Broadcasting Review

Example: Subtract the per-column mins from an ndarray:

Shape: $(\cancel{3},2)$. axis=0

result → shape: (2,)

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix} \qquad \text{mins} = \text{np.min}(A, \text{axis}=0) : \underline{[1, 2]}$$

Shape: (2,)

shape: (3,2) shape: (2,)

$\boxed{\text{Doing: } A - \text{mins}}$

1 subtracted from this col

2 subtracted from this col

Expected result: $\begin{bmatrix} 0 & 0 \\ 2 & 2 \\ 4 & 4 \end{bmatrix}$

$\boxed{\text{Numpy is a match-maker (for shapes :)}}$

Try #1: Broadcast across axis 0

	Shape
A :	(3, 2)
mins :	(2, ??)

Fail! mismatch

← "2 rows"

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix} - \begin{bmatrix} 1 \\ 2 \\ ?? \end{bmatrix}$$

Try #2: Broadcast across axis $\underline{1}$

Shape

A : $(3, 2)$

minus : $(1, 2)$

match

"1 row 2 cols"

"singleton dimension"

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix} - \begin{bmatrix} 1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 1-1 & 2-2 \\ 3-1 & 4-2 \\ 5-1 & 6-2 \end{bmatrix} = \boxed{\begin{bmatrix} 0 & 0 \\ 2 & 2 \\ 4 & 4 \end{bmatrix}}$$

Success!

Matrix Multiplication

Computing AB:

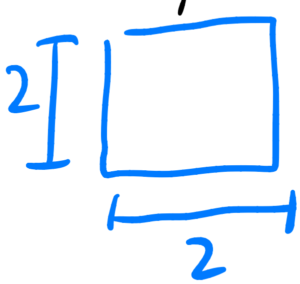
Matrix multiplication between matrix A & Matrix B.

1) Before doing AB:

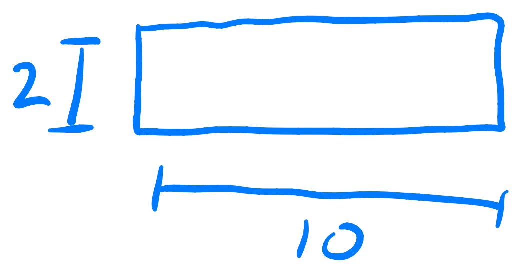
we check to see if the shapes are valid.

☆ Need inner dimensions of shapes to match:

e.g. A.shape = (2, 2)



B.shape = (2, 10)



$$AB = (2, 2) \times (2, 10)$$

Match! multiplication will work!

e.g. AC   C.shape = (3, 2)

$$AC = (2, 2) \times (3, 2)$$

mismatch! Can't do it!

- If we have a mismatch but need to do the multiplication, we can transpose a matrix to try to get matching inner dimensions.

numpy notation to do transpose

$C : (3, 2) \qquad C.T : (2, 3)$

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix} \xrightarrow{\text{Transpose}} \begin{bmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \end{bmatrix}$$

$\longleftrightarrow$

flip row and cols

$A\,C.T : (3, 2) \times (2, 3)$

will work!

2) If the shapes are compatible, we figure out the final output shape

$\Rightarrow$ Output.shape = (# rows of left mat, # cols of right mat)

$$\text{e.g. } AB = \overset{A}{(2, 2)} \times \overset{B}{(2, 10)}$$

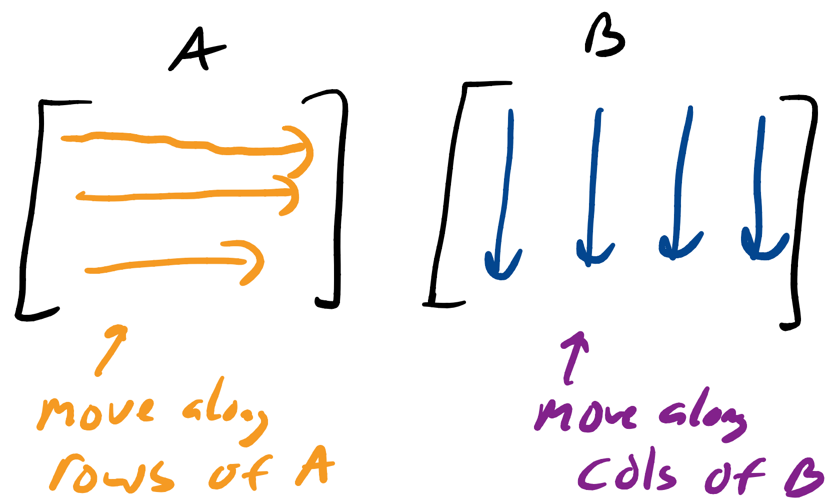
$$\text{answer.shape} = \underbrace{(2, 10)}_{\text{output shape}}$$

