Bitcoin price and its marginal cost of production: supporting evidence

Article *in* Applied Economics Letters · June 2018

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Bitcoin price and its marginal cost of production: support for a fundamental value

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Abstract This study back-tests a cost of production model proposed to value the digital currency bitcoin. Results from both conventional regression and vector autoregression (VAR) models show that the marginal cost of production plays an important role in explaining bitcoin prices, challenging recent allegations that bitcoins are essentially worthless. Even with markets pricing bitcoin in the thousands of dollars each, the valuation model seems robust. The data show that a price deviation may have begun in the summer of 2017 which is yet to be completely resolved.

Keywords: bitcoin, cost of production, valuation model, cryptocurrency
JEL classification: D4, E47, G12, C52

1. Introduction

The price of the digital currency bitcoin increased from $250 in June of 2015 to more than $10,000 by February of 2018, where it now commands a market capitalization in excess of $185 billion and sees many hundreds of millions of dollars' worth of transactions cross its system on a daily basis. And yet, Bitcoin functions entirely as its own decentralized computer network, without any central bank or government or regulatory body to back it.[1] This has led many to conclude that the price the cryptocurrency is undergoing a massive speculative bubble, with some researchers claiming that there is *no* fundamental underpinning to its value (e.g. Hanley 2013; Yermack 2013). Cheah and Fry (2015) echo recent comments made by Jamie Dimon, the CEO of investment bank JPMorgan Chase in asserting that the fundamental value of bitcoin is zero, and the entire pursuit is a fool's errand, or

[1] For consistency, Bitcoin with a capital 'B' refers to the general system, network and protocol, while bitcoin with a small 'b' refers to the digital currency itself or units thereof

worse a fraud.[2] Even the Wall Street Journal has opined that Bitcoin is "probably worth zero."[3] Against this backdrop, it is of growing concern to evaluate the basis for value of bitcoin.

Challenging the views of Mr. Dimon and the hypotheses of the above research, Hayes (2016) suggests that bitcoin does indeed have a quantifiable intrinsic value and formalizes a pricing model based on its marginal cost of production:[4] "mining," or the process of creating new bitcoins through concerted computational effort requires the consumption of electric power, which incurs a real monetary cost for mining participants, and thus the value of bitcoin is the embodied costs of production (on the margin).

This paper seeks to test the validity of this cost of production theory of value by back-testing the pricing model against the observed market price, going back in excess of four years. A simple OLS regression indicates that the model price explains approximately 80% of the observed market price. Following this up, a Granger test on the postestimation results of a subsequent vector autoregression (VAR) model is carried out, which strongly rejects the null hypothesis that the pricing model does not "cause" the market price. The Granger test is used here not asset causality, but to supports the notion that the modeled price and the observed price match up to a statistically significant degree.

2. The Cost of Production Model

The process and technical elaboration of bitcoin production ("mining") is described in-depth elsewhere (e.g. Kroll et al. 2013; Sapirshtein et al 2016; Nakamoto 2008). Suffice it to say that mining

[2] https://www.bloomberg.com/news/articles/2017-09-12/jpmorgan-s-ceo-says-he-d-fire-traders-who-bet-on-fraud-bitcoin
[3] https://www.wsj.com/articles/bitcoins-wild-ride-shows-the-truth-it-is-probably-worth-zero-1505760623
[4] Or at least, an expected lower bound to its market price

involves a competition among producers; with a novelty that the rate of new unit formation is fixed so that increased demand cannot induce a greater supply, and where this elasticity is manifest instead through increased difficulty in the production process itself that increases the system-wide marginal cost of production.

