Translation summary

To translate data variables by amounts $\vec{t}$, compute:

$$\boxed{A' = (T A.T).T}$$

Implementation tip: $T_3 = \begin{bmatrix} 1 & 0 & 0 & t_x \\ 0 & 1 & 0 & t_y \\ 0 & 0 & 1 & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$

Can be made by making a 4×4 identity matrix:

$$I_4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

then replacing values in last column with $\vec{t}$

[with an extra 1 appended at the end]

$$\begin{bmatrix} 1 & 0 & 0 & \\ 0 & 1 & 0 & \\ 0 & 0 & 1 & \\ 0 & 0 & 0 & \end{bmatrix} \qquad \vec{t} = \begin{bmatrix} t_x \\ t_y \\ t_z \\ 1 \end{bmatrix}$$

Why column of 1s in A?

Let's try getting rid of them:

$$\begin{array}{cc} T & A.T \\ \begin{bmatrix} 1 & 0 & 0 & -32 \\ 0 & 1 & 0 & -80 \\ 0 & 0 & 1 & 10 \\ 0 & 0 & 0 & 1 \end{bmatrix} & \begin{bmatrix} 32 & 6 \\ 105 & 85 \\ 50 & 40 \end{bmatrix} \end{array}$$

$\Rightarrow$ No way we can multiply T & A.T now!

$$(4,4) \times (3,2)$$

OK... what if we dropped the last col of T to try and make it work?

$$\begin{array}{cc} T & A.T \\ \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} & \begin{bmatrix} 32 & 6 \\ 105 & 85 \\ 50 & 40 \end{bmatrix} \end{array}$$

Where should the shifts go!?

$\Rightarrow$ Doesn't work $\Rightarrow$ we need the 1s.

★ The column of 1s added to A is like adding a "fake" variable or dimension — called <u>homogeneous coordinate</u>.

homogeneous coordinate = col of 1s aka "fake variable"

$$
\begin{array}{c}
\begin{array}{cccc} \text{ME} & \text{FL} & \text{DC} & \end{array} \\
\begin{bmatrix} 32 & 105 & 50 & 1 \\ 6 & 85 & 40 & 1 \end{bmatrix}
\end{array}
$$

★ Because A with homogeneous coordinate has $\underbrace{4}_{M+1}$ cols for data that has $\underbrace{3}_{M}$ variables,

T must be a $\underbrace{4 \times 4}_{(M+1,\, M+1)}$ matrix.

[otherwise matrix multiplication will fail].

Scaling

Let's learn about another transformation called <u>Scaling</u>

Replace $T \to S$:

$$\boxed{A' = (S @ A.T).T}$$

Instead of doing this on all 3 temperature variables, let's do it on ME and DC only:

$$\text{orig\_dataset} = \begin{array}{cccc} ME & FL & DC & \\ \left[\begin{array}{c} 32 \\ 6 \end{array}\right. & \begin{array}{c} 105 \\ 85 \end{array} & \begin{array}{c} 50 \\ 40 \end{array} & \left.\begin{array}{c} 1 \\ 1 \end{array}\right] \end{array}$$

