Physics 220 Fall 2024

Problem Set # 3 (due on Thu Oct 19th)

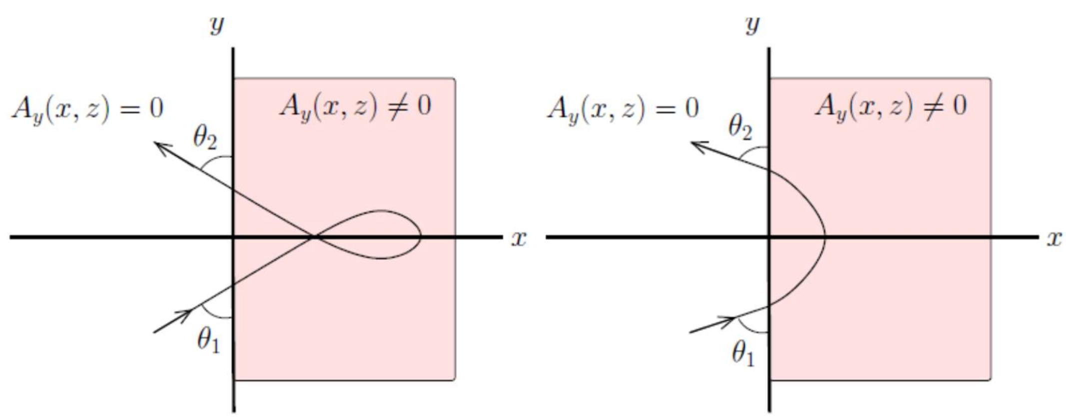
1) Consider the following time-dependent Lagrangian

$$L = e^{\frac{\gamma t}{m}}\left(\frac{m}{2}\dot{x}^2 - \frac{k}{2}x^2\right)$$

   a) Calculate the equation of motion of this system and write the general solutions
   b) Sketch in the $x, \dot{x}$ plane the trajectory of a particle starting at rest at the initial position x_0
   c) Calculate the Hamiltonian
   d) Is the canonical volume preserved by the solutions of the Hamilton equations for this system?

2) A non relativistic electron (charge e = -|e| and rest mass m) with mechanical momentum $\boldsymbol{p} = p_0(\widehat{\boldsymbol{x}}\ \sin\theta + \widehat{\boldsymbol{y}}\ \cos\theta)$ enters into a static magnetic field region (x>0) from a region of free space (zero magnetic field and zero vector potential) at x<0. The magnetic field has no y-component. It is due to a vector potential which has only a y-component A_y with (x,z) dependence, i.e. $\boldsymbol{A} = \widehat{\boldsymbol{y}}A_y(x,z)$
In addition, at z=0 the magnetic field is perpendicular to the x-y plane.

a) Starting from the Lagrangian of a charged particle in external electromagnetic fields, construct the Hamiltonian of the system and the canonical momentum of the particle.
b) Show that a trajectory of an electron located at z = 0 with its momentum in the x-y plane will stay in the x-y plane.
c) Obtain two conserved quantities for the problem above and show, assuming that the electron eventually leaves the static magnetic field region, that this system is indeed a mirror for trajectories in the x-y plane, namely an electrons with initial momentum **p** is reflected such that the angles that the incoming and outgoing trajectories make with the y-axis are equal in magnitude and opposite in sign (i.e. $\theta_1 = \theta_2$ in the picture below).
d) Find an equation for the depth the penetration (the furthest the electron reaches into the magnetic field region) and solve the resulting equation for the particular case of field **B** = G (($\widehat{\boldsymbol{x}}z - \widehat{\boldsymbol{z}}x$). Which sign of G corresponds to the trajectories shown in each figure of the figures below?



3) Consider a mass point which moves on a wire which rotates with constant angular velocity.

   a) By transforming to general coordinates which rotate with the wire show that the Lagrangian of the system is given by

$$L = \frac{m}{2}(\dot{r}^2 + \omega^2 r^2)$$

   b) Calculate the Hamiltonian of the system. Does it change with time ?

   c) Calculate the kinetic energy (which is the total energy in this case) of the mass point in terms of the generalized coordinate r. Is it the same as the Hamiltonian and is it conserved

4) Consider a transformation of the Lagrangian to an equivalent Lagrangian, i.e.

$$L(q,\dot{q},t) \to L(q,\dot{q},t) + \frac{d\Lambda}{dt}$$

a) How do the coordinates and canonical momenta change ?

b) How does the Hamiltonian change ?

c) Consider this change as a generalized coordinate change $(q,p) \to (q,^{I} p^{I})$. Show that the Poisson bracket for the new coordinates will have the same form as the old one.