An Analysis of the Effects of Neighborhood Radius and Dimensionality on Local Selection Algorithms

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Abstract

As Sarma and De Jong have previously demonstrated, both the shape and radius of neighborhoods have a pronounced impact on the rate of propagation of the best individual throughout a population using conventional selection methods. Although they had only demonstrated this in two dimensions, they speculated that these effects would continue into n dimensions. This paper intends to test that assertion as well as provide a closer examination of these effects in dimensions greater than two.

Sections

0 Introduction

As parallel computing has seen tremendous gains recently, it is advantageous to parallelize problems whenever possible. However, a single GA is inherently resistant to parallelization as all individual solutions must be able interact in traditional selection implementations. Thus the island model was developed, in which a single GA is run on each individual cores and the best inhabitants are passed between neighboring cores to help the average fitness increase across islands. One important variable in this model is which cores are set as neighboring cores, as this affects how information propagates throughout the islands.

Sarma and De Jong showed that in a 2 dimensional island setup the most important factor for selection pressure is the radius of an island's neighborhood (Sarma and De Jong, 1996). However, no mention was made of higher dimensional setups, despite the fact that their formulas were written for n-dimensional representation. Thus we have tested to see if radius is still the only determining factor in higher dimensional space.

1 Representation

We follow Sarma and De Jong's representation scheme by abstracting the island model to a single GA (Sarma and De Jong, 1996). To do so, each island is represented as a node containing a single solution and given a neighborhood of islands which it may reproduce with. These islands are placed in a toroidal n-hypercube lattice and all islands receive a neighborhood of similar shape and radius. Additionally, since we only want to observe how information propagates, crossover and mutation have been disabled. The only way to exchange information is through selection, which is accomplished by using linear ranking on an island's neighborhood. To observe selection pressure, we measure how many members of the population have converged to the optimal solution at a given generation.

2 GA Description

Each solution is represented as a value between 1 and 1000, with 1000 representing the optimal solution. Fitness is simply the current value, so selecting a solution for reproduction is a simple matter.

2.2 Neighborhoods

Each node is assigned a neighborhood defined by one of two shapes, linear or compact. A linear neighborhood is defined as all nodes adjacent to the original node along the cardinal directions whose distance is less than or equal to the radius r. A compact neighborhood is defined as all the nodes which can be contained within an n-hypersphere of radius r centered at the original node, including nodes which exist on the hypersphere's boundary. The neighborhood type is held constant across all nodes so all neighborhoods are of the same size and shape.

2.3 Lattice Structure

The structure of the lattice containing the nodes is defined as a toroidal n-hypercube whose length is determined in two ways. Firstly, we chose length based on a constant population size so that the population be similar regardless of dimensionality. Secondly, we chose a constant length for the n-hypercube and varied the population size accordingly. This allowed for us to the grow the population along with the neighborhood growth which occurs from increased dimensionality.

3 Experiments

We tested our representation using a variety of factors including different dimensions, differing dimension size, multiple neighborhood radii, and two neighborhood type. These tests spanned 2, 3, and 4 dimensions, neighborhood radii of 1 through 4, and linear and compact neighborhood types. We also varied the dimension size used, with our first set of tests using a constant population size of 4096, which was broken into (64 x 64) for 2 dimensions, (16 x 16 x 16) for 3 dimensions, and (8 x 8 x 8 x 8) for 4 dimensions. For our second set, we set the dimension size as 16 and instead varied the total population size, having a population of 256 for 2 dimension, 4096 for 3 dimensions, and 65,536 for 4 dimensions. While changing these factors one at a time, other parameters were held constant. Additionally, all experiments were ran using 40 generations for 10 runs using the same seed for random numbers each time. Averages were then calculated for every generation over all 10 runs.

