Sakurai 2.30

Use propagator K $\longrightarrow$ $\langle x'| e^{-\frac{i\hat{H}t}{\hbar}} |\psi\rangle$ where $\hat{H} = \frac{\hat{p}^2}{2m}$

$$K(x'', t; x', t_0) = \sum_{a'} \langle x'' | a' \rangle \langle a' | x' \rangle \exp\left(\frac{iE_{a'}(t-t_0)}{\hbar} \right)$$

what does a propagator do?

$$\psi(x'', t) = \int d^3x' \; K(x'', t; \vec{x}', t_0)\, \psi(x', t_0)$$

K is a kernel of the integral operator that gives the time evolution WF of a state given its Hamiltonian and its state at t_0.

initial WF

$$\psi(x', t_0) = \begin{cases} \sqrt{\kappa}\, e^{\kappa x} & x < 0 \\ \sqrt{\kappa}\, e^{-\kappa x} & x > 0 \end{cases} \qquad \kappa = \frac{mV_0}{\hbar}$$

when $t > t_0$, $\hat{H} = \frac{\hat{p}^2}{2m}$

$$\langle x'' | \psi(t) \rangle = \sum_{a'} \underbrace{\langle x'' | a' \rangle}_{C_{a'}(t)} \underbrace{\langle a' | \alpha, t_0 \rangle}_{=} \exp\left(-\frac{iE_a(t-t_0)}{\hbar} \right)$$

$$u_{a'}(\vec{x}) = \langle x' | a' \rangle$$

$$\langle a' | \psi(t) \rangle = \int d^3x' \; u^*_{a'}(\vec{x})\, \psi(\vec{x}, t_0)$$

$$K(x'', t; x', t_0) = \langle x'' | \exp\left(-i \frac{\hat{p}^2}{2m} \right) | x' \rangle$$

$$\psi(x'', t) = \langle x'' | \psi(t) \rangle = \langle x'' | \int dx' \exp\left(\frac{-i\hat{H}}{\hbar} t \right) | x' \rangle \langle x' | \psi \rangle$$

$t = t_0$

$$= \int dx' \langle x'' | x' \rangle \exp\left(-i \frac{\hat{p}^2}{2m\hbar} t \right) \psi(x', t_0)$$

$$= \int dx'' \exp\left(-i \frac{\hat{p}^2}{2m\hbar} t \right) \psi(x'', t_0)$$

$$\Psi(x,t) = \int dx \exp\left(-\frac{i\hat{p}^2}{2m\hbar} t\right) \psi(x, t_0)$$

$$-\frac{i\hat{p}^2}{2m\hbar} t = -\frac{i(-i\hbar)^2}{2m\hbar} t \cdot \frac{d^2}{dx^2} = i\frac{\hbar}{2m} t \frac{d^2}{dx^2}$$

$$\exp\left(-\frac{i\hat{p}^2}{2m\hbar} t\right) \psi(x, t_0) = \sum_{n=0}^{\infty} \frac{1}{n!} \left(i\frac{\hbar}{2m} t\right)^n \frac{d^{2n}\psi(x)}{dx^{2n}}$$

$$\Psi(x,t) = \sum_{n=0}^{\infty} \frac{1}{n!} \left(i\frac{\hbar}{2m} t\right)^n \int \frac{d^{2n}\psi(x)}{dx^{2n}} dx$$