0 \\ 0 & s_2 & \ddots & \vdots \\ \vdots & & & \\ & & s_M & \\ 0 & 0 \cdots & 0 & 1 \end{bmatrix}$$

Shape: $(M+1, M+1)$

Translation in general: T for M variables



Shape: $(m+1, m+1)$