3.2 Results

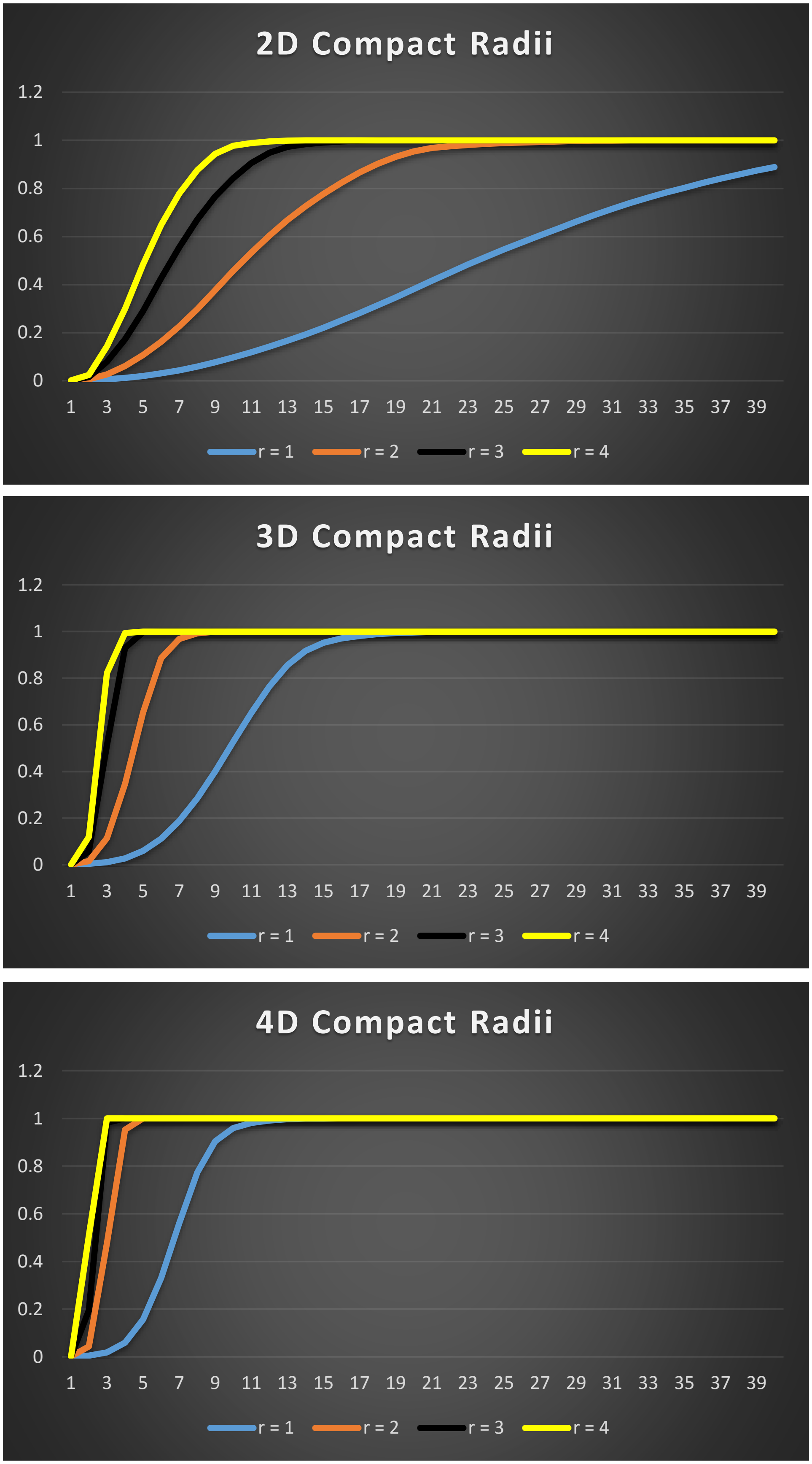

Figure 1. Shows the % of the population with the best fit individual on the y axis and the average at each generation on the x axis at various dimensions and neighborhood radii using the compact neighborhood type.

