CS189, HW6: Neural Nets

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Problem 1: Derivations

Let X be the matrix of sample points.
Let X^1 be the matrix of sample points with a column of a bias dimension augmented to the end.
Let x be a row of X (784 dimensions), and let x^1 be a row of X_1 (785 dimensions).
Similarly, let h be the hidden units (200 dimensions), and let h^1 be the hidden units with a bias unit appended to the end (201 dimensions).
Let z be the 26 dimension output layer.

Let V be the 200×784 matrix for the weights between x and h (without bias).
Let V^1 be the 200×785 matrix for the weights between x_1 and h (with bias).
Let W be the 26×200 matrix for the weights between h and z (without bias).
Let W^1 be the 26×201 matrix for the weights between h_1 and z (with bias).

Let y be the desired output (26 dimensions).
Let $\mathbb{1}$ denote a vector of ones, and $\circ$ be the Hadamard product.

$$\tanh(V^1 x^1) = h$$

$$s(W^1 h^1) = z$$

$$L(z, y) = -\sum_{j=1}^{26} y_j ln(z_j) + (1 - y_j) ln(1 - z_j)$$

$$\frac{\partial L}{\partial z_j} - \frac{y_j}{z_j} + \frac{1 - y_j}{1 - z_j} = \frac{z_j - y_j}{z_j(1 - z_j)}$$

$$\nabla_{W_j^1} L = \frac{\partial L}{\partial z_j} \nabla_{W_j^1} z_j = \frac{\partial L}{\partial z_j} z_j(1 - z_j) h^1 = (z_j - y_j) h^1 \Rightarrow \nabla_{W^1} L = (z - y){h^1}^T$$

$$\nabla_h L = \sum_{j=1}^{k} \frac{\partial L}{\partial z_j} \nabla_h z_j = \sum_{j=1}^{26} z_j(1 - z_j) \frac{\partial L}{\partial z_j} W_j = \sum_{j=1}^{26} (z_j - y_j) W_j = W^T(z - y)$$

$$h_i = \tanh(V_i^1 x_i^1) \Rightarrow \nabla_{V_i^1} h_i^1 = \tanh'(V_i^1 \cdot x) x = (1 - {h_i^1}^2) x$$

$$\nabla_{V_i^1} L = \frac{\partial L}{\partial h_i} \nabla_{V_i^1} h_i = \frac{\partial L}{\partial h_i} (1 - {h_i^1}^2) x^1 \Rightarrow \nabla_{V^1} L = ((\mathbb{1} - h \circ h) \circ \nabla_h L){x^1}^T$$

Suppose that we pick a learning rate of ϵ. Then our updates for V^1 and W^1 will be:

$$V^1 \leftarrow V^1 - \epsilon((\mathbb{1} - h \circ h) \circ W^T(z - y)){x^1}^T$$

$$W^1 \leftarrow W^1 - \epsilon(z - y){h^1}^T$$

Problem 2: Implementation

1. Any hyperparameters that you tuned.

   Solution: The only hyper-parameter I ended up tuning was epsilon. Trial and error led to the conclusion that a value of $\epsilon = .002$ worked.

2. Your training accuracy.

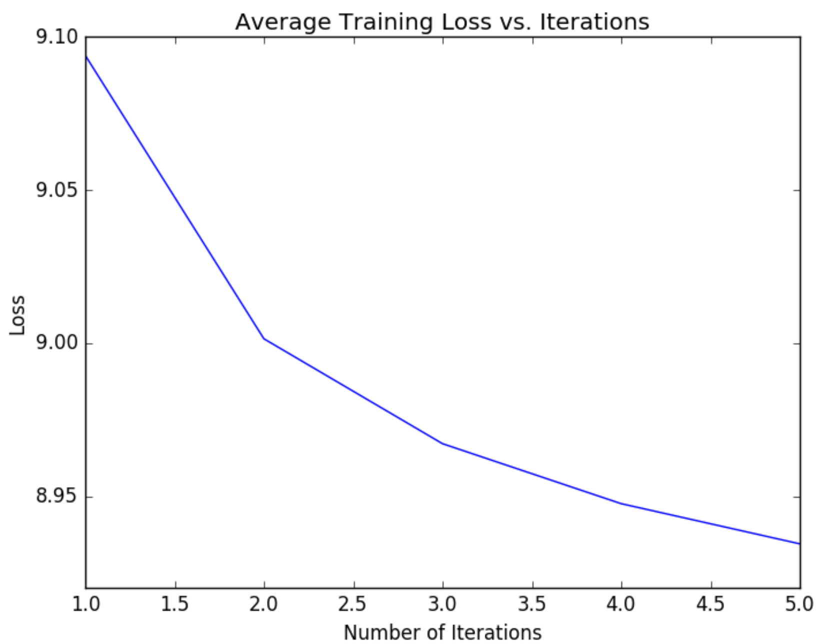
   Solution: Training accuracy is about .88336 after 5 epochs.