The primary ongoing cost for bitcoin production is that of electricity, measured in dollars per kilowatt-hour (kWh). Of course, different regions of the world will consume electricity at their local rates (which may vary by customer type, power generation source, and time of day) and in their local currencies, but for the sake of convenience it is a good working assumption that the average rate of electricity worldwide accounting for both residential and commercial rates is approximately USD $0.135 per kWh.[5]

Following Hayes (2016), The (marginal) cost of production per day, E_{day} per unit of mining power can be expressed as:

$$E_{day} = \left(\rho / 1000\right)\left(\$ / kWh \cdot WperGH / s \cdot hr_{day}\right) \tag{1}$$

where: E_{day} is the dollar cost per day for a producer, ρ is the hashpower (computational power) employed by a miner, $/ kWh is the dollar price per kilowatt-hour, and W per GH/s is the energy efficiency of the hardware, and hrs_{day} is the number of hours in a day.

In order to calculate the expected number of bitcoins the same miner can produce daily, the following equation is used to calculate the daily (marginal) product:

$$BTC / day^* = \left(\frac{\beta\rho \cdot \sec_{hr}}{\delta \cdot 2^{32}}\right) hr_{day} \tag{2}$$

[5] https://en.wikipedia.org/wiki/Electricity_pricing

where: BTC/day* is the expected level of daily bitcoin production when mining bitcoin, β is the block reward (expressed in units of BTC/block), ρ is the hashing power employed by a miner, and δ is the difficulty (expressed in units of GH/block) The constant sec_{hr} is the number of seconds in an hour, hr_{day} the number of hours in a day. Presently, the block reward is 12.5 BTC per block.

According to microeconomic theory, under conditions of competition, the marginal product should equate with its marginal cost, which should also equal its selling price. Because of this theoretical equivalence, and since cost per day is expressed in terms of $/day and production in BTC/day, the $/BTC price level is revealed as the ratio of (cost/day) divided by (BTC/day). This objective price of production, P^*, serves as a logical lower bound for the market price, below which a producer would operate at a marginal loss and presumably remove themselves from the network. P^* is expressed in dollars per bitcoin, given the difficulty and cost of production:

$$P^* = \frac{E_{day}}{BTC/day^*} \qquad (3)$$

3. Testing the Model Empirically

I backtest the above model using historical observed price data and compare that to what the model would have predicted. Observed market price and difficulty data were collected using blockchain.info, a reliable and transparent source of Bitcoin market and protocol data, at the dates of difficulty changes in the network (approximately once every two weeks) to consistently measure market price given a particular value of mining difficulty, from June 29, 2013 through September 18, 2017.

