Homework 2

1.15, 1.28, 3.11, 3.12, 3.14

1.15 A beam of spin $\frac{1}{2}$ atoms goes through a series of Stern–Gerlach type measurements as follows.

a. The first measurement accepts $s_z = \hbar/2$ atoms and rejects $s_z = -\hbar/2$ atoms.

b. The second measurement accepts $s_n = \hbar/2$ atoms and rejects $s_n = -\hbar/2$ atoms, where s_n is the eigenvalue of the operator $\mathbf{S}\cdot\hat{\mathbf{n}}$, with $\hat{\mathbf{n}}$ making an angle β in the xz-plane with respect to the z-axis.

c. The third measurement accepts $s_z = -\hbar/2$ atoms and rejects $s_z = \hbar/2$ atoms.

What is the intensity of the final $s_z = -\hbar/2$ beam when the $s_z = \hbar/2$ beam surviving the first measurement is normalized to unity? How must we orient the second measuring apparatus if we are to maximize the intensity of the final $s_z = -\hbar/2$ beam?

1.28 Construct the transformation matrix that connects the S_z diagonal basis to the S_x diagonal basis. Show that your result is consistent with the general relation

$$U = \sum_r |b^{(r)}\rangle\langle a^{(r)}|.$$

3.11 Use the triangle inequality (1.147) and the definition (3.100) of the density operator ρ to prove that $0 \le \mathrm{Tr}(\rho^2) \le 1$.

which does not depend on the particular observable A is factored out. This motivates us to define the **density operator** ρ as follows:

$$\rho \equiv \sum_i w_i |\alpha^{(i)}\rangle\langle\alpha^{(i)}|. \qquad (3.100)$$

3.12 A large collection of spin $\frac{1}{2}$ particles is in a mixture of the two states $|S_z;+\rangle$ and $|S_y;-\rangle$. The fraction of particles in the state $|S_z;+\rangle$ is a. Find the ensemble averages $[S_x]$, $[S_y]$, and $[S_z]$ in terms of a. Confirm that your expression gives the answers you expect for $a = 0$ and $a = 1$.

3.14 Consider a one-dimensional simple harmonic oscillator with frequency ω and eigenstates $|0\rangle$, $|1\rangle$, $|2\rangle$,.... A mixed ensemble is formed with equal parts of each of the three states

$$|\alpha\rangle \equiv \frac{1}{\sqrt{2}}[|0\rangle + |1\rangle], \qquad |\beta\rangle \equiv \frac{1}{\sqrt{2}}[|1\rangle + |2\rangle], \qquad \text{and} \qquad |2\rangle.$$

Lemma 1 *The Schwarz inequality*

$$\langle\alpha|\alpha\rangle\langle\beta|\beta\rangle \ge |\langle\alpha|\beta\rangle|^2, \qquad (1.147)$$

which is analogous to

$$|\mathbf{a}|^2|\mathbf{b}|^2 \ge |\mathbf{a}\cdot\mathbf{b}|^2 \qquad (1.148)$$

in real Euclidian space.