3. Your validation accuracy.

   Solution: Validation accuracy is about .86142 after 5 epochs.

4. A plot of the loss value versus the number of iterations. You may sample (i.e., compute the loss every x iterations).

   Solution:



5. Your Kaggle score and display name.

   Solution:

   Name: Commendable Fennel

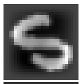
   Score: .85692

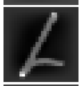
6. Your code (as an appendix at the end).

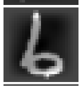
Expect validation accuracies of 85% or higher. Your TAs are able to achieve 87.3% validation accuracy after 1 minute of training on a MacBook Pro. With some tweaks (800 hidden units, ReLU hidden layer activation, softmax output layer activation), your TAs are able to get 90.17% validation accuracy after 3 minutes of training.

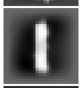
Problem 3: Visualization

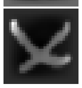
Correctly classified images:

 S

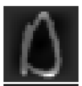
 L

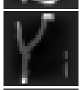
 B

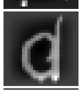
 I

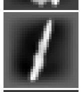
 X

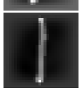
Incorrectly classified images:

 Classified as B instead of D

 Classified as R instead of Y

 Classified as U instead of D

 Classified as L instead of I

 Classified as L instead of I

Problem 4: Bells and Whistles

In lecture, we covered many variations and ways to improve the learning speed and accuracy of neural networks. After you have implemented the basic neural network as we have prescribed, you may go above and beyond to improve your network for your Kaggle submissions. As with Problem 2, you must implement any additional functionality yourself. (E.g., you **cannot** train a neural network with Tensorflow and use that.) In this section of your writeup, report any extra functionality or tricks you implemented. Some ideas:

- Use different learning rates for different layers, and have those rates decay over time. (Highly recommended. See Section 5.4.)
- Change the number of hidden layer units, or even add one or more additional hidden layers.
- Use different activation functions. ReLUs for the hidden layer and softmax output units can speed up training and improve accuracy.
- Implement mini-batch gradient descent. This will dramatically speed up your training time.
- Regularization: both L_2 regularization and dropout are surprisingly easy to implement, and tend to reduce overfitting.
- Momentum, different initialization schemes, an ensemble of neural networks, etc. See the "Implementation Tips" section for ideas.

Solution: I implemented mini-batch gradient descent using 40 sample points at a time to speed up training time.

