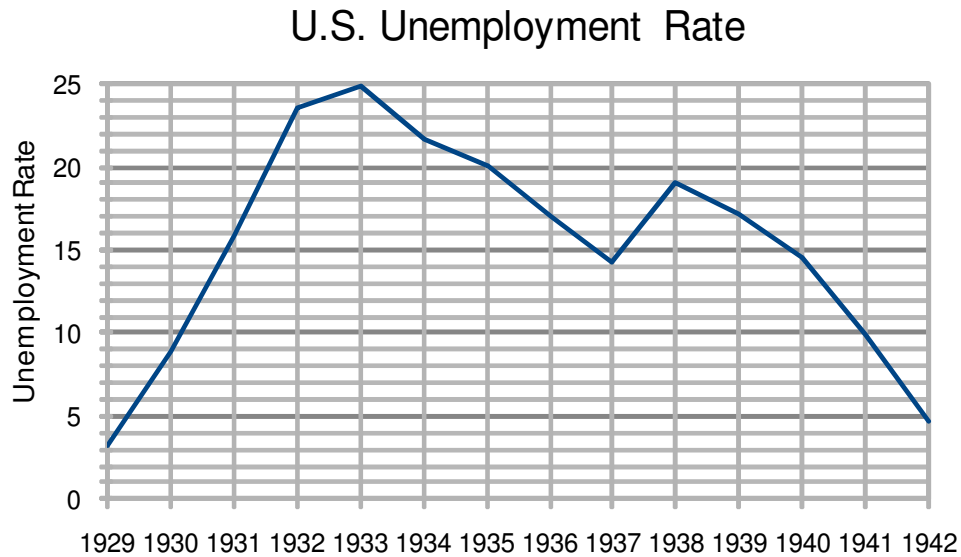


# Lesson 1.1 Variables

## Activity 1 Reading a Graph

The graph shows the U.S. unemployment rate as a percent of the labor force during the years surrounding the Great Depression.



- What was the unemployment rate in 1930?
- When did the unemployment rate first reach 15% ?
- When did the unemployment rate reach its highest value? What was the unemployment rate at that time?
- After 1930, when was the first time the unemployment rate fell below 10% ?
- During which year did the unemployment rate show the greatest increase?  
During which year did it show the greatest decrease?
- Complete the table:

Year	Unemployment Rate	Labor Force (Millions)	Number Unemployed	Year	Unemployment Rate	Labor Force (Millions)	Number Unemployed
1929		48.0		1936		53.3	
1930		48.8		1937		54.1	
1931		49.6		1938		54.9	
1932		50.3		1939		55.6	
1933		51.1		1940		56.2	
1934		51.9		1941		57.5	
1935		52.6		1942		60.4	

- During which year did the number of unemployed workers increase the most?

## Activity 2 Writing Mathematical Sentences

1. Barry lives with his aunt while he attends college. Every week he gives her \$20 from his paycheck to help pay for groceries. Fill in the table:

Barry's paycheck	45	60	75	100	125	$p$
Calculation	$45 - 20$					
Amount he keeps	<b>25</b>					

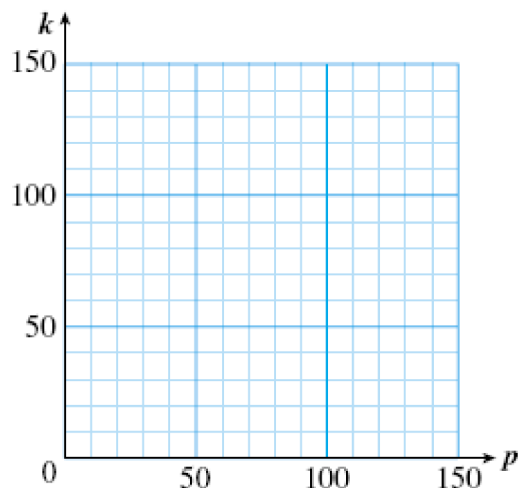
- a. Explain in words how to find the amount Barry keeps from his paycheck.

- b. Write your explanation as a mathematical sentence:

Amount he keeps =

- c. Let  $p$  stand for the amount of Barry's paycheck and  $k$  for the amount he keeps. Write an equation for  $k$  in terms of  $p$ .

- d. Plot the points from the table and connect them with a smooth curve.



2. Liz makes \$6 an hour as a tutor in the Math Lab. Her wages for the week depend on the number of hours she works. Fill in the table.

Hours worked	3	5	6	8	15	$h$
Calculation	$6 \times 3$					
Wages	<b>18</b>					

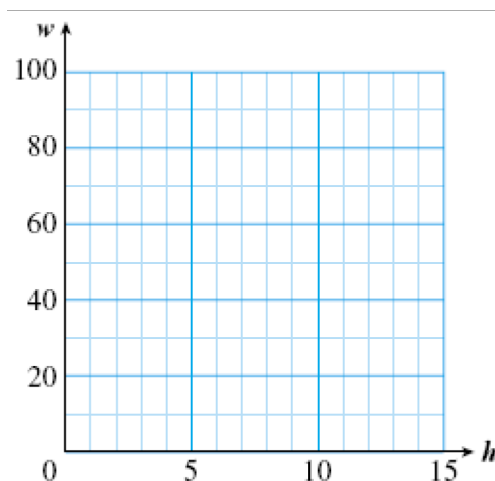
- a. Explain in words how to find Liz's wages for the week.

- b. Write your explanation as a mathematical sentence:

Wages =

- c. Let  $h$  stand for the number of hours Liz worked and  $w$  for her wages. Write an equation for  $w$  in terms of  $h$ .

- d. Plot the points from the table and connect them with a smooth curve.



## Wrap-Up

In this Lesson we practiced the following skills:

- Reading values from a graph
- Plotting points from a table of values
- Describing a relationship between two variables
- Writing an equation relating two variables

1. In Activity 1e, how do we calculate the increase in unemployment rate?
2. In Activity 1f, were more people unemployed in 1931 or in 1940?
3. In Activity 2, is the first row of the table plotted on the horizontal axis or the vertical axis?

## 1.1 Homework Preview

■ Write an equation for  $y$  in terms of  $x$ .

1. 

$x$	1	2	5	8
$y$	9	10	13	16

2. 

$x$	6	12	24	30
$y$	1	2	4	5

3. 

$x$	5	7	8	10
$y$	3	5	6	8

4. 

$x$	1	3	5	7
$y$	4	12	20	28

5. 

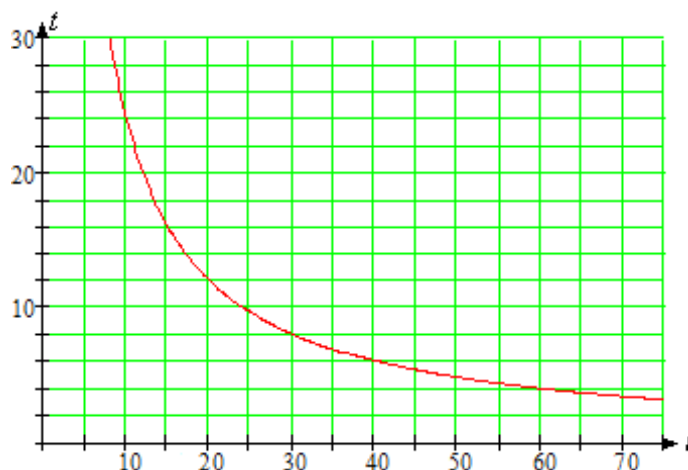
$x$	2	4	6	9
$y$	10	8	6	3

6. 

$x$	4	8	12	20
$y$	3	6	8	15

■ Matt wants to travel 240 miles to visit a friend over spring break. He is deciding whether to ride his bike or drive. If he travels at an average speed of  $r$  miles per hour, then the trip will take  $t$  hours. Use the graph to answer the questions.

7. If Matt can ride at an average speed of 15 miles per hour, how long will the trip take?
8. If Matt wants to arrive in 8 hours, what average speed will he need to maintain?
9. How long will the trip take if Matt drives at 60 miles per hour?
10. If Matt doubles his speed, what happens to his travel time?



## Answers

- |                                 |                   |                               |
|---------------------------------|-------------------|-------------------------------|
| 1. $y = x + 8$                  | 2. $y = x \div 6$ | 3. $y = x - 2$                |
| 4. $y = 4 \times x$             | 5. $y = 12 - x$   | 6. $y = \frac{3}{4} \times x$ |
| 7. 16 hrs                       | 8. 30 mph         | 9. 4 hrs                      |
| 10. Travel time is cut in half. |                   |                               |

## Lesson 1.2 Algebraic Expressions

### Activity 1 Writing Algebraic Expressions

**To write an algebraic expression:**

- Step 1** Identify the unknown quantity and write a short phrase to describe it.
- Step 2** Choose a variable to represent the unknown quantity.
- Step 3** Use mathematical symbols to represent the relationship described.

Study the Examples below, then write an expression for each Exercise.

**Example 1** Write an algebraic expression for each quantity.

- a. 3 feet more than the length of the rug
- b. Twice the age of the building

**Solutions a. Steps 1-2** The length of the rug is unknown.

**Length of the rug:**  $l$

**Step 3** The words "more than" indicate addition:  $l + 3$

**b. Steps 1-2** The age of the building is unknown.

**Age of the building:**  $a$

**Step 3** "Twice" means two times:  $2a$

**Exercise 1** Write an algebraic expression for each quantity.

- a. Ten more than the number of students  
Steps 1-2  
Step 3
- b. Five times the height of the triangle  
Steps 1-2  
Step 3
- c. 4% of the original price  
Steps 1-2  
Step 3
- d. Two and a quarter inches taller than last year's height.  
Steps 1-2  
Step 3

The next example involves subtraction and division.

**Example 2** Write an algebraic expression for each quantity.

- a. 5 square feet less than the area of the circle
- b. The ratio of your quiz score to 20

**Solutions a. Steps 1-2** The area of the circle is unknown.

**Area of circle:**  $A$

**Step 3** "Less than" indicates subtraction:  $A - 5$

**b. Steps 1-2** Your quiz score is unknown.

**Quiz score:**  $s$

**Step 3** A "ratio" is a fraction:  $\frac{s}{20}$

**Exercise 2** Write an algebraic expression for each quantity.

- a. \$60 less than first-class airfare

Steps 1-2

Step 3

- b. The quotient of the volume of the sphere and 6

Steps 1-2

Step 3

- c. The ratio of the number of gallons of alcohol to 20

Steps 1-2

Step 3

- d. The current population diminished by 50

Steps 1-2

Step 3

3. Each of the words listed refers to one of the four arithmetic operations. Group them under the correct operation.

times	take away	sum of	divided by
less than	reduced by	twice	(fraction) of
increased by	product of	more than	quotient of
exceeded by	deducted from	ratio of	total
difference of	split	minus	per

- a. addition      b. subtraction      c. multiplication      d. division

## Activity 2 Evaluating Algebraic Expressions

**Example** The Appliance Mart is having a store wide 15%-off sale. If the regular price of an appliance is  $P$  dollars, then the sale price  $S$  is given by

$$S = 0.85P$$

How much is a refrigerator that regularly sells for \$600?

**Solution** We substitute **600** for the regular price,  $P$ , in the expression.

$$\begin{aligned} S &= 0.85P \\ &= 0.85(\mathbf{600}) = 510 \end{aligned}$$

The sale price is \$510.

**Exercise 1** Evaluate the algebraic expression in the Example above to complete the table below showing the sale price for various appliances.

$P$	120	200	380	480	520
$S$					

Complete each table by evaluating the expression.

2.

$x$	1	2	10	100
$0.65x$				

3.

$n$	0.1	0.5	0.75	1
$n - 0.05$				

4.

$a$	6	8	$\frac{3}{4}$	$\frac{6}{5}$
$\frac{2}{3}a$				

5.

$h$	1	$\frac{5}{4}$	$2\frac{1}{2}$	3
$h - \frac{3}{4}$				

6.

$w$	0.1	0.5	0.75	1.2
$2 - w$				

7.

$p$	0.1	0.5	1	4
$\frac{p}{0.4}$				

### Activity 3 Using Algebraic Formulas

Write down the five algebraic formulas from the Reading, as algebraic expressions and in words.

	Formula	Meaning in Words
1.		
2.		
3.		
4.		
5.		

- For the Exercises below,
- Choose the appropriate formula and write an algebraic expression.
  - Evaluate the expression to answer the question.
- It cost Ariel \$380 to buy supplies and advertising for her pet-sitting business.
    - Write an expression for her profit from the business.
    - If Ariel earned \$820 in revenue, what was her profit?
  - Jamie cycled for  $2\frac{1}{2}$  hours this morning.
    - Write an expression for the distance Jamie cycled.
    - If Jamie's average speed was 9 miles per hour, how far did she cycle?
  - SaveOurPark collected \$2600 in donations from people in the neighborhood.
    - Write an expression for the average amount that each person donated.
    - If 142 people made a donation, what was the average donation?

## Wrap-Up

In this Lesson we practiced the following skills:

- Writing an algebraic expression
- Evaluating an algebraic expression
- Choosing an appropriate formula

1. What is the first step in evaluating an algebraic expression?
2. What does it mean to evaluate an algebraic expression?
3. Give definitions for the following terms:

revenue, principal, percentage rate, average, perimeter

## 1.2 Homework Preview

■ Write algebraic expressions.

1. The ratio of 6 to  $x$
2.  $x$  decreased by 6
3. 6% of  $x$
4. One-sixth of  $x$
5. 8 less than  $v$
6.  $H$  increased by 14

■ Choose a variable and write an algebraic expression.

7. The ratio of your quiz score to 20
8. 3 feet more than the length of the rug
9. Twice the age of the building
10. \$6 less than the sales price

■ Use a formula to write an expression.

11. Marla's revenue was  $D$  dollars, and her costs were \$100. What was her profit?
12. Gertrude earned  $P$  points on 5 quizzes. What was her average score?
13. 49.2% of the voters favored your candidate. If  $V$  people voted, how many favored your candidate?
14. The height of a triangle is 6 inches and its base is  $n$  inches. What is its area?

## Answers

- |                  |             |                   |                   |
|------------------|-------------|-------------------|-------------------|
| 1. $\frac{6}{x}$ | 2. $x - 6$  | 3. $0.06x$        | 4. $\frac{1}{6}x$ |
| 5. $v - 8$       | 6. $H + 14$ | 7. $\frac{Q}{20}$ | 8. $L + 3$        |
| 9. $2a$          | 10. $S - 6$ | 11. $D - 100$     | 12. $\frac{P}{5}$ |
| 13. $0.492V$     | 14. $3n$    |                   |                   |



## Lesson 1.3 Equations and Graphs

### To graph an equation:

- Step 1** Make a table of values.  
**Step 2** Choose scales for the axes.  
**Step 3** Plot the points and connect them with a smooth curve.

### Activity 1 Making a Graph

1. Laura takes her daughter Stefanie berry-picking at a local strawberry farm. Laura can pick three baskets of strawberries in the same time that Stefanie picks one basket.
  - a. Let  $L$  stand for the number of baskets Laura has picked and  $S$  for the number of baskets that Stefanie has picked. Write an equation relating the variables.

- b. Graph the equation.

**Step 1** Make a table of values. Choose some reasonable values for  $S$ , such as:

$S$	1	2	4	6	10
$L$					

Use the equation to find the corresponding values of  $L$ .

For example, when  $S = 1$ ,

$$L =$$

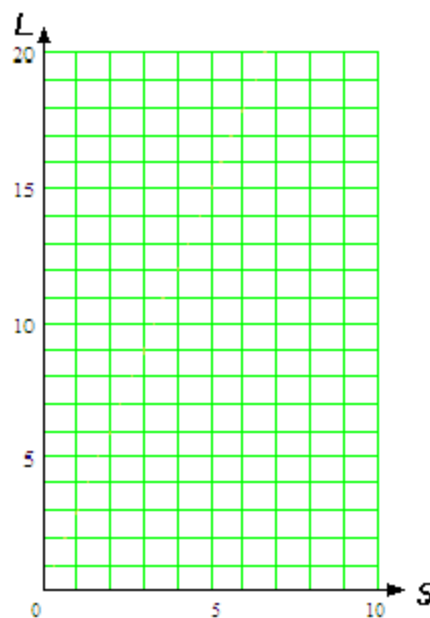
and when  $S = 2$ ,

$$L =$$

Complete the table.

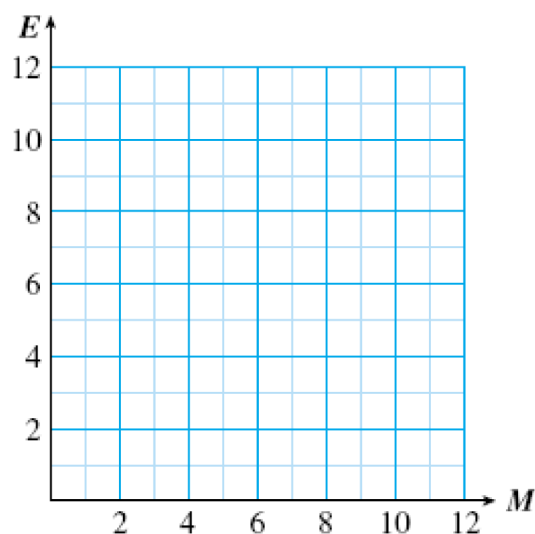
**Step 2** Label the horizontal axis with the input variable, and the vertical axis with the output variable. Then label the scales on the axes.

**Step 3** Plot the points in the table and connect them with a smooth curve. The points on this graph lie on a straight line.



2. Emily and Megan pledged to walk a total of 12 miles for their school's fundraising walkathon. Let  $E$  stand for the number of miles Emily walks, and  $M$  for the number of miles Megan walks.
- Write an equation for  $E$  in terms of  $M$ .
  - Make a table of values and graph the equation.

$M$				
$E$				



## Activity 2 Choosing Scales for the Axes

1. Corey's truck holds 20 gallons of gasoline and gets 18 miles to the gallon.
- Write an equation that relates the distance,  $d$ , that Corey can travel to the number of gallons of gas,  $g$ , in his truck.
  - Graph the equation.

**Step 1** Make a table of values.

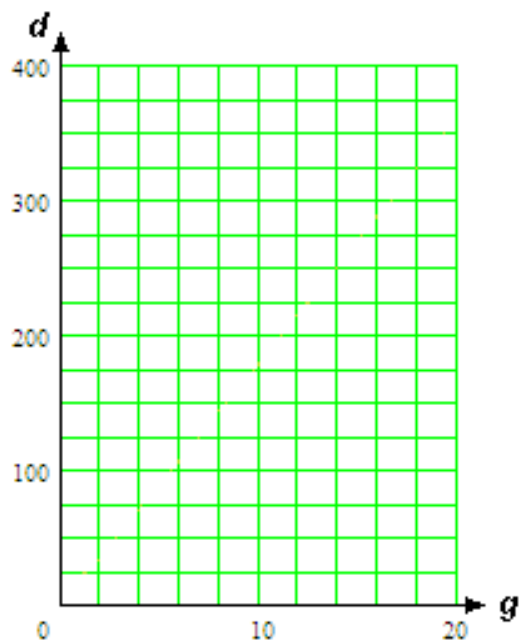
Choose values of  $g$  between 0 and 20, because Corey's truck holds 20 gallons of gas. Use the equation to calculate the values of  $d$ .

$g$	2	5	8	10	15	20
$d$						

**Step 2** Choose scales for the axes: Use 10 grid lines on the horizontal axis. The length of each interval is

$$20 \div 10 = \underline{\hspace{1cm}} \text{ units,}$$

so we scale the horizontal axis in intervals of 2. On the vertical axis, we use increments of 25 from 0 to 400, which gives us 16 grid lines.



**Step 3** Plot the points from the table of values. You will need to estimate the location of some of the points between tick marks.

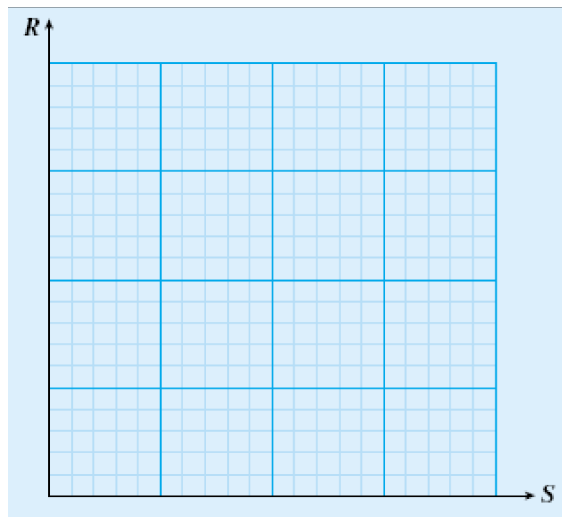
2. The Harris Aircraft company gave all its employees a 5% raise.
- Write an equation that expresses each employee's raise,  $R$ , in terms of his or her present salary,  $S$ .
  - Graph the equation.

**Step 1** Complete the table of values.

$S$	18,000	24,000	32,000	36,000
$R$				

**Step 2** Choose scales for the axes, and label them. (What are the largest values of  $S$  and  $R$  in your table?)

**Step 3** Plot points and draw the graph.



### Activity 3 Using a Graph

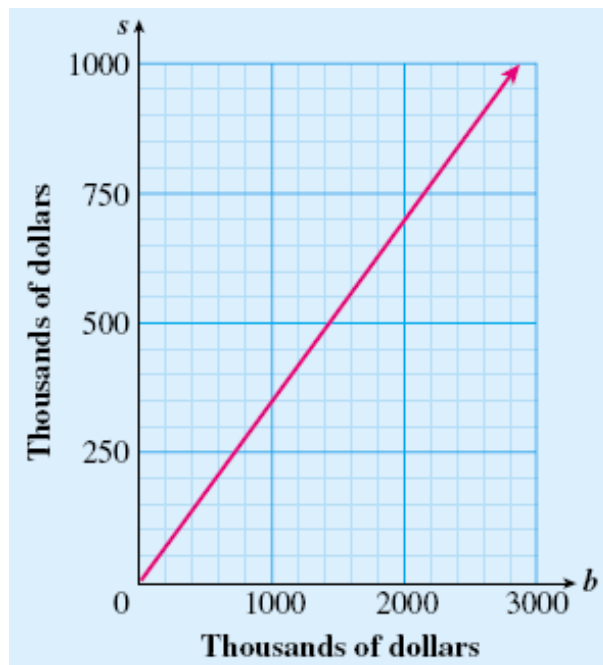
The Reedville City Council voted that 35% of the town's budget should be allotted to education.

- Write an equation for the amount budgeted for education,  $s$ , in terms of the total budget  $b$ .

Use your equation to answer the following questions:

- If Reedville's total budget for next year is \$1,800,000, how much will be allotted for education?
- If Reedville spent \$875,000 on education last year, what was its total budget?

Here is a graph of the equation from part (a). Both axes of the graph are scaled in *thousands* of dollars. Use the graph to estimate the answers to the parts (b) and (c). Show directly on the graph how you obtained your estimates.



Estimates: (b) \_\_\_\_\_

(c) \_\_\_\_\_

## Wrap-Up

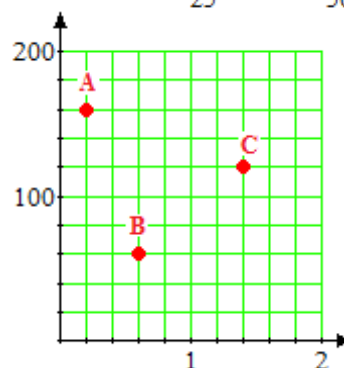
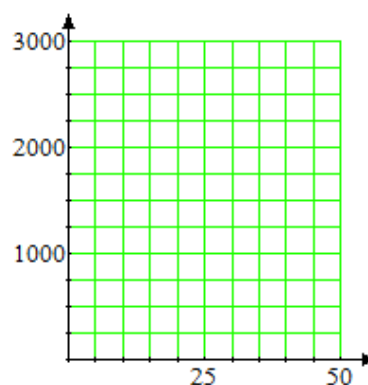
In this Lesson, we practiced the following skills:

- Making a table of values
- Choosing scales for the axes
- Plotting points and drawing a curve
- Using a graph to evaluate an expression or solve an equation

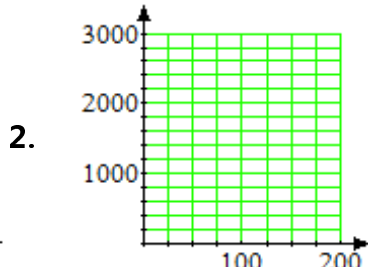
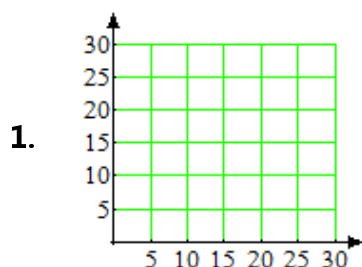
1. If you graph the equation  $Q = M + 12$ , which variable goes on the horizontal axis?
2. If the output values range from 0 to 6000, what would be a good interval to use for the scale on the vertical axis?
3. In Activity 3b, did we evaluate an expression or solve an equation?

## 1.3 Homework Preview

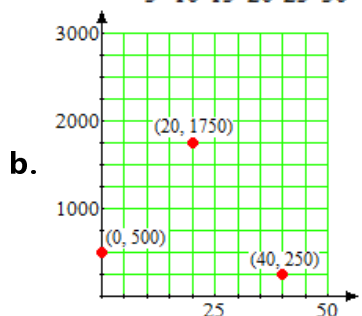
1. Draw axes and label both scales by 5's, from 0 to 30.
2. Draw axes and label the horizontal scale by 25's, from 0 to 200, and the vertical scale by 200's, from 0 to 3000.
3. a. What interval does each grid line represent on the horizontal axis? On the vertical axis?  
b. Plot the following points on the grid:  $(0, 500)$ ,  $(20, 1750)$ ,  $(40, 250)$ .
4. a. What interval does each grid line represent on the horizontal axis? On the vertical axis?  
b. Find the coordinates of each point.



## Answers



3a. horizontal: 5; vertical: 250



4a. horizontal: 0.2; vertical: 20  
b.  $A(0.2, 160)$ ,  $B(0.6, 60)$ ,  $C(1.4, 120)$

## Lesson 1.4 Solving Equations

### Activity 1 Opposite Operations

We can use opposite or inverse operations to "undo" an algebraic expression.

1. Francine is exactly four years older than Delbert, so

$$F = D + 4$$

where  $D$  stands for Delbert's age and  $F$  stands for Francine's age. Fill in the missing values in the table, and think about how you found each one.

$D$	5	7		10		18
$F$			12		19	

- a. When we are given a value of  $D$ , we \_\_\_\_\_ 4 to find the value of  $F$ .  
(Why?)
- b. When we are given a value of  $F$ , we \_\_\_\_\_ 4 to find Delbert's age.  
(Why?)
2. Fernando plans to share an apartment with three other students and split the rent equally.
- a. Let  $r$  stand for the rent on the apartment and  $s$  for Fernando's share. Write an equation for  $s$  in terms of  $r$ .
- b. Fill in the table.

$r$	260	300		360		480
$s$			80		105	

- c. Explain how you found the unknown values of  $s$ .

Explain how you found the unknown values of  $r$ .

## Activity 2 Solving Equations Algebraically

Follow the Examples below to write out the solution for each Exercise.

**Caution!** Do not do the problems in your head! Soon we will encounter equations that cannot be solved so easily. In order to learn the algebraic method, it is important for you to write down the steps of your solution.

1. Solve  $x + 6 = 11$       **Step 1** 6 is *added* to the variable.

$$\begin{array}{r} x + 6 = 11 \\ \underline{-6} \quad \underline{-6} \\ x = 5 \end{array}$$

**Step 2** We *subtract* 6 from both sides.

The solution is 5.  
Check:  $5 + 6 = 11$

**Exercise 1** Solve  $5 + y = 9$

2. Solve  $n - 17 = 32$       **Step 1** 17 is *subtracted* from the variable.

$$\begin{array}{r} n - 17 = 32 \\ \underline{+17} \quad \underline{+17} \\ n = 49 \end{array}$$

**Step 2** We *add* 17 to both sides.

The solution is 49.  
Check:  $49 - 17 = 32$

**Exercise 2** Solve  $x - 4 = 12$

3. Solve  $12x = 60$       **Step 1** The variable is *multiplied* by 12.

$$\begin{array}{r} \frac{12x}{12} = \frac{60}{12} \\ x = 5 \end{array}$$

**Step 2** We *divide* both sides by 12.

The solution is 5.  
Check:  $12(5) = 60$

**Exercise 3** Solve  $6z = 24$

**4.** Solve  $\frac{w}{7} = 21$       **Step 1** The variable is *divided* by 7.

**Step 2** We *multiply* both sides by 7.

$$7\left(\frac{w}{7}\right) = 7(21)$$

$$w = 147$$

The solution is 147.

Check:  $\frac{147}{7} = 21$

**Exercise 4** Solve  $\frac{w}{3} = 6$

### Activity 3 Using Formulas

The distance from Los Angeles to San Francisco is approximately 420 miles. How long will it take a car traveling at 60 miles per hour to go from Los Angeles to San Francisco? Follow the steps to solve the problem:

- 1) Write down the appropriate formula.
- 2) List the given values of the variables.
- 3) Which variable is unknown?
- 4) Substitute the known values into the formula:
- 5) Solve the equation for the unknown variable:

Answer:



## Activity 4 Writing Equations

In the following Exercises, concentrate on writing an equation for the problem. Use the hints to help you solve the problems.

1. A two-bedroom house costs \$20,000 more than a one-bedroom house in the same neighborhood. The two-bedroom house costs \$405,000. How much does the one-bedroom house cost?

**Step 1** Choose a variable for the unknown quantity.

Cost of the one-bedroom house: \_\_\_\_\_ .

**Step 2** Write an equation in terms of your variable.

$$\text{_____} + \text{_____} = \text{_____}$$

cost of one-  
bedroom house

cost of two-  
bedroom house

**Step 3** Solve your equation.

The one-bedroom house costs \_\_\_\_\_ .

2. A restaurant bill is divided equally by seven people. If each person paid \$8.50, how much was the bill?

**Step 1** Choose a variable for the unknown quantity. (What are we asked to find?)

**Step 2** Write an equation. Express each person's share in two different ways.

**Step 3** Solve your equation.

The bill was \_\_\_\_\_ .

3. Iris got a 6% raise. Her new salary is \$21 a week more than her old salary. What was her old salary?

**Step 1** Choose a variable for the unknown quantity.

**Step 2** Write an equation. Express Iris's raise in two different ways.

**Step 3** Solve your equation.

Iris's old salary was \_\_\_\_\_ .

### Wrap-Up

In this Lesson we practiced the following skills:

- Solving an equation algebraically
  - Using a formula to solve a problem
  - Writing an equation to model a problem
1. How can you check to see whether a given number is a solution of an equation?
  2. **a.** Is the statement  $3 + 4 = 12$  an equation? Why or why not?  
**b.** Is the statement  $x + 4 = 12$  an equation? Why or why not?
  3. Describe a two-step strategy for solving an equation algebraically.
  4. **a.** What is the inverse operation for subtraction?  
**b.** What is the inverse operation for division?
  5. In Activity 4, Problem 3, how do we write 6% as a decimal?

