Physics 221A Homework 6
Sakurai 2.27, 2.29, 2.30, 2.35, 2.36, 2.41, 2.44

2.27 Consider a particle of mass m subject to a one-dimensional potential of the following form:

$$V = \begin{cases} \frac{1}{2}kx^2 & \text{for} \quad x > 0 \\ \infty & \text{for} \quad x < 0. \end{cases}$$

a. What is the ground-state energy?
b. What is the expectation value $\langle x^2 \rangle$ for the ground state?

2.29 Consider a particle in one dimension bound to a fixed center by a δ-function potential of the form

$$V(x) = -\nu_0 \delta(x)$$

where ν_0 is real and positive. Find the wave function and the binding energy of the ground state. Are there excited bound states?

2.30 A particle of mass m in one dimension is bound to a fixed center by an attractive δ-function potential:

$$V(x) = -\lambda \delta(x) \quad (\lambda > 0).$$

At $t = 0$, the potential is suddenly switched off (that is, $V = 0$ for $t > 0$). Find the wave function for $t > 0$. (Be quantitative! But you need not attempt to evaluate an integral that may appear.)

2.35 Derive an expression for the density of free-particle states in *two* dimensions, normalized with periodic boundary conditions inside a box of side length L.

2.36 Use the WKB method to find the (approximate) energy eigenvalues for the one-dimensional simple harmonic oscillator potential $V(x) = m\omega^2 x^2/2$.

2.41 A particle of mass m moves along one of two "paths" through space and time connecting the points $(x,t)=(0,0)$ and $(x,t)=(D,T)$. One path is quadratic in time, i.e. $x_1(t)=\frac{1}{2}at^2$ where a is a constant. The second path is linear in time, i.e. $x_2(t)=vt$ where v is a constant. The correct classical path is the quadratic path, that is $x_1(t)$.

a. Find the acceleration a for the correct classical path. Use freshman physics to find the force $F=ma=-dV/dx$ and then the potential energy function $V(x)$ in terms of m, D, and T. Also find the velocity v for the linear (i.e. incorrect classical) path.

b. Calculate the classical action $S[x(t)]=\int_0^T\left[\frac{1}{2}m\dot{x}^2-V(x)\right]dt$ for each of the two paths $x_1(t)$ and $x_2(t)$. Confirm that $S_1\equiv S[x_1(t)]<S_2\equiv S[x_2(t)]$, and find $\Delta S=S_2-S_1$.

c. Calculate $\Delta S/\hbar$ for a particle which moves 1 mm in 1 ms for two cases. The particle is a nanoparticle made up of 100 carbon atoms in one case. The other case is an electron. For which of these would you consider the motion "quantum mechanical" and why?

2.44 a. Write down an expression for the classical action for a simple harmonic oscillator for a finite time interval.

b. Construct $\langle x_n,t_n|x_{n-1},t_{n-1}\rangle$ for a simple harmonic oscillator using Feynman's prescription for $t_n-t_{n-1}=\Delta t$ small. Keeping only terms up to order $(\Delta t)^2$, show that it is in complete agreement with the $t-t_0\to 0$ limit of the propagator given by (2.290).