3) To multiply matrices A and B we:

a) Go along the <u>rows</u> of the left matrix (A)

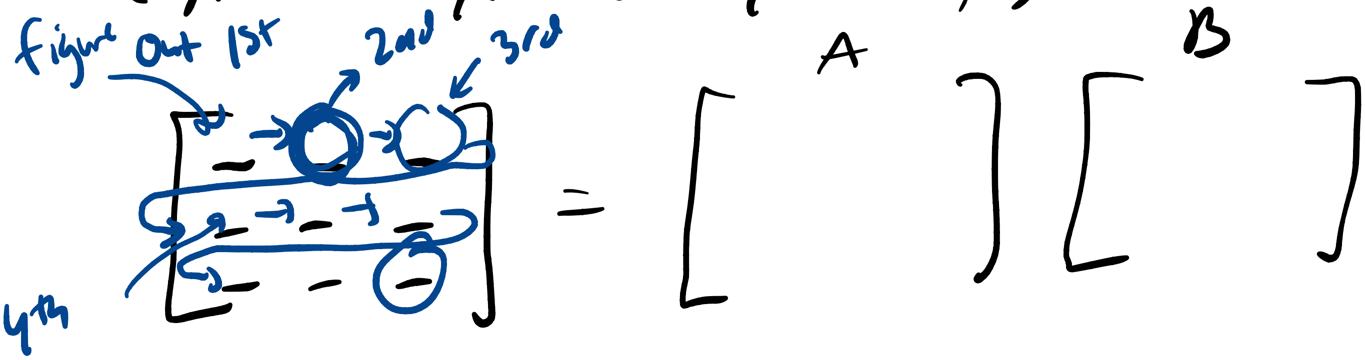
b) Go along the <u>cols</u> of the right matrix (B)

e.g. A B =



c) We fill in entries in "blank" output matrix one-by-one starting top-left, working rightward going row-by-row.

e.g. A.shape = B.shape = (3, 3)



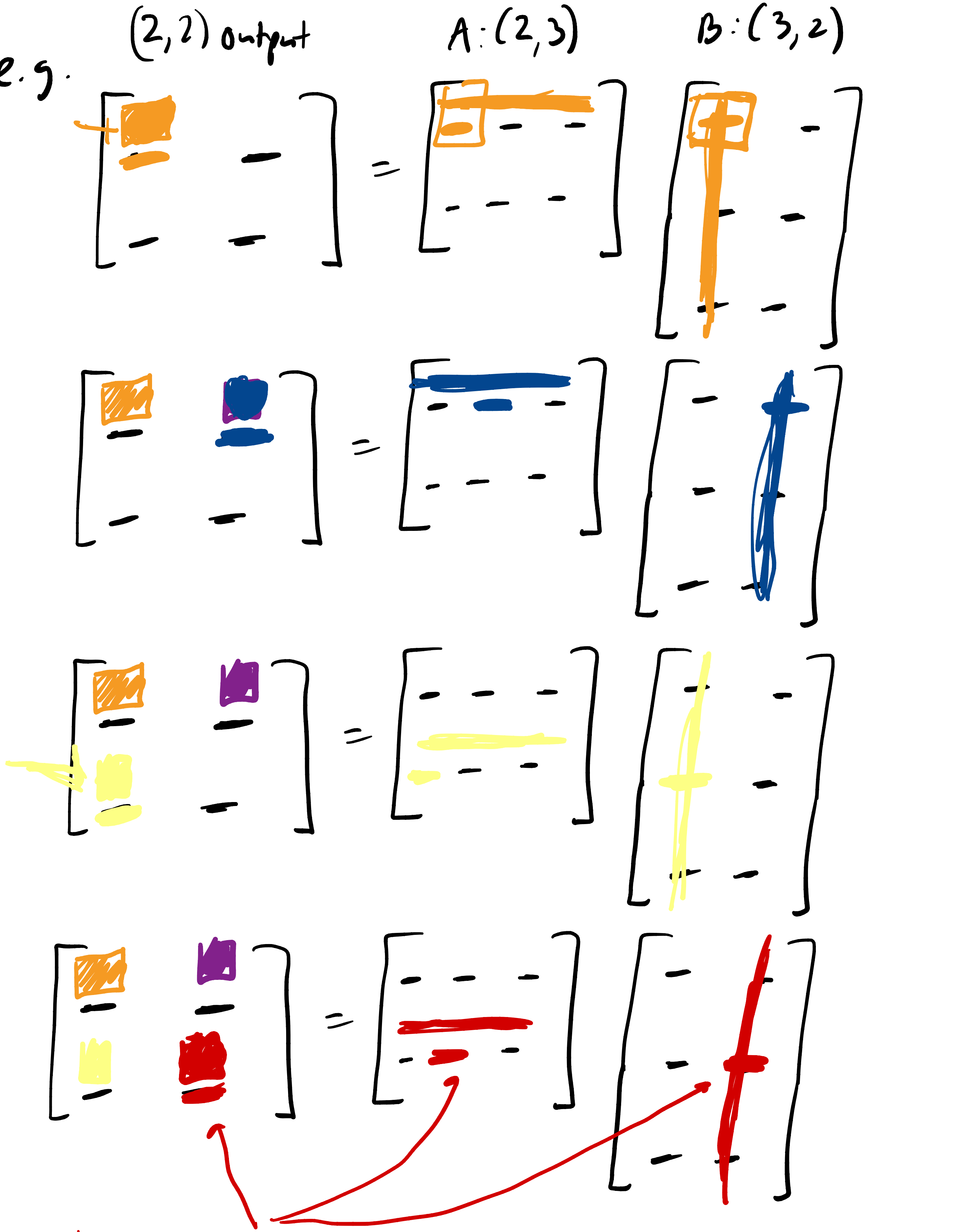
d) Each output matrix value determined by multiplying paired items in

