Physics 220 FALL 2024

Practice Midterm

1. Consider a 3-dimensional one-particle system whose potential energy in cylindrical coordinates ρ, θ, z is of the form $V(\rho, k\theta + z)$ for some constant k i.e. the Lagrangian can be written as

$$L = \frac{1}{2}m\left(\dot{z}^2 + \dot{\rho}^2 + \rho^2\dot{\theta}^2\right) - V(\rho, k\theta + z)$$

   a) Show that, for any potential function V, this system has two continuous symmetries, and derive the associated symmetry transformations on the coordinates.
   b) Derive the associated Noether charges.

2. A particle of mass m is constrained to move (without friction) on a parabola which at time $t=0$ satisfies the equation $z = cy^2$. The parabola rotates with angular velocity ω along the z-axis. The mass is subject to a uniform gravitational field acting in the z-direction with acceleration g.
   a) Write down the constraints.
   b) Write down the Lagrangian and the Euler-Lagrange equations
   c) Write down the Hamiltonian. Is the Hamiltonian a conserved quantity ? Is it equal to the energy of the motion ?

3. A system with two degrees of freedom is given by the Hamiltonian

$$H = {p_x}^2 + x^2 {p_y}^2 + x^2 y^2$$

   a) Write down the Hamilton-Jacobi equation for this system
   b) Solve the Hamilton-Jacobi equation by a separation ansatz.
   c) Use the solution of b) to find $x(t), y(t), p_x(t), p_y(t)$

4. (in case you don't like any of the previous problems) A mass point m_1 is constrained to move (without friction) along a horizontal line. It is connected via a massless string of length l to a second mass m_2. Both masses are subject to a uniform gravitational field g pointing in the vertical direction (see figure)
   a) Write down the Lagrangian using generalized coordinates which solve the constraints
   b) Show that the horizontal component of the total momentum is conserve.