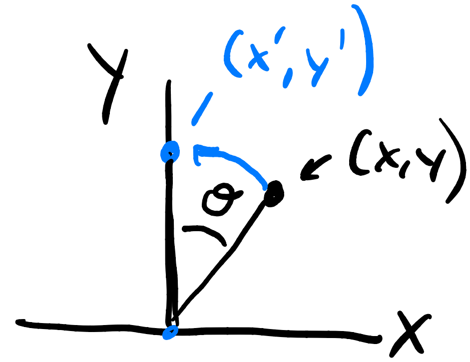
Lecture 10: 3D rotation

$$R(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



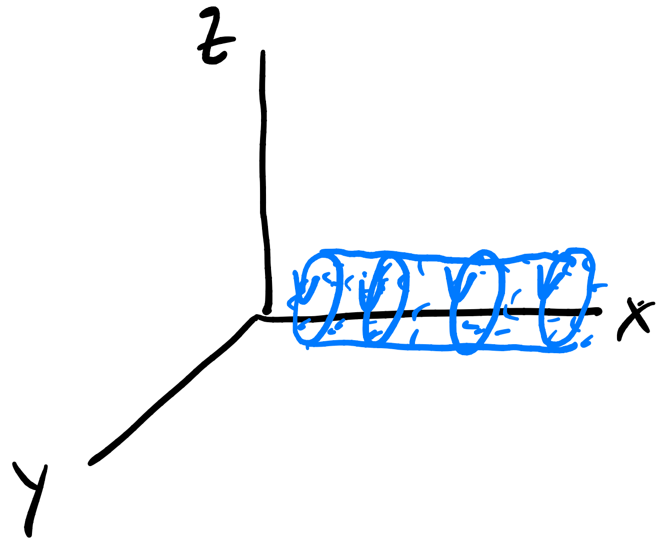
2 vars

$n = 2$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$\underbrace{A'}_{\text{rotated data}} = (R(\theta) \;@\; \underbrace{A}_{\text{initial data}}.T).T$$

3D: $M=3$



Rotation about x-axis

rotated data

$$x' = x$$

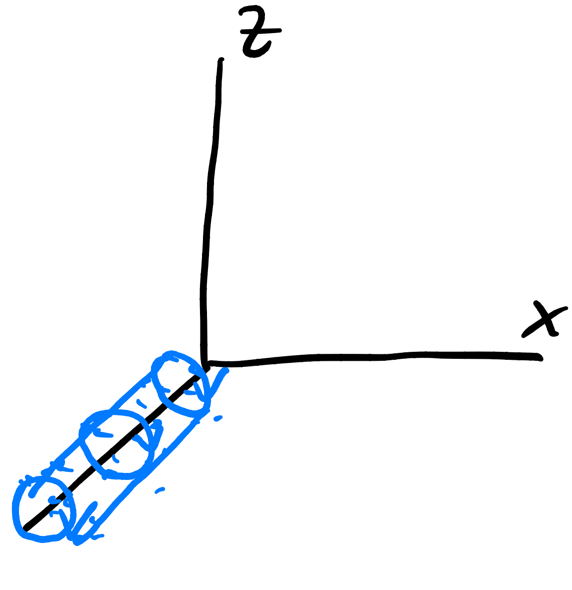
$$y' = y\cos\theta - z\sin\theta$$

$$z' = y\cos\theta + z\sin\theta$$

3D rotation matrix $R(\theta)$ about x-axis:

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Rotation about y-axis :

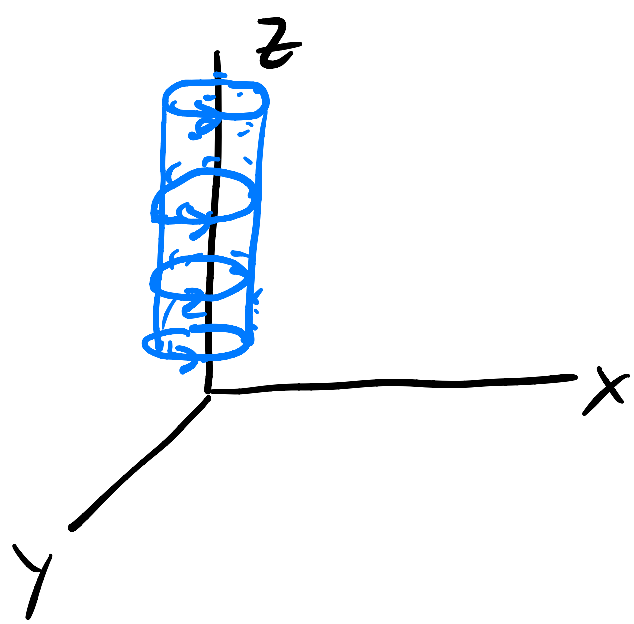


$y' = y$

R(θ) about y axis.

