Name (please print):____________________________________

UID:____________________________________

Lecture Section, 3 (1PM) or 4 (3PM):__________

**Instructions**

- You will independently be given points for your reasoning, your mathematical work, and for correctness of answers, so make sure to **show your reasoning and work** even if you're not certain you can compute the correct answer. Try to **convey your solution strategy even if you can't execute it in time**.
- If on a given problem you need some space to do scratch work, do that scratch work elsewhere. **Only the work that is included on this test packet in the space provided will be graded**.
- Place final numerical answers, if any, in the box at the end of the space provided for the problem.
- Don't worry about significant figures or precision of final numerical answers. Don't worry about roundoff errors. As long as numerical answers are reasonable close to the correct answer and your process is correct, you will get full credit for numerical answers.

**Academic Conduct Acknowledgement**

*I acknowledge that I have completed this quiz in the time allotted entirely on my own, without using anything but the exam, a writing utensil, eraser, and calculator. I have not posted the quiz question or an equivalent variant of it on the internet (for example on Chegg). I did not communicate with anyone about the quiz during the 24-hour submission period.*

Please sign here:__________________________________________

Faaizah is a medical researcher who needs to measure the speed $v$ of the blood flow in a certain section of a horizontally-oriented blood vessel of length $L = 10\,\text{cm}$ and cross-sectional radius $R = 2\,\text{mm}$. Having taken Physics 5B, she realizes that there's a way to determine the desired blood flow speed in terms of the pressure difference $\Delta p$ between the two ends of the blood vessel. By treating the flow as viscous but incompressible, she is able to derive the following relationship between $v$ and $\Delta p$ (recall that $\eta = 2.5 \times 10^{-3}\,\text{Pa} \cdot \text{s}$ is the viscosity of blood):

$$v = \alpha \Delta p, \qquad \text{were} \quad \alpha = \frac{R^2}{8\eta L} \tag{1}$$

**1**. Derive Faaizah's equation (1) using (i) Poiseuille's equation for viscous flow, (ii) an appropriate expression for volume flow rate, and (iii) a fact about circles. After deriving equation (1), use it to compute the numerical value of $\alpha$ (in SI units) for the blood vessel Faaizah is measuring.

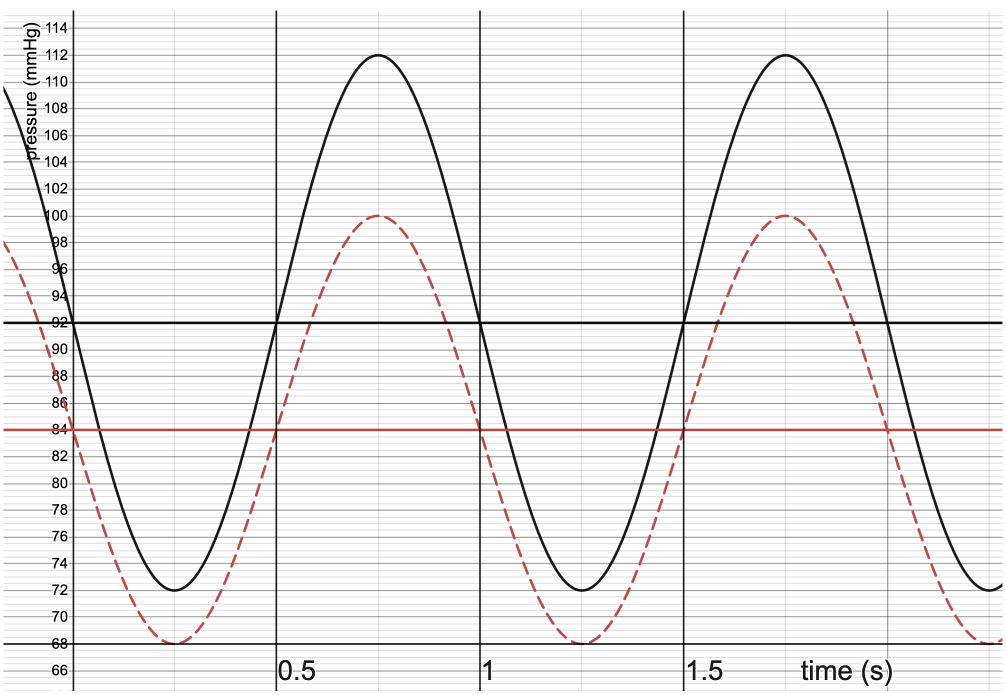
**Solution**. Combining Poiseuille's equation with the expression $Q = vA$ for volume flow rate and the expression $\pi R^2$ for the area of the circle yields the following:

$$\Delta p = \frac{8}{\pi}\frac{\eta L Q}{R^4} = \frac{8}{\pi}\frac{\eta L (vA)}{R^4} = \frac{8}{\pi}\frac{\eta L v(\pi R^2)}{R^4} = \frac{8\eta L}{R^2} v. \tag{2}$$