## 1.4 Homework Preview

- Choose the equation that best describes each situation. In each case,  $n$  represents the unknown quantity.

$$n + 5 = 30$$

$$n - 5 = 30$$

$$5n = 30$$

$$\frac{n}{5} = 30$$

1. Five less than a number is 30.
2. The quotient of a number and 5 is 30.
3. The product of a number and 5 is 30.
4. Five more than a number is 30.
5. The price of a concert ticket increased \$5 this year and is now \$30. How much did a ticket cost last year?
6. Amir spent 5 dollars and now has 30 dollars. How much did he have before he spent \$5?
7. Marty jogged the same course five days this week for a total of 30 miles. How far did he jog each day?
8. Five brothers split the cost of a new TV, each paying \$30. How much did the TV cost?

### Answers

- |                 |                       |              |                       |
|-----------------|-----------------------|--------------|-----------------------|
| 1. $n - 5 = 30$ | 2. $\frac{n}{5} = 30$ | 3. $5n = 30$ | 4. $n + 5 = 30$       |
| 5. $n + 5 = 30$ | 6. $n - 5 = 30$       | 7. $5n = 30$ | 8. $\frac{n}{5} = 30$ |

## Lesson 1.5 Order of Operations

### Activity 1 Which Operations Come First?

1. In the Reading assignment, we established the following rules.

1. In a string of additions and subtractions, we perform the operations **in order from left to right**.
2. Similarly, we perform multiplications and divisions **in order from left to right**.

**Exercise 1** Simplify each expression.

a.  $30 - 17 - 5 + 4$

b.  $72 \div 4 \cdot (-3) \div 6$

### 2. Combined Operations

**Always perform multiplications and divisions before additions and subtractions.**

**Exercise 2** Simplify.

a.  $12 - 6\left(\frac{1}{2}\right)$

b.  $2(3.5) + 10(1.4)$

### 3. Grouping into Terms

**Exercise 3** Simplify  $12 + 24 \div 4 \cdot 3 + 16 - 10 - 4$

## Activity 2 Parentheses and Fraction Bars

### 1. Parentheses

**Exercise 1** Simplify each expression.

a.  $28 - 3(12 - 2 \cdot 4)$

b.  $12 + 36 \div 4(9 - 2 \cdot 3)$

### 2. Fraction Bars

**Exercise 2** Simplify  $\frac{8 - 2(6 - 4)}{(8 - 2)6 - 4}$

**Step 1** Perform operations inside parentheses.

**Step 2** Simplify above the fraction bar — multiplication first.

**Step 3** Simplify below the fraction bar — multiplication first.

**Step 4** Reduce the fraction.

### 3. Summary

If an expression involves more than one type of grouping symbol (say, both parentheses and brackets), we start with the innermost grouping symbols and work outward.

**Exercise 3** Follow the steps to simplify  $6 + 2[3(12 - 5) - 4(7 - 3)]$

Subtract inside parentheses.  $6 + 2[3(\mathbf{12 - 5}) - 4(\mathbf{7 - 3})]$

Multiply inside the brackets.

Subtract inside the brackets.

Multiply, then add.

**Exercise 4** Follow the steps to simplify  $19 + 5 \left[ 4(22 - 19) - \frac{12}{2} \right]$

Subtract inside parentheses.  $19 + 5 \left[ 4(\mathbf{22} - \mathbf{19}) - \frac{12}{2} \right]$

Multiply inside the brackets,  
divide inside the brackets.

Subtract inside the brackets.

Multiply, then add.

### Activity 3 Algebraic Expressions

1. Choose the correct algebraic expression for each phrase.

a. 6 times the sum of  $x$  and 5

$$6x + 5 \quad \text{or} \quad 6(x + 5)$$

b.  $\frac{1}{2}$  the difference of  $p$  and  $q$

$$\frac{1}{2}(p - q) \quad \text{or} \quad \frac{1}{2}p - q$$

c. 4 less than the product of 6 and  $w$

$$6w - 4 \quad \text{or} \quad 4 - 6w$$

d. 2 less than the quotient of 10 and  $z$

$$\frac{10}{z} - 2 \quad \text{or} \quad 2 - \frac{10}{z}$$

Use the tables to evaluate each of the following expressions in two steps. (The first one is done for you.) Note especially how the order of operations differs in parts (a) and (b).

**2a.**  $8 + 3t$

**b.**  $3(t + 8)$

**3a.**  $6 + \frac{x}{2}$

**b.**  $\frac{6 + x}{2}$

$t$	$3t$	$8 + 3t$	$t$	$t + 8$	$3(t + 8)$
0	0	8	0	8	24
2			2		
7			7		

$x$	$\frac{x}{2}$	$6 + \frac{x}{2}$	$x$	$6 + x$	$\frac{6 + x}{2}$
4	2	8	4	10	5
8			8		
9			9		

## Activity 4 Using Your Calculator

Simplify each expression two ways: by hand, and with a calculator. Follow the order of operations.

1.  $9 + 2 \cdot 5 - 3 \cdot 4$

By hand:

With a calculator:

2.  $6(10 - 2 \cdot 4) \div 4$

By hand:

With a calculator:


3.  $2.4[25 - 3(6.7)] + 5.5$

By hand:

With a calculator:

**Caution!** Most calculators cannot use a fraction bar as a grouping symbol. Consider the expression  $\frac{24}{6-4}$ , which simplifies to  $\frac{24}{2}$ , or 12. If we enter the expression into a calculator as

$$24 \div 6 - 4$$

we get 0, which is not correct. This is because the calculator follows the order of operations and calculates  $24 \div 6$  first. 

If we use a calculator to compute  $\frac{24}{6-4}$ , we must tell the calculator that  $6-4$  should be computed first. To do this, we use parentheses and enter the expression as

$$24 \div (6 - 4)$$

We call this way of writing the expression the **in-line form**.

When using a calculator, we must enclose in parentheses any expression that appears above or below a fraction bar.

4. Use a scientific calculator to simplify the expression  $\frac{16.2}{(2.4)(1.5)}$

## Wrap-Up

In this Lesson, we practiced the following skills:

- Simplifying expressions by following the order of operations
  - Using a calculator to simplify expressions
1. Give examples to show that the associative laws do not hold for subtraction or division.
  2. Why should we separate an expression into its terms?
  3. True or false: always start simplifying from left to right.
  4. True or false: we should perform multiplications before divisions.
  5. How do we enter expressions with fraction bars into a calculator?

## 1.5 Homework Preview

■ Simplify.

- |                               |                                  |
|-------------------------------|----------------------------------|
| 1. a. $20 - 3(2)$             | b. $(20 - 3) \cdot 2$            |
| 2. a. $20 - 8 - 2$            | b. $20 - (8 - 2)$                |
| 3. a. $20 - 3(2 + 4)$         | b. $20 - (3 \cdot 2 + 4)$        |
| 4. a. $\frac{20 + 12}{4 + 2}$ | b. $\frac{20}{4} + \frac{12}{2}$ |
| 5. a. $\frac{25 - 8}{5}$      | b. $\frac{40}{8} - \frac{18}{6}$ |

## Answers

- 1a. 14    b. 34    2a. 10    b. 14    3a. 2    b. 10    4a.  $\frac{16}{3}$     b. 11    5a. 3.4    b. 2

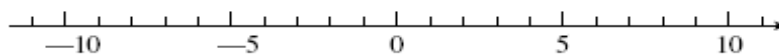


## Lesson 2.1 Signed Numbers

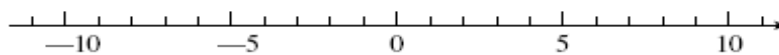
### Activity 1 Sums and Differences

1. Illustrate each sum on a number line, then give the answer.

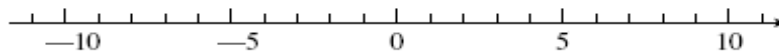
a.  $2 + 4 =$



b.  $(-4) + (-7) =$

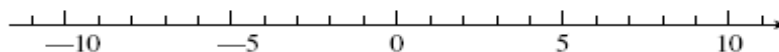


c.  $(-6) + (-3) =$

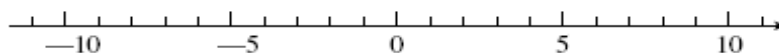


2. Illustrate each sum on a number line, then give the answer.

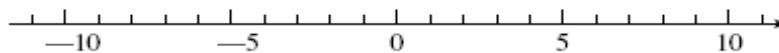
a.  $(+5) + (-3) =$



b.  $(-7) + (+2) =$



c.  $(-5) + (+9) =$



3. Fill in the blanks.

a. The sum of two positive numbers is \_\_\_\_\_.

b. The sum of two negative numbers is \_\_\_\_\_.

c. To add two numbers with opposite signs, \_\_\_\_\_ their absolute values. The sum has the same sign as the number with the \_\_\_\_\_ absolute value.

4. Find the following sums.

a.  $(-9) + (-9)$

b.  $(-14) + (-11)$

c.  $4 + (-12)$

d.  $15 + (-9)$

e.  $-6 + (-3)$

f.  $-8 + (+3)$

g.  $-7 + 19$

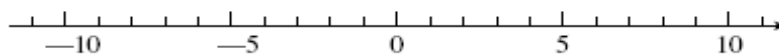
h.  $18 + (-10)$

i.  $5 + (-5)$

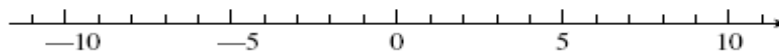
j.  $-5 + (-5)$

5. Illustrate each subtraction problem on the number line, then state the answer.

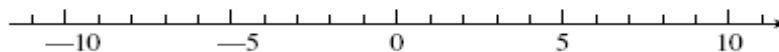
a.  $2 - (-6) =$



b.  $-7 - (-4) =$



c.  $-3 - (-7) =$



6. Rewrite each subtraction problem as an addition, then compute the answer.

a.  $3 - (-9)$

b.  $-4 - (-7)$

c.  $-8 - (-2)$

## Activity 2 Products and Quotients

1. Write the product  $5(-4)$  as a repeated addition, and compute the product.

2. Compute the product.

a.  $5(-4)$

b.  $\frac{-3}{4} \cdot \frac{1}{2}$

c.  $(-3)(-3)$

d.  $\frac{-5}{3} \cdot \frac{-3}{10}$

3. Use the relationship between products and quotients to complete each statement. No calculation is necessary!

a.  $\frac{8190}{26} = \underline{\hspace{2cm}}$  because  $26 \cdot 315 = 8190$

b.  $62 \cdot \underline{\hspace{2cm}} = 83.7$  because  $\frac{83.7}{62} = 1.35$

4. Find each quotient by rewriting the division as an equivalent multiplication fact.

a.  $\frac{6}{3} = \square$  because  $\underline{\hspace{2cm}}$

b.  $\frac{6}{-3} = \square$  because  $\underline{\hspace{2cm}}$

c.  $\frac{-6}{3} = \square$  because  $\underline{\hspace{2cm}}$

d.  $\frac{-6}{-3} = \square$  because  $\underline{\hspace{2cm}}$

5. Compute the following quotients.

a.  $\frac{-25}{-5}$

b.  $\frac{32}{-8}$

c.  $-27 \div 9$

d.  $-42 \div (-7)$

6. Fill in the blanks.

a. The product or quotient of two numbers with the same sign is  $\underline{\hspace{2cm}}$ .

b. The product or quotient of two numbers with opposite signs is  $\underline{\hspace{2cm}}$ .

7. Find the quotient, if it exists.

a.  $\frac{0}{18}$

b.  $\frac{13}{0}$

c.  $-9 \div 0$

d.  $0 \div (-2)$

8. a. Use your calculator to verify that  $-\frac{2}{5} = \frac{-2}{5} = \frac{2}{-5}$

b. Does  $-\frac{2}{5} = \frac{-2}{-5}$ ?

## Wrap-Up

In this Lesson we practiced the following skills:

- Illustrating sums and differences on a number line
- Performing operations on signed numbers
- Write a subtraction as an equivalent addition

1. Delbert says that "two negatives make a positive." For which operations is he correct, and for which is he incorrect?
2. How do we convert a subtraction of signed numbers into an addition? Give an example.
3. True or false:
  - a.  $\frac{2}{-2} = 0$
  - b.  $\frac{2}{0} = 0$
  - c.  $-2 - 2 = 0$
  - d.  $-2(-2) = 0$
4. Does the negative sign in a fraction such as  $-\frac{5}{2}$  apply to the numerator, the denominator, or both?

## 2.1 Homework Preview

■ Perform the operations.

1. a.  $8 + (-4)$       b.  $8 - (-4)$       c.  $-8 - 4$       d.  $-8 - (-4)$
2. a.  $8(-4)$       b.  $-8(-4)$       c.  $\frac{-8}{-4}$       d.  $\frac{8}{-4}$

■ Solve.

3.  $7 + x = -8$
4.  $-2x = 12$

## Answers

- |         |       |        |       |
|---------|-------|--------|-------|
| 1a. 4   | b. 12 | c. -12 | d. -4 |
| 2a. -32 | b. 32 | c. 2   | d. -2 |
| 3. -15  |       | 4. -6  |       |

## Lesson 2.2 Expressions and Equations

### Activity 1 Order of Operations

■ Follow the order of operations to simplify each expression.

1. a.  $5 - (+7) - 3 - (-2)$

b.  $-4 - (-9) - 3 - 8$

c.  $7(-3) - 2(-5)$

d.  $9 - 4(-6)$

2. a.  $-6(-2)(-5)$

b.  $-6(-2) - 5$

c.  $-6(-2 - 5)$

d.  $-6 - (2 - 5)$

3. a.  $[-5 - 8] - [7 - 10 - 4]$

b.  $-6 + [(4 - 8) - (-9)]$

c.  $28 - 3(-12 - 2 \cdot 4)$

d.  $12 - 36 \div 4(9 - 2 \cdot 3)$

## Activity 2 Writing Algebraic Expressions

1. Neda decides to order some photo albums as gifts. Each album costs \$12, and the total shipping cost is \$4. Neda would like an algebraic expression that describes the total cost of ordering  $a$  albums.

a. Consider some specific values for the variable:

What is Neda's bill if she orders 3 albums? If she orders 5 albums?

For 3 albums, the bill is:

$$12 \cdot 3 + 4 =$$

For 5 albums, the bill is:

b. Describe in words how you calculated your answers for specific values.

c. Replace the specific values in your calculations by a variable.

If Neda orders  $a$  albums, an expression for the bill is:

Let  $B$  stand for Neda's bill, and write an equation that gives Neda's bill,  $B$ , in terms of the number of albums she orders,  $a$ :

2. Megan would like to buy a kayak on sale. She calculates that the kayak she wants costs \$40 less than three weeks' salary.
- a. Write an expression for the price of the kayak if Megan makes \$280 per week.
- b. If Megan makes  $w$  dollars per week, write an expression for the price of the kayak.
3. Emily bought five rose bushes for her garden. Each rose bush cost \$9 plus tax.
- a. Write an expression for the total amount Emily paid if the tax on one rose bush is \$0.45.
- b. If the tax on one rose bush is  $t$ , write an expression for the total amount Emily paid.

### Activity 3 Evaluating Algebraic Expressions

1. When a company purchases a piece of equipment such as a computer or a copy machine, the value of the equipment depreciates over time. One way to calculate the value of the equipment uses the formula

$$V = C\left(1 - \frac{t}{n}\right)$$

where  $C$  is the original cost of the equipment,  $t$  is the number of years since it was purchased, and  $n$  stands for the useful lifetime of the equipment in years. Find the value of a 4-year-old copy machine if it has a useful lifetime of 6 years and cost \$3000 when new.

- a. List the values of the variables:

$$C = \underline{\hspace{2cm}}, \quad t = \underline{\hspace{2cm}}, \quad n = \underline{\hspace{2cm}}$$

- b. Substitute the values into the formula:

- c. Simplify the expression. Follow the order of operations.

Simplify inside the parentheses.

Multiply.

Answer:

2. Evaluate  $8(x + xy)$  for  $x = \frac{1}{2}$  and  $y = -6$ .

Substitute the values:

$$8\left(\frac{1}{2} + \frac{1}{2}(-6)\right)$$

Follow the order of operations.

3. Evaluate  $3a(a - b)$  for  $a = -4$  and  $b = -6$

4. Evaluate  $-2(a + b) - ab$  for  $a = 6$ ,  $b = -3$

## Wrap-Up

In this Lesson, we practiced the following skills:

- Simplifying expressions with the order of operations
  - Writing algebraic expressions with two or more operations
  - Evaluating algebraic expressions at signed numbers
1. Explain what is wrong with this calculation:  $9 - 4(-6) = 5(-6) = -30$
  2. For "5 dollars less than the price of 3 shirts," Delbert writes  $5 - 3s$ . What is wrong with his expression?
  3. In problems 2 and 3 of Activity 2, which expression required parentheses? Why?
  4. When we evaluate  $ab$  for  $a = 6$ ,  $b = -3$ , why should we enclose  $-3$  in parentheses?

## 2.2 Homework Preview

■ Write algebraic expressions.

1. Boyer's history book is 600 pages long, and he reads 20 pages per night. How many pages does he have left to read after  $t$  nights?
2. Kristi deposits \$50 from her paycheck into savings, and then gives herself 15% of the rest for spending money. If her paycheck is  $p$  dollars, how much spending money does she get?
3. The area of a pyramid is one-third the product of its length, its width, and its height.
4. The difference of a number  $B$  and twice its reciprocal.

■ Evaluate.

5.  $2m(m + p)(m - p)$  for  $m = -5$  and  $p = -8$

6.  $(z + 2)(2z - 1)$  for  $z = \frac{-3}{4}$

## Answers

1.  $600 - 20t$

2.  $0.15(p - 50)$

3.  $\frac{1}{3}lwh$

4.  $B - \frac{2}{B}$

5. 390

6.  $\frac{-25}{8}$



# Lesson 2.3 Graphs of Linear Equations

## Activity 1 Graphing Equations

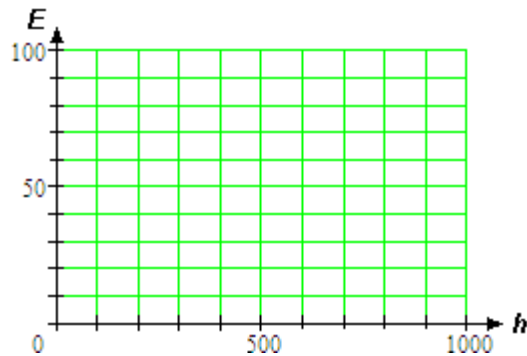
Jasmine's electricity company charges her \$6 per month plus \$0.10 per kilowatt hour (kWh) of energy she uses.

- Write an equation for Jasmine's electric bill,  $E$ , if she uses  $h$  kWh of electricity.
- Graph your equation.

Make a table of values.

$h$	$E$
100	
400	
800	

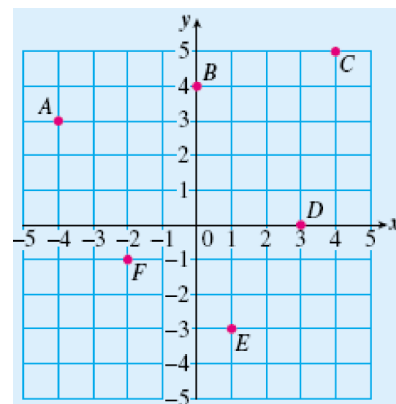
Plot the points and draw the graph.



## Activity 2 Cartesian Coordinate System

- Give the coordinates of each point shown in the figure below.

A \_\_\_\_\_  
 B \_\_\_\_\_  
 C \_\_\_\_\_  
 D \_\_\_\_\_  
 E \_\_\_\_\_  
 F \_\_\_\_\_



- Graph the equation  $y = -2x + 6$ 
  - Choose values for  $x$  and make a table of values. Choose both positive and negative  $x$ -values, as in the suggested table below.

$x$	$y$
-3	
-1	
0	
2	
4	

$$y = -2(-3) + 6 =$$

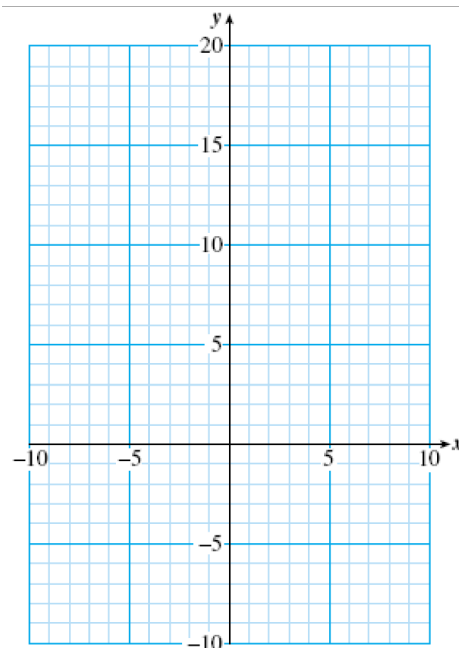
$$y = -2(-1) + 6 =$$

$$y = -2(0) + 6 =$$

$$y = -2(2) + 6 =$$

$$y = -2(4) + 6 =$$

- Plot the points and connect them with a straight line.



### Activity 3 Using a Graph

1. Use the graph in Activity 2 to answer parts (a) and (b). Label the point on the graph that gives the answer.

a. Evaluate the expression  $-2x + 6$  for  $x = -5$

b. Solve the equation  $-2x + 6 = 10$

2. Francine borrowed money from her mother, and she owes her \$750 right now. She has been paying off the debt at a rate of \$50 per month.

a. Write an equation for Francine's financial status,  $F$ , in terms of  $m$ , months from now.

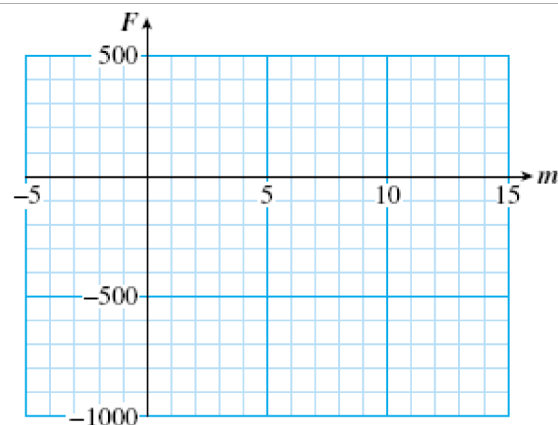
b. Fill in the table. Negative values of  $m$  mean months in the past. (Francine's current financial status is  $-\$750$ .)

$m$	-5	-2	0	2	6	10	12
$F$							

c. Graph your equation, using the values in the table.

Use your graph to answer the questions, and label the point on the graph that gives the answer:

d. What will Francine's financial status be 7 months from now?



e. When was Francine's financial status  $-\$900$ ?

### Wrap-Up

In this Lesson we practiced the following skills:

- Graphing a linear equation
- Plotting points on a Cartesian coordinate system
- Using a graph to answer questions about a model

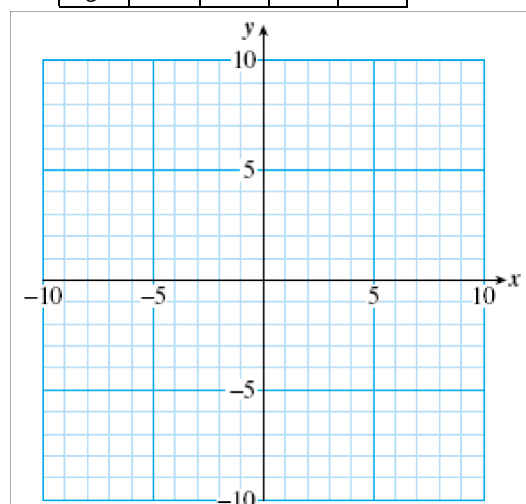
1. In Activity 1, what are the intervals represented by each grid line on the axes?
2. In Activity 2, what is the  $x$ -coordinate of the point with  $y$ -coordinate 16?
3. In Activity 3, problem 2, if you increase the value of  $m$ , does  $F$  increase or decrease?

## 2.3 Homework Preview

■ Complete the table of values and graph the equation.

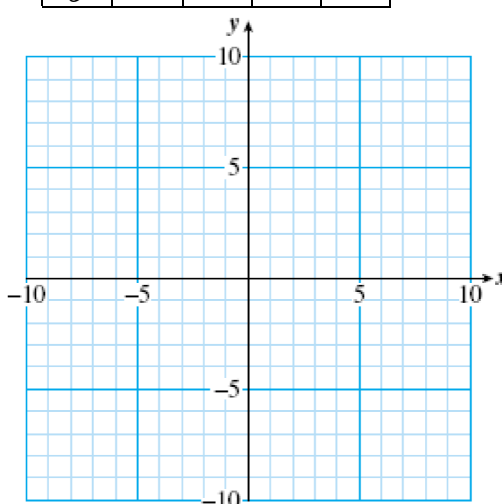
1.  $y = 4 - 2x$

$x$	-3	0	2	5
$y$				



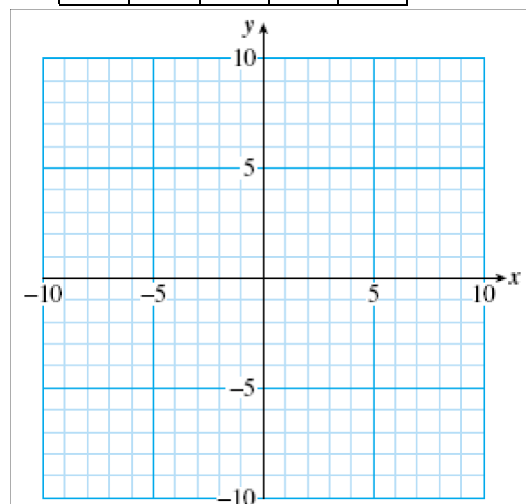
2.  $y = -6 + 3x$

$x$	-1	0	2	4
$y$				



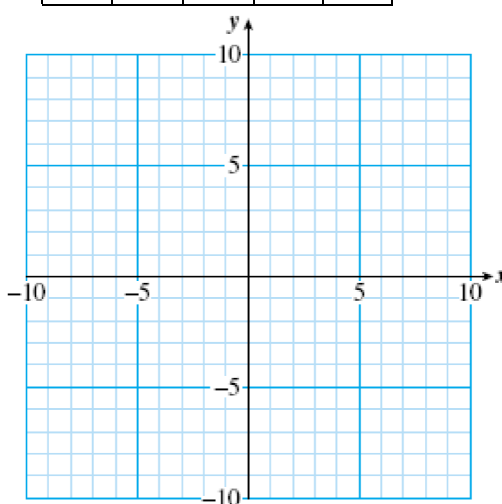
3.  $y = -2 + \frac{4}{3}x$

$x$	-3	0	3	6
$y$				

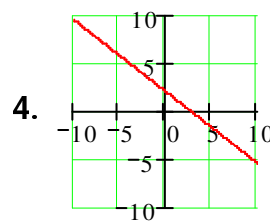
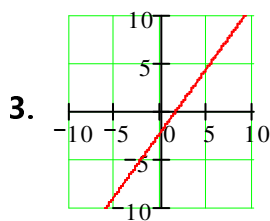
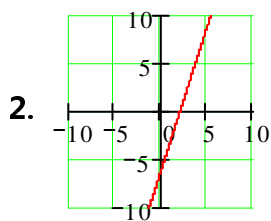
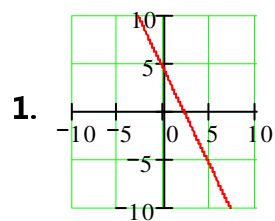


4.  $y = 2 - \frac{3}{4}x$

$x$	-8	-4	0	8
$y$				



## Answers



## Lesson 2.4 Linear Equations and Inequalities

### Activity 1 Order of Operations

Study each example, then try the corresponding exercise.

1. Use the order of operations to analyze the expression containing the variable and to plan your approach.
2. Carry out the steps of the solution.

**Example 1** Solve  $4x - 5 = 7$

Operations performed on $x$	Steps for solution
1. Multiply by 4	1. Add 5
2. Subtract 5	2. Divide by 4

$$4x - 5 = 7 \quad \text{Add 5 to both sides.}$$

$$4x - 5 + 5 = 7 + 5$$

$$4x = 12 \quad \text{Divide both sides by 4.}$$

$$\frac{4x}{4} = \frac{12}{4}$$

$$x = 3$$

The solution is 3. **Check:**  $4(3) - 5 = 7$

**Exercise 1** Solve  $3x + 2 = 17$

**Example 2** Solve  $\frac{-3t}{4} = 6$

Operations performed on $x$	Steps for solution
1. Multiply by $-3$	1. Multiply by 4
2. Divide by 4	2. Divide by $-3$

$$\frac{-3t}{4} = 6 \quad \text{Multiply both sides by 4.}$$

$$4\left(\frac{-3t}{4}\right) = (6)4$$

$$-3t = 24 \quad \text{Divide both sides by } -3.$$

$$\frac{-3t}{-3} = \frac{24}{-3}$$

$$t = -8$$

The solution is  $-8$ . **Check:**  $\frac{-3(-8)}{4} = 6$

**Exercise 2** Solve  $\frac{-4a}{5} = -12$

**Example 3** Solve  $5(z - 6) = 65$

<i>Operations performed on <math>x</math></i>	<i>Steps for solution</i>
1. Subtract 6	1. Divide by 5
2. Multiply by 5	2. Add 6

$$5(z - 6) = 65 \quad \text{Divide both sides by 5.}$$

$$\frac{5(z - 6)}{5} = \frac{65}{5}$$

$$z - 6 = 13 \quad \text{Add 6 to both sides.}$$

$$z - 6 + 6 = 13 + 6$$

$$z = 19$$

The solution is 19. **Check:**  $5(19 - 6) = 65$

**Exercise 3** Solve  $\frac{b + 3}{4} = 6$

**Exercise 4** Follow the steps to solve the equation  $-6 - \frac{2x}{3} = 8$

Add 6 to both sides.

Rewrite the fraction  $-\frac{2x}{3}$  in standard form.

Multiply both sides by 3.

Divide both sides by  $-2$ .

Check your solution.

## Activity 2 Problem Solving

1. Mitch bought a Blu-Ray player for \$269 and a number of discs at \$14 each.
  - a. Write an equation for Mitch's total bill,  $B$ , in terms of the number of discs,  $d$ , he bought.
  - b. If the total bill before tax was \$367, how many discs did Mitch buy?

Substitute 367 for  $B$  and solve for  $d$ :

Write your answer in a sentence.

2. Home Station had a promotion offering \$4 off on a gallon of house paint.
  - a. Write an equation for the cost,  $C$ , of 15 gallons of paint in terms of the regular price,  $p$ .
  - b. Randall bought 15 gallons and paid \$480 before tax. What is the regular price of a gallon of paint?

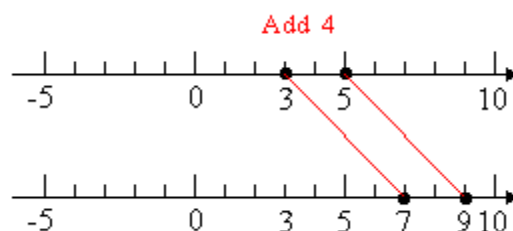
## Activity 3 Inequalities

### 1. Properties of inequalities.

Start with an inequality that we know is true, say  $3 < 5$ . Which of the following operations keeps the inequality true?