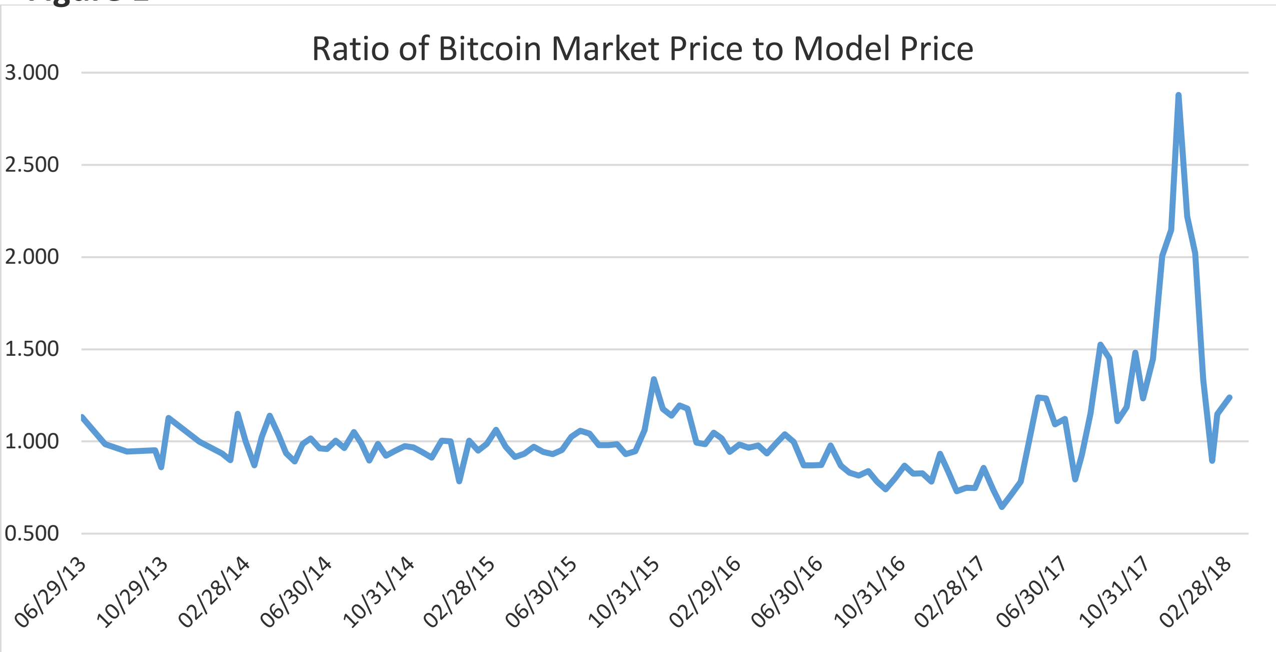
The model requires as an input the average energy efficiency of the mining network. This information was ecextracted from Bitcoin mining hardware manufacturer websites and checked against a dedicated wiki page that catalogues the efficiency of current mining hardware (https://en.bitcoin.it/wiki/Mining_hardware_comparison). This data was then collected for each date of difficulty change using the internet archive's *wayback machine* (https://web.archive.org/web/20170215000000*/https://en.bitcoin.it/wiki/Mining_hardware_comparison). For simplicity, I hold electricity costs constant at 13.5 cents per kilowatt-hour.[6] Table 1, which appears in the Appendix, describes these data points along with estimated model price for each difficulty change date.

3.1 Conventional Regression Analysis

As a first pass, I compared the ratio of observed price to modeled price over time, from June 2013 through February 2018. As Figure 1 shows, since June 2013, the market price has tended to fluctuate about the price estimated by the model. In the chart, a y-axis value of 1.00 indicates that the market price and model price are identical. Values above 1.00 indicate a premium in the market relative to the model and below 1.00 a relative discount. Over the long-run (4+ years), the average ratio is 1.04, $\sigma = 0.30$, which is striking in its accuracy. This suggests that the market for bitcoin has been quite efficient from a production standpoint, if not volatile, contradicting assertions that this market is consistently inefficient (e.g. Urquhart 2016). There is evidence of increased volatility from approximately September 2017 through January 2018, indicating that the market had deviated from the model, but did ultimately converge once again. This spike indicates the emergence and reconciliation of a price bubble, however, even so such bubbles do not indicate a zero value, only that prolonged departures from the modeled price can exist, but which ultimately resolve.

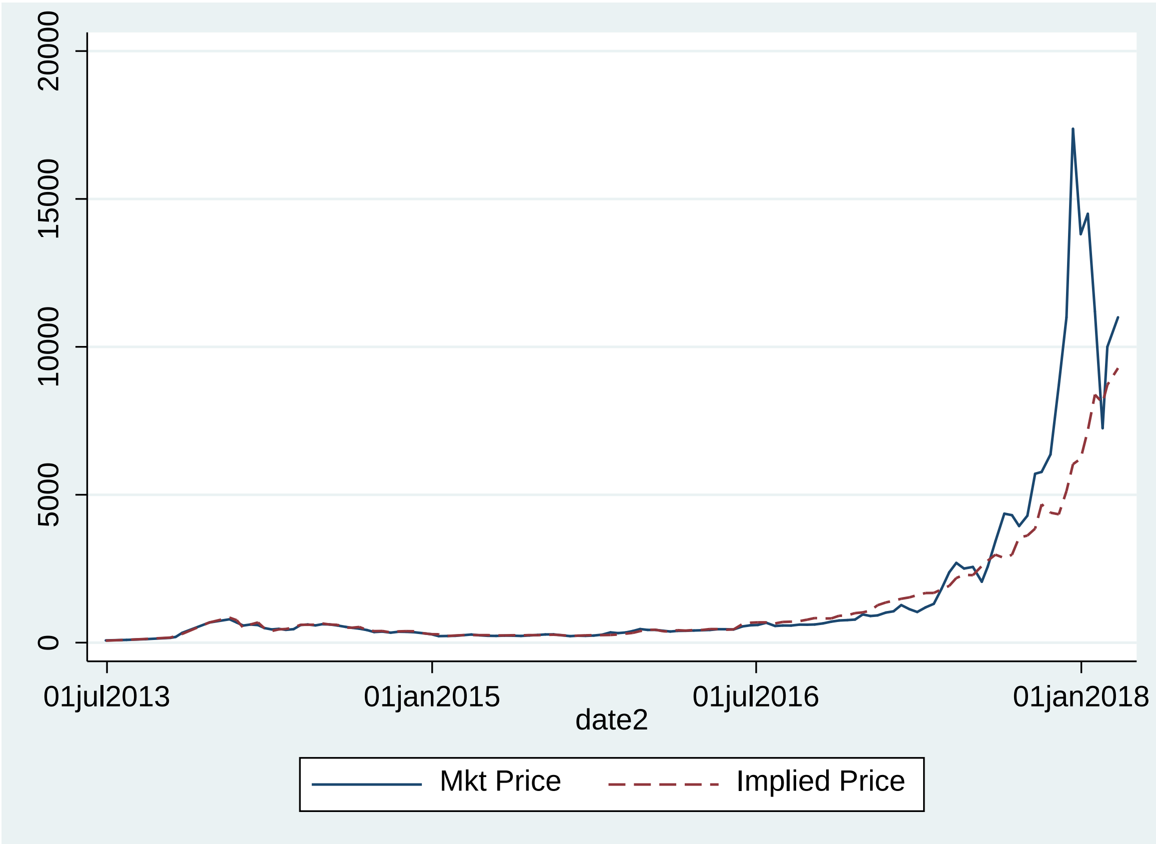
[6] I have modeled a dynamic energy cost that grows with inflation, but the results are largely similar.

Figure 1



This initial result is suggestive, and so a more rigorous analysis was undertaken to test the "fit" of the pricing model against observed historical data. The valuation model output and observed prices appear in Figure 2, with what amounts to two time series for comparison. A conventional OLS regression was first carried out to obtain a proxy for model fit and to judge how much of the market price is "explained" by the model; which produces $R^2 = 0.845$, telling us that nearly 85% of the observed market price can be explained by the marginal cost of production model. Next, I conduct a second OLS regression on the log transformations of each time series, yielding an $R^2 = 0.968$, suggesting that nearly all of the marginal change in market price can be explained by the marginal change in cost.

Figure 2: Observed Market Price vs. Implied (ex-Post) Model Price (June 2013 - Feb 2018)



3.2 VAR Granger Analysis

Next, in order to compare these two time series directly to each other in a methodologically rigorous way, I estimate a multivariate vector autoregression (VAR) with two lags each on the log transformation of market price and implied model price.[7] The purpose of the VAR is primarily to test the postestimation results using a Granger test. Typically used to suggest temporal causality, I use the test here instead to test the post-hoc predictive power of the cost of production pricing model. The test considers two null hypotheses:

$H_0$1: The market price does *not* “cause” the model price; and

$H_0$2: The model price does *not* “cause” the market price.

[7] Testing for autocorrelation suggests this is the appropriate number of lags.

As Table 1 shows, $H_0$1 cannot be rejected, which is to be expected: the model is supposed to describe the market and not the other way around. $H_0$2 however, is strongly rejected, and the alternative hypothesis that the model price implies the market price is given a large degree of support ($p < 0.000$). This key finding lends credibility that the marginal cost of production of bitcoin describes its price and disputes those who claim that bitcoin is worthless.

Table 1: Granger Test on VAR Postestimation Equations

Granger Causality Wald Tests

H0	chi2	df	Prob >chi2
1: Market price does not Granger cause the model price	4.704	2	0.095
2: Model price does not Granger cause the market price	14.785	2	0.001

Note: H01 cannot be rejected, which is to be expected: the model is supposed to describe the market and not the other way around. H02 however, is strongly rejected, and the alternative hypothesis that the model price implies the market price is given a large degree of support (p ¡ 0.000). This key finding lends credibility that the marginal cost of production of bitcoin describes its price and disputes those who claim that bitcoin is worthless.

4. Conclusion

The marginal cost of production was proposed as a model to value bitcoin (Hayes 2016). In this paper, the cost of production model was back-tested using historical data showing that the market price of bitcoin tends to fluctuate around the model price, and with the model price predicting the market price in a statistically significant manner.

This finding is striking given the volume of recent media accounts and research projects that have supposed no fundamental value at all for bitcoin (e.g. Cheah and Fry 2015). Moreover, it suggests that attempts to find a correlation between the price of bitcoin and exogeneous factors may be misguided (e.g. Ciaian et al. 2016; Kristoufek 2015; Polasik et al 2015).

The data do suggest that despite a significant deviation in price to the upside from the Fall of 2017, the model remained resilient, and the model price did converge with the market price. The pricing model leads us to expect that during periods of excess demand (aka a price bubble), either the market price will fall and/or the mining difficulty will increase to resolve the discrepancy.

It is important to note that this analysis applies primarily to bitcoin and does not necessarily extend to other cryptocurrencies such as Ethereum or Litecoin (although a similar analysis might). However, with Bitcoin still dominating the digital currency market, both in scale and scope, it is a worthwhile pursuit to understand why it has value.

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APPENDIX

Table 1: Data for analysis

Date	Difficulty	Energy Efficiency (Observed)	Block Reward (BTC)	Market Price (USD, Observed)	Implied Price (USD, Calculated)
06/29/13	21,335,329	25	500	77.87	68.73
08/03/13	37,392,766	25	400	94.95	96.36
09/04/13	86,933,018	25	225	119.10	126.01
10/16/13	267,731,249	25	100	164.08	172.48
10/25/13	390,928,788	25	90	195.05	226.67
11/05/13	510,929,738	25	90	333.93	296.25
12/21/13	1,180,923,195	25	90	683.69	684.72
01/24/14	2,193,847,870	25	60	792.02	848.03
02/05/14	2,621,404,453	25	45	682.21	759.97
02/16/14	3,129,573,175	25	25	580.01	504.05
02/28/14	3,815,723,799	25	25	614.46	614.57
03/13/14	4,250,217,920	25	25	595.75	684.55
03/24/14	5,006,860,589	25	15	495.02	483.85
04/05/14	6,119,726,089	25	10	449.62	394.26
04/17/14	6,978,842,560	25	10	468.89	449.61
04/29/14	8,000,872,136	25	9	434.67	463.91
05/12/14	8,853,416,309	25	9	457.65	513.34
05/24/14	10,455,720,138	25	9	598.07	606.24
06/05/14	11,756,551,917	25	8	616.04	605.93
06/18/14	13,462,580,115	25	7	584.68	607.12
06/29/14	16,818,461,371	25	6	623.82	650.11
07/12/14	17,336,316,979	25	5.5	616.36	614.29
07/25/14	18,736,441,558	25	5	581.91	603.54
08/08/14	19,729,645,941	25	4	534.26	508.43
08/19/14	23,844,670,039	25	3.3	501.86	506.94
08/31/14	27,428,630,902	25	3	474.96	530.12
09/13/14	29,829,733,124	25	2.25	426.66	432.40
09/25/14	34,661,425,924	25	1.75	360.23	390.78
10/09/14	35,002,482,026	25	1.75	375.10	394.63
10/23/14	35,985,640,265	25	1.5	338.82	347.75
11/05/14	39,603,666,252	25	1.5	370.64	382.72
11/18/14	40,300,030,328	25	1.5	366.80	389.45
12/02/14	40,007,470,217	25	1.5	352.88	386.62
12/17/14	39,457,671,307	25	1.25	319.05	317.76
12/30/14	40,640,955,017	25	1.1	288.21	288.01
01/12/15	43,971,662,056	25	1	221.97	283.29
01/27/15	41,272,837,895	25	0.85	226.81	226.01
02/09/15	44,455,415,962	25	0.85	231.28	243.44
02/22/15	46,684,376,317	25	0.85	252.45	255.65
03/08/15	47,427,554,951	25	0.85	276.01	259.72
03/22/15	46,717,549,645	25	0.85	248.65	255.83
04/05/15	49,446,390,688	25	0.8	233.50	254.84
04/19/15	47,610,564,513	25	0.8	228.89	245.38
05/03/15	47,643,398,018	25	0.8	238.70	245.55
05/17/15	48,807,487,245	25	0.8	237.34	251.55

