Lecture 14: Polynomial regression

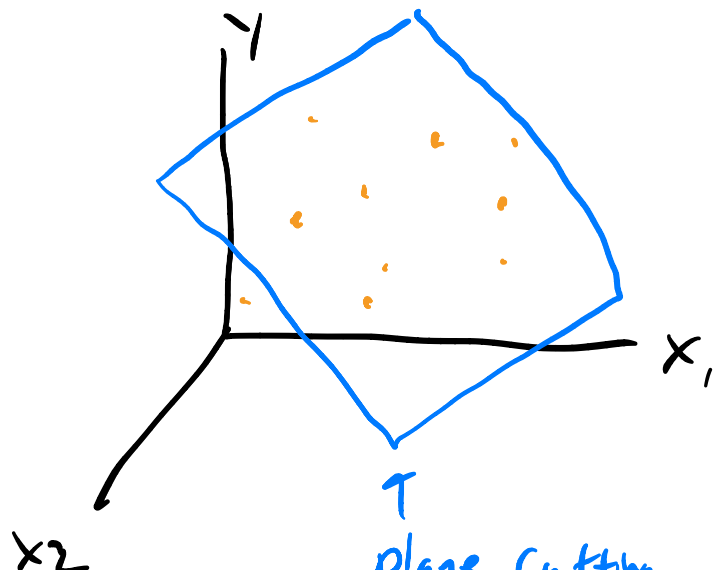
So far: Simple linear regression — One Y, One X

Multiple linear regression — One Y, >1 X

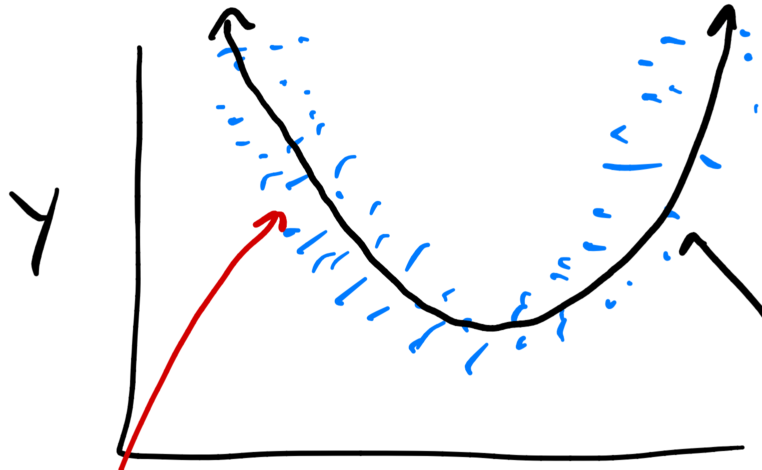
[waffle house 2 ind vars x_1, x_2]

$z = x + y$] plane equation

$y = x_1 + x_2$



plane cutting thru 3D Scatterplot



$$\underbrace{X_1}_{\text{years smoking}}$$

← line fit never will be good here

we want to use linear regession to fit this curve.

quadratic model to fit that data

$$\boxed{y = \overbrace{C_0}^{\text{intercept}} + C_1 X_1 + C_2 X_1^2}$$

unique/diff coefficients for each power term

Cubic fit: $y = C_0 + C_1 X_1 + C_2 X_1^2 + C_3 X_1^3$

Still want $\vec{y} = A\vec{c}$

$\boxed{\text{Main modification for polynomial regression is } A}$

e.g. Cubic equation example

$$A = \begin{bmatrix} 1 & \vec{x}_1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & \vec{x}_1 & \vec{x}_1^{\,2} & \vec{x}_1^{\,3} \end{bmatrix}$$

$$A: \begin{bmatrix} 1 & 8 \\ 1 & 2 \\ 1 & -3 \\ \vdots & \vdots \end{bmatrix} \rightarrow \begin{bmatrix} 1 & x_{1,1} & x_{1,1}^2 & x_{1,1}^3 \\ 1 & x_{1,2} & x_{1,2}^2 & x_{1,2}^3 \\ 1 & x_{1,3} & x_{1,3}^2 & x_{1,3}^3 \\ \vdots & \vdots & \vdots & \vdots \\ 1 & x_{1,N} & x_{1,N}^2 & x_{1,N}^3 \end{bmatrix}$$

$\Rightarrow$ we plug into lstsqr (solver) $\Rightarrow$ get $\vec{c}$

$= [c_0, c_1, c_2, c_3]$

Example:

$$\overbrace{\vec{x}_1}^{\text{years smoking}} = \begin{bmatrix}2\\4\\1\\3\\5\end{bmatrix} \qquad \overbrace{\vec{y}}^{\text{cancer rate}} = \begin{bmatrix}15\\5\\2\\11\\4\end{bmatrix}$$

polynomial regression model equation:

$$y = c_0 + c_1 x_1 + c_2 x_1^2 + c_3 x_1^3$$

Set up: $\vec{y} = A\vec{c}$

$$\overbrace{\begin{bmatrix}15\\5\\2\\11\\4\end{bmatrix}}^{\vec{y}} = \overbrace{\begin{bmatrix}1 & 2 & 4 & 8\\1 & 4 & 16 & 64\\1 & 1 & 1 & 1\\1 & 3 & 9 & 27\\1 & 5 & 25 & 125\end{bmatrix}}^{A} \overbrace{\begin{bmatrix}c_0\\c_1\\c_2\\c_3\end{bmatrix}}^{\vec{c}}$$

$\Rightarrow$ c ___ = lstsqr(A, y)

HW3: Linear Regression

1) List ind and dep vars:

$$\overbrace{Y}^{\text{dep}} = C_0 + C_1 \overbrace{X_1}^{1} + C_2 \overbrace{X_2}^{2} + C_3 \overbrace{X_3}^{3}$$

2) $$Y = C_0 X_0 + C_1 X_1 + C_2$$

$$X_0 = \begin{bmatrix} 0 \\ 1 \\ 2 \\ 3 \end{bmatrix} \qquad X_1 = \begin{bmatrix} 3 \\ 4 \\ 5 \\ 6 \end{bmatrix} \qquad Y = \begin{bmatrix} 2.09 \\ 4.55 \\ 9.22 \\ 12.04 \end{bmatrix}$$

$$Y = Ac$$

$$\overbrace{\begin{bmatrix} 2.09 \\ 4.55 \\ 9.22 \\ 12.04 \end{bmatrix}}^{\vec{y}} = \overbrace{\begin{bmatrix} 0 & 3 & 1 \\ 1 & 4 & 1 \\ 2 & 5 & 1 \\ 3 & 6 & 1 \end{bmatrix}}^{A} \overbrace{\begin{bmatrix} C_0 \\ C_1 \\ C_2 \end{bmatrix}}^{\vec{c}}$$