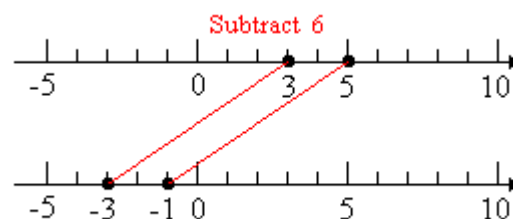
- a. Add a positive number, say 4, to both sides of the inequality, to get

$$\begin{aligned} 3 + 4 &< 5 + 4 \\ 7 &< 9 \end{aligned}$$



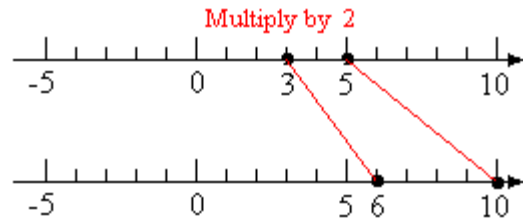
- b. Subtract the same quantity, say  $-6$ , from both sides, to get

$$\begin{aligned} 3 - 6 &< 5 - 6 \\ -3 &< -1 \end{aligned}$$



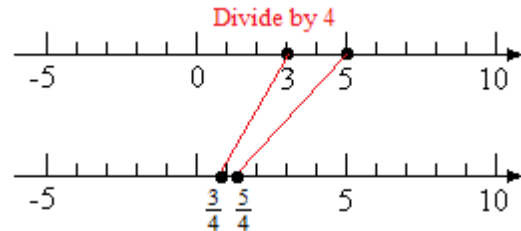
- c. Multiply both sides of the inequality by a positive number, say 2, to get

$$2(3) < 2(5) \\ 6 < 10$$



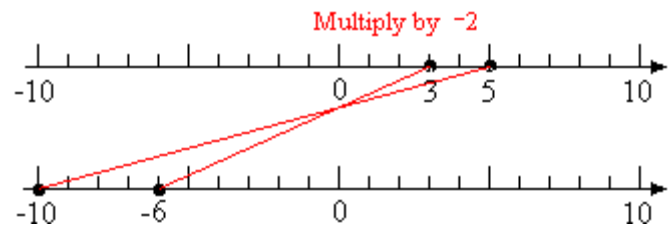
- d. Divide both sides by a positive number, say 2, to get

$$\frac{3}{4} < \frac{5}{4}$$



- e. Multiply both sides by a negative number, say  $-2$ .

$$-2(3) < -2(5) \\ -6 < -10 \quad \text{False !}$$



2. Complete the box:

If we \_\_\_\_\_ or \_\_\_\_\_ both sides of an inequality by the same \_\_\_\_\_ quantity, we must \_\_\_\_\_ the direction of the inequality.

3. Fill in the correct symbol,  $>$  or  $<$ , in each statement.

- a. If  $x > 8$ , then  $x - 7$  \_\_\_\_\_ 1.  
 b. If  $x < -4$ , then  $3x$  \_\_\_\_\_  $-12$ .  
 c. If  $x > -2$ , then  $-9x$  \_\_\_\_\_ 18.

#### 4. Solving inequalities

a.  $\frac{x}{4} \geq -2$

What should you do to isolate  $x$ ?

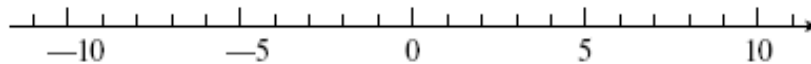
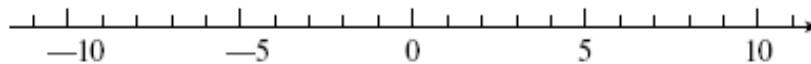
Should you reverse the direction of the inequality?

**Check:** Is your answer reasonable:  
 Is  $-12$  a solution?  
 Is  $-5$  a solution?

b.  $-5x > 20$

What should you do to isolate  $x$ ?

Should you reverse the direction of the inequality?

**Check:** Is your answer reasonable:Is  $-10$  a solution?Is  $2$  a solution?c. Solve  $5 - 2x \leq -3$  and graph the solutions on the number line.d. Solve  $-8 < 4 - 3x < 10$  and graph the solutions on the number line.

## Wrap-Up

In this Lesson, we practiced the following skills:

- Solving equations with two or more operations
- Writing equations to model applied problems
- Solving inequalities

1. To solve an equation, in what order do we undo the operations?
2. Which operation does a fraction bar represent?
3. When we evaluate the expression  $5(z - 6)$ , which operation do we perform first?
4. When should you reverse the direction of an inequality?



## 2.4 Homework Preview

■ Solve.

1.  $12 = \frac{x}{6}$

2.  $\frac{5a}{3} = 20$

3.  $2(10z + 18) = 96$

4.  $8 - b = -3$

5.  $-2t + 18 = -4$

6.  $7 - \frac{2m}{3} = -9$

7.  $8 - 4x > -2$

8.  $-6 \leq \frac{4 - x}{3} < 2$

## Answers

1. 72

2. 12

3. 3

4. 11

5. 11

6. 24

7.  $x < \frac{5}{2}$

8.  $22 \geq x > -2$

## Lesson 2.5 Like Terms

### Activity 1 Equivalent Expressions

1. a. Show that the expressions  $6 + 2x$  and  $8x$  are equal if  $x = 1$ :

$$\begin{array}{l} 6 + 2x = \underline{\hspace{2cm}} \\ 8x = \underline{\hspace{2cm}} \end{array}$$

- b. Show that the expressions  $6 + 2x$  and  $8x$  are not equal if  $x = 2$ :

$$\begin{array}{l} 6 + 2x = \underline{\hspace{2cm}} \\ 8x = \underline{\hspace{2cm}} \end{array}$$

- c. Are the expressions  $6 + 2x$  and  $8x$  equivalent?

2. Explain the following phrases, and give an example for each.

a. equivalent expressions

b. like terms

c. numerical coefficient

3. Combine like terms by adding or subtracting.

a.  $-2x + 6x - x$

b.  $3bc - (-4bc) - 8bc$

4. Simplify by removing parentheses and combining like terms.

a.  $-5u - 6uv + 8uv + 9u$

b.  $(32h - 26) + (-3 + 2h)$

c.  $(3a - 2) - 2a - (5 - 2a)$

## Activity 2 Solving Equations

### Steps for Solving Linear Equations

1. Combine like terms on each side of the equation.
2. By adding or subtracting the same quantity on both sides of the equation, get all the variable terms on one side and all the constant terms on the other.
3. Divide both sides by the coefficient of the variable to obtain an equation of the form  $x = a$ .

1. Solve  $4 - 5x = 28 + 3x$

2. Solve  $10 - 6d - 3d \geq 2 - 5d$

## Activity 3 Like Terms and Parentheses

1. One angle of a triangle is three times the smallest angle, and the third angle is  $20^\circ$  greater than the smallest angle.
  - a. If the smallest angle is  $x$ , write expressions for the other two angles.
  - b. Write an expression for the sum of the three angles in terms of  $x$ .
2. The StageLights theater group plans to sell T-shirts to raise money. It will cost them  $5x + 60$  dollars to print  $x$  T-shirts, and they will sell the T-shirts at \$12 each.
  - a. Write and simplify an expression for their profit from selling  $x$  T-shirts.
  - b. How many T-shirts must they sell in order to make a profit of \$500?

## Wrap-Up

In this Lesson, we practiced the following skills:

- Combining like terms
- Solving linear equations
- Writing equations for applied problems

1. Does  $6 + 2x = 8x$  ? Why or why not?
2. Explain how to simplify each side of an equation before beginning to solve.
3. What is wrong with this expression for the profit in Activity 3, problem 2?

$$12x - 5x + 60$$

## 2.5 Homework Preview

■ Simplify.

1.  $-2x - 16 - (-5x) - 4$

2.  $-3ab + 8a - (-5ab) - 6a$

3.  $(6bx + 8x) - (2x - 2bx) + (-5 - 3bx)$

■ Solve.

4.  $6p - 8 = -3p - 26$

5.  $3x - 5 < -6x + 7$

6.  $12c - (3 + 6c) = 3c + 4 - (8 - 2c)$

## Answers

1.  $3x - 20$     2.  $2ab + 2a$     3.  $5bx + 6x - 5$     4.  $-2$     5.  $x < \frac{4}{3}$     6.  $-1$

## Lesson 3.1 Intercepts

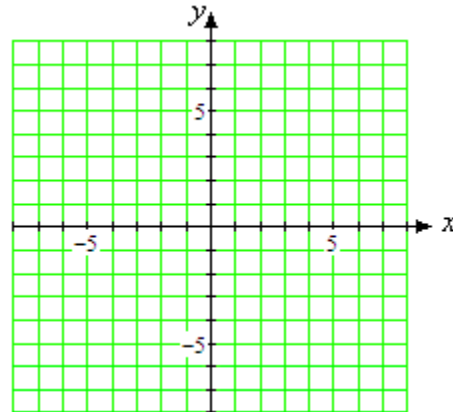
### Activity 1 Intercept Method of Graphing

1. a. Find the  $x$ - and  $y$ -intercepts of the graph of

$$3x - 2y = 12$$

- b. Graph the equation by the intercept method.

$x$	$y$
0	
	0



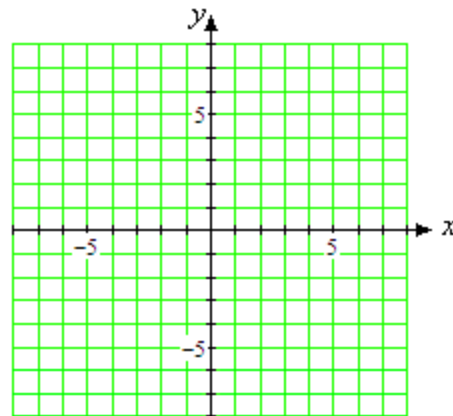
- c. Plot a third point as a check.

2. a. Find the  $x$ - and  $y$ -intercepts of the graph of

$$2x = 5y - 10$$

- b. Graph the equation by the intercept method.

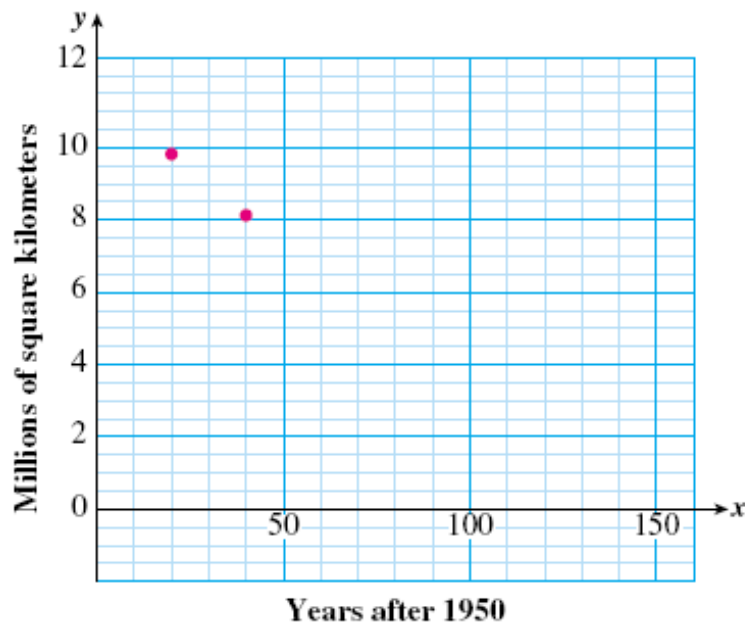
$x$	$y$
0	
	0



- c. Plot a third point as a check.

### Activity 2 Interpreting the Intercepts

1. Around 1950, people began cutting down the world's rain forests to clear land for agriculture. In 1970, there were about 9.8 million square kilometers of rain forest left, and by 1990 that figure had been reduced to 8.2 million square kilometers. These two data points are shown on the graph on the next page. The horizontal axis displays the year,  $x$ , and the vertical axis shows the amount of rainforest remaining,  $y$  (in millions of square kilometers).
- a. If we continue to clear the rainforests at the same rate, the graph will be a straight line. Draw a straight line through the two points in the figure. Make your line long enough to cross both axes.



b. Estimate the coordinates of the  $x$ - and  $y$ -intercepts of your graph.

$y$ -intercept: \_\_\_\_\_

$x$ -intercept: \_\_\_\_\_

Use the intercepts to answer the following questions:

c. How many million square kilometers of rainforest were present initially?

d. If we continue to clear the rainforest at the same rate, when will it be completely demolished?

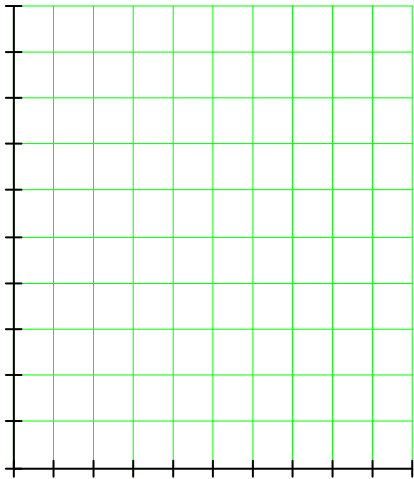
2. Sheri bought a bottle of multivitamins for her family. The number of vitamins left in the bottle after  $d$  days is given by

$$N = 300 - 5d$$

a. Find the intercepts and use them to make a graph of the equation.

$d$	$N$
0	
	0

b. Explain what each intercept tells us about the vitamins.



## Wrap-Up

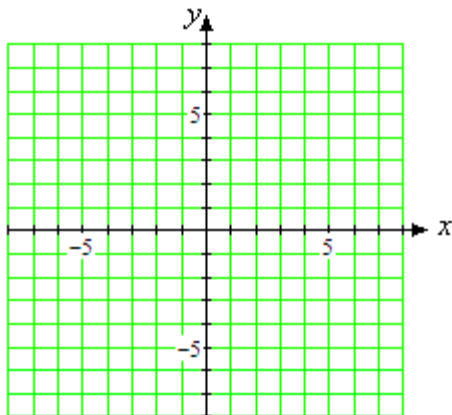
In this Lesson, we practiced the following skills:

- Finding the intercepts of the a graph
  - Graphing a line by the intercept method
  - Interpreting the intercepts in context
1. In Activity 1, Problem 1, Delbert says that the intercepts are  $(4, -6)$ . What is wrong with his statement?
  2. In Activity 2, Problem 1, how did you find the answer to part (c)?
  3. In Activity 2, Problem 2, what intervals did you use to scale each axis?

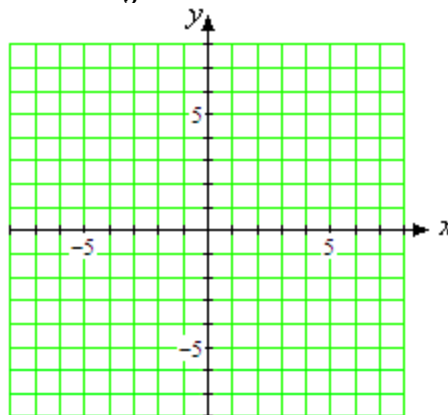
## 3.1 Homework Preview

- a. Find the  $x$ - and  $y$ -intercepts of the line.
- b. Use the intercept method to graph the line.

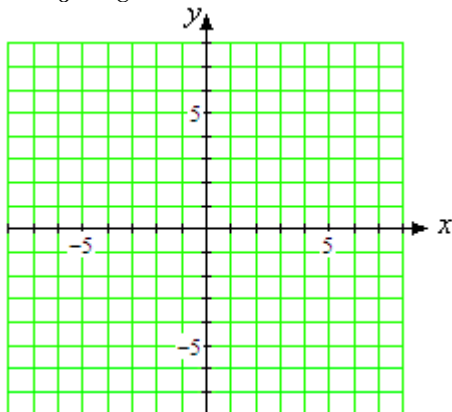
1.  $3x - 5y = 15$



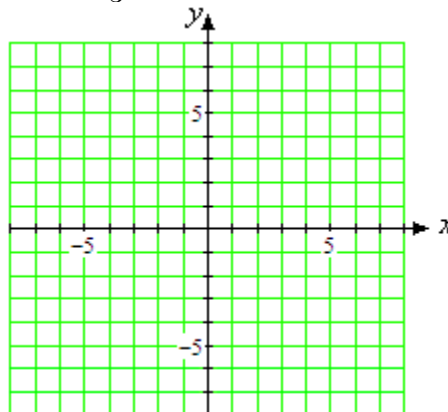
2.  $y = \frac{-4}{3}x + 8$



3.  $\frac{x}{6} + \frac{y}{8} = -1$



4.  $x - \frac{2}{3}y - 4 = 0$



## Answers

1.  $(5, 0)$ ,  $(0, -3)$
2.  $(6, 0)$ ,  $(0, 8)$
3.  $(-6, 0)$ ,  $(0, -8)$
4.  $(4, 0)$ ,  $(0, -6)$

## Lesson 3.2 Ratio and Proportion

### Activity 1 Ratios and Rates

1. On a Saturday evening, a restaurant found that 84 of its customers asked for the nonsmoking section, and 35 customers preferred the smoking section.
  - a. What was the ratio of smokers to nonsmokers?
  
  
  
  
  
  - b. Write the ratio in simplest form.
  
2. Major Motors budgeted \$5.6 million for research and development (R&D) next year and \$3.5 million for advertising. What is the ratio of the amount budgeted for advertising to the amount budgeted for R&D?
  
  
  
  
  
3. Bitu made \$240 for 25 hours of work last week. Express her rate of pay as a ratio, and then as a rate.

### Activity 2 Proportions

1. Solve  $\frac{q}{3.2} = \frac{1.25}{4}$
  
  
  
  
  
  
  
  
  
  
2. If Sarah can drive 390 miles on 15 gallons of gas, how much gas will she need to travel 800 miles?

Assign a variable:

Write a proportion:

Solve the proportion:

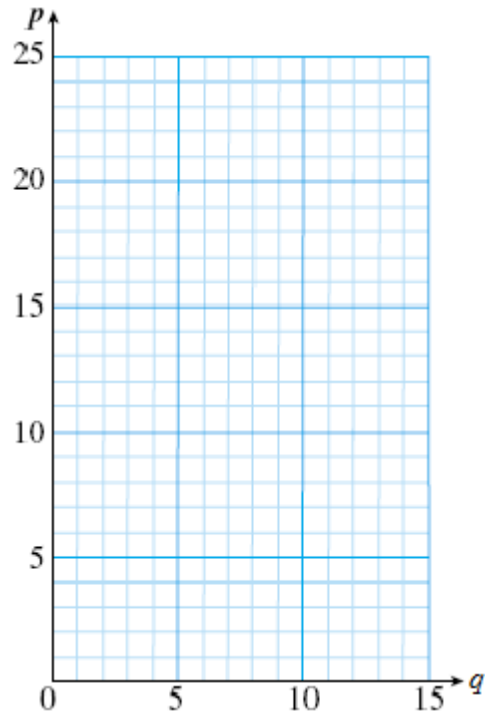
Solution:



### Activity 3 Proportional Variables

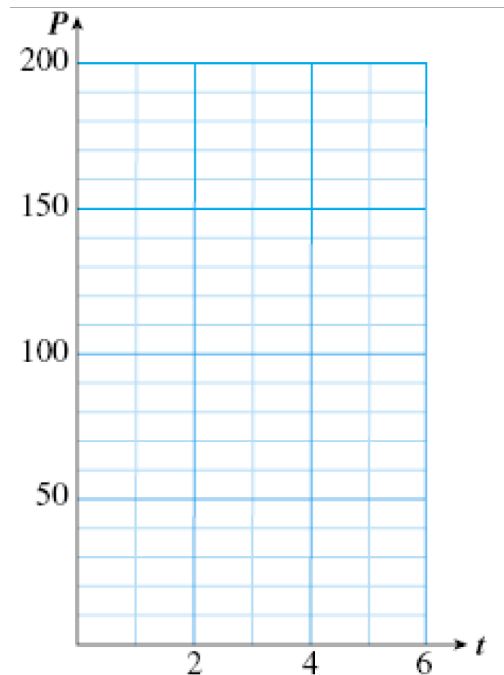
- The table below shows the price,  $p$ , for  $q$  quarts of SereniTea. Plot the data on the grid. Then compute the ratio  $\frac{\text{Price}}{\text{Quarts}}$  for each data point.

Quarts	Total Price	$\frac{\text{Price}}{\text{Quarts}}$
4	\$6.00	$\frac{6.00}{4} = ?$
6	\$9.00	$\frac{9.00}{6} = ?$
9	\$13.50	
12	\$18.00	
15	\$22.50	



- The table shows the population,  $P$ , of a new suburb  $t$  years after it was built. Plot the data on the grid. Then compute the ratio  $\frac{\text{People}}{\text{Year}}$  for each data point.

Years	Population	$\frac{\text{People}}{\text{Year}}$
1	10	$\frac{10}{1} = ?$
2	20	$\frac{20}{2} = ?$
3	40	
4	80	
5	160	



- At this point, can you make a conjecture (educated guess) about the graphs of proportional variables? To help you decide if your conjecture is true, continue with the graphs in parts 4 and 5.

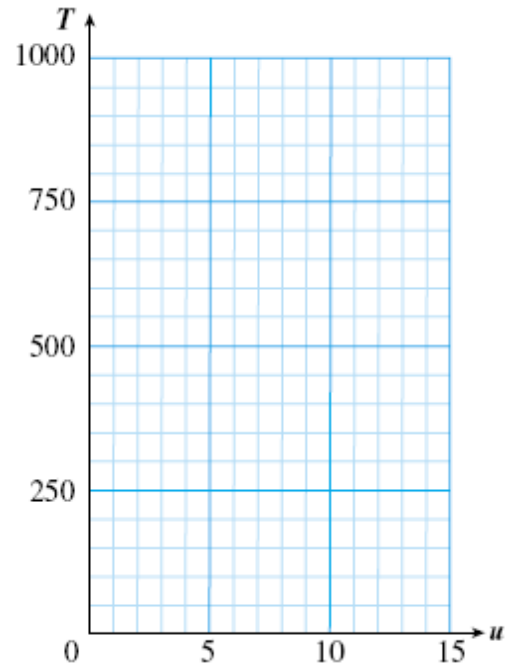
**Conjecture:**

4. Tuition at Woodrow University is \$400 plus \$30 per unit.
- Write an equation for tuition,  $T$ , in terms of the number of units,  $u$ .

$$T =$$

- Use your equation to fill in the second column of the table.
- Graph the equation on the grid.
- Are the variables proportional? Compute their ratios to decide.

Units	Tuition	Tuition Unit
3		
5		
8		
10		
12		

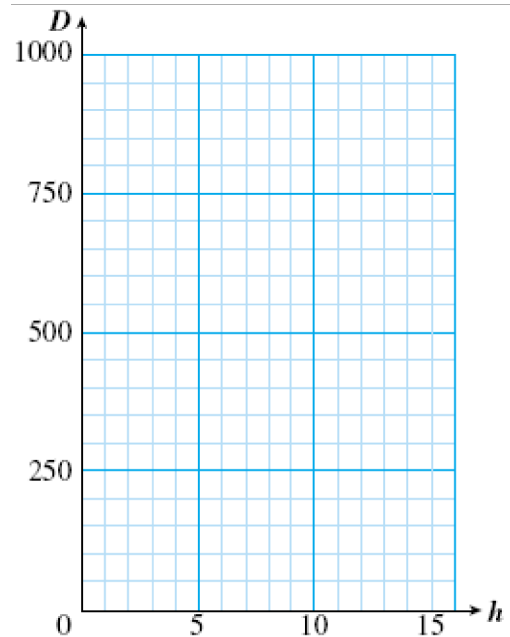


5. Anouk is traveling by train across Alaska at 60 miles per hour.
- Write an equation for the distance,  $D$ , Anouk has traveled in terms of hours,  $h$ .

$$D =$$

- Use your equation to fill in the table.
- Graph the equation on the grid.
- Are the variables proportional? Compute their ratios to decide.

Hours	Distance	Distance Hour
3		
5		
8		
10		
16		



6. Revise your conjecture about the graphs of proportional variables so that it applies to the graphs on this page as well. **Hint:** Look at the two graphs of proportional variables. What is the  $y$ -intercept of both graphs?

**Revised Conjecture:**

## Wrap-Up

In this Lesson, we practiced the following skills:

- Comparing two quantities with a ratio or a rate
  - Solving a proportion
  - Writing a proportion to model a problem
  - Recognizing proportional variables
1. In Activity 1, Problem 2, did you express the ratio as a common fraction or as a decimal fraction? Why?
  2. In Activity 2, Problem 3, did you use miles or gallons in the numerators of your ratios? Could you have used the other unit?
  3. In Activity 3, what did you learn about the graphs of proportional variables?

### 3.2 Homework Preview

1. A cake recipe asks for  $2\frac{1}{2}$  cups of flour and  $\frac{3}{4}$  cup of brown sugar. What is the ratio of flour to brown sugar?

2. Decide whether the two variables are proportional:

$x$	2	5	10	16
$y$	3.2	8	16	25.6

3. Write and solve a proportion to answer the question:

On a map of Chesterfield County, 1 inch represents  $4\frac{1}{2}$  miles. If Richmond and Petersburg are  $6\frac{2}{3}$  inches apart on the map, what is the actual distance between the two towns?

### Answers

1.  $\frac{10}{3}$

2. Yes

3.  $\frac{1}{9/2} = \frac{20/3}{x}$ ; 30 mi

## Lesson 3.3 Slope

### Activity 1 Calculating Slope

To calculate slope, we choose two points on the graph and compute the ratio

$$\frac{\text{change in vertical coordinate}}{\text{change in horizontal coordinate}}$$

Be sure to include units with your ratios!

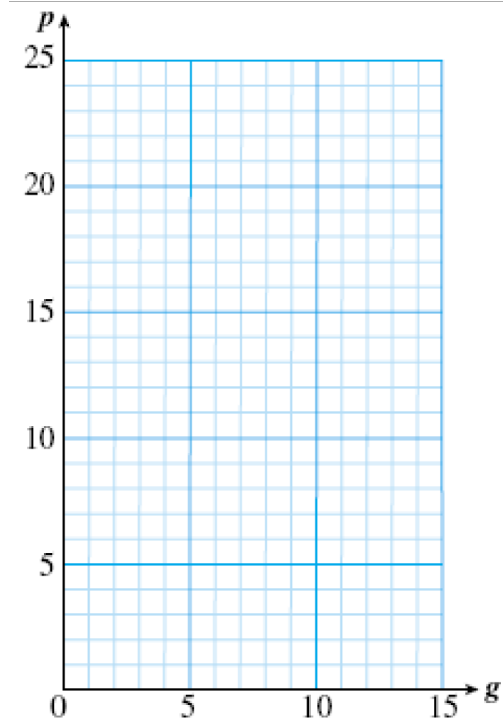
1. The table shows the price,  $p$ , for  $g$  gallons of gasoline at the pump.

Gallons	Total Price
4	\$6.00
6	\$9.00
9	\$13.50
12	\$18.00
15	\$22.50

- a. Plot the data on the grid.
- b. Choose two points from the graph and use them to compute the slope.

First point:

Second point:



Change in vertical coordinates  $\Delta p =$

Change in horizontal coordinates  $\Delta g =$

Slope  $\frac{\Delta p}{\Delta g} =$

- c. Illustrate the slope on the graph.
- d. Write the slope as a rate of change, including units. What does the slope tell you about the variables?

2. The table shows the growth in population,  $P$ , of a new suburb  $t$  years after it was built.

Years	Population
1	10
2	20
3	40
4	80
5	160

- a. Plot the data on the grid.  
b. Choose two points from the graph and use them to compute the slope.

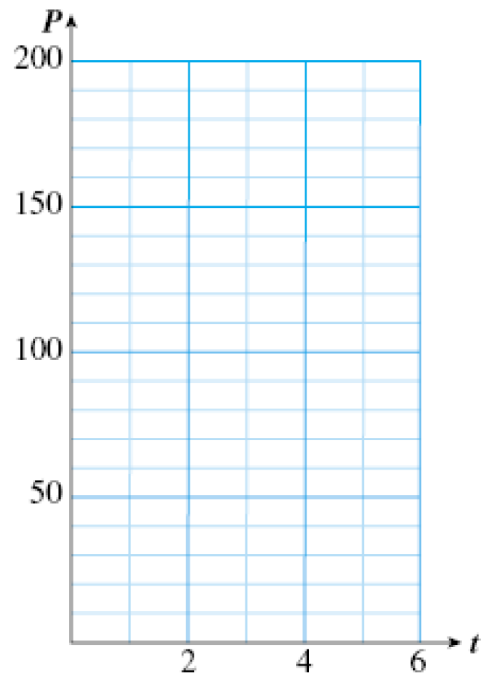
First point:

Second point:

Change in vertical coordinates  $\Delta P =$

Change in horizontal coordinates  $\Delta t =$

Slope  $\frac{\Delta P}{\Delta t} =$



- c. Illustrate the slope on the graph. Is the slope the same between all points?  
d. Write the slope as a rate of change, including units. What does the slope tell you about the variables?

3. Tuition at Woodrow University is \$400 plus \$30 per unit.

- a. Write an equation for tuition,  $T$ , in terms of the number of units,  $u$ .

$T =$

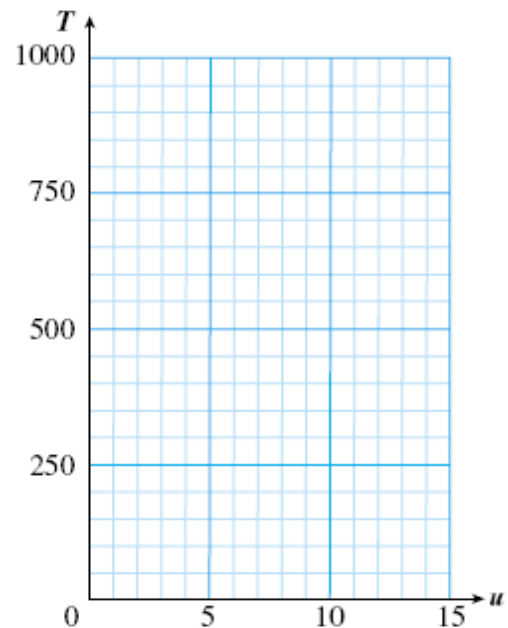
- b. Use your equation to fill in the table.

Units	3	5	8	10	12
Tuition					

- c. Graph the equation on the grid.  
d. Choose two points from the graph and use them to compute the slope.

First point:

Second point:



$$\begin{array}{ll} \text{Change in vertical coordinates} & \Delta T = \\ \text{Change in horizontal coordinates} & \Delta u = \\ \text{Slope} & \frac{\Delta T}{\Delta u} = \end{array}$$

- Illustrate the slope on the graph.
- Write the slope as a rate of change, including units. What does the slope tell you about the variables?

4. Anouk is traveling by train across Alaska at 60 miles per hour.

- Write an equation for the distance,  $D$ , Anouk has traveled in terms of hours,  $h$ .

$$D =$$

- Use your equation to fill in the table.