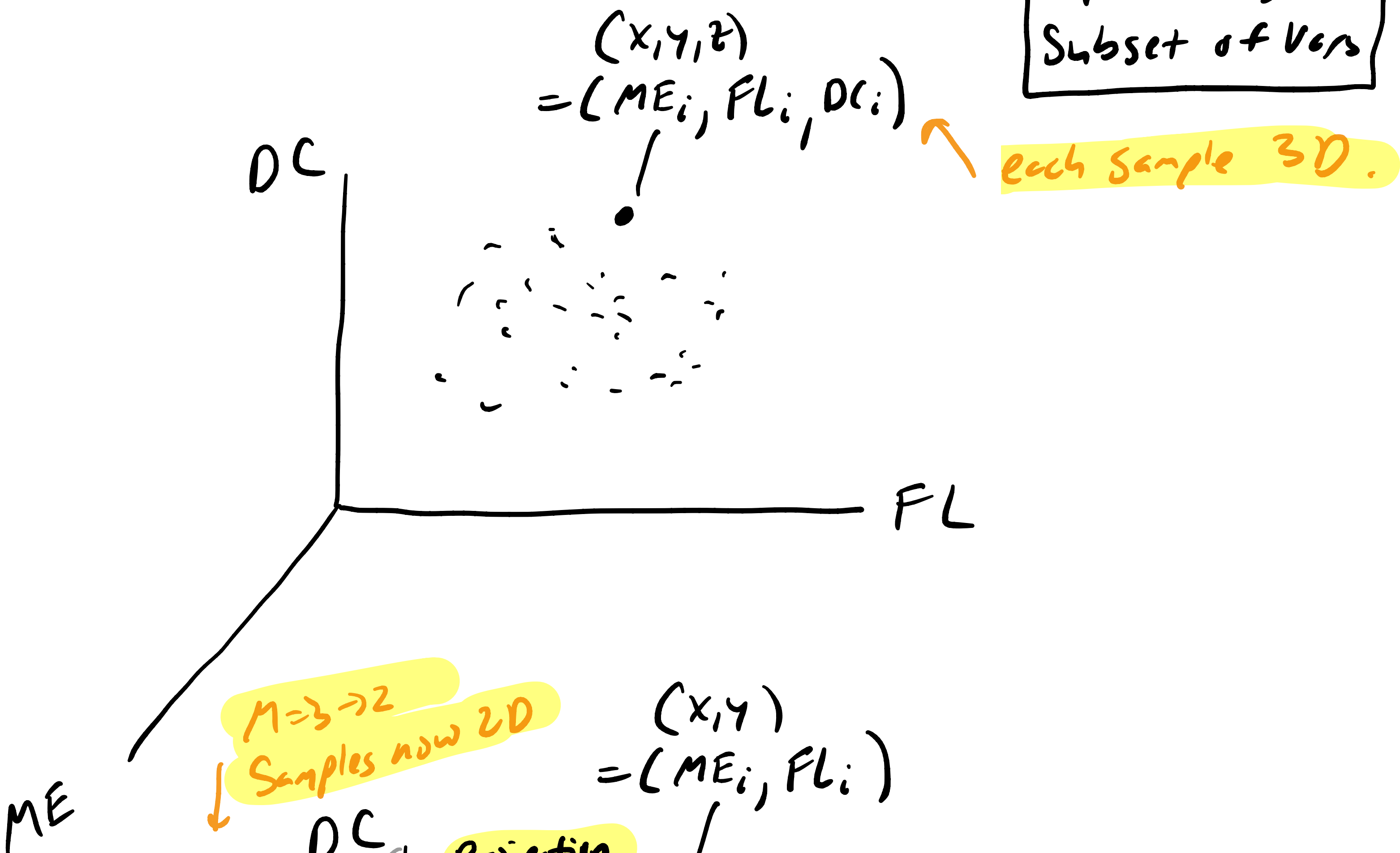
$$A = \text{dataProj} = \begin{array}{ccc} ME & DC & \\ \left[\begin{array}{c} 32 \\ 6 \end{array}\right. & \begin{array}{c} 50 \\ 40 \end{array} & \left.\begin{array}{c} 1 \\ 1 \end{array}\right] \end{array}$$