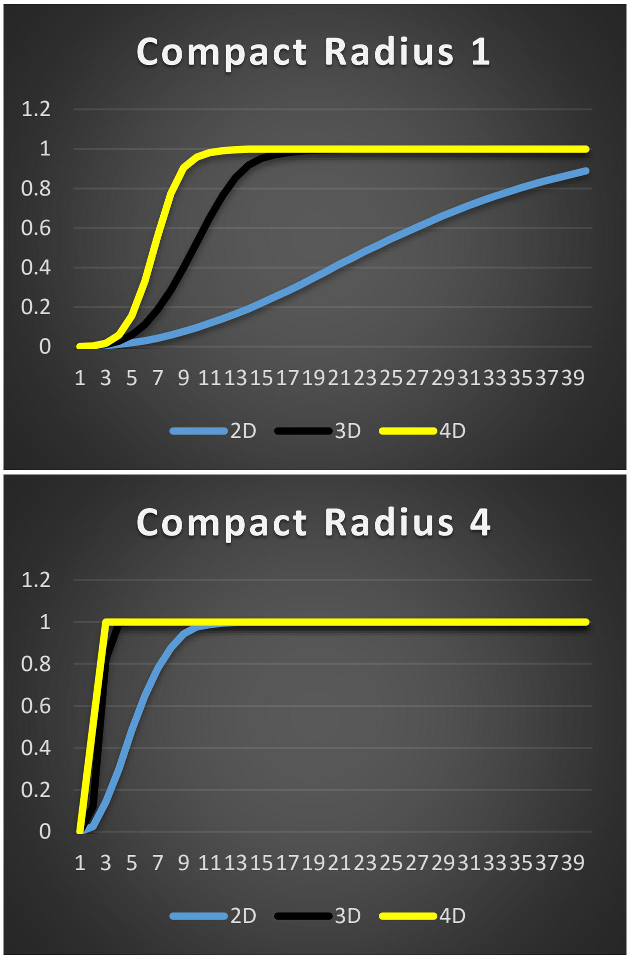

Figure 2. The compact radius graphs show the % of the population with the best fit individual on the y axis and the average for each generation on the x axis for the compact neighborhood type with a fixed radius across different dimensions.

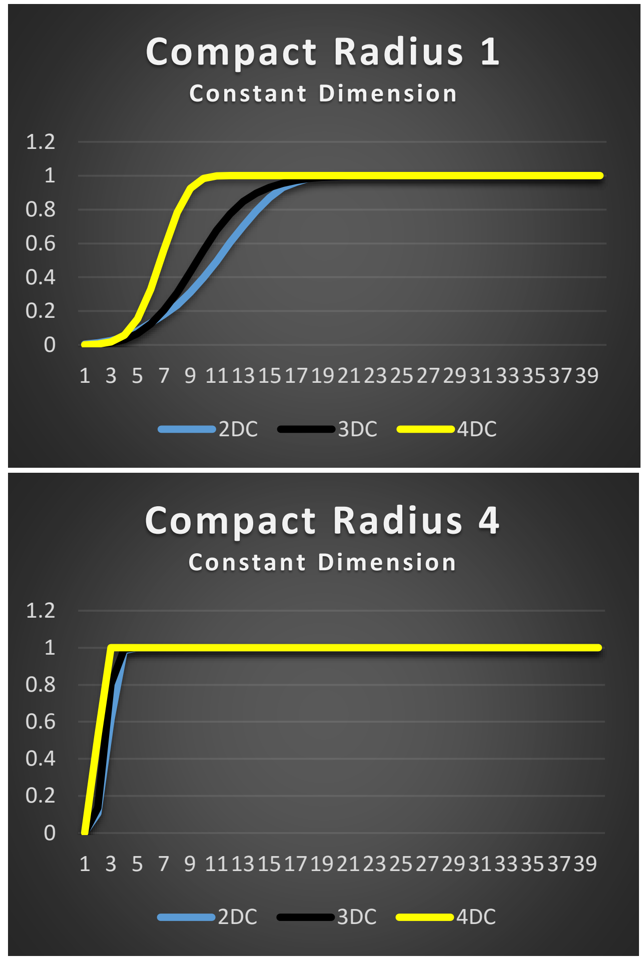

Figure 3. The compact radius constant dimension graphs show the % of the population with the best fit individual on the y axis and the average for each generation on the x axis for the compact neighborhood type with a fixed radius across different dimensions using a fixed dimension size.

