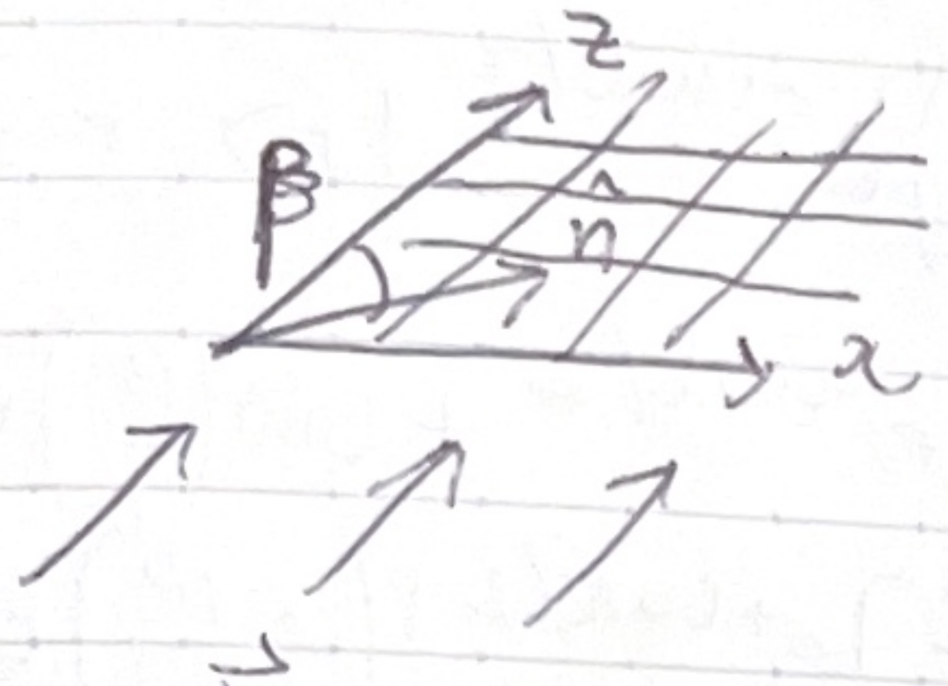
Sakurai 2.3

Time-Independent Hamiltonian

$B\hat{z} = \vec{B}$ $\qquad S\cdot\hat{n}$

$|\psi\rangle = \cos\left(\frac{\theta}{2}\right)|\uparrow\rangle + \sin\left(\frac{\theta}{2}\right)\exp(i\phi)|\downarrow\rangle$



$\phi = 0$ (in xz plane)

$\theta = \beta$ (measured from $\hat{z}$)

$\vec{B} = B\hat{z}$

$\longrightarrow |\psi\rangle = \cos\left(\frac{\beta}{2}\right)|\uparrow\rangle + \sin\left(\frac{\beta}{2}\right)|\downarrow\rangle$

$|\psi(t)\rangle = \sum_n c_n(t)|E_n\rangle = c_0(t)|\uparrow\rangle + c_1(t)|\downarrow\rangle$

$\cancel{\hat{H}|\psi\rangle = \omega\hat{S}_z|\psi\rangle} = \cos\left(\frac{\beta}{2}\right)e^{-i\omega t/2}|\uparrow\rangle$

$+ \sin\left(\frac{\beta}{2}\right)e^{+i\omega t/2}|\downarrow\rangle$

$|{}_x\langle\uparrow|\psi(t)\rangle|^2 = \left|\frac{1}{\sqrt{2}}(\langle\uparrow| + \langle\downarrow|)\left[\cos\left(\frac{\beta}{2}\right)e^{-i\omega t/2}|\uparrow\rangle\right.\right.$

$\left.\left. \cancel{+\sin\left(\frac{\beta}{2}\right)|\downarrow\rangle]} \; e^{+i\omega t/2}|\downarrow\rangle\right]\right|^2 = \left|\frac{1}{\sqrt{2}}\left[\cos\left(\frac{\beta}{2}\right)e^{-i\omega t/2}\langle\uparrow|\uparrow\rangle + \right.\right.$

$\left.\sin\left(\frac{\beta}{2}\right)e^{+i\omega t/2}\langle\downarrow|\downarrow\rangle\right|^2$

$= \frac{1}{2}\cancel{\left[\cos^2(\beta/2) + \sin^2(\beta/2) + \sin(\beta/2)\cos(\beta/2)\right.}$

$= \frac{1}{2}\left(\cos\frac{\beta}{2}e^{-i\omega t/2} + \sin\frac{\beta}{2}e^{i\omega t/2}\right)\left(\cos\frac{\beta}{2}e^{i\omega t/2} + \sin\frac{\beta}{2}e^{-i\omega t/2}\right)$

$= \frac{1}{2}\left(1 + 2\sin\frac{\beta}{2}\cos\frac{\beta}{2}\left(e^{+i\omega t} + e^{-i\omega t}\right)\right)$

Date

(a) $= \frac{1}{2}\left(1 + \cos\frac{\beta}{2}\sin\frac{\beta}{2}\left(e^{+i\omega t} + e^{-i\omega t}\right)\right.$