- A rows
- B cols

Then adding them up.

A Result from each computation = <u>Scalar always</u>

e.g.



row/col of output entry matches intersection of current row in A, col in B.

example with numbers:

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \qquad B = \begin{bmatrix} 0 & 2 \\ 4 & 6 \\ 1 & 1 \end{bmatrix}$$

shape = (2, 2) shape = (3, 2)

mismatch!

$AB \Rightarrow (2, 2) \times (3, 2)$

Assume we actually still want to multiply A + B.

Try: B.T : $(3,2) \downarrow (2,3)$ $= \begin{bmatrix} 0 & 4 & 1 \\ 2 & 6 & 1 \end{bmatrix}$

$A\,B.T = (2, 2) \times (2, 3)$

Output shape =

$$\begin{bmatrix} \underline{*1} & \underline{*2} & \underline{*3} \\ \underline{*4} & \underline{*5} & \underline{*6} \end{bmatrix} = \overbrace{\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}}^{A} \overbrace{\begin{bmatrix} 0 & 4 & 1 \\ 2 & 6 & 1 \end{bmatrix}}^{B.T}$$

shape: (2, 3)

$*1 = 1 \cdot 0 + 2 \cdot 2 = 0 + 4 = 4$

$*2 = 1 \cdot 4 + 2 \cdot 6 = 4 + 12 = 16$

$*3 = 1 \cdot 1 + 2 \cdot 1 = 1 + 2 = 3$

$*4 = 3 \cdot 0 + 4 \cdot 2 = 8$

$*5 = 3 \cdot 4 + 4 \cdot 6 = 12 + 24 = 36$

$*6 = 3 \cdot 1 + 4 \cdot 1 = 7$

$$AB.T = \begin{bmatrix} 4 & 16 & 3 \\ 8 & 36 & 7 \end{bmatrix} \Rightarrow \text{Done!}$$

e) Order matters with <u>matrix multiplication</u>, unlike Scaler multiplication:

$a = 1$
$b = 2$ $\Rightarrow$ $\overset{1}{a} * \overset{2}{b} = \overset{2}{b} * \overset{1}{a} = 2$ ✓

<u>Not</u> true for matrices: $\boxed{AB \mathrel{!}= BA}$

e.g. $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ $B = \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}$

$$AB = \begin{bmatrix} 1\cdot 5 + 2\cdot 7 & 1\cdot 6 + 2\cdot 8 \\ 3\cdot 5 + 4\cdot 7 & 3\cdot 6 + 4\cdot 8 \end{bmatrix} = \begin{bmatrix} 19 & 22 \\ 43 & 50 \end{bmatrix}$$

$$!=$$

$$BA = \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 5\cdot 1 + 6\cdot 3 & 5\cdot 2 + 6\cdot 4 \\ 7\cdot 1 + 8\cdot 3 & 7\cdot 2 + 8\cdot 4 \end{bmatrix} = \begin{bmatrix} 23 & 34 \\ 31 & 46 \end{bmatrix}$$

Now let's see how to do Matrix multiplication with Numpy