Hours	3	5	8	10	16
Distance					

- Graph the equation on the grid.

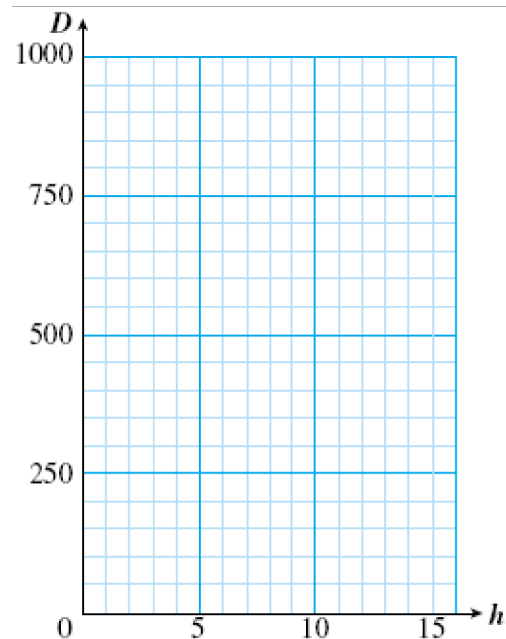
- Choose two points from the graph and use them to compute the slope.

First point:

Second point:

$$\begin{array}{ll} \text{Change in vertical coordinates} & \Delta D = \\ \text{Change in horizontal coordinates} & \Delta h = \\ \text{Slope} & \frac{\Delta D}{\Delta h} = \end{array}$$

- Illustrate the slope on the graph.
- Write the slope as a rate of change, including units. What does the slope tell you about the variables?



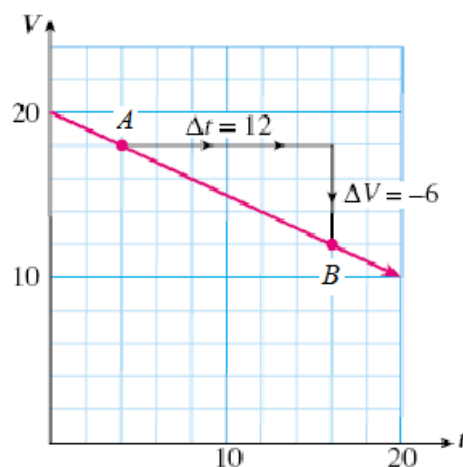
- Which of the four graphs will give different values for the slope, depending on which points you choose?
  - What is different about this graph, compared to the other three graphs?

## Activity 2 Negative Slopes

An increasing quantity has a positive rate of change, or slope, and a decreasing quantity has a negative rate of change.

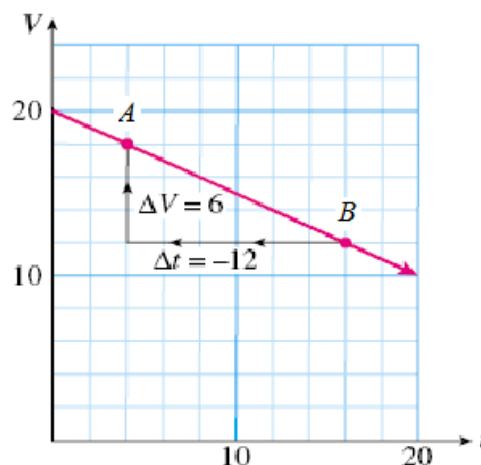
When we move to the right on a graph,  $\Delta x$  is positive.  
 When we move to the left,  $\Delta x$  is negative.  
 When we move up on a graph,  $\Delta y$  is positive.  
 When we move down,  $\Delta y$  is negative.

1. The value of new office equipment decreases, or depreciates, over time. The graph shows the value,  $V$ , in thousands of dollars, of a large copy machine  $t$  years after it was purchased.



- a. Compute the slope of the line by moving from point  $A$  to point  $B$ .
- b. What does the slope tell you about the value of the machine?

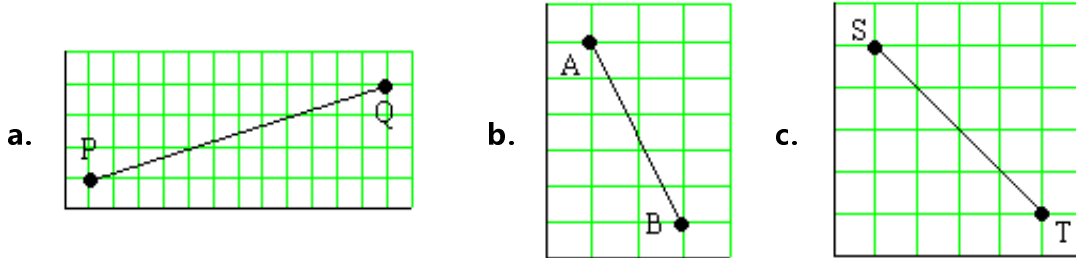
- c. Compute the slope of the line by moving from point  $B$  to point  $A$ .



It doesn't matter which direction we move along a line to compute its slope; the answer will be the same.



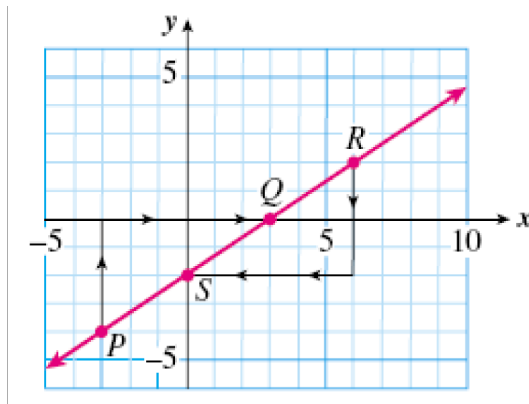
2. Find the slope of each line segment. Verify that you get the same answer if you move in the opposite direction. (Each square counts for one unit.)



### Activity 3 Lines Have Constant Slope

How do we know which two points to choose when we want to compute the slope of a line? It turns out that any two points on the line will do.

1. Calculate the slope of the line  $y = \frac{2}{3}x - 2$  shown in the figure:



- a. By using the points  $P(-3, -4)$  and  $Q(3, 0)$ .
- b. By using the points  $R(6, 2)$  and  $S(0, -2)$ .
- c. Do you get the same value for the slope in each case?

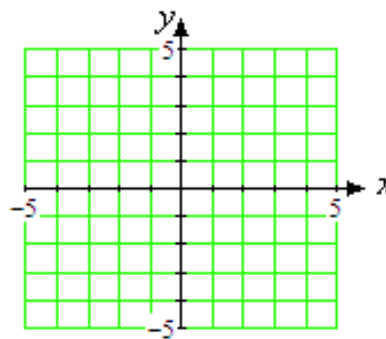
No matter which two points we use to calculate the slope of a line, we will always get the same result.

2. a. Graph the line

$$4x - 2y = 8$$

by finding the  $x$ - and  $y$ -intercepts.

$x$	$y$
0	
	0



- b. Compute the slope of the line using the  $x$ -intercept and  $y$ -intercept.

$$m = \frac{\Delta y}{\Delta x} =$$

- c. Compute the slope of the line using the points  $(4, 4)$  and  $(1, -2)$ .

$$m = \frac{\Delta y}{\Delta x} =$$

## Wrap-Up

In this Lesson, we practiced the following skills:

- Computing the slope of a line
- Interpreting the slope as a rate of change

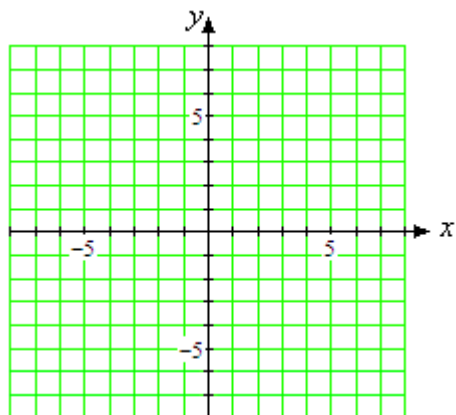
1. In Activity 1, which of the four graphs was not a straight line?
2. What does the sign of the slope tell you about the graph?
3. Delbert says that if both intercepts of a line are positive, then the slope is positive also. Is he correct? Explain why or why not.

### 3.3 Homework Preview

■ Use the intercepts to find the slope of the line. Illustrate the slope on the graph.

1.  $5x - 3y = 15$

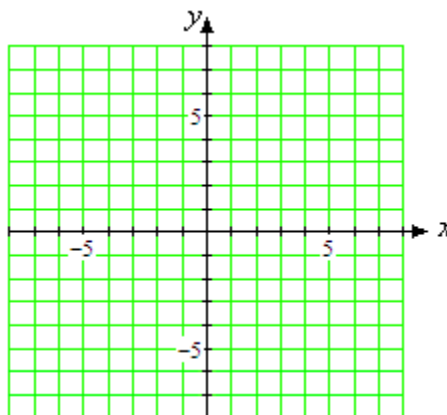
$x$	0	
$y$		0



$m =$

2.  $y = \frac{3}{4}x - 6$

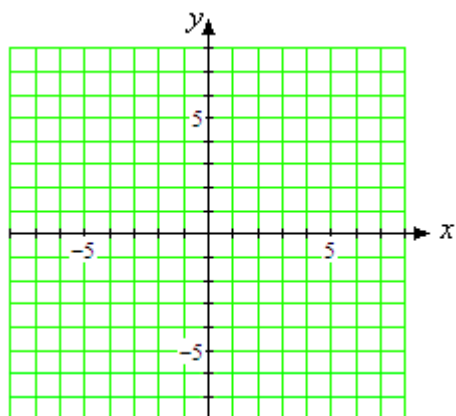
$x$	0	
$y$		0



$m =$

3.  $\frac{x}{6} + \frac{y}{8} = 1$

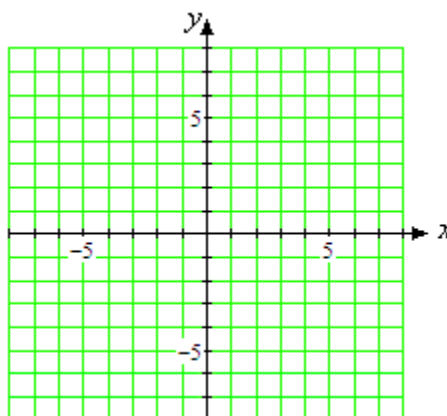
$x$	0	
$y$		0



$m =$

4.  $x + \frac{2}{3}y + 4 = 0$

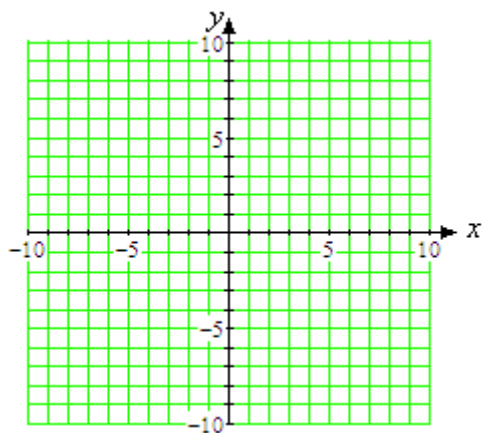
$x$	0	
$y$		0



$m =$

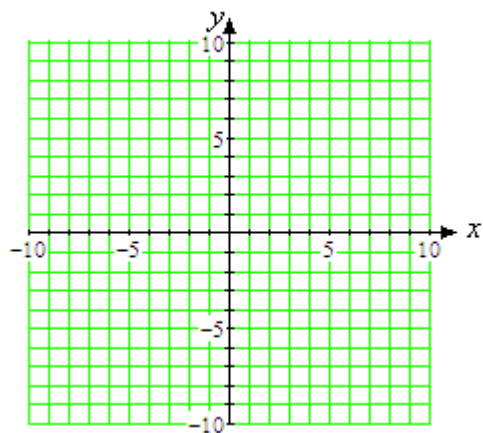
■ Plot the points and find the slope of the line between them.

5.  $(-2, 8)$  and  $(4, -6)$



$m =$

6.  $(-7, -3)$  and  $(5, 9)$



$m =$

7. If  $m = \frac{4}{5}$ , and  $\Delta x = -6$ , find  $\Delta y$ .

8. If  $m = \frac{-3}{2}$ , and  $\Delta y = 6$ , find  $\Delta x$ .

### Answers

1.  $m = \frac{5}{3}$

2.  $m = \frac{3}{4}$

3.  $m = \frac{-4}{3}$

4.  $m = \frac{-3}{2}$

5.  $m = \frac{-7}{3}$

6.  $m = 1$

7.  $\Delta y = \frac{-24}{5}$

8.  $\Delta x = -4$

# Lesson 3.4 Slope-Intercept Form

## Activity 1 The Slope and the $y$ -intercept

1. a. Tuition at Woodrow University is \$400 plus \$30 per unit. Write an equation for tuition,  $W$ , in terms of the number of units,  $u$ , and fill in the table below.

$$W =$$

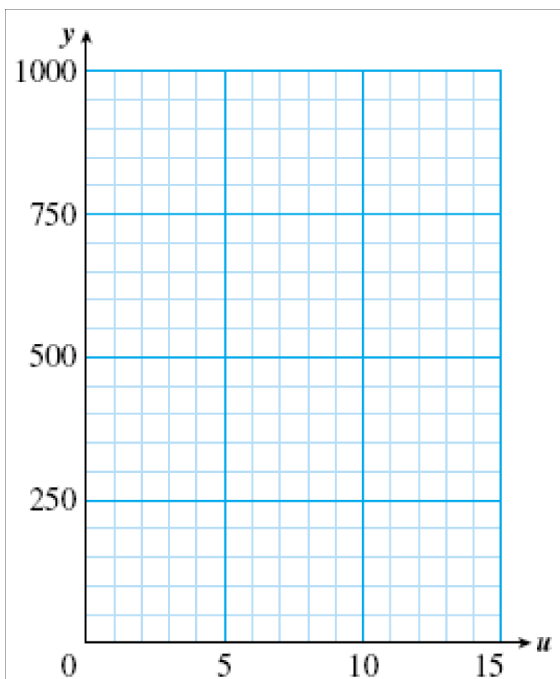
- b. At Xavier College, the tuition,  $X$ , is \$200 plus \$30 per unit. Write an equation for  $X$  and fill in the table.

$$X =$$

- c. At the Yardley Institute, the tuition,  $Y$ , is \$30 per unit. Write an equation for  $Y$  and fill in the table.

$$Y =$$

- d. Graph all three equations on the grid.



$u$	$W$	$X$	$Y$
3			
5			
8			
10			
12			

- e. Find the slope and the  $y$ -intercept for each equation.

$$W: \text{ slope } = \quad y\text{-intercept } =$$

$$X: \text{ slope } = \quad y\text{-intercept } =$$

$$Y: \text{ slope } = \quad y\text{-intercept } =$$

- f. How are your results from part (e) reflected in the graphs of the equations?

2. a. Anouk is traveling by train across Alaska at 60 miles per hour. Write an equation for the distance,  $A$ , Anouk has traveled in terms of hours,  $h$ , and fill in the table below.

$A =$

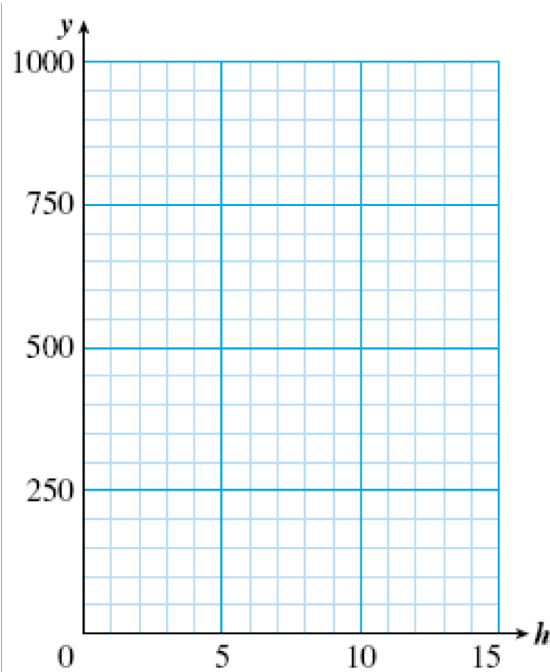
- b. Boris is traveling by snowmobile at 30 miles per hour. Write an equation for Boris' distance,  $B$ , and fill in the table.

$B =$

- c. Chaka is traveling in a small plane at 100 miles per hour. Write an equation for Chaka's distance,  $C$ , and fill in the table.

$C =$

- d. Graph all three equations on the grid.



$h$	$A$	$B$	$C$
3			
5			
8			
10			
16			

- e. Find the slope and the  $y$ -intercept for each equation.

$A$ : slope =                       $y$ -intercept =

$B$ : slope =                       $y$ -intercept =

$C$ : slope =                       $y$ -intercept =

- f. How are your results from part (e) reflected in the graphs of the equations?

## 3. Let's summarize the results of the Activity.

In part 1 we graphed three equations:

$$W = 30u + 400$$

$$X = 30u + 200$$

$$Y = 30u$$

All of these equations have the same \_\_\_\_\_, namely 30, but different  $y$ -intercepts. In each case the  $y$ -intercept is the same as the \_\_\_\_\_ term in the equation. (This makes sense, because we find the  $y$ -intercept by setting  $u = 0$ .)

In part 2 we graphed

$$A = 60h$$

$$B = 30h$$

$$C = 100h$$

All of these equations pass through the origin, so their  $y$ -intercepts are all \_\_\_\_\_, but each has a different \_\_\_\_\_. In each case, the slope is the same as the coefficient of  $h$  in the equation. This also makes sense if you think about it: if we increase  $h$  by one hour, then  $A$  increases by 60 miles,  $B$  increases by 30 miles, and  $C$  increases by 100 miles.

## Activity 2 Slope-Intercept Form

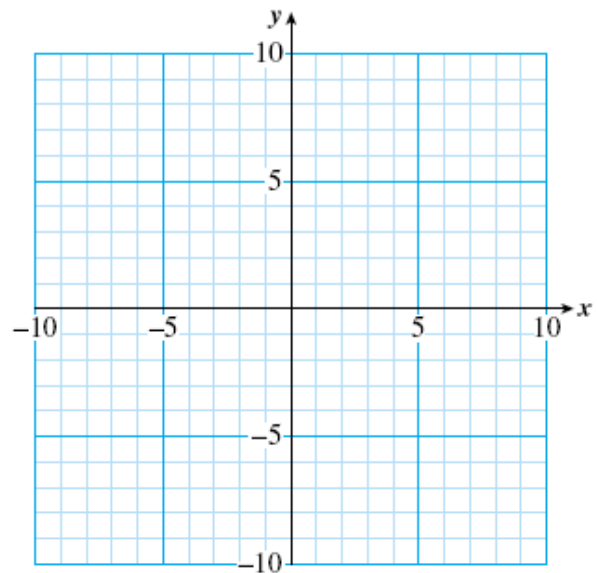
1. a. Write an equation for the line whose  $y$ -intercept is  $(0, -3)$  and whose slope is  $-\frac{2}{3}$ .

- b. Use the slope-intercept method to graph the equation.

Plot the  $y$ -intercept.

Use the slope  $m = \frac{\Delta y}{\Delta x} = \frac{-2}{3}$  to plot another point.

Use the slope  $m = \frac{\Delta y}{\Delta x} = \frac{2}{-3}$  to plot another point.

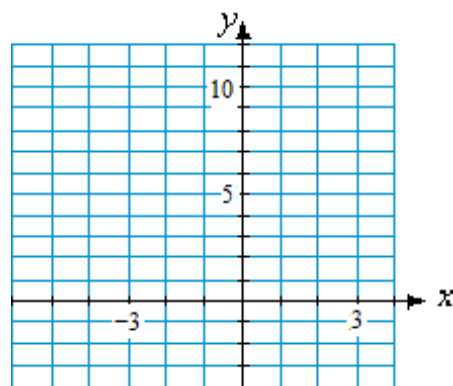


2. a. State the slope and  $y$ -intercept of the equation  $y = -3x + 4$

- b. Graph the equation by the slope-intercept method.

Hint: Write the slope as a fraction,

$$m = \frac{\Delta y}{\Delta x} = \frac{-3}{1} = \frac{3}{-1}$$

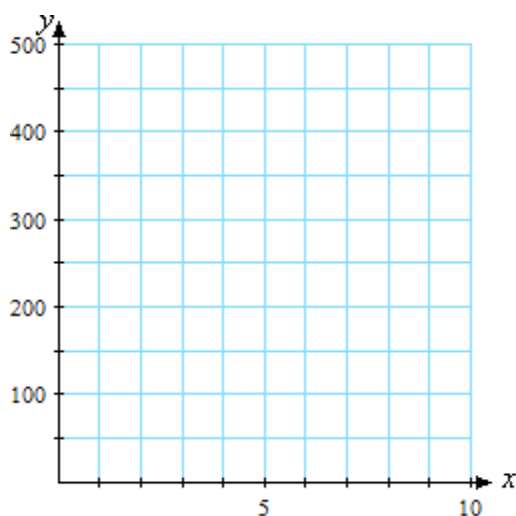


3. On Memorial Day weekend, Arturo drives from his home to a cabin on Diamond Lake. His distance in miles from Diamond Lake after  $x$  hours of driving is given by the equation  $y = 450 - 50x$ .

- a. What are the slope and the  $y$ -intercept of the graph of this equation?

- b. Graph the equation.

- c. What does the  $y$ -intercept tell you about the problem?



What does the slope tell you about the problem?

## Wrap-Up

In this Lesson, we practiced the following skills:

- Writing an equation in slope-intercept form
- Identifying the slope and  $y$ -intercept of a line from its equation
- Graphing a linear equation by the slope-intercept method
- Interpreting the slope and  $y$ -intercept in context

1. In Activity 1, Problem 1, what do the slopes of the lines represent?
2. What do the slopes of the lines represent in Activity 1, Problem 2?
3. In Activity 2 part 1, does the negative sign in front of  $\frac{2}{3}$  apply to the numerator, the denominator, or both?
4. In Activity 2 part 3, what does the  $x$ -intercept of the line represent?
5. In Activity 2, Problem 3, why does it make sense that the slope is negative?

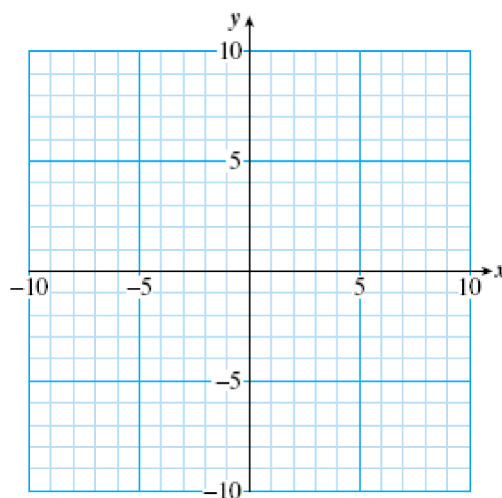
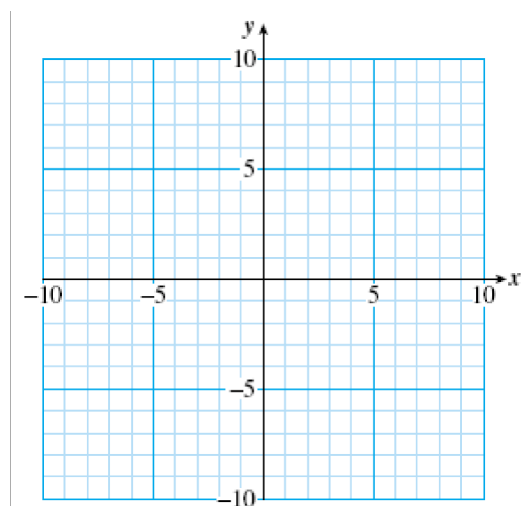


### 3.4 Homework Preview

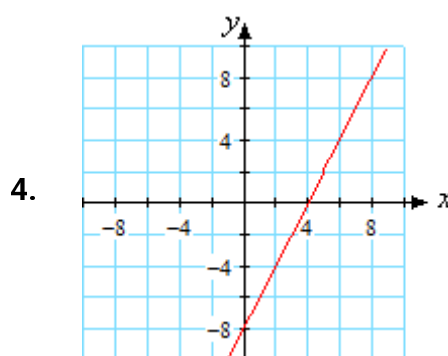
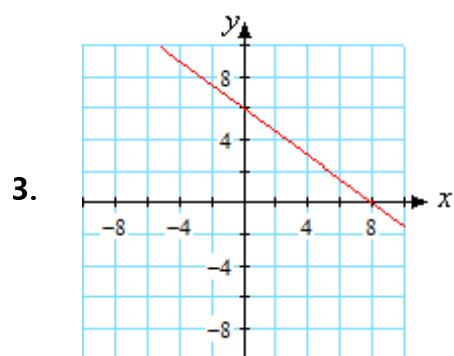
- a. Put the equation in slope-intercept form.  
b. Graph the line by the slope-intercept method.

1.  $12x - 8y = 16$

2.  $4x + 3y = 0$



- Write the equation of the line in slope-intercept form.



### Answers

1.  $y = \frac{3}{2}x - 2$

2.  $y = -\frac{4}{3}x$

3.  $y = -\frac{3}{4}x + 6$

4.  $y = 2x - 8$

## Lesson 3.5 Properties of Lines

### Activity 1 Parallel and Perpendicular Lines

1. a. Put each equation into slope-intercept form.

$$l_1 : 2x - 3y + 3 = 0$$

$$l_2 : 2x + 3y - 6 = 0$$

$$l_3 : 3x + 2y - 2 = 0$$

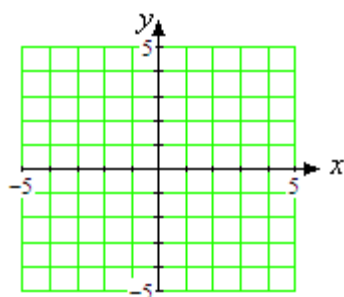
$$l_4 : 2x - 3y - 2 = 0$$

- b. Which of the four lines in part (a) are parallel? How do you know?
- c. Which of the four lines in part (a) are perpendicular? How do you know?

### Activity 2 Horizontal and Vertical Lines

1. a. Sketch a graph of the vertical line passing through  $(-4, -1)$ , then find its equation. What is the slope of the line?

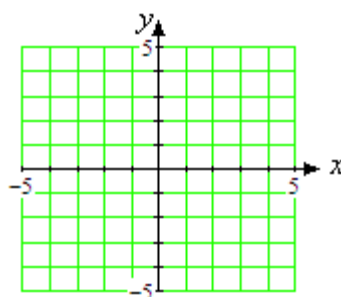
Grid for part (a)



Equation:

Slope:

Grid for part (b)



Equation:

Slope:

- b. Sketch a graph of the horizontal line passing through  $(-4, -1)$ , then find its equation. What is the slope of the line?

### Activity 3 Slope

1. Compute the slope of the line segment joining  $A$  and  $C$  in two ways:

**a. Using the graph.**

Draw the line through  $A$  and  $C$ . Use point  $B$  to find  $\Delta y$  and  $\Delta x$ .

$$\Delta y = \underline{\hspace{2cm}}, \quad \Delta x = \underline{\hspace{2cm}}.$$

$$m = \frac{\Delta y}{\Delta x} =$$

**b. Using coordinates.**

**Step 1** Write down the coordinates of  $A$ .

$A$  \_\_\_\_\_

Write down the coordinates of  $C$ .

$C$  \_\_\_\_\_

**Step 2** Compute  $\Delta y$  and  $\Delta x$ .

$$\Delta x = \underline{\hspace{2cm}}$$

final – initial

$$\Delta y = \underline{\hspace{2cm}}$$

final – initial

**Step 3** Compute the slope:  $m = \frac{\Delta y}{\Delta x} =$

Do you get the same answers for parts (a) and (b)? You should!

2. Follow the steps to compute the slope of the line segment joining  $H$  and  $K$ .

**Step 1** Let  $H$  be the first point and  $K$  the second point. Write down their coordinates.

$$H(x_1, y_1) =$$

$$K(x_2, y_2) =$$

**Step 2** Fill in the blanks:

$$y_2 = \underline{\hspace{2cm}}, \quad y_1 = \underline{\hspace{2cm}}$$

$$x_2 = \underline{\hspace{2cm}}, \quad x_1 = \underline{\hspace{2cm}}$$

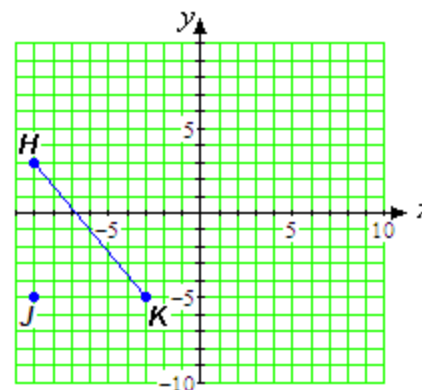
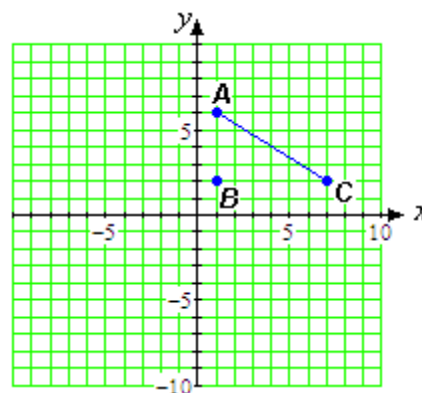
**Step 3** Compute  $\Delta y$  and  $\Delta x$ .

$$\Delta y = y_2 - y_1 =$$

$$\Delta x = x_2 - x_1 =$$

**Step 4** Compute the slope:  $m = \frac{y_2 - y_1}{x_2 - x_1} =$

Illustrate  $\Delta y$  and  $\Delta x$  on the graph. Is your value for the slope reasonable?



3. Use the formula  $m = \frac{y_2 - y_1}{x_2 - x_1}$  to compute the slope of the line joining the points  $(-4, -7)$  and  $(2, -3)$ .

### Wrap-Up

In this Lesson we practiced the following skills:

- Finding equations for parallel or perpendicular lines
  - Finding equations for horizontal or vertical lines
  - Using the slope formula
1. How can you decide if two lines are parallel, perpendicular, or neither?
  2. How many points do you need to find the equation of a horizontal or a vertical line?
  3. Explain the difference between the equation of a horizontal or vertical line and the slope of the line.
  4. Explain why the two-point formula for slope is the same as our old formula,  
 $m = \frac{\Delta y}{\Delta x}$ .

