1.21

$|\psi\rangle = |+\rangle_z = \begin{pmatrix}1\\0\end{pmatrix}$

$\langle S_x^2\rangle = (1\ 0)\left[\frac{\hbar}{2}\begin{pmatrix}0&1\\1&0\end{pmatrix}\right]^2\begin{pmatrix}1\\0\end{pmatrix} = \frac{\hbar^2}{4}(1\ 0)\begin{pmatrix}0&1\\1&0\end{pmatrix}\begin{pmatrix}0&1\\1&0\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix} = \frac{\hbar^2}{4}(1\ 0)\begin{pmatrix}0&1\\1&0\end{pmatrix}\begin{pmatrix}0\\1\end{pmatrix} = \frac{\hbar^2}{4}(1\ 0)\begin{pmatrix}1\\0\end{pmatrix}$

$= \frac{\hbar^2}{4}$

$(\langle S_x\rangle)^2 = \left[(1\ 0)\frac{\hbar}{2}\begin{pmatrix}0&1\\1&0\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix}\right]^2 = \frac{\hbar^2}{4}\left[(1\ 0)\begin{pmatrix}0\\1\end{pmatrix}\right]^2 = 0$

$\langle(\Delta S_x)^2\rangle = \frac{\hbar^2}{4} - 0 = \frac{\hbar^2}{2}$

$\langle(\Delta S_y)^2\rangle = \langle S_y^2\rangle - \langle S_y\rangle^2$

$\langle S_y^2\rangle = \left[(1\ 0)\frac{\hbar^2}{4}\begin{pmatrix}0&-i\\i&0\end{pmatrix}\begin{pmatrix}0&-i\\i&0\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix}\right] = \frac{\hbar^2}{4}(1\ 0)\begin{pmatrix}0&-i\\i&0\end{pmatrix}\begin{pmatrix}i\\0\end{pmatrix} = \frac{\hbar^2}{4}(1\ 0)\begin{pmatrix}0\\-1\end{pmatrix} = 0$

$\langle S_y\rangle^2 = \left[(1\ 0)\frac{\hbar}{2}\begin{pmatrix}0&-i\\i&0\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix}\right]^2 = \frac{\hbar^2}{4}\left[(1\ 0)\begin{pmatrix}i\\0\end{pmatrix}\right]^2 = -\frac{\hbar^2}{4}$

$\langle S_y^2\rangle - \langle S_y\rangle^2 = 0 - \left(-\frac{\hbar^2}{4}\right) = \frac{\hbar^2}{4}$

$[S_x, S_y] = \frac{\hbar^2}{4}\begin{pmatrix}2i&0\\0&-2i\end{pmatrix}$

$\frac{1}{4}\left|\left\langle\frac{\hbar^2}{4}\begin{pmatrix}2i&0\\0&-2i\end{pmatrix}\right\rangle\right|^2 = \frac{1}{4}\frac{\hbar^4}{16}\left|(1\ 0)\begin{pmatrix}2i&0\\0&-2i\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix}\right|^2 = \frac{\hbar^4}{64}|2i|^2 = \frac{\hbar^4}{16}$

$\frac{\hbar^2}{4}\cdot\frac{\hbar^2}{4} \geq \frac{\hbar^4}{16}$

with $|\psi\rangle = |+\rangle_x = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}$

$\langle(\Delta S_x)^2\rangle = \langle S_x^2\rangle - \langle S_x\rangle^2 = 0$

$\langle(\Delta S_y)^2\rangle = \frac{\hbar^2}{4}$

$\frac{1}{4}|\langle[S_x, S_y]\rangle|^2 = \frac{1}{4}\left|\frac{1}{2}(1\ 1)\begin{pmatrix}2i&0\\0&-2i\end{pmatrix}\begin{pmatrix}1\\1\end{pmatrix}\right|^2 = 0$

$\langle(\Delta S_x)^2\rangle\langle(\Delta S_y)^2\rangle = 0 \geq \frac{1}{4}|\langle[S_x, S_y]\rangle|^2 = 0$