05/31/15	47,589,591,154	25	0.8	228.49	245.27
06/14/15	49,692,386,355	25	0.8	244.30	256.11
06/28/15	49,402,014,931	25	0.8	261.20	254.62
07/11/15	51,076,366,303	25	0.8	278.70	263.25
07/25/15	52,278,304,846	25	0.8	280.99	269.44
08/08/15	52,699,842,409	25	0.75	249.33	254.64
08/22/15	54,256,630,328	25	0.65	222.53	227.20
09/04/15	56,957,648,455	25	0.65	235.00	238.52
09/17/15	59,335,351,234	25	0.65	231.26	248.47
10/01/15	60,813,224,039	25	0.65	241.15	254.66
10/15/15	60,883,825,480	25	0.65	270.82	254.96
10/29/15	62,253,982,450	25	0.65	348.63	260.69
11/11/15	65,848,255,180	25	0.65	324.50	275.75
11/24/15	72,722,780,643	25	0.65	346.90	304.53
12/06/15	79,102,380,900	25	0.65	396.00	331.25
12/18/15	93,448,670,796	25	0.65	461.00	391.33
12/31/15	103,880,340,815	25	0.65	432.00	435.01
01/13/16	113,354,299,801	25	0.6	432.00	438.17
01/26/16	120,033,340,651	25	0.5	405.00	386.65
02/07/16	144,116,447,847	25	0.4	377.35	371.39
02/19/16	163,491,654,909	25	0.4	397.83	421.31
03/04/16	158,427,203,767	25	0.4	401.64	408.26
03/18/16	165,496,835,118	25	0.4	411.96	426.48
04/01/16	166,851,513,283	25	0.4	420.71	429.97
04/14/16	178,678,307,672	25	0.4	430.07	460.45
04/28/16	178,659,257,773	25	0.4	456.00	460.40
05/11/16	194,254,820,283	25	0.35	455.00	438.02
05/24/16	199,312,067,531	25	0.35	448.00	449.42
6/8/16	196,061,423,940	12.5	0.25	550.00	631.56
6/21/16	209,453,158,595	12.5	0.25	587.00	674.70
7/4/16	213,398,925,331	12.5	0.25	600.00	687.41
7/18/16	213,492,501,108	12.5	0.25	673.00	687.71
8/2/16	201,893,210,853	12.5	0.25	565.00	650.34
8/15/16	217,375,482,757	12.5	0.25	581.00	700.22
8/29/16	220,755,908,330	12.5	0.25	579.00	711.10
9/12/16	225,832,872,179	12.5	0.25	611.00	727.46
9/25/16	241,227,200,230	12.5	0.25	608.00	777.05
10/08/2016	258,522,748,405	12.5	0.25	616.00	832.76
10/22/2016	253,618,246,641	12.5	0.25	653.00	816.96
11/05/2016	254,620,187,304	12.5	0.25	712.00	820.19
11/18/16	281,800,917,193	12.5	0.25	750.00	907.74
12/02/16	286,765,766,821	12.5	0.25	764.00	923.74
12/15/16	310,153,855,703	12.5	0.25	782.00	999.08
12/28/16	317,688,400,354	12.5	0.25	955.00	1,023.35
01/10/17	336,899,932,796	12.5	0.25	902.00	1,085.23
01/22/17	392,963,262,344	12.5	0.25	923.00	1,265.82
02/05/17	422,170,566,884	12.5	0.25	1,018.00	1,359.91
02/18/17	440,779,902,287	12.5	0.25	1,061.00	1,419.85
03/03/17	460,769,358,091	12.5	0.25	1,272.00	1,484.24