Solving for $v$ gives Faizaah's result. The numerical value of $\alpha$ for the vessel Faizaah is considering is

$$\alpha = \frac{R^2}{8\eta L} = \frac{(2\,\text{mm})^2}{8(2.5 \times 10^{-3}\,\text{Pa} \cdot \text{s})(10\,\text{cm})} = 2 \times 10^{-3}\,\text{m}/(\text{Pa} \cdot \text{s}) \tag{3}$$

**2**. Suppose that the blood is flowing from the left end of the blood vessel to its right end and that $p_1(t)$ and $p_2(t)$ are the pressures at the left and right ends respectively. The following plot shows the graphs of these pressures as functions of time. The black solid line is $p_1(t)$ and the red dotted line is $p_2(t)$. Carefully note the values of the tick marks and labels on the vertical and horizontal axes.




The plotted pressures $p_1(t)$ and $p_2(t)$ can be written in variables in the following form:

$$p_1(t) = P_1 + A_1 \cos(\omega t + \phi_0), \qquad p_2(t) = P_2 + A_2 \cos(\omega t + \phi_0) \tag{4}$$

Based on the graphs above, what are the SI values (with units) of the variables $P_1$, $P_2$, $A_1$, $A_2$, $\omega$, and what is the value of $\phi_0$ in radians? Recall that $1\,\text{mmHg} \approx 133\,\text{Pa}$.

**Solution.** The solid black and solid red horizontal lines are the pressures at which the cosine term in $p_1$ and $p_2$ is zero (halfway between the max and min), and this corresponds to the variables $P_1$ and $P_2$ which therefore can be read off the graph. The amplitude of each graph is the difference between that horizontal line and the maximum on the graph which can also be read off the graph. Lastly $\omega = 2\pi f = 2\pi/T$ where $T$ is the duration of a full cycle, which for both graphs is 1 s, and $\phi_0$ is obtained by noting that the given pressure graphs are equivalent to cosine graphs shifted over to the left by a quarter cycle which corresponds to an angle of $\pi/2$. From all of this, it follows that

$$P_1 = 92\,\mathrm{mmHg} = 12{,}236\,\mathrm{Pa} \tag{5}$$
$$P_2 = 84\,\mathrm{mmHg} = 11{,}172\,\mathrm{Pa} \tag{6}$$
$$A_1 = 20\,\mathrm{mmHg} = 2{,}660\,\mathrm{Pa} \tag{7}$$
$$A_2 = 16\,\mathrm{mmHg} = 2{,}128\,\mathrm{Pa} \tag{8}$$
$$\omega = \frac{2\pi}{1\,\mathrm{s}} \approx 6.3\,\mathrm{rad/s} \tag{9}$$
$$\phi_0 = \pi/2 \tag{10}$$

**3.** Using equations (1) and (4) it can be shown that the speed as a function of time $v(t)$ of the blood in the vessel can be written as follows:

$$v(t) = \alpha(P_1 - P_2) + \alpha(A_1 - A_2)\cos(\omega t + \phi_0) \tag{11}$$

You can take this expression for granted. Use this expression and some intermediate calculations on scratch paper to plot $v(t)$ in meters per second versus $t$ in seconds on the grid below. Make sure to include tick marks with values and axis labels with units on your plot. Try to make it as numerically accurate as you can given that it's being drawn by hand. Start the plot at $t = 0$, and show at least two full cycles of the oscillation on the plot.

**Solution.** Using the value of $\alpha$ computed in the first part, one computes $\alpha(P_1 - P_2) = 2.128\,\mathrm{m/s}$ and $\alpha(A_1 - A_2) = 1.064\,\mathrm{m/s}$ which yields the following graph: