Physics 220 Fall 2024

Problem Set # 6 (due on Fri Nov 15th)

1) Two relativistic particles with rest masses m_1 and m_2 are observed to move along the observer's z-axis towards each other with velocities v_1 and v_2 respectively. Upon collision they are observed to coalesce into one particle of rest mass m, moving with velocity v relative to the observer.
   a) Find m and v in terms of m_1, m_2, v_1, v_2.
   b) Would it be possible for the resultant particle to be a photon with mass $m = 0$ assuming that $m_1, m_2 > 0$?
   c) Same question as in (b) but when one or both of the masses vanish

2) A π^+ meson (rest mass $m_\pi = 139$ MeV collides with a neutron (rest mass $m_n = 939$ MeV), which is at rest in the laboratory, to produce a K+ meson (rest mass $m_K = 494$ MeV) and a Λ baryon (rest mass $m_\Lambda = 1115$ MeV). What is the threshold total energy (in the laboratory frame) of the π^+ for this reaction to proceed?

3) Consider the relativistic one dimensional harmonic oscillator with mass m and potential energy $U = \frac{1}{2}kx^2$. The maximal amplitude of the oscillator is xmax = a.
   a) Find the Lagrangian for this system.
   b) Find the Euler-Lagrange equation for this system.
   c) Find the conserved energy for this system.
   d) Find an integral expression for the period τ of the oscillation.
   e) Determine the first relativistic correction to the period by expanding the integral expression for the period found in d) to first order in ka2 mc2 and performing the integrations.

4) Consider the motion of a relativistic rocket, on which no external forces act. The rocket is propelled by expelling gasses at a constant velocity u with respect to the rocket. As a result of this propulsion, the mass m of the rocket, and its velocity v, will change over time.

   a) Obtain the differential equation for the change in the rocket velocity v as a function of the change in its mass m, as a function of m, v and u.

   b) Solve this equation for v as a function of m