03/17/17	475,705,205,062	12.5	0.25	1,131.00	1,532.35
03/30/17	499,635,929,817	12.5	0.25	1,037.00	1,609.44
04/13/17	520,808,749,422	12.5	0.25	1,193.00	1,677.64
04/27/17	521,974,519,554	12.5	0.25	1,314.00	1,681.40
05/10/17	559,970,892,891	12.5	0.25	1,821.00	1,803.79
05/23/17	595,921,917,085	12.5	0.25	2,379.00	1,919.60
06/04/17	678,760,110,083	12.5	0.25	2,698.00	2,186.44
06/17/17	711,697,198,174	12.5	0.25	2,507.00	2,292.54
07/02/17	708,659,466,230	12.5	0.25	2,561.00	2,282.75
07/17/17	804,525,194,568	12.5	0.25	2,059.00	2,591.56
07/27/17	860,221,984,436	12.5	0.25	2,570.00	2,770.97
08/09/17	923,233,068,449	12.5	0.25	3,424.00	2,973.94
08/24/17	888,171,856,257	12.5	0.25	4,363.00	2,861.00
09/06/17	922,724,699,726	12.5	0.25	4,311.00	2,972.30
09/18/17	1,103,400,932,964	12.5	0.25	3,943.00	3,554.30
10/02/17	1,123,863,285,133	12.5	0.25	4,293.00	3,620.22
10/15/17	1,196,792,694,099	12.5	0.25	5,711.00	3,855.14
10/26/17	1,452,839,779,146	12.5	0.25	5,773.00	4,679.92
11/10/17	1,364,422,081,125	12.5	0.25	6,363.00	4,395.11
11/24/17	1,347,001,430,559	12.5	0.25	8,707.00	4,339.00
12/07/17	1,590,896,927,258	12.5	0.25	11,000.00	5,124.64
12/18/17	1,873,105,475,221	12.5	0.25	17,373.00	6,033.70
12/31/17	1,931,136,454,487	12.5	0.25	13,812.00	6,220.63
01/12/18	2,227,847,638,503	12.5	0.25	14,500.00	7,176.40
01/24/18	2,603,077,300,218	12.5	0.25	11,200.00	8,385.10
02/06/18	2,860,000,000,000	12.5	0.22	7,250.00	8,107.18

Note: Source for difficulty and market price: blockchain.info.
Source for energy efficiency:
https://web.archive.org/web/20170215000000*/https://en.bitcoin.it/wiki/Mining_hardware_comparison
Block reward halves approximately every four years, this is part of the protocol.
Implied price is the calculated model price given the relevant inputs for each date.
Market price is the observed market price at the time of difficulty changes
Energy cost is held constant at 13.5 cents per kWh