4 Conclusions

The only experiment that did not converge within 40 generations was using a neighborhood radius of 1 in 2 dimensions. It is important to note that when using a radius of 1, both types were linear as the diagonal individuals in compact neighborhoods were a distance of 1.2 away. However increasing the neighborhood radius to 2 in 2 dimensions led to convergence by around generation 27 for both neighborhood types. With the sole exception of a radius of 1 (for the aforementioned reason), compact neighborhood types consistently outperformed linear neighborhood types, all other parameters being equal. That being said, linear neighborhoods never lagged far behind compact ones, with the gap being on average within a handful of generations slower, although the gap tended to be more pronounced at higher dimensions and less contingent on radius.

Contrary to Sarma and De Jong's results, the radius does not seem to be the only factor in determining selection pressure (Sarma and De Jong, 1996). Rather, both higher dimensions and larger neighborhood radii led to faster convergence in all cases, with the fastest being 4 dimensions with a radius of 4 and of compact type. Under those settings convergence happened on average by generation 3. Though increasing the neighborhood radius significantly increased convergence, increasing the dimensionality had an even more drastic effect in general. For example, an increase from 1 to 2 in neighborhood radius in 2 dimensions led to convergence within roughly 27 generations, while using a radius of one and changing from 2 dimensions to 3 dimensions led to convergence within around 17 generations. It is worth noting though that as one increases both dimensions and neighborhood radius, both exhibit diminishing returns the higher one goes.

Additionally, we found that the percentage of total nodes found in the neighborhood is not a major determining factor in convergence speed. In our set of tests which held the population constant, it can be easily observed that as dimensions increase, each neighborhood contains a higher percentage of the total population. This would be a reasonable explanation for why higher dimensions cause faster convergence except for the results of the tests where dimension size was held constant. In these tests the percentage of total nodes a neighborhood contains decreases as the dimension grows, opposite to the other set of tests. However, we observed that both sets of tests have near identical convergence times. This correlates with the results which Sarma and De Jong obtained showing that neighborhood size in 2 dimensions is not a determining factor of the rate of convergence.

5 Extensions

As we have shown with our paper that the link between radius and selection pressure does not hold in higher dimensions, the question remains of what factor is driving convergence. This will require far more extensive study to understand and we have a few ideas for possible areas to investigate. The first idea would be creating a neighborhood which maintains its ratio of neighborhood size to population size regardless of dimension. While we have shown that the percent of nodes a solution can reach does not seem to be a defining factor of convergence rate, such a neighborhood would allow more rigorous analysis of the concept.

Secondly, all neighborhoods used here have some degree of regularity and high symmetry, it would be interesting to see how non-uniform neighborhoods and neighborhoods of irregular shape perform. For instance, neighborhoods whose size was dependent on their placement would be an interesting test as it would bias certain nodes to converge to an optimal solution first.

Thirdly it would be interesting to see how disconnected neighborhoods or neighborhoods not containing the node perform. This would greatly decrease the maximum distance between nodes and may allow for information to spread much more quickly. Also not including a node in its own neighborhood would be interesting as it would guarantee that no nodes could self-replicate, possibly increasing diversity in the population.

Lastly experimenting with the size and shape of the lattice itself could yield interesting results, as a toroidal hypercube was chosen primarily based of Sarma and De Jong's work (Sarma and De Jong, 1996). Other shapes such a sphere where boundary points connect to elsewhere on the boundary may allow for a more uniform distribution of information between nodes. These are just a few of the many possible aspects of the problem which bear investigating, and much more rigorous analysis must be performed to find a truly generalized formula for island selection pressure.

References

1. Sarma, J., De Jong, K.: An Analysis of the Effects of Neighborhood Size and Shape on Local Selection Algorithms. Lecture Notes in Computer Science, Vol. 1141: Parallel Problem Solving from Nature – PPSN IV. Springer, Berlin (1996) 236-244