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 & \sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Rotation about z axis :



$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \underbrace{\begin{bmatrix} \boxed{\begin{matrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{matrix}} & \begin{matrix} 0 & 0 \\ 0 & 0 \end{matrix} \\ \begin{matrix} 0 & 0 \\ 0 & 0 \end{matrix} & \begin{matrix} 1 & 0 \\ 0 & 1 \end{matrix} \end{bmatrix}}_{R(\theta) \text{ about } z\text{-axis}} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

2D rotation matrix

R(θ) about z-axis

<u>Note</u>: How do we know which direction we rotate when we rotate with ⊕ θ value.

<u>Right-hand rule</u>: Tells us which direction of rotation (θ) is ⊕

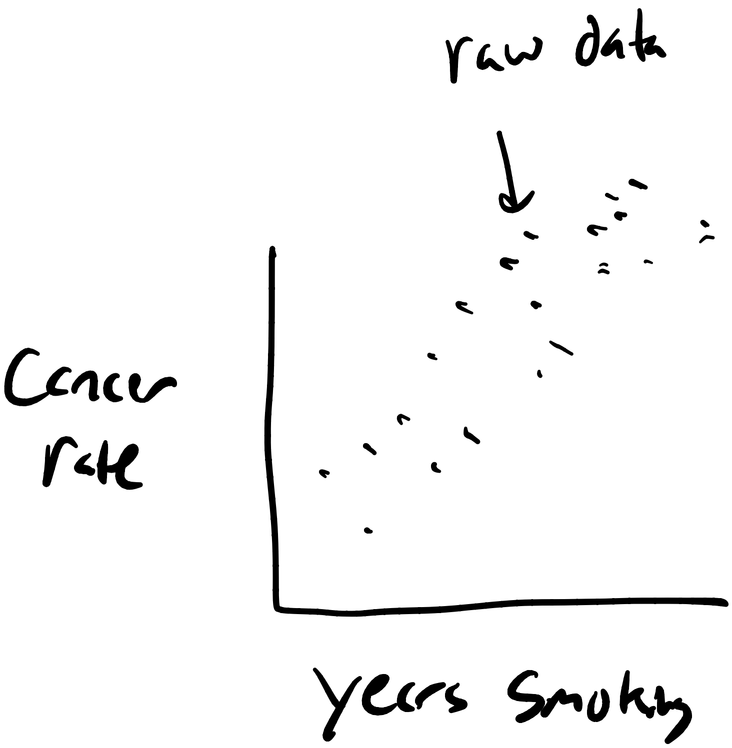
1) Figure out axis you are rotating about

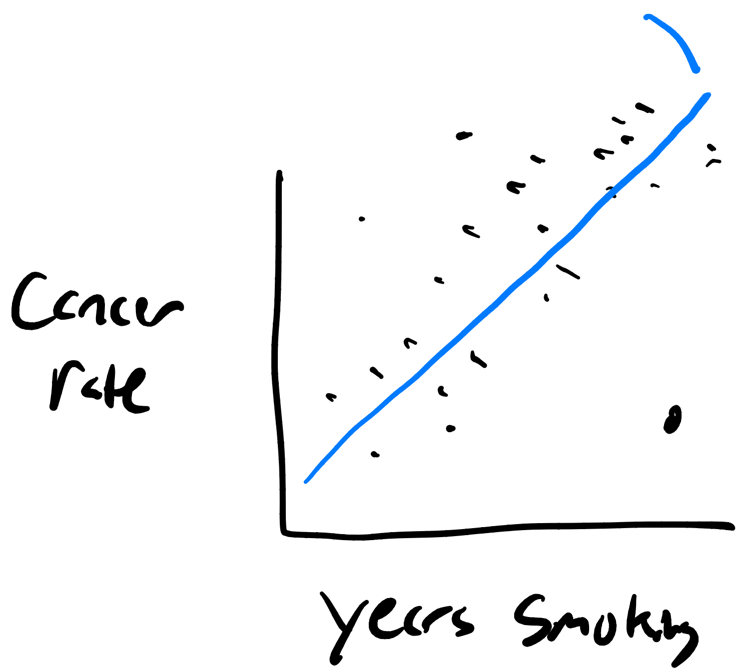
2) make a fist with <u>right hand</u>, thumb out.

3) Align right thumb w/ ⊕ axis you are rotating about.

4) ⊕ θ rotation happens in direction in which your fingers curl.

Linear regression: "go beyond" looking at our data — **quantify** strength of association between 2+ vars





linear regression: fit "line" to data in way that tolerates noise and measurement error