### 3.5 Homework Preview

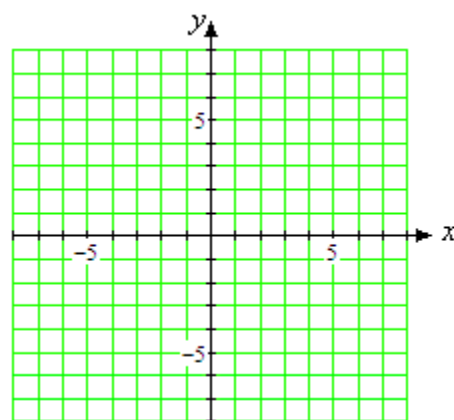
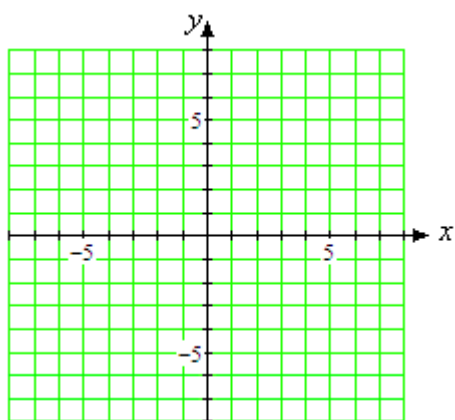
1. Are the lines parallel, perpendicular, or neither?

$$3x - 4y = 6 \quad \text{and} \quad 4x - 3y = 2$$

2. Sketch a graph of each equation, label the coordinates of its intercept, and state the slope of the line.

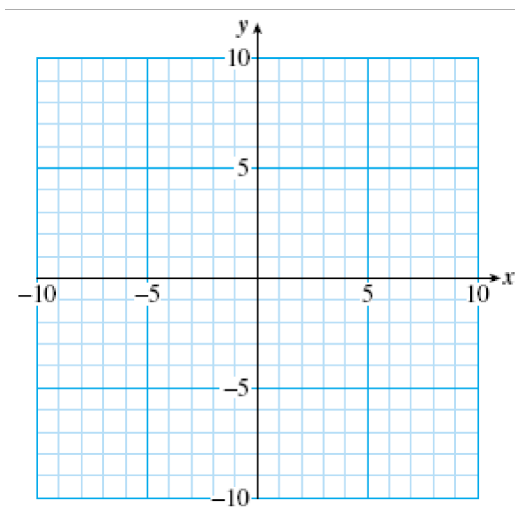
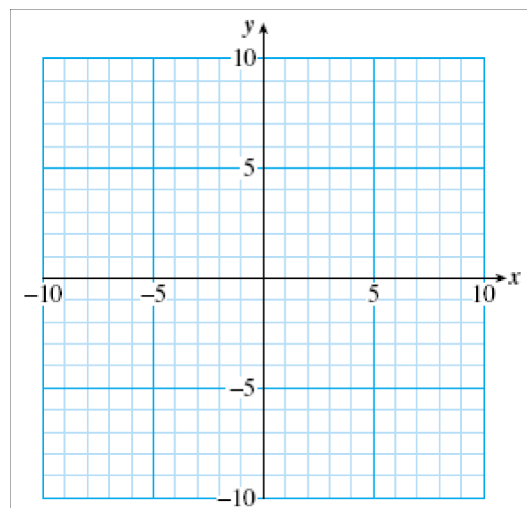
a.  $y = -6$

b.  $x = -2$



3. Find the slope of the line through  $(-2, 3)$  and  $(1, -5)$ . Graph the line.

$m =$



4. Find the slope of the line through  $(5, -4)$  and  $(-2, -4)$ . Graph the line.

$m =$

### Answers

1. Neither

2a.  $(0, -6)$ ,  $m = 0$

b.  $(-2, 0)$ ,  $m$  is undefined

3.  $m = \frac{-8}{3}$

4.  $m$  is undefined

## Lesson 4.1 The Distributive Law

### Activity 1 Simplifying Expressions

1. Simplify each product.

a.  $6(-2b)$

b.  $-4(-7w)$

2. Use the distributive law to simplify each expression.

a.  $8(3y - 6)$

b.  $-3(7 + 5x)$

### Activity 2 Solving Equations

#### Steps for Solving Linear Equations

1. Use the distributive law to remove any parentheses.
2. Combine like terms on each side of the equation.
3. By adding or subtracting the same quantity on both sides of the equation, get all the variable terms on one side and all the constant terms on the other.
4. Divide both sides by the coefficient of the variable to obtain an equation of the form  $x = a$ .

1. Solve  $2n + (4 - 3n) \geq 6 - (3 - 2n)$

2. Solve  $6 - 3(x - 2) = 4x - (x + 8)$

### Activity 3 Problem Solving

1. Shalia runs a landscaping business. She has a budget of \$385 to buy 20 rose bushes for one of her clients. Hybrid tea roses cost \$21 each, and shrub roses cost \$16.
  - a. If Shalia buys  $x$  tea roses, write an expression for the number of shrub roses she needs.
  - b. Write expressions for the cost of the tea roses and the cost of the shrub roses.
  - c. Write and simplify an expression for the total cost of the roses.
2. One angle of a triangle is three times the smallest angle, and the third angle is  $20^\circ$  greater than the smallest angle. Find the degree measure of each angle.

**Step 1** Let  $x$  represent the smallest angle, and write expressions for the other two angles.

Second angle:

Third angle:

**Step 2** Write an equation, using the fact that the sum of the three angles of a triangle is  $180^\circ$ .

**Step 3** Solve the equation. Begin by simplifying the left side.

Write your answer in a sentence:

## Wrap-Up

In this Lesson, we practiced the following skills:

- Applying the distributive law
  - Solving linear equations
  - Writing equations for applied problems
1. Explain the difference between  $6(-2b)$  and  $6(-2 + b)$ . Which law or property do you apply to simplify each expression?
  2. Explain how to simplify each side of an equation before beginning to solve.
  3. What formula did you use in Problem 2 of Activity 3?

## 4.1 Homework Preview

■ Simplify.

1.  $8 - 3(2x + 4) - 3x - 2$

2.  $3a - 4 - 3(2a - 5)$

3. a.  $-4(-6t) + 2$

b.  $-4(-6t + 2)$

c.  $-4 - 6t - 2$

d.  $(-4 - 6t)(-2)$

■ Solve.

4.  $-9 + 2(9 + 4z) = -23$

5.  $35 = 4(2w + 5) - 3w$

6.  $4(2 - 3w) = 9 - 3(2w - 1)$

## Answers

1.  $-9x - 6$

2.  $-3a + 11$

3a.  $24t + 2$

b.  $24t - 8$

c.  $-6 - 6t$

d.  $8 + 12t$

4.  $-4$

5.  $3$

6.  $\frac{-2}{3}$



## Lesson 4.2 Systems of Linear Equations

### Activity 1 Solving Systems by Graphing

A biologist wants to know the average weights of two species of birds in a wildlife preserve. She sets up a feeder whose platform is actually a scale, and mounts a camera to monitor the feeder. She waits until the feeder is occupied only by members of the two species she is studying, robins and thrushes. Then she takes a picture, which records the number of each species on the scale, and the total weight registered.

From her two best pictures, she obtains the following information. The total weight of three thrushes and six robins is 48 ounces, and the total weight of five thrushes and two robins is 32 ounces.

- We begin by assigning variables to the two unknown quantities:

Average weight of a thrush:  $t$

Average weight of a robin:  $r$

Write two equations about the weights of the birds:

(weight of thrushes) + (weight of robins) = total weight

Eqn. (1)

Eqn. (2)

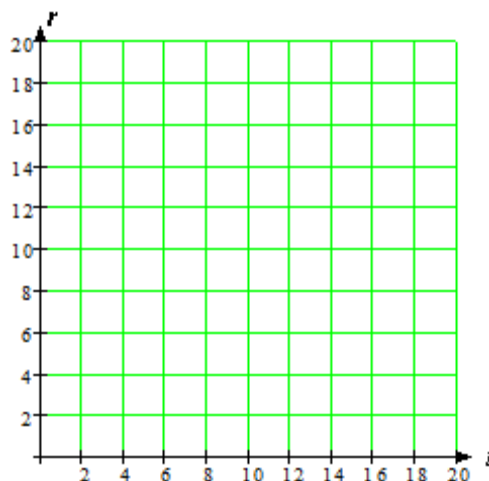
- Use the intercept method to graph each equation on the grid at right.

Eqn. (1)

$t$	$r$
0	
	0

Eqn. (2)

$t$	$r$
0	
	0



- Locate the point where the two graphs intersect. What are its coordinates?

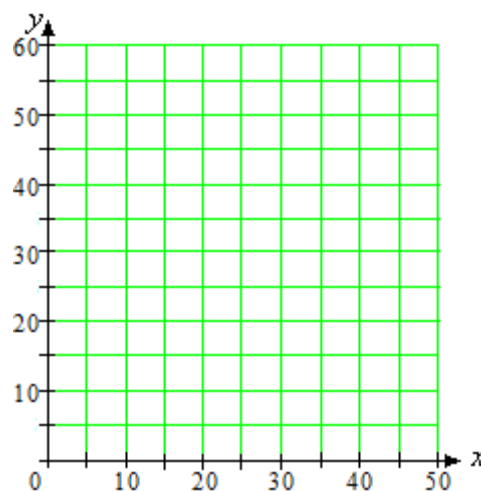
Answer the question posed by the biologist.

## Activity 2 Problem Solving

The manager for Books for Cooks plans to spend \$300 stocking a new diet cookbook. The paperback version costs her \$5, and the hardback costs \$10. She finds that she will sell three times as many paperbacks as hardbacks. How many of each should she buy?

Let  $x$  represent the number of hardbacks and  $y$  the number of paperbacks she should buy.

- Write an equation about the cost of the books.
- Write a second equation about the number of each type of book.
- Graph both equations on the grid and solve the system. Then answer the question in the problem.



## Activity 3 Inconsistent and Dependent Systems

Robert and Ruth are moving from Los Angeles to Baltimore. Robert is driving a rental truck at an average speed of 50 miles per hour. Ruth leaves one day later in their car, and averages 65 miles per hour. When Ruth set out, Robert had already traveled 300 miles. When will Ruth catch up with Robert?

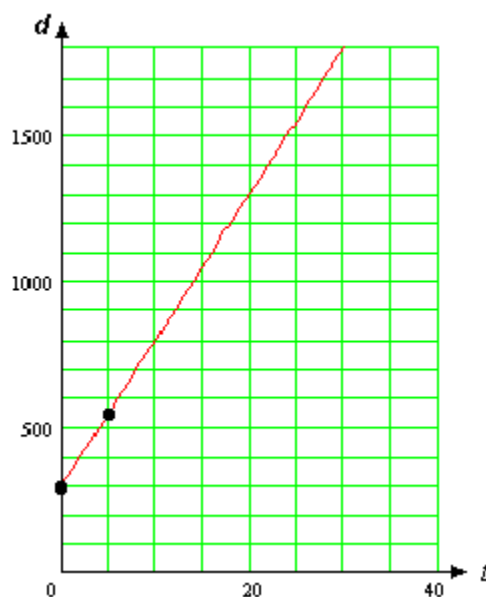
- Let  $t$  stand for the number of hours that Ruth has traveled. When she catches up with Robert, they will both have traveled the same distance, so we begin by writing equations for the distance,  $d$ , each has traveled after  $t$  hours.
  - Ruth travels at 65 miles per hour, so an equation for the distance she has traveled is  
     Ruth:
  - Robert's speed is 50 miles per hour, but he has already traveled 300 miles when Ruth starts, so his distance is given by  
     Robert:
  - Together, the two equations form a system:

2. Next, we graph both equations on the same axes.

- The graph of Robert's distance is shown in the figure. The  $d$ -intercept of the graph is 300 and its slope is 50.
- To graph the equation for Ruth's distance, it is probably easiest to plot a few points. Fill in the table and graph the equation for Ruth's distance on the same grid with Robert's distance.

$t$	0	10	20
$d$			

- Locate the point where the two graphs intersect. What are its coordinates?



3. Finally, we interpret the solution.

- What does the  $t$ -coordinate of the intersection point tell you about the problem?
- What does the  $d$ -coordinate of the point tell you?
- Verify that the intersection point is a solution of both equations in the system.

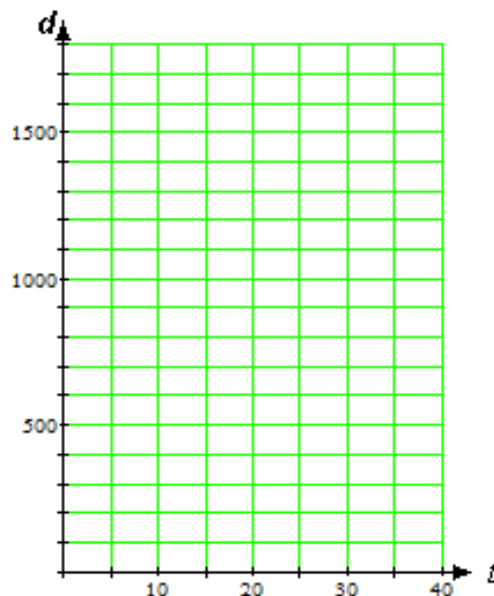
4. Now let's change the problem and suppose instead that Ruth and Robert both drive at an average speed of 50 miles per hour.

- Write a system of equations for this problem:

Robert:  $d =$

Ruth:  $d =$

- Graph the equations on the grid.
- What is the solution of this system?  
What is this type of system called?

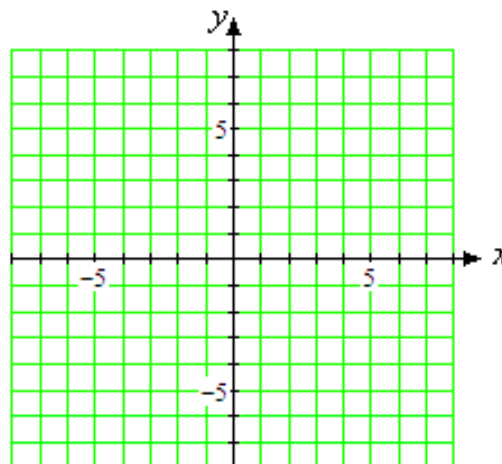


5. a. Graph the system of equations.

$$3x = 2y + 6$$

$$y = \frac{3}{2}x - 3$$

**Hint:** Use the intercept method to graph the first equation, and the slope-intercept method to graph the second equation.



- b. Is the system inconsistent or dependent?

## Wrap-Up

In this Lesson we practiced the following skills:

- Deciding whether an ordered pair is a solution of a system
- Solving a system of equations by graphing
- Identifying a system as consistent, inconsistent, or dependent
- Writing a system of equations to solve an applied problem

1. In Activity 1, how far has Robert traveled at  $t = 0$ ?
2. In Activity 2, Problem 1, explain why it makes sense that the system has no solution.
3. In Activity 3, which equation is correct:  $x = 3y$  or  $y = 3x$ ? Why?
4. In Activity 3, how much did the manager spend on hardback books? How much on paperback books?

## 4.2 Homework Preview

1. Is  $(-3, 5)$  a solution of the system?  
Explain how you know.

$$2x + 3y = 9$$

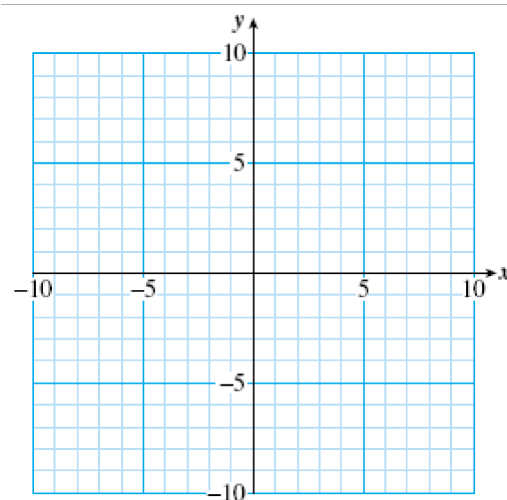
$$2x - y = 1$$

2. Solve the system by graphing.

$$y = \frac{2}{3}x - 4$$

$$2x - 3y = 12$$

**Hint:** graph the first equation by the slope-intercept method, and the second equation by the intercept method.



3. Write a system of equations for the problem:  
Kathy has two cats, Miso and Nori. Together they weigh 18 pounds. Miso's weight is 3 pounds more than half of Nori's weight. How much does each cat weigh?
4. Decide whether the system is inconsistent or dependent.

$$6x - 4y = 8$$

$$x = \frac{2}{3}y + 2$$

## Answers

1. No      2.  $(3, -2)$       3.  $x + y = 18$   
 $y = \frac{1}{2}x + 3$       4. Inconsistent

## Lesson 4.3 Algebraic Solution of Systems

### Activity 1 Substitution Method

1. Solve the system algebraically:

$$\begin{aligned}d &= 65t \\d &= 300 + 50t\end{aligned}$$

2. Follow the suggested steps to solve the system by substitution:

$$\begin{aligned}3y - 2x &= 3 \\x - 2y &= -2\end{aligned}$$

Step 1 Solve the second equation for  $x$  in terms of  $y$ .

Step 2 Substitute your expression for  $x$  into the first equation.

Step 3 Solve the equation you got in Step 2.

Step 4 Substitute the  $y$ -value into your result from Step 1 to find  $x$ .

Check Verify that your solution values satisfy both equations in the system.

### Activity 2 Elimination Method

1. Follow the suggested steps to solve the system by elimination.

$$\begin{aligned}2x - 3y &= 6 \\4x - 5y &= 8\end{aligned}$$

For this problem, we will eliminate the  $x$ -terms, so we arrange for their coefficients to be opposites.

Step 1 Multiply each term of the first equation by  $-2$ .

Step 2 Add the new equations and solve the result for  $y$ .

Step 3 Substitute your value for  $y$  into the first equation and solve for  $x$ .

Check Verify that your solution values satisfy both equations in the system.

2. Follow the suggested steps to solve the system by elimination.

$$\begin{aligned}3x &= 2y + 13 \\ 3y - 15 &= -7x\end{aligned}$$

For this problem, we will eliminate the  $y$ -terms.

Step 1 Write each equation in the form  $Ax + By = C$ .

Step 2 Find the LCM of the  $y$ -coefficients. Multiply each equation by an appropriate constant.

Step 3 Add the new equations and solve the result for  $x$ .

Step 4 Substitute your value for  $x$  into the second equation and solve for  $y$ .

Check Verify that your solution values satisfy both equations in the system.

3. a. Solve the system by elimination.

$$\begin{aligned}x + 3y &= 6 \\ 2x - 12 &= -6y\end{aligned}$$

b. Is the system dependent, inconsistent, or consistent and independent?

Activity 3 Applications

1. A train ticket from Camarillo to San Diego costs \$31 in coach and \$47 for business class. On Tuesday, there were 42 passengers on the morning train, and Amtrak took in \$1494 in fares. How many coach passengers took the morning train, and how many business class passengers?

Choose variables:     $x =$   
                                  $y =$

Complete the table:

	Number of passengers	Price per ticket	Total revenue
Coach			
Business			

Write an equation about the number of passengers:

Write an equation about revenue:

Solve your system.

Answer:

2. The perimeter of a rectangular playground is 197 yards, and its length is 5 yards less than twice its width. Find the dimensions of the playground.

Choose variables:     $x =$   
                                  $y =$

Write an equation about the perimeter of the playground:

Write an equation relating the length and the width:

Solve your system.

Answer:



## Wrap-Up

In this Lesson we practiced the following skills:

- Solving systems by the substitution method
  - Solving systems by the elimination method
  - Identifying a system as consistent, inconsistent, or dependent
  - Writing a system of equations to solve an applied problem
1. In Activity 1, Problem 2, why did we choose to solve the second equation for  $x$  in terms of  $y$ ?
  2. In Activity 2, Problem 1, why did we multiply the first equation by  $-2$ ?
  3. In Activity 2, Problem 2, what was the LCM of the  $y$ -coefficients?
  4. In Activity 3, Problem 1, what were the two equations about?

## 4.3 Homework Preview

1. Solve the system by substitution:

$$\begin{aligned}3x - y &= 5 \\ 2x - 3y &= 8\end{aligned}$$

2. Solve the system by elimination:

$$\begin{aligned}2x - 9y &= 3 \\ 4x - 5y &= -7\end{aligned}$$

3. Solve the system by elimination:

$$\begin{aligned}5x + 2y &= 5 \\ 4x + 3y &= -3\end{aligned}$$

4. Decide whether the system is dependent or inconsistent:

$$\begin{aligned}3x - 4y &= 2 \\ 2y + 1 &= \frac{3}{2}x\end{aligned}$$

## Answers

1.  $(1, -2)$
2.  $(-3, -1)$
3.  $(3, -5)$
4. Dependent

# Lesson 4.4 Problem Solving with Systems

## Activity 1 Interest

Jerry invested \$2000, part at 4% interest and the remainder at 9%. His yearly income from the 9% investment is \$37 more than his income from the 4% investment. How much did he invest at each rate?

**Step 1** Choose variables for the unknown quantities

Fill in the table.

	Principal	Interest Rate	Interest
First investment			
Second investment			

**Step 2** Write two equations; one about the principal, and one about the interest.

Principal:  
Interest:

**Step 3** Solve the system. (Which method seems easiest?)

**Step 4** Answer the question in the problem.

## Activity 2 Mixtures

Polls conducted by Senator Quagmire's campaign manager show that he can win 60% of the rural vote in his state but only 45% of the urban vote. If 1,200,000 citizens vote, how many voters from rural areas and how many from urban areas must vote in order for the Senator to win 50% of the votes?

**Step 1** Let  $x$  represent the number of rural voters and  $y$  the number of urban voters. Fill in the table.

	Number of Voters ( $W$ )	Percent for Quagmire ( $r$ )	Number for Quagmire ( $P$ )
Rural			
Urban			
Total			

**Step 2** Add down the first and third columns to write a system of equations.

Number of voters:

Number for Quagmire:

**Step 3** Solve your system.

**Step 4** Answer the question in the problem.

Activity 3 Motion

A river steamer requires 3 hours to travel 24 miles upstream and 2 hours for the return trip downstream. Find the speed of the current and the speed of the steamer in still water.

Step 1 Choose variables: Speed of the current:  
Speed of the steamer:

Fill in the table about the steamer. **Hint:** When you are traveling downstream, the current adds to the speed of the boat. When you are traveling upstream, the current subtracts from the speed of the boat.

	Rate	Time	Distance
Upstream			
Downstream			

Step 2 Write two equations about the steamer.

Step 3 Solve your system.

Step 4 Answer the question in the problem.

Wrap-Up

In this Lesson we practiced the following skills:

- Using a system to solve problems about interest
- Using a system to solve problems about mixtures
- Using a system to solve problems about motion

1. In Activity 1, is \$2000 part of the equation about the principal or the equation about the interest?
2. In Activity 2, what did  $x$  and  $y$  represent?
3. In Activity 3, what did  $x + y$  and  $x - y$  represent?

## 4.4 Homework Preview

- a. State the unknown quantities in each problem.
  - b. Write a system of equations for the problem.
1. Mischa invested \$2000 in two accounts: a CD that earns 4% interest, and a savings account that earns  $2\frac{1}{2}\%$  interest. At the end of the year, he had earned \$72.50 in interest. How much did he invest in each account?
  2. Bran flakes cost \$1.40 per cup, and raisins cost \$2.20 per cup. A 3-cup box of raisin bran costs \$4.60. How many cups of raisins and how many cups of bran flakes are in the box?
  3. EnergyBurst powdered lemonade is 50% sugar by weight. LemonLite is 20% sugar. How much of each powder should you use to make 12 ounces of a mixture that is 30% sugar?
  4. Jada and Charee participate in Walk for the Cure. Jada walks at 4 miles per hour, and Charee jogs at 6 miles per hour. It takes Jada  $2\frac{1}{2}$  hours longer than Charee to complete the course. How long is the course?

## Answers

- |   |                                       |
|---|---------------------------------------|
| 1. $x + y = 2000$<br>$0.04x + 0.025y = 72.50$ | 2. $x + y = 3$<br>$1.4x + 2.2y = 4.6$ |
| 3. $x + y = 12$<br>$0.5x + 0.2y = 0.3(12)$    | 4. $4x = 6y$<br>$x = y + 2.5$         |

## Lesson 4.5 Point-Slope Form

### Activity 1 Point-Slope Formula

1. a. Graph the line of slope  $\frac{-1}{2}$  that passes through the point  $(-3, 2)$ .

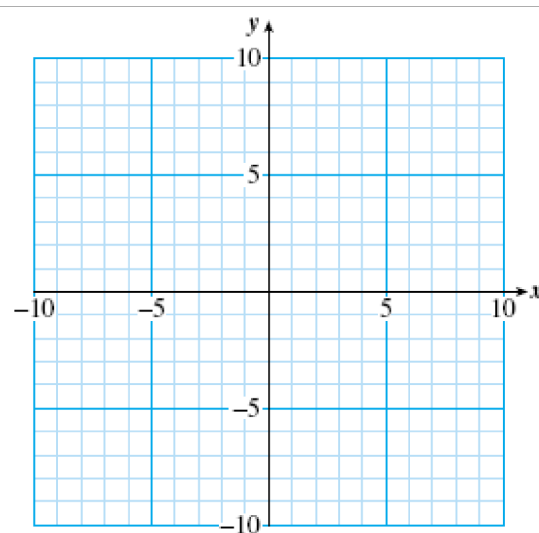
Plot the point  $(-3, 2)$ .

Use the slope to find a second point.

$$m = \frac{\Delta y}{\Delta x} = \frac{-1}{2}$$

Use the slope to find a third point.

$$m = \frac{\Delta y}{\Delta x} = \frac{1}{-2}$$



- b. Find an equation for the line.

Step 1 Use the point-slope formula  $\frac{y - y_1}{x - x_1} = m$

Step 2 Cross-multiply to simplify the equation.

Step 3 Solve for  $y$ .

2. a. Graph the line of slope 3 that passes through the point  $(-4, -6)$ .

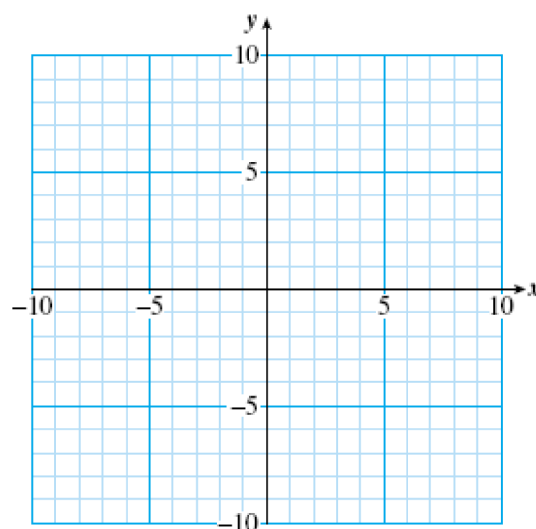
Plot the point  $(-4, -6)$ .

Use the slope to find a second point.

$$m = \frac{\Delta y}{\Delta x} = \frac{3}{1}$$

Use the slope to find a third point.

$$m = \frac{\Delta y}{\Delta x} = \frac{-3}{-1}$$



- b. Find an equation for the line.

Step 1 Use the point-slope formula  $\frac{y - y_1}{x - x_1} = m$

Step 2 Cross-multiply to simplify the equation.

Step 3 Solve for  $y$ .

## Activity 2 Using the Point-Slope Formula

1. Find an equation for the line that passes through  $(-1, 4)$  and  $(3, -2)$ .

Step 1 Compute the slope of the line.

Step 2 Apply the point-slope formula.

2. Around 1950, people began cutting down the world's rain forests to clear land for agriculture. In 1970 there were about 9.8 million square kilometers of rain forest left, and in 1990 there were about 8.2 million square kilometers.
- a. Use these data points to find a linear equation for the number of million square kilometers,  $y$ , of rain forest left  $x$  years after 1950.

Data points:

Compute the slope:

Apply the point-slope formula:

- b. If we continue to clear the rain forest at the same rate, when will it be completely destroyed?
3. a. What is the slope of a line that is parallel to  $x + 4y = 2$  ?
- b. Find an equation for the line that is parallel to  $x + 4y = 2$  and passes through  $(2, 3)$ .
4. a. What is the slope of a line that is perpendicular to  $x + 4y = 2$  ?
- b. Find an equation for the line that is perpendicular to  $x + 4y = 2$  and passes through  $(2, 3)$ .



## Wrap-Up

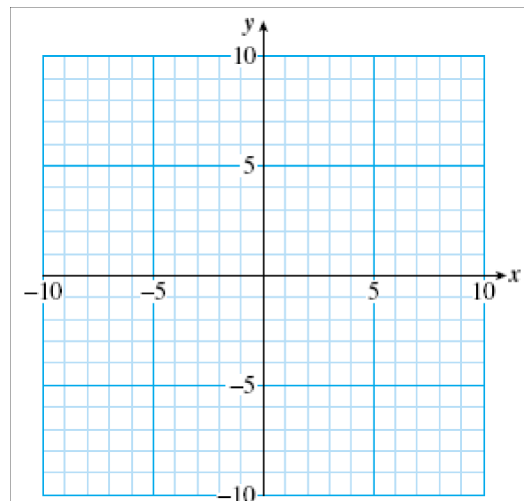
In this Lesson, we practiced the following skills:

- Using the point-slope formula to find the equation of a line
  - Using the point-slope formula to graph a line
  - Using the point-slope formula in applications
1. In graphing the line in Activity 1, explain how the 2 in the point  $(-3, 2)$  and the 2 in the slope  $\frac{-1}{2}$  are used differently.
  2. In Problem 2 of Activity 2, what were the two data points?
  3. What was different in the solutions of Problems 2 and 3 of Activity 2?

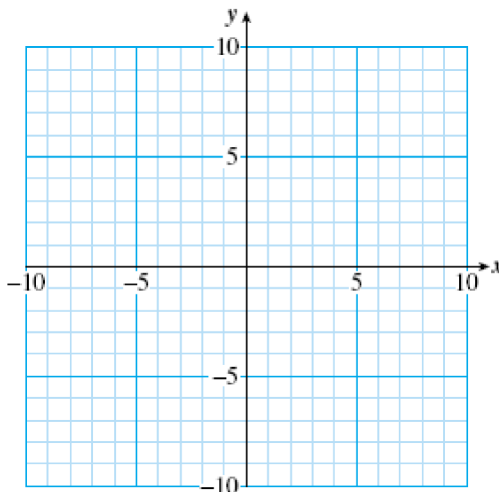
## 4.5 Homework Preview

- a. Graph the line.
- b. Find the equation of the line.

1.  $m = -3$ ,  $(-6, 8)$



2.  $m = \frac{2}{5}$ ,  $(7, 4)$



3. Find an equation of the line that goes through  $(-6, -4)$  and  $(2, 8)$ .

4. Find an equation for the line that is perpendicular to  $3x - 5y = 1$  and passes through  $(0, 4)$ .

## Answers

1.  $y = -3x - 10$       2.  $y = \frac{2}{5}x + \frac{6}{5}$       3.  $y = \frac{3}{2}x + 5$       4.  $y = \frac{-5}{3}x + 4$

## Lesson 5.1 Exponents

### Activity 1 Exponents

1. a. Compute the squares of all the integers from one to ten.

$1^2$	$2^2$	$3^2$	$4^2$	$5^2$	$6^2$	$7^2$	$8^2$	$9^2$	$10^2$

- b. Compute the cubes of all the integers from one to ten.

$1^3$	$2^3$	$3^3$	$4^3$	$5^3$	$6^3$	$7^3$	$8^3$	$9^3$	$10^3$

2. a. Compute all the powers of 2 up to  $2^{10}$ .