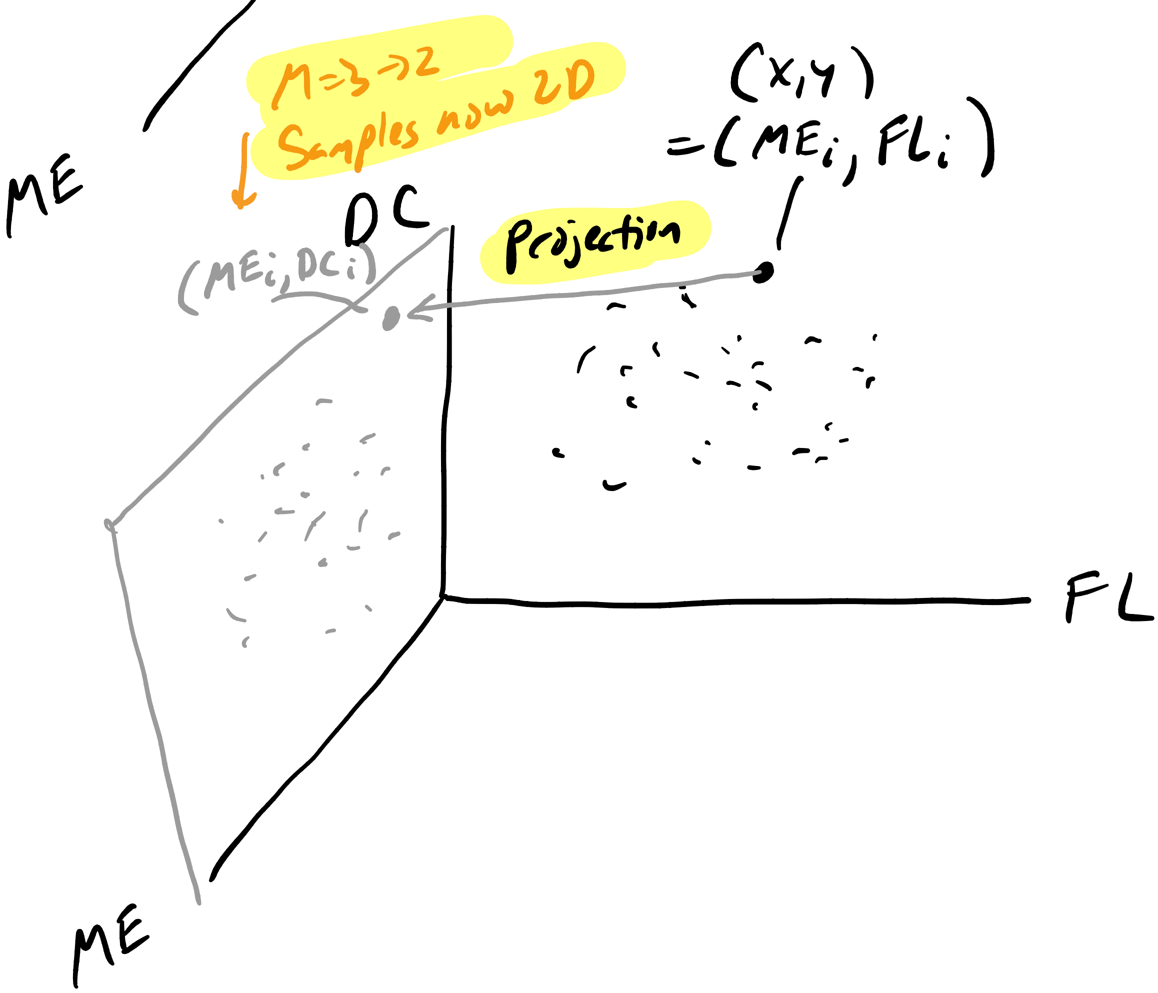
Now $M = 2$

This process of dropping 1+ variables from original dataset called orthographic projection

$\Rightarrow$ Fancy name from simple idea

$M = 3 \to 2$
by selecting
Subset of Vars





Back to Scaling:

$$A.T = \begin{bmatrix} 32 & 6 \\ 50 & 40 \\ 1 & 1 \end{bmatrix}$$

$$S = \begin{bmatrix} 0.5 & 0 & 0 \\ 0 & 0.1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

S@A.T : Output shape = $(3,3) \times (3,2) = (3,2)$

$$\begin{bmatrix} *1 & *2 \\ *3 & *4 \\ *5 & *6 \end{bmatrix} = \begin{bmatrix} 0.5 & 0 & 0 \\ 0 & 0.1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 32 & 6 \\ 50 & 40 \\ 1 & 1 \end{bmatrix}$$

$*1 = 0.5 \cdot 32 + 0 \cdot 50 + 0 \cdot 1 = 16$

$*2 = 0.5 \cdot 6 + 0 \cdot 40 + 0 \cdot 1 = 3$

$*3 = 0 \cdot 32 + 0.1 \cdot 50 + 0 \cdot 1 = 5$

$*4 = 0 \cdot 6 + 0.1 \cdot 40 + 0 \cdot 1 = 4$

$*5 = 0 \cdot 32 + 0 \cdot 50 + 1 \cdot 1 = 1$

$*6 = 0 \cdot 6 + 0 \cdot 40 + 1 \cdot 1 = 1$

$$S@A.T = \begin{bmatrix} 16 & 3 \\ 5 & 4 \\ 1 & 1 \end{bmatrix}$$

$$(S@A.T).T = \begin{bmatrix} 16 & 5 & 1 \\ 3 & 4 & 1 \end{bmatrix}$$

Compare with

$$A = \begin{bmatrix} 32 & 50 & 1 \\ 6 & 40 & 1 \end{bmatrix}$$

What happened to ME values? $\boxed{\text{1/2 the original value}}$

What happened to DC values? $\boxed{\text{1/10 the original value}}$

How did S do this?

Scale factor for ME values (Var 1)

$$S = \begin{bmatrix} 0.5 & 0 & 0 \\ 0 & 0.1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Scale factor for DC values (Var 2)

More formally:

Scale factors

$$S_2(\vec{s}) = \begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \text{where } \vec{s} = (\underset{\text{var 1}}{S_x}, \underset{\text{var 2}}{S_y})$$

homogeneous coordinate

is the **Scaling vector** for each variable.

S_2 is $\underbrace{3 \times 3}_{(m+1,\, m+1)}$ for data with $\underbrace{2}_{m}$ variables.

- How would we write scaling matrix for 3 data variables — (x, y, z)?

$$S_3(\vec{s}) = \underbrace{\begin{bmatrix} S_x & 0 & 0 & 0 \\ 0 & S_y & 0 & 0 \\ 0 & 0 & S_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{4 \times 4}$$

In general: M data variables

$$S_M(\vec{s}) = \underbrace{\left.\begin{bmatrix} s_1 & 0 & \cdots & & \\ 0 & s_1 & & & \\ \vdots & & \ddots & & \\ \vdots & & & s_M & \\ 0 & \cdots & & & 1 \end{bmatrix}\right\} M+1}_{M+1}$$

$$\vec{s} = \underbrace{(\overset{\text{Scale factor for variable 1}}{s_1}, s_2, \ldots, s_M)}_{\text{length } M}$$

Scale factor for variable M

Circling back to translation, how would we write T for M variables?

$$T_M(\vec{f}) = \underbrace{\left.\begin{bmatrix} 1 & 0 & \cdots & & f_1 \\ 0 & 1 & & & f_2 \\ & & \ddots & & \vdots \\ \vdots & \vdots & & & f_M \\ 0 & 0 & & & 1 \end{bmatrix}\right\} M+1}_{M+1}$$

$$\vec{f} = \underbrace{(\underset{\text{Shift for var 1}}{f_1}, f_2, \ldots, f_M)}_{\text{length } M}$$

Shift for var M