Appendix

The following code was used for everything:

```
import numpy as np
import scipy.io
import matplotlib.pyplot as plt
import csv
import scipy.misc
import sys

traindatafilename = "hw6_data_dist/letters_data"
data = scipy.io.loadmat(traindatafilename)

TRAINING_FRACTION = .8

traindata = data['train_x']
trainlabels = data['train_y']

NUM_FEATURES = traindata.shape[1]
NUM_SAMPLES = traindata.shape[0]

testdata = data['test_x']

#Shuffle the training data
temp = np.hstack((traindata, trainlabels))
np.random.shuffle(temp)
X = temp[:,:NUM_FEATURES]
y = temp[:,NUM_FEATURES:]

#Add bias terms and normalize all data
"""
temp = np.vstack((X, testdata))
temp = np.hstack((temp, np.ones(temp.shape[0]).reshape(temp.shape[0], 1)))
temp = temp / (np.linalg.norm(temp, axis=0)[np.newaxis, :] + .000000001)
temp = temp - np.mean(temp, axis=0)
X = temp[:NUM_SAMPLES]
testdata = temp[NUM_SAMPLES:]
"""

column_means = np.mean(X, axis=0)

X = X - column_means
testdata = testdata - column_means

column_stds = np.std(X, axis=0) + .000000001
X = X / column_stds
testdata = testdata / column_stds

X = np.hstack((X, np.ones(X.shape[0]).reshape(X.shape[0], 1)))
testdata = np.hstack((testdata, np.ones(testdata.shape[0]).reshape(testdata.shape[0], 1)))

#Increment features by 1 to account for bias
NUM_FEATURES += 1

#Create training and validation sets
TRAINING_SIZE = int(TRAINING_FRACTION * NUM_SAMPLES)
VALIDATION_SIZE = NUM_SAMPLES - TRAINING_SIZE
sys.stdout.flush()

X_train = X[:TRAINING_SIZE]
y_train = y[:TRAINING_SIZE]

X_validation = X[TRAINING_SIZE:]
y_validation = y[TRAINING_SIZE:]

#END PREPROCESSING DATA
```

```python
#Initialize constants and matrices
EPSILON = .002
decay_factor = .85
V = np.random.normal(0, .01, (200, 785))
W = np.random.normal(0, .01, (26, 201))

y_array = [None]
for i in range(26):
    temp = np.ones((26, 1)) * .1
    temp[i] = .9
    y_array.append(temp)

def sigmoid(num):
    return 1 / (1 + np.exp(-num))

def classify(sample):
    h = np.tanh(np.dot(V, sample)).reshape((200, 1))
    h1 = np.vstack((h, 1))
    z = sigmoid(np.dot(W, h1))
    return np.argmax(z) + 1

def caclulate_validation_accuracy():
    right = 0
    for i in range(VALIDATION_SIZE):
        sample = X_validation[i]
        guess = classify(sample)
        label = y_validation[i]
        if guess == label:
            right += 1
    return right / VALIDATION_SIZE

def calculate_training_accuracy():
    right = 0
    for i in range(TRAINING_SIZE):
        sample = X_train[i]
        guess = classify(sample)
        label = y_train[i]
        if guess == label:
            right += 1
    return right / TRAINING_SIZE

def calculate_training_loss():
    total_loss = 0
    for i in range(TRAINING_SIZE):
        x = X_train[i]
        h = np.tanh(np.dot(V, x).reshape((200, 1)))
        h1 = np.vstack((h, 1))
        z = sigmoid(np.dot(W, h1))
        y = y_array[int(y_train[i])]
        total_loss += loss(z, y)
    return float(total_loss / TRAINING_SIZE)

def loss(z, y):
    l = 0
    for i in range(26):
        zi, yi = z[i], y[i]
        l -= yi*np.log(zi) + (1-yi)*np.log(1-zi)
    return l

def plot_loss(loss_array):
    iterations = [i + 1 for i in range(len(loss_array))]
    plt.figure()
    plt.plot(iterations, loss_array)
    plt.title("Average Training Loss vs. Iterations")
    plt.ylabel("Loss")
    plt.xlabel("Number of Iterations")
    plt.show()
```

```python
def save_classified_images():
    right = 0
    wrong = 0
    index = 0
    while right < 5 or wrong < 5:
        sample = X_validation[index]
        guess = classify(sample)
        sample = sample[:784].reshape((28, 28))
        label = y_validation[index]
        if guess == label:
            if right < 5:
                s = 'right' + str(right) + '.png'
                scipy.misc.imsave(s, sample)
                right += 1
                print(s, guess)
        else:
            if wrong < 5:
                s = 'wrong' + str(wrong) + '.png'
                scipy.misc.imsave(s, sample)
                wrong += 1
                print(s, guess, label)
        index += 1

def classify_test_data():
    TEST_SIZE = testdata.shape[0]
    guesses = []
    for i in range(TEST_SIZE):
        sample = testdata[i]
        guesses.append(classify(sample))
    with open('submission_1.csv', 'w', newline='') as csvfile:
        writer = csv.writer(csvfile)
        writer.writerow(['Id', 'Category'])
        i = 1
        for g in guesses:
            writer.writerow([i, g])
            i += 1


loss_array = []
for epoch in range(5):
    for i in range(0, TRAINING_SIZE, 40):
        x = np.transpose(X_train[i:i+40])
        h = np.tanh(np.dot(V, x))
        h1 = np.vstack((h, np.ones(40)))
        z = sigmoid(np.dot(W, h1))
        y = []
        for j in range(40):
            y.append(y_array[int(y_train[i+j])].reshape(26))
        y = np.transpose(y)
        grad_w = np.dot(z-y, np.transpose(h1))
        temp1 = np.ones((200, 40)) - h * h
        temp2 = np.dot(np.transpose(W)[:200], z - y)
        grad_v = np.dot(temp1 * temp2, np.transpose(x))
        V = V - grad_v * EPSILON
        W = W - grad_w * EPSILON
    EPSILON *= decay_factor
    print("Epochs:", epoch + 1)
    training_loss = calculate_training_loss()
    print("Training loss:", training_loss)
    loss_array.append(training_loss)
    sys.stdout.flush()

print("Training accuracy:", calculate_training_accuracy())
print("Validation accuracy:", caclulate_validation_accuracy())

plot_loss(loss_array)
classify_test_data()
save_classified_images()
```