$2^1$	$2^2$	$2^3$	$2^4$	$2^5$	$2^6$	$2^7$	$2^8$	$2^9$	$2^{10}$

- b. Compute all the powers of 3 up to  $3^{10}$ .

$3^1$	$3^2$	$3^3$	$3^4$	$3^5$	$3^6$	$3^7$	$3^8$	$3^9$	$3^{10}$

3. a. Compute all the powers of 10 up to  $10^5$ .

- b. Describe how to write down the number  $10^{24}$ .

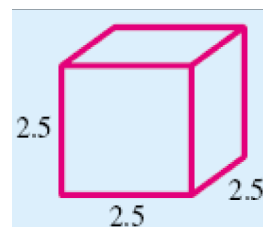
4. a. Explain the difference between  $4(3)$  and  $3^4$ .

- b. Explain the difference between  $4x$  and  $x^4$ .

### Activity 2 Computing Powers

1. a. Find the area of a square whose side is  $2\frac{1}{2}$  centimeters long.

- b. Find the volume of a cube whose base is the square in part (a).



2. Compute each power.

a.  $(-2)^1$

d.  $(-2)^4$

b.  $(-2)^2$

e.  $(-2)^5$

c.  $(-2)^3$

f.  $(-2)^6$

Do you see a pattern developing? What is the connection between the exponent and the sign of the answer?

g. The sign of the power is \_\_\_\_\_ if the exponent is odd .

The sign of the power is \_\_\_\_\_ if the exponent is even .

3. Use a calculator to compute each power.

a.  $(-1.1)^4$

b.  $(-2.3)^3$

### Activity 3 Order of Operations

1. a. Write each expression as a repeated addition or multiplication.

$$a^5 = \underline{\hspace{2cm}}$$

$$5a = \underline{\hspace{2cm}}$$

- b. Evaluate each expression above for  $a = 3$ .

2. Compare the expressions by writing them without exponents.

a.  $3ab^4 =$

$$3(ab)^4 =$$

b.  $a + 5b^2 =$

$$(a + 5b)^2 =$$

3. Simplify  $9 - 5 \cdot 2^3$

4. Evaluate each expression for  $t = -3$ .

a.  $-3t^2 - 3$

b.  $-t^3 - 3t$

### Activity 4 Like Terms

1. Combine like terms if possible.

a.  $12v - 6v^2$

b.  $-4h^2 - 4h^2$

2. Simplify the expression by adding or subtracting like terms.

a.  $(2m^4 - 4m^3 + 3m - 1) + (m^3 + 4m^2 - 6m + 2)$

b.  $(4x^2 + 2x - 5) - (2x^2 - 3x - 2)$  (Be careful with the signs!)

3. It costs The Cookie Company  $200 + 2x$  dollars to produce  $x$  bags of cookies per week, and they earn  $8x - 0.01x^2$  dollars from the sale of  $x$  bags of cookies.

a. Write an expression for the profit earned by The Cookie Company on  $x$  bags of cookies.

b. Find the company's profit on 300 bags of cookies, and on 600 bags.

## Wrap-Up

In this Lesson we practiced the following skills:

- Computing powers
- Simplifying expressions involving exponents
- Evaluating expressions involving exponents
- Combining like terms

1. In Activity 1, which is bigger,  $10^2$  or  $2^{10}$ ? What is the smallest value of  $n$  for which  $2^n$  is bigger than  $n^2$ ?
2. In Activity 2, what are the units of area? What are the units of volume?
3. Why is it incorrect to evaluate  $t^2$  for  $t = -3$  by entering  $-3^2$  into your calculator?
4. In Activity 4, what is wrong with this statement?

$$(4x^2 + 2x - 5) - (2x^2 - 3x - 2) = 4x^2 + 2x - 5 - 2x^2 - 3x - 2$$

## 5.1 Homework Preview

1. Simplify.

a.  $-4 - 2^3$

b.  $-4(-2)^3$

c.  $(-4 - 2)^3$

2. Evaluate for  $x = -3$ ,  $y = -2$ .

a.  $-2x^2 + y^3$

b.  $5(x - y^2)^3$

c.  $4(x - y)(x + 2y)$

3. Simplify if possible.

a.  $3x^2 - 5x^3$

b.  $3x^3 - 5x^3$

4. Combine like terms.

a.  $2a^3 + 6a^2 - 4a - (a^3 - 3a + 4a^2)$

b.  $5xt^2 - 3xt - (-2x^2t) + (-xt - xt^2)$

## Answers

1. a.  $-12$

b.  $32$

c.  $-216$

2. a.  $-1715$

c.  $28$

3. a. cannot be simplified

b.  $-2x^3$

4. a.  $a^3 + 2a^2 - a$

b.  $4xt^2 - 4xt + 2x^2t$

## Lesson 5.2 Square Roots and Cube Roots

### Activity 1 Roots and Radicals

- Find two square roots for each number.
  - 225
  - $\frac{4}{9}$
- Simplify each radical, if possible.
  - $\sqrt{64}$
  - $\sqrt{-64}$
  - $-\sqrt{64}$
  - $\pm\sqrt{64}$
- Evaluate each cube root. (Use a calculator if necessary.)
  - $\sqrt[3]{8}$
  - $\sqrt[3]{-125}$
  - $\sqrt[3]{-1}$
  - $\sqrt[3]{50}$
- If  $p = \sqrt{d}$ , then  $d =$  \_\_\_\_\_.
  - If  $v^2 = k$ , then  $v =$  \_\_\_\_\_.
- Explain why the square root of a negative number is undefined.
  - If  $x > 0$ , explain the difference between  $\sqrt{-x}$  and  $-\sqrt{x}$ .
  - Explain why you can always simplify  $\sqrt{x}\sqrt{x}$ , as long as  $x$  is non-negative.
  - Explain why you can always simplify  $\frac{x}{\sqrt{x}}$ , as long as  $x$  is positive.
- Make a list of the squares of all the integers from 1 to 20. These are the first 20 perfect squares. Now make a list of square roots for these perfect squares.
  - Make a list of the cubes of all the integers from 1 to 10. These are the first 10 perfect cubes. Now make a list of cube roots for these perfect cubes.

### Activity 2 Rational Numbers

- Find the decimal form for each rational number. Does it terminate?
  - $\frac{2}{3}$
  - $\frac{5}{2}$
  - $\frac{13}{27}$
  - $\frac{962}{2000}$



2. Give a decimal equivalent for each radical, and identify it as rational or irrational. If necessary, round your answers to three decimal places.

$\sqrt{1} = \underline{\hspace{2cm}}$

$\sqrt{6} = \underline{\hspace{2cm}}$

$\sqrt{2} = \underline{\hspace{2cm}}$

$\sqrt{7} = \underline{\hspace{2cm}}$

$\sqrt{3} = \underline{\hspace{2cm}}$

$\sqrt{8} = \underline{\hspace{2cm}}$

$\sqrt{4} = \underline{\hspace{2cm}}$

$\sqrt{9} = \underline{\hspace{2cm}}$

$\sqrt{5} = \underline{\hspace{2cm}}$

$\sqrt{10} = \underline{\hspace{2cm}}$

3. True or false.
- If a number is irrational, it cannot be an integer.
  - Every real number is either rational or irrational.
  - Irrational numbers do not have an exact location on the number line.
  - We cannot find an exact decimal equivalent for an irrational number.
  - 2.8 is only an approximation for  $\sqrt{8}$ ; the exact value is 2.828427125.
  - $\sqrt{17}$  appears somewhere between 16 and 18 on the number line.

### Activity 3 Order of Operations

1. A radical symbol acts like parentheses to group operations. Any operations that appear under a radical should be performed before evaluating the root.
- Simplify  $\sqrt{6^2 - 4(3)}$

b. Approximate your answer to part (a) to three decimal places.

c. Explain why the following is **incorrect**:

$$\begin{aligned}\sqrt{6^2 - 4(3)} &= \sqrt{6^2} - \sqrt{4(3)} \\ &= \sqrt{36} - \sqrt{12} \\ &\approx 6 - 3.464 = 2.536\end{aligned}$$

2. a. Simplify  $5 - 3\sqrt{16 + 2(-3)}$

b. Approximate your answer to part (a) to three decimal places.

3. Evaluate  $2x^2 - \sqrt{9 - x}$  for  $x = -3$ .

## Wrap-Up

In this Lesson we practiced the following skills:

- Computing square roots and cube roots
- Using radical notation
- Distinguishing between rational and irrational numbers
- Distinguishing between exact values and approximations
- Simplifying expressions involving radicals

1. Explain why you don't need a calculator to evaluate  $\frac{23}{\sqrt{23}}$ .
2. Explain why you cannot write down an exact decimal form for  $\sqrt{6}$ .
3. Explain why  $\sqrt{3^2 + 4^2} \neq 3 + 4$ .

## 5.2 Homework Preview

1. Simplify.

a.  $8 - 3\sqrt{25}$

b.  $(3\sqrt{16})(-2\sqrt{81})$

c.  $\frac{3 - \sqrt{36}}{6}$

2. Find a decimal approximation rounded to two places.

a.  $-3 + \sqrt{34}$

b.  $\sqrt{6^2 - 4(3)}$

c.  $\sqrt[3]{\frac{18}{5}}$

3. Simplify.

a.  $2\sqrt{x}(8\sqrt{x})$

b.  $\frac{6m}{3\sqrt{m}}$

c.  $\sqrt[3]{H}(\sqrt[3]{H})(\sqrt[3]{H})$

4. Evaluate for  $x = 3$ ,  $y = 5$ . Round to hundredths.

a.  $\sqrt{x^2 + y^2}$

b.  $(x + y)^2 - (x^2 + y^2)$

c.  $\sqrt{x} + \sqrt{y} - \sqrt{x + y}$

### Answers

1. a.  $-7$

b.  $-216$

c.  $-\frac{1}{2}$

2. a.  $2.83$

b.  $4.90$

c.  $1.53$

3. a.  $16x$

b.  $2\sqrt{m}$

c.  $H$

4. a.  $5.83$

b.  $30$

c.  $1.14$

## Lesson 5.3 Using Formulas

### Activity 1 Volume and Surface Area

■ Illustrate each problem with a sketch.

1.
  - a. Find the area of a square that measures 8 inches on a side.
  - b. Find the volume of a cube whose base is the square in part (a).
2.
  - a. Find the area of a square that measures 0.1 meter on a side.
  - b. Find the volume of a cube whose base is the square in part (a).
3. How many one-centimeter cubes will fit inside a box that measures 16 centimeters by 20 centimeters by 8 centimeters?
4. How many cubic feet of dirt came out of a hole that is 12.5 feet wide, 15 feet long, and 8.2 feet deep?
5. The side of a cube is 20 centimeters long.
  - a. Find the area of one face of the cube.
  - b. How many faces does the cube have?  
Find the total surface area (the sum of the areas of all the faces) of the cube.
  - c. Find the volume of the cube.

6. a. How many square feet are in a square yard?
- b. How many square inches are in a square foot?
- c. How many cubic feet are in a cubic yard?
- d. How many cubic inches are in a cubic foot?

## Activity 2 Pythagorean Theorem

1. The two short sides of a right triangle are 9 meters and 40 meters.
- a. Sketch and label a figure.
- b. What is the hypotenuse?
2. A baseball diamond is a square whose sides are 90 feet.
- a. Sketch and label a figure.
- b. How far is it from first base to third base (which are on opposite corners of the square)?

### Activity 3 Formulas

1. a. The box at right has length  $l$ , and its width and height are both  $w$ . The box has 6 sides. Write expressions for the area of each face of the box.

Top:

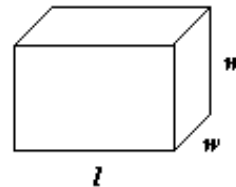
Bottom:

Front:

Back:

Left end:

Right end:



- b. Add the six areas together to get a formula for the surface area of the box.

$$S =$$

- c. Francine would like to wrap a package that is 4 feet long and whose width and height are both 1.5 feet. She has 24 square feet of wrapping paper. Is that enough paper?

2. Delbert would like to build a box whose width and height are both 1.5 feet, and whose surface area is no more than 24 square feet. What is the longest box that Delbert can build? (Use the formula you found in Problem 1.)

3. Solve the formula  $S = 2w^2 + 4lw$  for  $l$ .

## Wrap-Up

In this Lesson we practiced the following skills:

- Using formulas to compute volume and surface area
  - Using the Pythagorean theorem
  - Solving a formula for one variable in terms of the others
1. Why are the units of area called square units? Why are the units of volume called cubic units?
  2. What is the Pythagorean theorem used for?
  3. Explain how to find the surface area of a box.

## 5.3 Homework Preview

1. Find the volume of a spherical bubble with diameter 5 cm.
2. A 6-foot tall conical tank has volume 14.137 cubic feet. What is the radius of the base of the cone?
3. The hypotenuse of a right triangle is 4.5 miles, and one leg is 1.8 miles long. How long is the other leg?
4. Solve for  $h$ :  $V = \pi r^2 h + \pi r^3$

## Answers

1.  $65.45 \text{ cm}^3$
2. 1.5 ft
3. 4.12 mi
4.  $\frac{V - \pi r^3}{\pi r^2}$

# Lesson 5.4 Products of Binomials

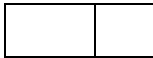
## Activity 1 Areas of Rectangles

1. Use the distributive law to find the products. Illustrate each product as the area of a rectangle.

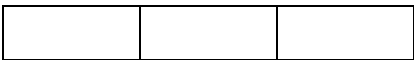
a.  $2a(6a - 5)$



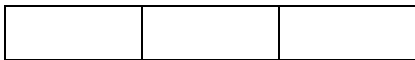
b.  $-4v(2v - 3)$



c.  $-5x(x^2 - 3x + 2)$



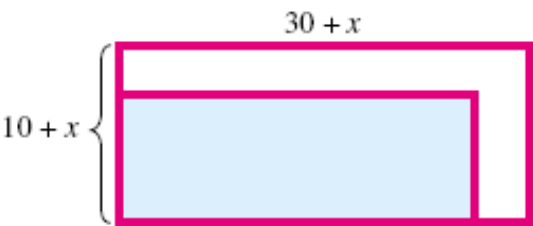
d.  $-3y(4y^2 - 2y + 2x)$



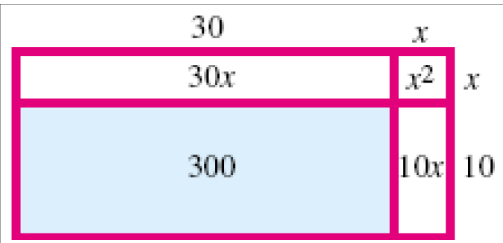
2. The City Council plans to install a 10-foot by 30-foot reflecting pool in front of City Hall. When the cost estimate comes in, they realize they can afford to enlarge the pool. They decide to increase both the length and the width by  $x$  feet. Write an equation for the new area,  $A$ , of the pool in terms of  $x$ .

a. Look at the drawing of the pool. Both dimensions of the original pool have been increased by  $x$ . The area of the enlarged pool is thus

$A = \text{length} \times \text{width}$   
 $=$



b.



We can partition the new pool into four sub-rectangles and compute the area of each. Adding these areas give us the following expression for the area of the new pool.

$A =$   
 $=$  Combine like terms.

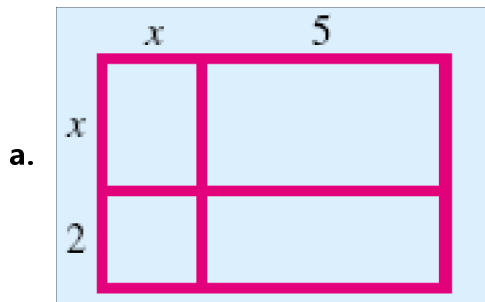
c. The two expressions for the area are equivalent. Thus,

$=$

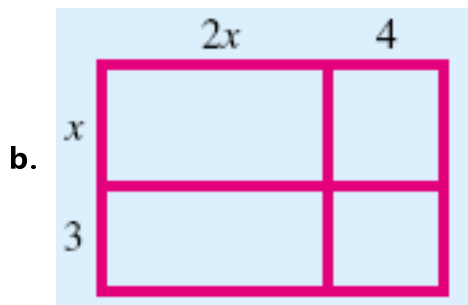


3. Write the area of each rectangle in two different ways: as the sum of four small areas, and then as one large rectangle, using the formula

$$\text{Area} = \text{length} \times \text{width}$$



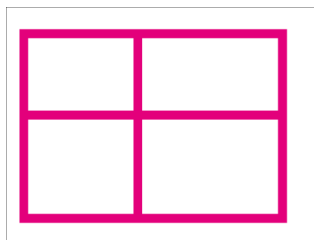
sum of four small areas = one large area



sum of four small areas = one large area

## Activity 2 Products of Binomials

1. a. Use a rectangle to represent the product  $(3x - 2)(x - 5)$ .

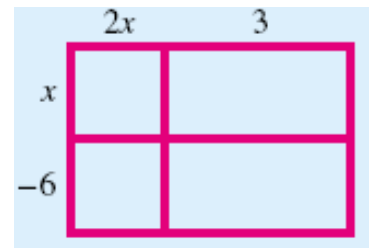


- b. Write the product as a quadratic trinomial.

2. a. Find the linear term in the product

$$(x - 6)(2x + 3)$$

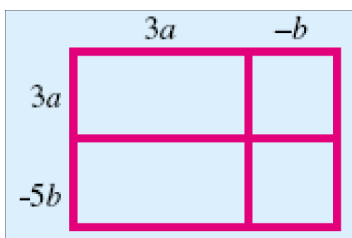
Use the diagram to help you.



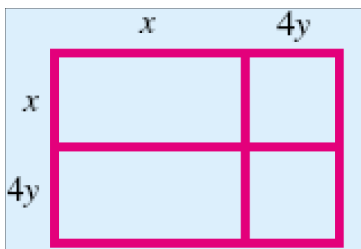
- b. Which of the four smaller rectangles make up the linear term?

3. Compute the area of each rectangle, then write each product as a quadratic trinomial in two variables.

a.  $(3a - 5b)(3a - b)$



b.  $(x + 4y)^2$



## Wrap-Up

In this Lesson we practiced the following skills:

- Computing the area of a rectangle
  - Representing the product of two binomials as the area of a rectangle
  - Computing the product of two binomials
1. Explain the difference between simplifying  $3a + a$  and simplifying  $3a(a)$ .
  2. Explain the difference between simplifying  $6t - 4t$  and simplifying  $6t(-4t)$ .
  3. In Activity 1, Problem 2, by how much did the area of the pool increase?
  4. In Activity 2, does  $(x + 4y)^2 = x^2 + 16y^2$ ?

## 5.4 Homework Preview

■ Compute the products.

**1a.**  $(5t)(-3t^2)$

**b.**  $5t(t^2 - 3t)$

**2a.**  $-3x(4x - 5)$

**b.**  $(6 - 2a)(-4a)$

**3a.**  $(2b - 8)(b + 4)$

**b.**  $(3c - 1)(2c - 3)$

**4a.**  $(5y - 2)^2$

**b.**  $(1 - 4w)^2$

### Answers

**1a.**  $-15t^3$

**b.**  $5t^3 - 15t^2$

**2a.**  $-12x^2 + 15x$

**b.**  $-24a + 8a^2$

**3a.**  $2b^2 - 32$

**b.**  $6c^2 - 11c + 3$

**4a.**  $25y^2 - 10y + 4$

**b.**  $1 - 8w + 16w^2$

## Lesson 6.1 Extracting Roots

### Activity 1 Quadratic Equations

1. Which of the following equations are quadratic?

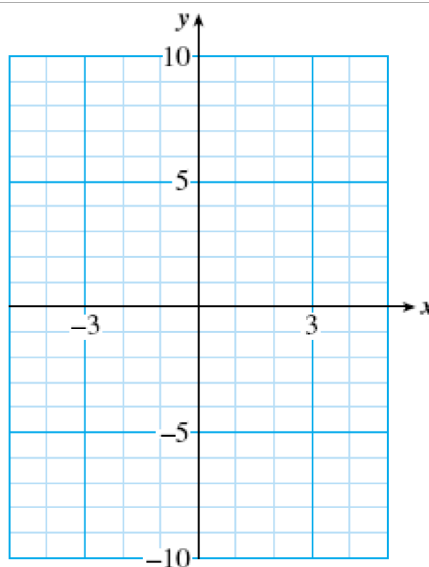
a.  $3x + 2x^2 = 1$

b.  $4z^2 - 2z^3 + 2 = 0$

c.  $36y - 16 = 0$

d.  $v^2 = 6v$

2. a. Make a table and graph  $y = 9 - x^2$ .



b. Use your graph to solve the equation  $9 - x^2 = -7$ .

### Activity 2 Extracting Roots

1. Solve each equation without using a calculator.

a.  $p^2 = 625$

b.  $u^2 = \frac{16}{49}$

c.  $n^2 = 0.25$

d.  $t^2 = (\sqrt{7})^2$

2. Solve by extracting roots.

a.  $4a^2 + 25 = 169$

First isolate  $a^2$ .

Take the square root of both sides.

b.  $\frac{3x^2 - 8}{4} = 10$

First isolate  $x^2$ .

Take the square root of both sides.

3. Solve by extracting roots. Round your answers to tenths.

a.  $3b^2 - 18 = 42$

First isolate  $b^2$ .

Take the square root of both sides.

Use your calculator to find decimal approximations for the solutions.

b.  $4\pi r^2 = 90$

First isolate  $r^2$ .

Take the square root of both sides.

Use your calculator to find decimal approximations for the solutions.

## Wrap-Up

In this Lesson we practiced the following skills:

- Identifying a quadratic equation
- Graphing a parabola by plotting points
- Using a graph to solve a quadratic equation
- Solving a quadratic equation by extracting roots
- Solving a quadratic formula for one variable

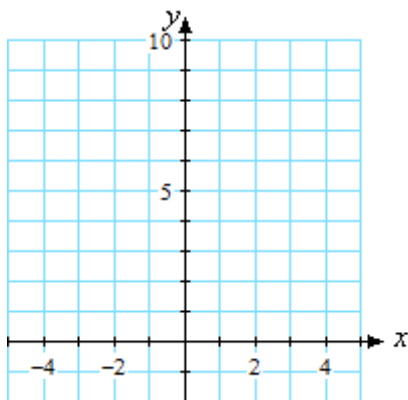
1. In Activity 1, Problem 2, Delbert evaluated  $4 - x^2$  for  $x = -1$  by entering  $4 - (-1^2)$  into his calculator and getting 5. What did he do wrong?
2. In Activity 2, Problem 1b, explain how to take the square root of a fraction.
3. Explain the difference between an exact solution and a decimal approximation.

## 6.1 Homework Preview

- a. Complete the table of values and sketch the graph.  
b. Use the graph to solve the equation.

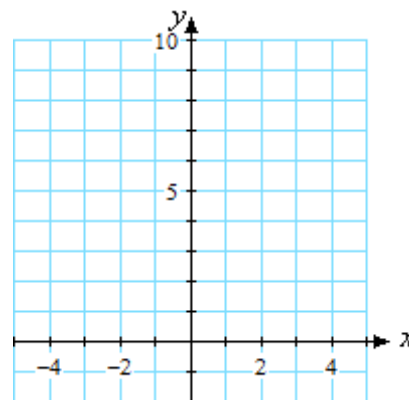
1. a.  $y = 2x^2$   
b.  $2x^2 = 8$

$x$	$y$
0	
1	
2	
3	
-1	
-2	
-3	



2. a.  $y = 8 - x^2$   
b.  $-1 = 8 - x^2$

$x$	$y$
0	
1	
2	
3	
-1	
-2	
-3	



- Solve. Give exact values for your answers.

3.  $x^2 - 8 = 4$

4.  $3x^2 - 4 = 11$

- Solve. Round your answers to hundredths.

5.  $2.1x^2 + 3.8 = 23.2$

6.  $5x^2 - 6 = 2x^2 + 9$

## Answers

1.  $\pm 2$       2.  $\pm 3$       3.  $\pm\sqrt{12}$       4.  $\pm\sqrt{5}$       5.  $\pm 3.04$       6.  $\pm 2.24$

## Lesson 6.2 Some Quadratic Models

### Activity 1 Revenue

Raingear manufactures and sells umbrellas. They would like to know how the price they charge for an umbrella affects their revenue. They conduct some market research, and discover that if they charge  $p$  dollars per umbrella, they will sell  $150 - 5p$  umbrellas per month.

1. a. Complete the table showing Raingear's revenue at various prices,  $p$ .

Price per umbrella $p$	No. of umbrellas sold $150 - 5p$	Revenue $R$
2		
4		
6		
8		
10		
12		
14		

- b. Use the formula for revenue to write an equation that gives Raingear's monthly revenue from umbrellas,  $R$ , in terms of  $p$ .

$$R =$$

Simplify your equation by applying the distributive law:

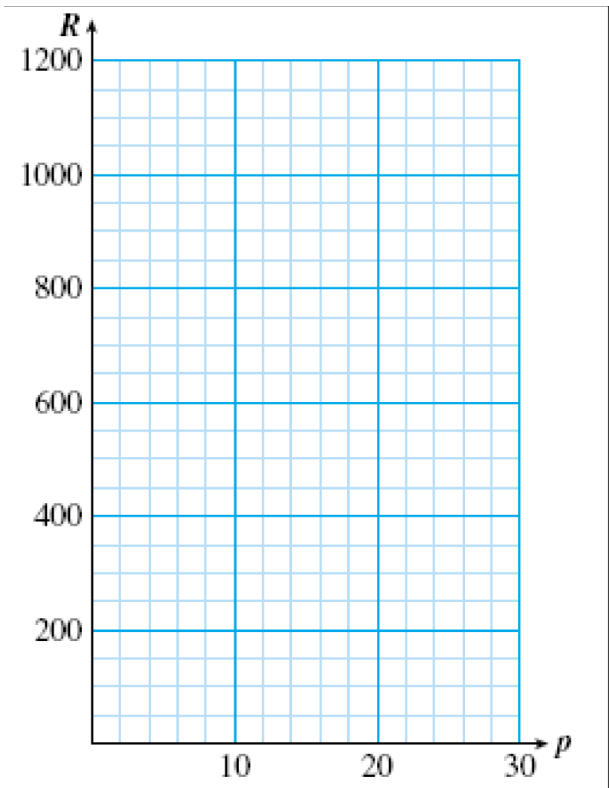
$$R =$$

- c. Write an equation to answer the question: How much should Raingear charge for each umbrella if they would like to make \$1000 monthly revenue?
- d. Can you use extraction of roots to solve the equation in part (c)? Why or why not?
- e. Use the table in part (a) to look for a solution to the equation in part (c).

2. a. Extend your table of values for Raingear's revenue by completing the table below.

Price per umbrella $p$	Number of umbrellas sold $150 - 5p$	Revenue $R$
14		
16		
18		
20		
22		
24		
26		
28		
30		

- b. On the grid at right, plot the values of  $R$  and  $p$  from your tables and draw a smooth graph of the equation for revenue,  $R$ , in terms of  $p$ .
- c. Locate the point on your graph that corresponds to a monthly revenue of \$1000. What are its coordinates?
- d. Find another point on the graph that corresponds to \$1000 monthly revenue. What price per umbrella produces this revenue? How many umbrellas will be sold at this price?
- e. What is the maximum monthly revenue that Raingear can earn from umbrellas?
- f. What price per umbrella should Raingear charge in order to earn the maximum revenue?





## Activity 2 Factoring Out a Common Factor

1. Factor out the largest common factor from each expression.

a.  $2x^2 + 8x = \underline{\hspace{2cm}} (\underline{\hspace{2cm}} + \underline{\hspace{2cm}})$

b.  $-3n^2 + 18n =$

2. Solve each quadratic equation by factoring.

a.  $2x^2 + 8x = 0$

b.  $-3n^2 + 18n = 0$

## Activity 3 Zero-Factor Principle

1. Find the solutions of each quadratic equation.

a.  $(x - 2)(x + 5) = 0$

b.  $y(y - 4) = 0$

2. Explain why each solution is incorrect.

a.  $(n - 3)(n + 5) = 3$

$n - 3 = 3, \quad n + 5 = 3$

$n = 6, \quad n = -2$

b.  $x^2 + 4x = 0$

$x^2 = -4x$

$x = -4$

## Wrap-Up

In this Lesson we practiced the following skills:

- Plotting points to graph a parabola
- Using the formula for revenue
- Factoring out a common factor
- Using the Zero-Factor Principle to solve quadratic equations

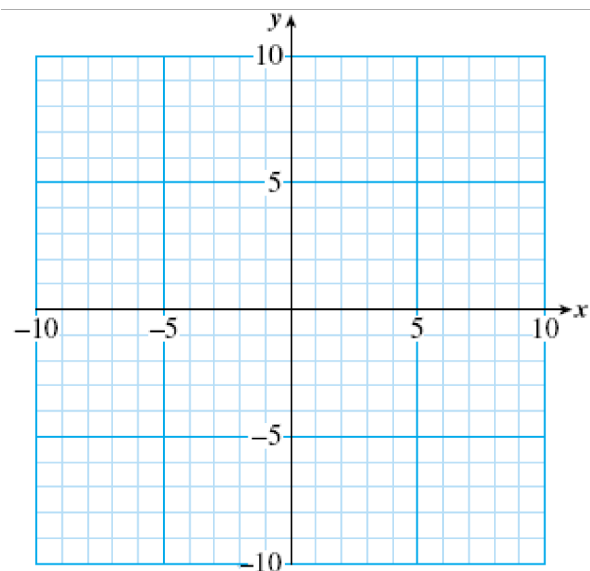
1. Factoring is the opposite of which operation on polynomials?
2. In Activity 2, Problem 2b, did you factor out a positive factor or a negative factor? Does it make a difference to the solutions of the equation?
3. What is the difference between solving a quadratic equation and factoring a quadratic trinomial?

## 6.2 Homework Preview

1. a. Complete the table of values and sketch a graph of

$$y = \frac{1}{2}x^2 - 4x$$

$x$	$y$
-2	
0	
2	
4	
6	
8	
10	



- b. Use the graph to solve the equation

$$\frac{1}{2}x^2 - 4x = -\frac{7}{2}$$

- c. Use the graph to solve the equation

$$\frac{1}{2}x^2 - 4x = \frac{9}{2}$$

2. Find the largest common factor for the monomials.

a.  $6x, 3x^2, 9xy$

b.  $12ab^2, 4ab, 6a^2b$

3. Factor out the largest common factor.

a.  $25t^2 - 15t$

b.  $-12k^2 - 9k$

4. Solve the equation.

a.  $18w^2 + 24w = 0$

b.  $21z^2 = 35z$

## Answers

1b. 1, 7   c. -1, 9   2a.  $3x$    b.  $2ab$    3a.  $5t(5t - 3)$    b.  $-3k(4k + 3)$  or  $3k(-4k - 3)$

4a.  $-\frac{3}{4}, 0$    b.  $0, \frac{5}{7}$

## Lesson 6.3 Solving Quadratic Equations by Factoring

### Activity 1 Factoring a Quadratic Trinomial

#### Case 1 Positive coefficients: $x^2 + bx + c$

If all the coefficients in the quadratic trinomial are positive, then  $p$  and  $q$  are also positive.