$= \frac{1}{2} + \cos\frac{\beta}{2}\sin\frac{\beta}{2}\cos(\omega t)$

$\cancel{\frac{1}{2}\left|\frac{1}{i}\right.}$ $\left|\frac{1}{\sqrt{2}}(\langle +| + \langle -|)\left(\cos\left(\frac{\beta}{2}\right)e^{-i\omega t/2}|+\rangle + \sin\left(\frac{\beta}{2}\right)e^{+i\omega t/2}\right.\right.$

$\left.\left.|-\rangle\right)\right|^2 = \frac{1}{2}\left|\cos\left(\frac{\beta}{2}\right)e^{-i\omega t/2} + \sin\left(\frac{\beta}{2}\right)e^{+i\omega t/2}\right|^2$

$= \frac{1}{2}\left(\cos\left(\frac{\beta}{2}\right)e^{-i\omega t/2} + \sin\left(\frac{\beta}{2}\right)e^{+i\omega t/2}\right)\left(\cos\left(\frac{\beta}{2}\right)e^{+i\omega t/2}\right.$

$\left.+ \sin\left(\frac{\beta}{2}\right)e^{-i\omega t/2}\right) = \frac{1}{2}\left(\cos^2\left(\frac{\beta}{2}\right) + \sin^2\left(\frac{\beta}{2}\right)\right.$

$\left.+ \cos\left(\frac{\beta}{2}\right)\sin\left(\frac{\beta}{2}\right)e^{-i\omega t} + \cos\left(\frac{\beta}{2}\right)\sin\left(\frac{\beta}{2}\right)e^{+i\omega t}\right)$

$= \frac{1}{2}\left(1 + \cos\left(\frac{\beta}{2}\right)\sin\left(\frac{\beta}{2}\right)\cos(\omega t)\right)$

$\boxed{= \frac{1}{2}\left(1 + \sin(\beta)\cos(\omega t)\right)}$

(b) $\langle S_x\rangle = \left(\cos\frac{\theta}{2}e^{+i\omega t/2}\langle +| + \sin\frac{\theta}{2}e^{-i\omega t/2}\langle -|\right)\frac{\hbar}{2}\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}$

$\left(\cos\frac{\theta}{2}e^{-i\omega t/2}|+\rangle + \sin\frac{\theta}{2}e^{+i\omega t/2}|-\rangle\right)$

$= \left(\cos\theta/2\, e^{i\omega t/2} \quad \sin\theta/2\, e^{-i\omega t/2}\right)\frac{\hbar}{2}\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}\begin{pmatrix}\cos\frac{\theta}{2}e^{-i\omega t/2}\\ \sin\frac{\theta}{2}e^{+i\omega t/2}\end{pmatrix}$

$= \frac{\hbar}{2}\left(\cos\frac{\theta}{2}e^{i\omega t/2} \quad \sin\frac{\theta}{2}e^{-i\omega t/2}\right)\begin{pmatrix}\sin\frac{\theta}{2}e^{+i\omega t/2}\\ \cos\frac{\theta}{2}e^{-i\omega t/2}\end{pmatrix}$

$= \frac{\hbar}{2}\sin\frac{\theta}{2}\cos\frac{\theta}{2}\left(e^{+i\omega t} + e^{-i\omega t}\right) = \boxed{\frac{\hbar}{2}\sin\theta\cos(\omega t)}$

Date 10 · 28 · 24

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~~Physics 220 HW 5~~

(c) $\langle S_x \rangle = \frac{\hbar}{2} \sin\beta \cos(\omega t)$

$|{}_x\langle\uparrow|\psi(t)\rangle|^2 = \frac{1}{2}(1+\sin\beta\cos(\omega t))$

when $\beta = 0$,

$\langle S_x \rangle = 0$

$|{}_x\langle\uparrow|\psi(t)\rangle|^2 = \frac{1}{2}$

when $\beta = \pi/2$

$\langle S_x \rangle = \frac{\hbar}{2}\cos(\omega t)$

$|{}_x\langle\uparrow|\psi(t)\rangle|^2 = \frac{1}{2}(1+\cos(\omega t))$

2.6

$$[[\hat{H},\hat{x}],x] + [[\hat{x},\hat{H}],\hat{x}] + [[\hat{x},\cancel{\hat{x}}],\hat{H}] = 0$$

$$[\hat{H},\hat{x}] = \frac{1}{2m}[\hat{p}^2,\hat{x}] + [V(x),\hat{x}]$$

$$= -\frac{1}{2m}[\hat{x},\hat{p}^2] + [V(x),\hat{x}]$$

$$= -\frac{1}{2m} i\hbar \frac{\partial}{\partial p} p^2 + [V(x),\hat{x}]$$

$$= -i\hbar\frac{p}{m} + \cancel{[V(x),\hat{x}]} = -i\hbar\frac{p}{m}$$

$$|\psi\rangle = \sum_{a'}|a'\rangle\langle a'|\psi\rangle = \sum_{a'}\langle a'|\psi\rangle e^{-iE_{a'}t/\hbar}|a'\rangle$$