**Example 1** Factor  $t^2 + 9t + 18$  as a product of two binomials

$$t^2 + 9t + 18 = (\underline{\quad} + \underline{\quad})(\underline{\quad} + \underline{\quad})$$

**Step 1** The product of the **F**irst terms must be  $t^2$ , so we place  $t$ 's in the First spots.

$$t^2 + 9t + 18 = (t + \underline{\quad})(t + \underline{\quad})$$

**Step 2** The product of the **L**ast terms is 18, so  $pq = 18$ . There are three possibilities for  $p$  and  $q$ :

1 and 18, 2 and 9, or 3 and 6

We use the middle term of the trinomial to decide which possibility is correct.

**Step 3** The sum of **O**utside plus **I**nside is the middle term,  $9t$ . We check each possibility to see which gives the correct middle term.

$$(t + 1)(t + 18) \quad O + I = t + 18t = 19t$$

$$(t + 2)(t + 9) \quad O + I = 2t + 9t = 11t$$

$$(t + 3)(t + 6) \quad O + I = 3t + 6t = 9t$$

The last choice is correct, so the factorization is

$$t^2 + 9t + 18 = (t + 3)(t + 6)$$

1. Factor each trinomial.

a.  $x^2 + 8x + 15$

b.  $y^2 + 14y + 49$

#### Case 2 Negative linear coefficient: $x^2 - bx + c$

If the coefficient of the linear term is negative and the constant term is positive, what are the signs of  $p$  and  $q$ ?

**Example 2** Factor  $x^2 - 12x + 20$  as a product of two binomials.

**Step 1** We look for a product of the form

$$x^2 - 12x + 20 = (x + p)(x + q)$$

The numbers  $p$  and  $q$  must satisfy two conditions:

- (1) Their product,  $pq$ , is the **L**ast term, and so must equal 20.
- (2) Their sum,  $p + q$ , equals  $-12$  (because **O** + **I** =  $px + qx = -12x$ .)

These two conditions tell us that  $p$  and  $q$  must both be negative.

**Step 2** We list all the ways to factor 20 with negative factors :

$$-1 \text{ and } -20, \quad -2 \text{ and } -10, \quad -4 \text{ and } -5$$

**Step 3** We check each possibility to see which one gives the correct middle term:

$$\begin{array}{ll} (x-1)(x-20) & O + I = -20x - x = -21x \\ (x-2)(x-10) & O + I = -2x - 10x = -12x \\ (x-4)(x-5) & O + I = -5x - 4x = -9x \end{array}$$

The second possibility is correct, so the factorization is

$$x^2 - 12x + 20 = (x-2)(x-10)$$

2. Factor each trinomial.

a.  $m^2 - 10m + 24$

b.  $m^2 - 11m + 24$

### Case 3 Negative constant term: $x^2 + bx - c$ or $x^2 - bx - c$

The linear term can be either positive or negative.

**Example 3** Factor  $x^2 + 2x - 15$ .

**Step 1** We look for a factorization of the form

$$x^2 + 2x - 15 = (x+p)(x+q)$$

The product  $pq$  of the two unknown numbers must be negative,  $-15$ . This means that  $p$  and  $q$  must have opposite signs, one positive and one negative. How do we decide which is positive and which is negative? We guess! If we make the wrong choice, we can easily fix it by switching the signs.

**Step 2** There are only two ways to factor 15, either 1 times 15 or 3 times 5. We just guess that the second factor is negative, and check  $O + I$  for each possibility:

$$\begin{array}{ll} (x+1)(x-15) & O + I = -15x + x = -14x \\ (x+3)(x-5) & O + I = -5x + 3x = -2x \end{array}$$

**Step 3** The second possibility gives a middle term of  $-2x$ . This is not quite correct, because the middle term we want is  $2x$ . We fix this by switching the signs on the factors of  $-15$ : instead of using  $+3$  and  $-5$ , we change to  $-3$  and  $+5$ . You can check that the correct factorization is

$$x^2 + 2x - 15 = (x-3)(x+5)$$

3. Factor each trinomial.

a.  $t^2 + 8t - 48$

b.  $t^2 - 8t - 48$

The sign patterns we have discovered for factoring quadratic trinomials are summarized below. **The order of the terms is very important.** For these strategies to work, the trinomial must be written in **descending powers** of the variable; that is, the quadratic term must come first, then the linear term, and finally the constant term, like this:

$$\begin{array}{ccc} x^2 & + bx & + c \\ \text{quadratic term} & \text{linear term} & \text{constant term} \end{array}$$

### Sign Patterns for Factoring Quadratic Trinomials

Assume that  $b$ ,  $c$ ,  $p$ , and  $q$  are positive integers. Then

1.  $x^2 + bx + c = (x + p)(x + q)$

If all the coefficients of the trinomial are positive, then both  $p$  and  $q$  are positive.

2.  $x^2 - bx + c = (x - p)(x - q)$

If the linear term of the trinomial is negative and the other two terms positive, then  $p$  and  $q$  are both negative.

3.  $x^2 \pm bx - c = (x + p)(x - q)$

If the constant term of the trinomial is negative, then  $p$  and  $q$  have opposite signs.

## Activity 2 Solving Quadratic Equations

Solve each quadratic equation.

1.  $a^2 - 13a + 30 = 0$

2.  $u^2 - 6u = 16$

3.  $3x^2 + 3 = 6x$

4.  $9x^2 - 18x = 0$

### Activity 3 Application

The town of Amory lies due north of Chester, and Bristol lies due east of Chester. If you drive from Amory to Bristol by way of Chester, the distance is 17 miles, but if you take the back road directly from Amory to Bristol, you save 4 miles. You would like to know the distance from Bristol to Chester.

- a. What is the unknown quantity? Call it  $x$ .
- b. Make a sketch and label it with distances.
- c. Write an equation about the variable.
- d. Solve your equation.
- e. How far is it from Bristol to Chester?

### Wrap-Up

In this Lesson we practiced the following skills:

- Using the zero-factor principle
- Factoring quadratic trinomials
- Solving a quadratic equation by factoring

1. Which technique would you use to solve each equation below? Explain your choice in each case.

a.  $x^2 - 6 = 0$

b.  $x^2 - 6x = 0$

2. In Activity 1, what can you say about the signs of  $p$  and  $q$  if  $c$  is positive?
3. In Activity 2, how did you know which type of factoring to use for each equation?
4. If  $p$  and  $q$  have opposite signs, how do we know which one is positive?

## 6.3 Homework Preview

■ Factor.

1.  $x^2 + 15x + 56$

2.  $m^2 - 9m - 52$

3.  $t^2 - 24t + 144$

■ Solve.

4.  $3x^2 - 48 = 0$

5.  $3x^2 - 48x = 0$

6.  $x^2 - 19x + 48 = 0$

7.  $x^2 - 8x = 48$

8.  $3x^2 + 18x = 48$

## Answers

1.  $(x + 7)(x + 8)$

2.  $(m - 13)(m + 4)$

3.  $(t - 12)(t - 12)$

4.  $\pm 4$

5.  $0, 16$

6.  $3, 16$

7.  $-8, 2$

8.  $-4, 12$

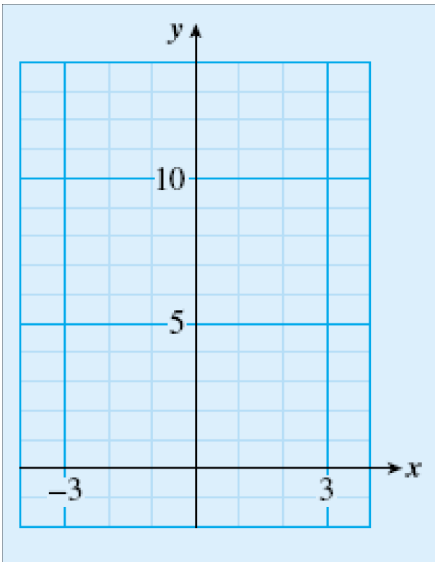
# Lesson 6.4 Graping Quadratic Equations

## Activity 1 Parabolas

- a. Complete the table of values and graph each quadratic equation.
- b. Find the  $x$ -intercepts and the vertex of each graph.

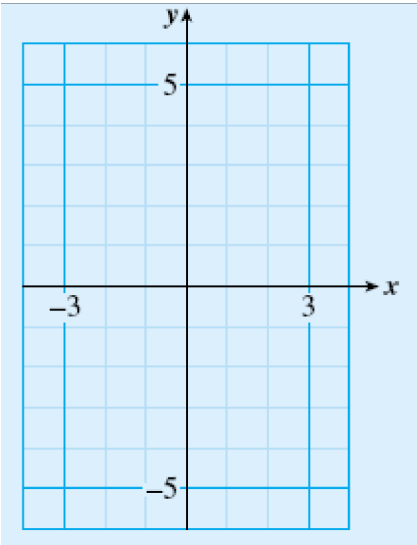
1.  $y = x^2 + 4$

$x$	-3	-2	-1	0	1	2	3
$y$							



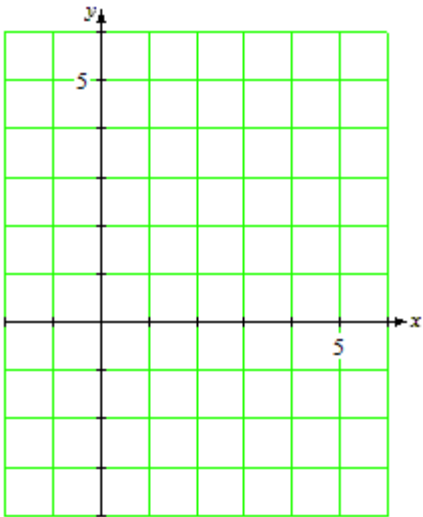
2.  $y = x^2 - 4$

$x$	-3	-2	-1	0	1	2	3
$y$							



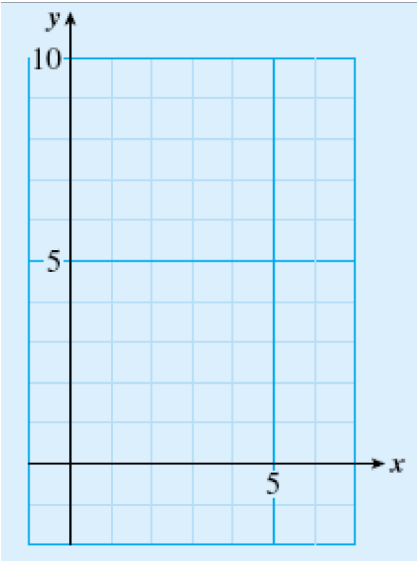
3.  $y = 4x - x^2$

$x$	-1	0	1	2	3	4	5
$y$							



4.  $y = (x - 4)^2$

$x$	1	2	3	4	5	6	7
$y$							





## Activity 2 Intercepts and Vertex

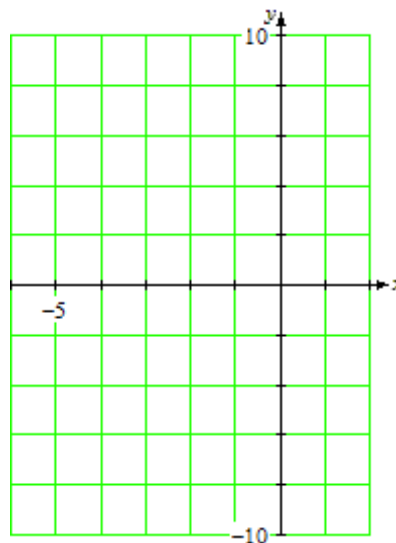
1. a. Find the  $x$ -intercepts of the graph of

$$y = 2x^2 + 8x$$

- b. Find the vertex of the graph.

- c. Find the  $y$ -intercept of the graph.

- d. Sketch the graph.



2. a. Find the  $x$ -intercepts of the graph of

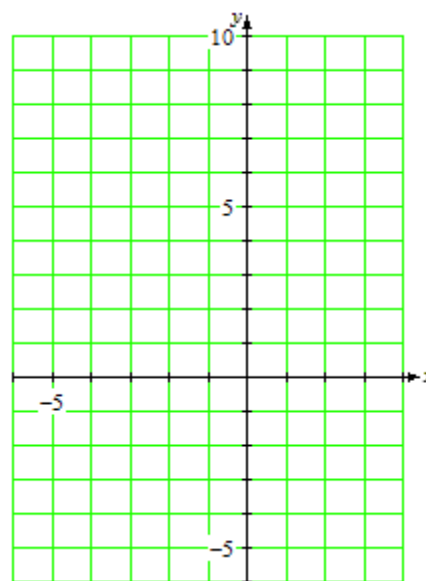
$$y = -x^2 - 2x + 8$$

(Hint: Factor out  $-1$  from the quadratic trinomial first.)

- b. Find the vertex of the graph.

- c. Find the  $y$ -intercept of the graph.

- d. Sketch the graph.




## Wrap-Up

In this Lesson we practiced the following skills:

- Finding the  $x$ -intercepts of a parabola
  - Finding the vertex of a parabola
  - Finding the  $y$ -intercept of a parabola
  - Sketching the graph of a quadratic equation
1. In Activity 1, Problems 1 and 2, how were the graphs different from the basic parabola?
  2. In Activity 2, Problem 2, how does the  $-1$  coefficient of  $x^2$  affect the graph?
  3. In Activity 2, Problem 2, does factoring  $-1$  from the equation affect the  $x$ -intercepts of the graph?

## 6.4 Homework Preview

-  a. Find the  $x$ -intercepts of the graph.  
b. Find the  $y$ -intercept of the graph.  
c. Find the vertex of the graph.

1.  $y = x^2 - 16$

2.  $y = 2x^2 + 16x$

3.  $y = x^2 - 10x + 16$

4.  $y = 16 + 6x - x^2$

## Answers

- |                     |             |              |
|---------------------|-------------|--------------|
| 1a. (4, 0), (-4, 0) | b. (0, -16) | c. (0, -16)  |
| 2a. (-8, 0), (0, 0) | b. (0, 0)   | c. (-4, -32) |
| 3a. (2, 0), (8, 0)  | b. (0, 16)  | c. (5, -9)   |
| 4a. (-2, 0), (8, 0) | b. (0, 16)  | c. (3, 25)   |

## Lesson 6.5 The Quadratic Formula

### Activity 1 Using the Quadratic Formula

1. a. Use the quadratic formula to solve  $2x^2 = 7 - 4x$ .

Write the equation in standard form.

Identify the coefficients.  $a = \underline{\hspace{2cm}}$ ,  $b = \underline{\hspace{2cm}}$ ,  $c = \underline{\hspace{2cm}}$ .

Substitute the values of  $a$ ,  $b$ , and  $c$  into the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} =$$

Simplify the expression.

- b. Find decimal approximations to two decimal places for the solutions.

Use a calculator to approximate each solution.

2. a. Solve the equation  $x^2 - \frac{x}{3} = \frac{4}{3}$  by factoring.  
(Hint: Clear the fractions first!)

- b. Solve the same equation using the quadratic formula.

## Activity 2 Application to Graphing

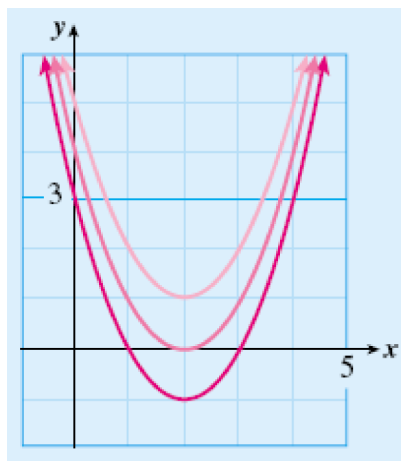
1. a. Here are equations for three parabolas. Factor each formula if possible.

**I**  $y = x^2 - 4x + 3$

**II**  $y = x^2 - 4x + 4$

**III**  $y = x^2 - 4x + 5$

- b. Match each equation from part (a) with one of the graphs shown. Explain your reasoning.



2. Find the  $x$ -intercepts of each parabola.

a.  $y = -x^2 + 2x - 1$

b.  $y = 2x^2 + 4x + 3$

### Wrap-Up

In this Lesson we practiced the following skills:

- Using the quadratic formula to solve equations
- Solving applied problems using the quadratic formula
- Using the quadratic formula to find the  $x$ -intercepts of a parabola
- Determining how many  $x$ -intercepts the graph has

1. In Activity 1, can we simplify  $\sqrt{b^2 - 4ac}$  to  $\sqrt{b^2} - \sqrt{4ac}$ ?
2. In Activity 1, Problem 2, why is it helpful to clear fractions from the equation before applying the quadratic formula?
3. In Activity 2, Problem 2, how can we find the  $x$ -intercepts if the equation does not factor?

## 6.5 Homework Preview

■ Solve. Give your answers as:

- a. exact values.      b. approximations to three decimal places.

1.  $x^2 + 2x - 2 = 0$

2.  $2x^2 - 3x = 1$

3.  $\frac{x^2}{4} - \frac{2x}{3} + \frac{1}{3} = 0$

4.  $3x^2 + 8x + 6 = 0$

### Answers

1a.  $\frac{-2 \pm \sqrt{12}}{2}$       b. -2.732, 0.732

3a.  $2, \frac{2}{3}$       b.  $2, \frac{2}{3}$

2a.  $\frac{3 \pm \sqrt{17}}{4}$       b. -0.281, 1.781

4. No real solutions

## Lesson 7.1 Polynomials

### Activity 1 Polynomials

1. Which of the following expressions are polynomials?

a.  $t^2 - 7t + 3$

b.  $2 + \frac{6}{x}$

c.  $-a^3b^2c^4$

d.  $3w^2 - w + 5\sqrt{w}$

2. Give the degree of each polynomial.

a.  $5x^9 - 1$

b.  $3t^2 - t - 6t^4 + 8$

3. Write the polynomial  $3x - 8 - 6x^3 - x^6$  in descending powers of  $x$ .

4. Evaluate  $2x^3y^2 - 3xy^3 - 3$  for  $x = -2$ ,  $y = -3$ .

### Activity 2 Adding and Subtracting Polynomials

1. Simplify by combining any like terms.

$$2a^2b - 5ba + 4ba^2 - (-3ab)$$

2. Add the polynomials:  $(2m^4 - 4m^3 + 3m - 1) + (m^3 + 4m^2 - 6m + 2)$

3. Use a vertical format to add  $5y^2 - 3y + 1$  to  $-3y^2 + 6y - 7$

4. Subtract the polynomials:  $(2m^4 - 4m^3 + 3m - 1) - (m^3 + 4m^2 - 6m + 2)$

5. Use a vertical format to subtract  $2x^2 + 5 - 2x$  from  $7 - 3x - 4x^2$

6. Simplify  $2a^2(3 - a + 4a^2) - 3a(5a - a^2)$

### Activity 3 Applications

1. The length of a box is twice its width,  $w$ , and its height is 3 inches less than its width. Write a polynomial for the volume of the box.
  - a. Sketch a picture of the box, and label its sides.
  - b. Write a polynomial for the volume of the box.
  - c. Write a polynomial for the surface area of the box.
  
2. Milly's Muffins can sell  $x$  muffins at the farmers' market if she charges  $6 - 0.02x$  dollars for a muffin. It costs Milly  $50 + 1.5x$  dollars to make  $x$  muffins.
  - a. Write a polynomial for Milly's revenue from selling muffins.  
**Hint:** Revenue = (number of items sold)  $\times$  (price per item)
  - b. Write a polynomial for Milly's profit.
  - c. Find Milly's profit if she sells 50 muffins, 100 muffins, and 150 muffins.

### Wrap-Up

In this Lesson we practiced the following skills:

- Identifying a polynomial
- Identifying degree and number of terms
- Writing a polynomial in descending powers
- Evaluating a polynomial
- Adding and subtracting polynomials
- Writing simple polynomial expressions

1. Name two places that a variable cannot appear in a polynomial.
2. What do you suppose "ascending powers" means?
3. What is wrong with this statement:

$$5x - (2x + 3x^2) = 5x - 2x + 3x^2$$

## 7.1 Homework Preview

1. Explain why each expression is not a polynomial.

a.  $4x^3 - x^2 + \frac{3}{x}$

b.  $2x^2 - 3x\sqrt{x} + 5x$

2. Evaluate  $2x^3 - 4x^2 + 3x - 6$  for  $x = -3$ .

3. Evaluate  $\frac{5}{4}x^2 + \frac{1}{2}xy - 2y^2$  for  $x = -4$ ,  $y = 2$ .

4. Add  $(5x^3 - 3x + 4x^2 - 1) + (3 + 2x - x^3 - 6x^2)$

5. Subtract  $(t^3 - 2t^2 - 3t + 5) - (2t^3 + 4t - 3 - t^2)$

6. Simplify  $ab(3ab - b) - b^2(2a^2 - ab)$

### Answers

1a. It has a variable in the denominator. b. It has a variable under a radical.

2.  $-105$       3.  $8$       4.  $4x^3 - 2x^2 - x + 2$

5.  $-t^3 - t^2 - 7t + 8$       6.  $a^2b^2 - ab^2 + ab^3$



## Lesson 7.2 Products of Polynomials

### Activity 1 Products of Monomials

1. Use the first law of exponents to find each product.

a.  $k^2 \cdot k^8$

b.  $y^3(y^3)$

c.  $a^4 \cdot a^9$

2. Multiply

a.  $2x^2y(-3x^2y^2)$

b.  $-3a^4b(-4a^3b)$

### Activity 2 Products of Polynomials

1. Multiply  $(3x + 2)(3x^2 + 4x - 2)$

2. Multiply  $s^2t^2(2s + 1)(3s - 1)$

3. Multiply  $(x + 2)(3x - 2)(2x - 1)$

## Wrap-Up

In this Lesson we practiced the following skills:

- Applying the first law of exponents
  - Multiplying a polynomial by a monomial
  - Computing the product of two or more polynomials
1. What is wrong with the statement  $5^7 \cdot 5^8 = 25^{15}$ ?
  2. What is the difference between the expressions  $6x^4(-2x^4)$  and  $6x^4 - 2x^4$ ?
  3. In Activity 2 Problem 2, which of the three factors did you multiply together first?
  4. In Activity 2 Problem 3, which of the three factors did you multiply together first?

## 7.2 Homework Preview

1. Simplify.

a.  $2b^2(-b^2) + b(3b - b^2)$

b.  $2b^2 - b^2(b + 3b) - b^2$

c.  $(2b - b^2)(b + 3b)(-b^2)$

■ Multiply.

2.  $-6x^2y(3y^2 + 2xy - 5y)$

3.  $(3z - 5t)(2z + 4t)(z - 2t)$

4.  $-2x^3(3x - 1)(2x - 7)$

## Answers

- 1a.  $3b^2 - 3b^3$     b.  $b^2 - 3b^3$     c.  $4b^5 - 8b^4$     2.  $-18x^2y^3 - 12x^3y^2 + 30x^2y^2$   
 3.  $6z^3 - 10z^2t - 24zt^2 + 40t^3$     4.  $-12x^5 + 46x^4 - 14x^3$

## Lesson 7.3 More About Factoring

### Activity 1 Quotients and the GCF

1. Use the second law of exponents to find each quotient.

a.  $\frac{x^6}{x^2}$

b.  $\frac{b^2}{b^3}$

2. Divide  $\frac{8x^2y}{12x^5y^3}$

3. Find the greatest common factor for  $15x^2y^2 - 12xy + 6xy^3$

4. Factor  $15x^2y^2 - 12xy + 6xy^3$

5. Factor  $9(x^2 + 5) - x(x^2 + 5)$

### Activity 2 Quadratic Trinomials

1. Factor  $3x^2 + 2x - 5$  by the guess-and-check method.

2. a. Compute the product  $(2x - 5)(3x - 4)$  using the area of a rectangle.

b. Verify that the products of the diagonal entries are equal.

3. Solve  $2x^2 = 7x + 15$

Write the equation in standard form.

Factor the left side.

Set each factor equal to zero.

Solve each equation.

### Activity 3 Factoring Completely

1. Factor completely  $4a^6 - 10a^5 + 6a^4$

3. Factor completely.

a.  $2a^3b - 24a^2b^2 - 90ab^3$

b.  $3x^2 - 8xy + 4y^2$

### Activity 4 The Box Method

Follow the steps to factor  $4x^2 + 4x - 3$ .

**Step 1** Enter the correct terms on the diagonal.

**Step 2** Compute the diagonal product:

$$D =$$



**Step 3** List all possible factors of  $D$ , and compute the sum of each pair of factors. The factors must have opposite signs. (Complete each pair of factors below.)

Factors of $D =$	Sum of Factors
$-x$ _____	
$-2x$ _____	
$-3x$ _____	

The correct factors are:

**Step 4** Enter the correct factors into the rectangle.

**Step 5** Factor the top row of the rectangle, and write the result at the top. Finally, factor the bottom row, and write the result on the left. The correct factorization is


$$4x^2 + 4x - 3 =$$

## Wrap-Up

In this Lesson we practiced the following skills:

- Factoring a quadratic trinomial  $ax^2 + bx + c$  by guess-and-check
- Factoring a quadratic trinomial  $ax^2 + bx + c$  by the box method
- Factoring a quadratic trinomial in two variables
- Factoring out a binomial common factor
- Combining techniques to factor completely

1. When is the guess-and-check method easier to use than the box method?
2. When we use the box method, where do the factors of the trinomial appear?
3. In Activity 3, Problem 1, what is the common factor?
4. In Activity 3, Problem 3a, what is the common factor?

## 7.3 Homework Preview

1. Factor  $3z^2 - 2z - 5$  by the guess-and-check method..

2. Factor completely:

a.  $2a^3 + 10a^2 - 72a$

b.  $6x^4 + 27x^3y - 54x^2y^2$

■ Factor using the box method.

3.  $2x^2 - x - 36$

4.  $4t^2 - 21t - 18$

### Answers

1.  $(3z - 5)(z + 1)$

3.  $(2x - 9)(x + 4)$

2a.  $2a(a + 9)(a - 4)$

4.  $(4t + 3)(t - 6)$

b.  $3x^2(2x - 3y)(x + 6y)$

## Lesson 7.4 Special Products and Factors

### Activity 1 Special Products

1. Simplify each square.

a.  $(-6h^6)^2$

b.  $(12st^8)^2$

2. Find a monomial whose square is given

a.  $64b^6$

b.  $169a^4b^{16}$

3. Use a formula to expand each product.

a.  $(4 - 3t)^2$

b.  $(6s + 2t)^2$

c.  $(5x^4 + 4)(5x^4 - 4)$

### Activity 2 Special Factorizations

1. Use one of the three formulas to factor each polynomial.

a.  $25y^2 - w^2$

b.  $m^6 - 18m^3 + 81$

c.  $4h^2 + 36hk + 81k^2$

2. Factor completely.

a.  $x^6 - 16x^2$

b.  $3b^9 + 18b^6 + 27b^3$

c.  $2x^5y - 20x^3y^3 + 50xy^5$

## Wrap-Up

In this Lesson we practiced the following skills:

- Identifying squares of monomials
  - Using the formulas for squares of binomials and difference of squares
  - Factoring squares of binomials and difference of squares
  - Knowing that the sum of squares cannot be factored
1. Explain the difference between squaring  $3b^3$  and squaring  $3 + b^3$ .
  2. How can you check whether a trinomial might be the square of a binomial?
  3. Explain why we can factor the difference of two squares, but we cannot factor the sum of two squares.

## 7.4 Homework Preview

■ Expand each product.

1.  $(4a - 5b)^2$

2.  $(4a + 5b)^2$

3.  $(4a + 5b)(4a - 5b)$

■ Factor.

4.  $36x^2 + 96x + 64$

5.  $36x^2 - 96x + 64$

6.  $36x^2 - 64$

## Answers

1.  $16a^2 - 40ab + 25b^2$

4.  $4(3x + 4)^2$

2.  $16a^2 + 40ab + 25b^2$

5.  $(6x - 8)^2$

3.  $16a^2 - 25b^2$

6.  $(6x + 8)(6x - 8)$



## Lesson 8.1 Algebraic Fractions

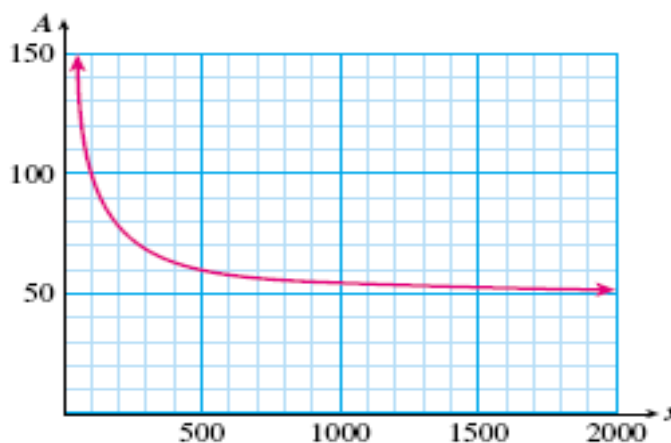
### Activity 1 Algebraic Fractions

1. a. Evaluate  $\frac{z-1}{2z+3}$  for  $z = -3$ .

b. For what value of  $z$  is the fraction undefined?

2. Use the table and graph of average cost from Example 2 to help you answer the questions.

$x$	$A$
50	150
100	100
200	75
400	62.50
500	60
1000	55
1250	54
2000	52.50



- Approximately how many filters should Envirogreen produce for the average cost to be \$75?
- What happens to the average cost per filter as  $x$  increases?
- As  $x$  increases, the average cost appears to be approaching a limiting value. What is that value?
- A market analysis concludes that Envirogreen can sell 1250 filters this year. How much should they charge for one filter if they would like to make a total profit of \$100,000?  
(Hint: Use the formula **Profit = Revenue - Cost**)

## Activity 2 Reducing Fractions

1. Explain why each calculation is incorrect.

a.  $\frac{x-6}{x-9} \rightarrow \frac{x-2}{x-3}$

b.  $\frac{2x+5}{2} \rightarrow x+5$

2. a. Reduce  $\frac{12x-18}{16x-24}$

b. If you evaluate  $\frac{12x-18}{16x-24}$  for  $x = -17$ , what do you expect to get?

3. Reduce  $\frac{x^2-x-6}{x^2-9}$

4. Find the opposite of each binomial.

a.  $2a + 3b$

b.  $-x + 1$

5. Reduce if possible.

a.  $\frac{-x+1}{1-x}$

b.  $\frac{1+x}{1-x}$

c.  $\frac{2a-3b}{3b-2a}$

d.  $\frac{2a-3b}{2b-3a}$

### Wrap-Up

In this Lesson we practiced the following skills:

- Evaluating algebraic fractions
- Reducing algebraic fractions

1. In Activity 1, Problem 2d, what was the average cost per filter?
2. In Activity 2, Problem 2, can we cancel the  $x^2$ 's? Why or why not?
3. In Activity 2, Problem 4, which numerator is the opposite of its denominator?

## 8.1 Homework Preview

1. The chess club will have 4 men and an unknown number  $w$  of women. What fraction of the club will be men?

2. a. Evaluate  $\frac{z^2 - 4}{z - 4}$  for  $z = -2$ .

b. For what values of  $z$  is  $\frac{z^2 - 4}{z - 4}$  undefined?

3. Reduce  $\frac{20s^3t^2}{12st^3}$

4. Reduce  $\frac{x^2 - 4x}{2x}$

5. Reduce  $\frac{a^2 - 9}{3a - 9}$

6. Reduce  $\frac{b - 2}{2b^2 - 3b - 2}$

### Answers

1.  $\frac{4}{4 + w}$     2a. 0    b. 4    3.  $\frac{5s^2}{3t}$     4.  $\frac{x - 4}{2}$     5.  $\frac{a + 3}{3}$     6.  $\frac{1}{2b + 1}$

## Lesson 8.2 Operations on Fractions

### Activity 1 Multiplying and Dividing Fractions

■ For Problems 1-5, review the Example, then complete the Exercise.

<b>1.</b> $\frac{5}{8} \cdot \frac{3}{4} = \frac{5 \cdot 3}{8 \cdot 4} = \frac{15}{32}$	Multiply the numerators together; multiply the denominators together.
---	--

**Exercise 1a.**  $\frac{2}{5} \cdot \frac{2}{3} =$

**b.**  $\frac{2}{w} \cdot \frac{z}{3} =$

<b>2.</b> $\frac{3}{4} \cdot \frac{5}{6} = \frac{\cancel{3}}{4} \cdot \frac{5}{2 \cdot \cancel{3}} = \frac{5}{8}$	Divide out common factors first, then multiply.
---	--

**Exercise 2a.**  $\frac{5}{4} \cdot \frac{8}{9} =$

**b.**  $\frac{5}{2a} \cdot \frac{4a}{9} =$

<b>3.</b> $\frac{3}{4} \cdot 6 = \frac{3}{4} \cdot \frac{6}{1} = \frac{3}{2 \cdot \cancel{2}} \cdot \frac{\cancel{2} \cdot 3}{1} = \frac{9}{2}$	Write 6 as $\frac{6}{1}$ .
---	----------------------------

**Exercise 3a.**  $\frac{7}{3} \cdot 12 =$

**b.**  $\frac{7}{x} \cdot 4x =$

<b>4.</b> $\frac{12}{5} \div \frac{8}{5} = \frac{12}{5} \cdot \frac{5}{8}$ $= \frac{\cancel{4} \cdot 3}{\cancel{5}} \cdot \frac{\cancel{5}}{\cancel{4} \cdot 2} = \frac{3}{2}$	Take the reciprocal of the second fraction; then change to multiplication.
---	---

**Exercise 4a.**  $\frac{8}{3} \div \frac{2}{9} =$

**b.**  $\frac{4a}{3b} \div \frac{2a}{3} =$

<b>5.</b> $\frac{3}{5} \div 6 = \frac{3}{5} \cdot \frac{1}{6} = \frac{\cancel{3}}{5} \cdot \frac{1}{\cancel{3} \cdot 2} = \frac{1}{15}$	
---	--

**Exercise 5a.**  $\frac{2}{3} \div 4 =$

**b.**  $\frac{2}{3y} \div 4y =$

6. Multiply:

a.  $\frac{-5}{b}a = \frac{-5}{b} \cdot \frac{a}{1} =$

b.  $6x\left(\frac{2}{x^2 - x}\right) = \frac{6x}{1} \cdot \frac{2}{x^2 - x} =$

7. Divide:  $\frac{6ab^2}{2a + 3b} \div (4a^2b)$

## Activity 2 Like Fractions

1a.  $\frac{13}{6} - \frac{5}{6} =$

b.  $\frac{13k}{n} - \frac{5k}{n} =$

2. Add:  $\frac{2n}{n - 3} + \frac{n + 2}{n - 3}$

3. Subtract:  $\frac{3}{x^2 + 2x + 1} - \frac{2 - x}{x^2 + 2x + 1}$

### Wrap-Up

In this Lesson we practiced the following skills:

- Multiplying algebraic fractions
- Dividing algebraic fractions
- Adding and subtracting like fractions

1. When we multiply a fraction by 2, do we multiply the top by 2, the bottom by 2, or both?
2. When we add like fractions, do we add the numerators together, the denominators together, or both?
3. Explain how a fraction bar acts like parentheses when we subtract fractions.

## 8.2 Homework Preview

1. Multiply:  $\frac{x^2 - 1}{2x + 2} \cdot \frac{6x + 12}{3x - 3}$

2. Divide:  $\frac{b^2 - b - 2}{2b} \div (4b - 8)$

3. Add:  $\frac{a - 2}{a^2 - 2} + \frac{a^2}{a^2 - 2a}$

4. Subtract:  $\frac{2n + 3}{n - 1} - \frac{n - 4}{n - 1}$

5. Divide:  $\frac{6t^6 - 4t^3 + t}{2t^2}$

### Answers

1.  $x + 2$     2.  $\frac{b + 1}{8b}$     3.  $\frac{a^2 + a - 2}{a^2 - 2}$     4.  $\frac{n + 7}{n - 1}$     5.  $3t^4 - 2t + \frac{1}{2t}$

## Lesson 8.3 Lowest Common Denominator

### Activity 1 Unlike Fractions

1. Add:  $\frac{5x}{6} + \frac{3-2x}{4}$

Step 1 Find the LCD

Step 2 Build each fraction.

Step 3 Subtract like fractions.

Step 4 Reduce if possible.

2. Follow the steps below to subtract  $\frac{2}{x+2} - \frac{3}{x-2}$

Step 1 Find the LCD

Step 2 Build each fraction.

Step 3 Subtract like fractions.

Step 4 Reduce if possible.

### Activity 2 Finding the LCD

1. Find the LCD for  $\frac{3}{4x^2} + \frac{5}{6xy}$

Factor each denominator.

Choose the correct factors.

LCD =

2. Subtract:  $\frac{b}{3b+9} - \frac{b-1}{2b^2+6b}$

Step 1 Find the LCD

Factor each denominator.

Choose the correct factors.

LCD =

Step 2 Build each fraction.

Step 3 Subtract like fractions.

Step 4 Reduce if possible.

## Wrap-Up

In this Lesson we practiced the following skills:

- Finding an LCD
- Adding and subtracting unlike fractions

1. In Activity 1, Problem 2, what was the LCD?
2. In Activity 1, Problem 2, what was the building factor for the first fraction?
3. In Activity 2, Problem 2, explain how to find the building factor for each fraction.



## 8.3 Homework Preview

1. Add:  $\frac{x+1}{x} + \frac{2x+1}{x+2}$

2. Find the LCD for  $\frac{1}{3a^2-6a}$  and  $\frac{1}{3a^2+3a-6}$

3. Subtract:  $\frac{x+1}{2x-4} - \frac{2x}{x^2-4}$

### Answers

1.  $\frac{3x^2+4x+2}{x(x+2)}$       2.  $3a(a-1)(a+2)(a-2)$       3.  $\frac{x^2-x+2}{2(x^2-4)}$

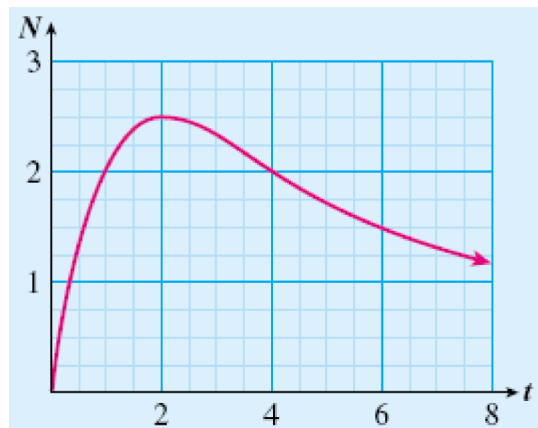
## Lesson 8.4 Equations with Fractions

### Activity 1 Using a Graph

A new health club opened up, and the manager kept track of the number of active members over its first few months of operation. The equation below gives the number,  $N$ , of active members, in hundreds,  $t$  months after the club opened.

$$N = \frac{10t}{4 + t^2}$$

The graph of this equation is shown at right.



- a. Use the equation to find out in which months the club had 200 active members.

- b. Verify your answers on the graph.

### Activity 2 Using Algebra

1. Solve  $\frac{x^2}{2} + \frac{5x}{4} = 3$  by first clearing the fractions.

2. Solve  $\frac{1}{x-2} + \frac{2}{x} = 1$

Step 1 Find the LCD for all the fractions in the equation

Step 2 Multiply each term of the equation by the LCD, and simplify.

Step 3 You now have an equation without fractions. Solve as usual.

3. Follow the steps below to solve  $\frac{15}{x^2 - 3x} + \frac{4}{x} = \frac{5}{x - 3}$

Step 1 Factor each denominator and find the LCD for the fractions in the equation.

Step 2 Multiply each term of the equation by the LCD, and simplify.

Step 3 You should now have an equation with no fractions. Solve as usual.

Step 4 Check for extraneous solutions.

4. Solve the formula  $\frac{w}{q} = 1 - \frac{T}{H}$  for  $q$ .

Activity 3 Work Problem

In this application, the work being done is the water flowing into (or out of) the reservoir, so the work rate is the flow rate of water through the pipes.

The city reservoir was completely emptied for repairs, and is now being refilled. Water flows in through the intake pipe at a steady rate that can fill the reservoir in 120 days. However, water is also being drained from the reservoir as it is used by the city, so that it actually takes 150 days to fill. After the reservoir is filled, the water supply is turned off, but the city continues to use water at the same rate. How long will it take to drain the reservoir dry?

- a. Let  $d$  stand for the number of days for the outflow pipe to drain the full reservoir.

Fraction of the reservoir that is drained in one day: \_\_\_\_\_  
This is the rate at which water leaves the reservoir.

- b. Now imagine that the intake pipe is open, but the outflow pipe is closed.

Fraction of the reservoir that is filled in one day: \_\_\_\_\_  
This is the rate at which water enters the reservoir.

- c. Consider the time period described in the problem when both pipes are open. Fill in the table.

	Flow rate	Time	Fraction of reservoir
Water entering			
Water leaving			

- d. Write an equation:

$$\left( \begin{array}{c} \text{Fraction of reservoir} \\ \text{filled} \end{array} \right) - \left( \begin{array}{c} \text{Fraction of reservoir} \\ \text{drained} \end{array} \right) = \text{One whole reservoir}$$

- e. Solve your equation.

## Wrap-Up

In this Lesson we practiced the following skills:

- Solving equations with fractions
- Identifying extraneous solutions
- Solving motion problems
- Solving work problems

1. In Activity 1, part (b), explain how you used the graph.
2. In Activity 2, Problem 2, what was the LCD?
3. In Activity 2, Problem 4, what was the LCD?

## 8.4 Homework Preview

1. Solve  $\frac{3x}{4} - 1 = 2x + \frac{9}{2}$

2. Solve  $\frac{3}{x+4} - 7 = \frac{-4}{x+4}$

3. Solve  $\frac{1}{a^2+a} = 2 + \frac{1}{a}$

4. Solve  $R = \frac{cd}{c+d}$  for  $c$

## Answers

1.  $x = \frac{-22}{5}$     2.  $x = -3$     3.  $a = \frac{-3}{2}$  (0 is extraneous)    4.  $c = \frac{dR}{d-R}$

## Lesson 9.1 Laws of Exponents

### Activity 1 Laws of Exponents

1. Use the laws of exponents to simplify.

a.  $(4x^4)(5x^5)$

b.  $\frac{8x^8}{4x^4}$

c.  $\frac{6a^3(a^4)}{9a^6}$

2. Use the laws of exponents to simplify.

a.  $(y^2)^5$

b.  $(5^3)^6$

c.  $(n^4)^2$

3. Simplify using the laws of exponents. State the law you used in each case.

a.  $(5^4)^2$

b.  $(5^4)(5^2)$

4. Use the laws of exponents to simplify.

a.  $(6a^3q^6)^3$

b.  $\left(\frac{n^3}{k^4}\right)^8$

c.  $\frac{3^2a^3(3a^2)}{2^3a(2^3a^4)}$

### Activity 2 Simplifying Expressions

1. Recall that  $-x$  means  $-1 \cdot x$ . Use this idea to simplify each expression if possible.

a.  $-x^4$

b.  $(-x)^4$

c.  $(-2)^4$

d.  $-x^5$

e.  $(-x)^5$

f.  $(-2)^5$

g. In general, an odd power of a negative number is \_\_\_\_\_, and an even power of a negative number is \_\_\_\_\_.

2. Follow the steps to simplify  $\left(\frac{-2ab^4}{3c^5}\right)^3$

**Step 1** Apply the fifth law: raise numerator and denominator to the third power.

**Step 2** Apply the fourth law: raise each factor to the third power.

**Step 3** Apply the third law: simplify each power of a power.

3. Follow the steps to simplify  $3x(xy^3)^2 - xy^3 + 4x^3(y^2)^3$

**Step 1** Identify the three terms of the expression by underlining each separately.

**Step 2** Simplify the first term: apply the fourth law, then the third law.

**Step 3** Simplify the last term: apply the third law.

**Step 4** Add or subtract any like terms.

## Wrap-Up

In this Lesson we practiced the following skills:

- Applying the laws of exponents
- Simplifying expressions involving powers

1. In Activity 1, explain the difference between  $\frac{8}{4}$  and  $\frac{x^8}{x^4}$ .
2. Explain the difference between  $4^2$  and  $(x^4)^2$ .
3. Explain why  $(5^4)(5^2)$  is not equal to  $25^6$ .

## 9.1 Homework Preview

1. Simplify.

a.  $3^6 \cdot 3^9$

b.  $\frac{3^6}{3^9}$

c.  $(3^6)^9$

2. Simplify.

a.  $2ab^2(2ab)^2$

b.  $\frac{4ab^2}{(4ab)^2}$

c.  $\frac{(3a^3b^2)^3}{3ab^3(-ab^3)}$

3. Simplify.

a.  $6 - 2np^3 + np(p^2 - n^2)$

b.  $4np(-2np) - 2np(-2np)^2$

### Answers

1a.  $3^{15}$    b.  $\frac{1}{3^3}$    c.  $3^{54}$    2a.  $8a^3b^4$    b.  $\frac{1}{4a^2}$    c.  $-9a^7$

3a.  $6 - np^3 - n^3p$    b.  $-8n^2p^2 - 8n^3p^3$



## Lesson 9.2 Negative Exponents and Scientific Notation

### Activity 1 Negative Exponents

Consider the two lists below, and fill in the unknown values by following the pattern. Notice that as we move down the list, we can find each new entry by dividing the previous entry by the base.

$$2^4 = 16$$

Divide by 2

$$2^3 = 8$$

Divide by 2

$$2^2 = 4$$

$$2^1 =$$

$$2^0 =$$

$$2^{-1} =$$

$$2^{-2} =$$

$$2^{-3} =$$

$$5^4 = 625$$

Divide by 5

$$5^3 = 125$$

Divide by 5

$$5^2 = 25$$

$$5^1 =$$

$$5^0 =$$

$$5^{-1} =$$

$$5^{-2} =$$

$$5^{-3} =$$

Answer the following questions about your lists:

1.
  - a. What did you find for the values of  $2^0$  and  $5^0$ ?
  - b. If you make a list with another base (say, 3, for example), what will you find for the value of  $3^0$ ?
  - c. Explain why this is true.
2.
  - a. Compare the values of  $2^3$  and  $2^{-3}$ . Do you see a relationship between them?
  - b. What about the values of  $5^2$  and  $5^{-2}$ ? Try to state a general rule about powers with negative exponents.
  - c. Use your rule to guess the value of  $3^{-4}$ .

3. Write each expression without exponents.

a.  $-6^2$

b.  $6^{-2}$

c.  $(-6)^{-2}$

4. Write each expression without negative exponents.

a.  $4t^{-2}$

b.  $(4t)^{-2}$

c.  $\left(\frac{x}{3}\right)^{-4}$

## Activity 2 Laws of Exponents

1. Simplify by using the first or second law of exponents.

a.  $3^{-3} \cdot 3^{-6}$

b.  $\frac{b^{-7}}{b^{-3}}$

2. Write without negative exponents and simplify.

a.  $\frac{1}{15^{-2}}$

b.  $\frac{3k^2}{m^{-4}}$

3. Simplify by using the third or fourth law of exponents.

a.  $(3y)^{-2}$

b.  $(a^{-3})^{-2}$

4. Explain the simplification of  $\frac{(3z^{-4})^{-2}}{2z^{-3}}$  shown below. State the law of exponents or other property used in each step.

Explanation:

$$\frac{(3z^{-4})^{-2}}{2z^{-3}} = \frac{3^{-2}(z^{-4})^{-2}}{2z^{-3}} \quad (1)$$

$$= \frac{3^{-2}z^8}{2z^{-3}} \quad (2)$$

$$= \frac{3^{-2}z^{8-(-3)}}{2} \quad (3)$$

$$= \frac{z^{11}}{3^2 \cdot 2} = \frac{z^{11}}{18} \quad (4)$$

### Activity 3 Scientific Notation

1. Compute each product.

a.  $1.47 \times 10^5$

b.  $5.2 \times 10^{-2}$

2. Fill in the correct power of 10 for each factored form.

a.  $0.00427 = 4.27 \times \underline{\hspace{2cm}}$

b.  $4800 = 4.8 \times \underline{\hspace{2cm}}$

3. Write each number in scientific notation.

a. The largest living animal is the blue whale, with an average weight of 120,000,000 grams.

b. The smallest animal is the fairy fly beetle, which weighs about 0.000 005 gram.

4. Perform the following calculations on your calculator. Write the results in scientific notation.

a.  $6,565,656 \times 34,567$

b.  $0.000123 \div 98,765$

5. An adult human brain weighs about 1350 grams. If one neuron weighs  $1.35 \times 10^{-8}$  gram on average, how many neurons are there in a human brain?

6. Use scientific notation to find the quotient

$$0.000\,000\,84 \div 0.000\,4$$

Write each number in scientific notation.

Combine the decimal numbers and the powers of 10 separately.

## Wrap-Up

In this Lesson we practiced the following skills:

- Writing expressions using negative exponents
- Simplifying expressions using the laws of exponents
- Converting between standard and scientific notation
- Performing computations using scientific notation

1. Explain why  $3^0 = 1$ .
2. Does  $a^6 a^{-2} = \frac{a^6}{a^2}$ ? Explain why or why not.
3. Your calculator gives a result of  $4 \text{ E } 12$ . What does this mean?

## 9.2 Homework Preview

1. Write each expression without using negative exponents.

a.  $3x^{-4}$

b.  $(3x)^{-4}$

c.  $\frac{1}{3x^{-4}}$

d.  $\left(\frac{3}{x}\right)^{-4}$

2. Simplify. Write your answers without using negative exponents.

a.  $n^{-5}n^3$

b.  $\frac{n^3}{n^{-5}}$

c.  $(n^3)^{-5}$

d.  $(5n)^{-3}$

3. Write in scientific notation.

a. 48,700,000,000

b. 0.000 000 000 52

4. Use a calculator to compute

$$\frac{0.000\,000\,006}{(428,000,000,000)(0.000\,000\,000\,003\,2)}$$

## Answers

1a.  $\frac{3}{x^4}$

b.  $\frac{1}{81x^4}$

c.  $\frac{x^4}{3}$

d.  $\frac{x^4}{81}$

3a.  $4.87 \times 10^{10}$

b.  $5.2 \times 10^{-10}$

2a.  $\frac{1}{n^2}$

b.  $n^8$

c.  $\frac{1}{n^{15}}$

d.  $\frac{1}{125n^3}$

4.  $4.38 \times 10^{-9}$

## Lesson 9.3 Properties of Radicals

### Activity 1 Properties of Radicals

Use the examples to decide whether each statement below is true for all nonnegative values of  $a$  and  $b$ .

1. Is it true that  $\sqrt{a+b} = \sqrt{a} + \sqrt{b}$ ?

a. Does  $\sqrt{9+16} = \sqrt{9} + \sqrt{16}$ ?

b. Does  $\sqrt{2+7} = \sqrt{2} + \sqrt{7}$ ?

2. Is it true that  $\sqrt{a-b} = \sqrt{a} - \sqrt{b}$ ?

a. Does  $\sqrt{100-36} = \sqrt{100} - \sqrt{36}$ ?

b. Does  $\sqrt{12-9} = \sqrt{12} - \sqrt{9}$ ?

3. Is it true that  $\sqrt{ab} = \sqrt{a}\sqrt{b}$ ?

a. Does  $\sqrt{4 \cdot 9} = \sqrt{4} \cdot \sqrt{9}$ ?

b. Does  $\sqrt{3 \cdot 5} = \sqrt{3} \cdot \sqrt{5}$ ?

4. Is it true that  $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$ ?

a. Does  $\sqrt{\frac{100}{4}} = \frac{\sqrt{100}}{\sqrt{4}}$ ?

b. Does  $\sqrt{\frac{20}{3}} = \frac{\sqrt{20}}{\sqrt{3}}$ ?

5. Decide whether the statement is true or false, and then verify with your calculator.

a.  $\sqrt{12} = \sqrt{4}\sqrt{3}$

b.  $\sqrt{12} = \sqrt{8} + \sqrt{4}$

6. Decide whether each statement is true or false.

a.  $\sqrt{(2a+b)^2} = 2a+b$

b.  $\sqrt{4a^2+b^2} = 2a+b$

c.  $\sqrt{1+25r^2} = 1+5r$

d.  $\sqrt{4s^2-1} = 2s-1$

e.  $\sqrt{9q^2+36} = 3\sqrt{q^2+4}$

f.  $\sqrt{m^2+\frac{1}{4}} = \frac{1}{2}\sqrt{4m^2+1}$

## Activity 2 Simplifying Radicals

1. Simplify  $\sqrt{75}$
2. Find the square root of each power.
  - a.  $\sqrt{y^4}$
  - b.  $\sqrt{a^{16}}$
3. Simplify  $\sqrt{b^9}$
4. Simplify  $\sqrt{72u^6v^9}$

## Activity 3 Sums and Differences

1. Write each expression as a single term.
  - a.  $13\sqrt{6} - 8\sqrt{6}$
  - b.  $5\sqrt{3x} + \sqrt{3x}$
2. Simplify  $2\sqrt{48} - \sqrt{75}$

### Wrap-Up

In this Lesson we practiced the following skills:

- Applying the product and quotient rules for radicals
- Simplifying square roots
- Adding and subtracting like radicals

1. Does  $\sqrt{x^2 - 4} = x - 2$ ? Does  $\sqrt{x^2 + 4} = x + 2$ ?
2. Explain how simplifying a radical is different from evaluating a radical.
3. Explain why  $\sqrt{16} = 4$  but  $\sqrt{a^{16}} = a^8$ .
4. What is wrong with the statement  $3\sqrt{6} + 2\sqrt{6} = 5\sqrt{12}$ ?

## 9.3 Homework Preview

■ Decide whether each statement is true or false, then verify with your calculator.

1. a.  $\sqrt{4} + \sqrt{16} = \sqrt{20}$

b.  $\sqrt{4} \cdot \sqrt{25} = \sqrt{100}$

c.  $\frac{\sqrt{36}}{\sqrt{9}} = \sqrt{4}$

d.  $\sqrt{100} - \sqrt{64} = \sqrt{36}$

2. a.  $\sqrt{3} + \sqrt{5} = \sqrt{8}$

b.  $\sqrt{6} \cdot \sqrt{5} = \sqrt{30}$

c.  $\frac{\sqrt{18}}{\sqrt{3}} = \sqrt{6}$

d.  $\sqrt{12} - \sqrt{3} = \sqrt{9}$

■ Simplify each square root.

3.  $\sqrt{54}$

4.  $\sqrt{b^9}$

5.  $\sqrt{20x^2y^3}$

6.  $3\sqrt{\frac{12x^4}{25}}$

■ Combine like terms.

7.  $2\sqrt{3} + 3\sqrt{5} - 8\sqrt{3} - \sqrt{5}$

8.  $3\sqrt{12} - 2\sqrt{18} - 4\sqrt{8}$

## Answers

1a. False   b. True   c. True   d. False   2a. False   b. True   c. True   d. False

3.  $3\sqrt{6}$    4.  $b^4\sqrt{b}$    5.  $2xy\sqrt{5y}$    6.  $\frac{6x^2\sqrt{3}}{5}$    7.  $-6\sqrt{3} + 2\sqrt{5}$    8.  $6\sqrt{3} - 14\sqrt{2}$

## Lesson 9.4 Operations on Radicals

### Activity 1 Products and Quotients

1. Find the product and simplify  $(3x\sqrt{6x})(y\sqrt{15xy})$

2. Multiply  $2a(2\sqrt{3} - \sqrt{a})$

3. Multiply  $3\sqrt{x}(\sqrt{2x} - 3x)$

4. Expand and simplify  $(\sqrt{2} - 2\sqrt{b})^2$

5. Simplify  $\frac{3ab\sqrt{75a^3b}}{\sqrt{6ab^5}}$

### Activity 2 Fractions with Radicals

1. Add  $\frac{\sqrt{x}}{3} + \frac{\sqrt{2}}{x}$

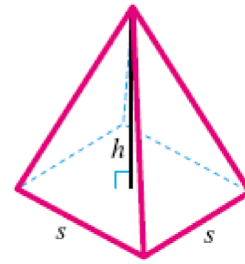
2. Rationalize the denominator of  $\frac{8}{\sqrt{x}}$



### Activity 3 Application

In this Activity we use a formula for the surface area of a pyramid with a square base, as shown in the figure at right.

The pyramid has five faces, namely, the square base and four triangular sides. We know the height,  $h$ , of the pyramid, and the length,  $s$ , of the base.



To calculate the surface area of the pyramid, we add the area of its base and the areas of the four triangular faces.

$$\begin{aligned}
 S &= (\text{area of base}) + 4(\text{area of triangular face}) \\
 &= s^2 + 4 \cdot \frac{1}{2}s\sqrt{4h^2 + s^2} \\
 &= s^2 + s\sqrt{4h^2 + s^2}
 \end{aligned}$$

Use the formula to find the surface area of the Sun Pyramid in Mexico. The pyramid is 210 feet high and has a square base 689 feet on each side.



### Wrap-Up

In this Lesson we practiced the following skills:

- Simplifying products involving radicals
- Simplifying quotients of radicals
- Rationalizing the denominator

1. In Activity 1 Problem 5, why don't we start by rationalizing the denominator?
2. Explain why  $\frac{\sqrt{6x}}{3}$  is not equivalent to  $\sqrt{2x}$ .
3. In Activity 3, why don't we simplify  $\sqrt{4h^2 + s^2}$  to  $2h + s$ ?

## 9.4 Homework Preview

■ Find the product and simplify.

1.  $(3a\sqrt{12a})(2b\sqrt{6ab^3})$

2.  $5t(3t^3 - 2t\sqrt{2t})$

3.  $4\sqrt{3x}(2x\sqrt{6x} + 3\sqrt{x})$

4.  $(3 - \sqrt{k})(4 + 2\sqrt{k})$

■ Find the quotient and simplify.

5.  $\frac{\sqrt{10}}{\sqrt{45}}$

6.  $\frac{\sqrt{72x^3}}{\sqrt{36x}}$

7. Subtract  $\frac{4 + \sqrt{2}}{3} - \frac{\sqrt{2}}{2}$

8. Rationalize the denominator  $\frac{3}{\sqrt{12x}}$

### Answers

1.  $36a^2b^2\sqrt{2b}$     2.  $15t^4 - 10t^2\sqrt{2t}$     3.  $24x^2\sqrt{2} + 12x\sqrt{3}$     4.  $12 + 2\sqrt{k} - 2k$   
5.  $\frac{\sqrt{2}}{3}$     6.  $x\sqrt{2}$     7.  $\frac{8 - \sqrt{2}}{6}$     8.  $\frac{\sqrt{3x}}{2x}$

## Lesson 9.5 Equations with Radicals

### Activity 1 Solving Radical Equations

After solving a radical equation, remember to check for extraneous solutions.

1. Solve  $\sqrt{x-6} = 2$

2. Solve  $\sqrt{x-3} + 5 = x$

3. Solve  $3\sqrt[3]{4x-1} = -15$

### Activity 2 Roller Coasters

You have been commissioned to design a new roller coaster for an amusement park. The roller coaster should be more spectacular than all existing roller coasters, and in particular it should include a vertical loop. In order to stay on the track through the loop, the cars must travel at a speed given (in miles per hour) by

$$v = \sqrt{89.3r}$$

where  $r$  is the radius of the loop in feet.

Here are the three roller coasters with the tallest vertical loops:

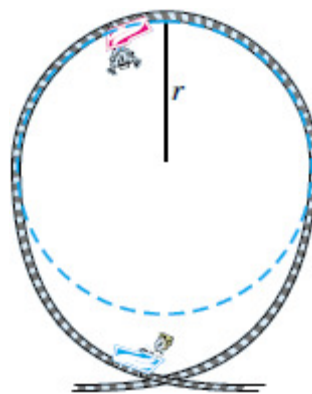
Height	Roller Coaster	Location
188 feet	Viper	Six Flags Magic Mountain, Valencia, California
173 feet	Gash	Six Flags Great Adventure, Jackson, New Jersey
170 feet	Shockwave	Six Flags Great Adventure, Gurnee, Illinois

1. Evaluate the formula to find the speed the cars must reach on each roller coaster. Because the vertical loops used in roller coasters are not perfect circles, the total height of the loop is about 2.5 times its radius.

Viper:

Gash:

Shockwave:



2. You would like your roller coaster to have a vertical loop that is 200 feet tall. How fast must the cars travel?
3. Now suppose you know the maximum speed possible for the cars on a particular roller coaster. Can you calculate the height of the tallest vertical loop the cars can negotiate? Here are the speeds of the world's three fastest roller coasters.

Speed	Roller Coaster	Location
86 mph	Fujiyama	Fujikyu Highland Park, Japan
80 mph	Steel Phantom	Kennywood, West Mifflin, Pennsylvania
79 mph	Desperado	Buffalo Bill's, Jean, Nevada

Calculate the maximum loop height possible for each roller coaster.

Desperado:

Steel Phantom:

Fujiyama:

### Activity 3 Quadratic Equations

1. Solve  $(2x + 1)^2 = 8$

2. Solve  $3x^2 - 6x + 2 = 0$ , and simplify the solutions.

### Wrap-Up

In this Lesson we practiced the following skills:

- Solving radical equations
- Checking for extraneous solutions
- Simplifying solutions of quadratic equations

1. What operation might introduce extraneous solutions?
2. What is wrong with this strategy: To solve an equation with a variable under a square root, square each term of the equation.
3. How many solutions does the equation  $x^2 = 64$  have? How many solutions does  $x^3 = -64$  have?

## 9.5 Homework Preview

■ Solve.

1.  $5\sqrt{2x-4} - 7 = 23$

2.  $4\sqrt[3]{6-x} + 5 = -3$

■ Solve, and check for extraneous solutions.

3.  $\sqrt{2x-3} = -5$

4.  $t - 5 = \sqrt{2t+5}$

5. Solve by extracting roots  $3(2x-3)^2 = 15$

6. Solve  $2x^2 - 4x - 5 = 0$  and simplify your answer.

## Answers

1.  $x = 20$    2.  $x = 14$    3. No solution   4.  $t = 10$    5.  $x = \frac{3 \pm \sqrt{5}}{2}$    6.  $x = \frac{2 \pm \sqrt{14}}{2}$