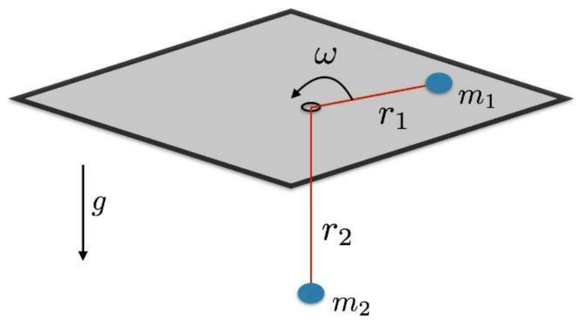
Physics 220 Fall 2024

Problem Set #2 (due on Thu Oct 10th)

1) (2013 Comprehensive exam) A mass m_1 is moving without friction on a desk in the presence of a constant gravitational field. It is attached by a massless rope through a hole in the desk to a second mass m_2. The first mass can rotate on the desk plane.
   a) Find the constraints and the Lagrangian for this system
   b) Calculate the equations of motion
   c) Find conserved quantities of the system to reduce the equations of motion to a single differential equation of first order
   d) Find the initial conditions for which mass m_1 moves on a circle and calculate the tension of the string for this solution.



2) A particle of mass m_1 is constrained to move on a circle of radius r. A second particle of mass m_2 is constrained to move on a second circle of same radius but with its center displaced by distance a in the x-direction. The two masses are connected by a massless spring with constant k (i.e. there is a potential which takes the form $V = -kd^2$, where d is the Euclidean distance between the masses).

   a) Identify the two generalized coordinates and express the Lagrangian in terms of these.
   b) Write down the equations of motion following from the Lagrangian
   c) For which value of a is there an additional conserved quantity?
   d) For the special case *c*) reduce the equation of motion to a first order equation.

3) Consider the following Lagrangian which describes the motion of a charged particle in a uniform constant electric field E

$$L_1 = \frac{1}{2}m\left(\frac{dx}{dt}\right)^2 + q\vec{E}\cdot\vec{x}$$

a) Even though the Lagrangian does not seem invariant under spatial translation, show that it transform under translations into an equivalent Lagrangian (i.e. only differing by a total time derivative)
b) Using Noether's theorem, calculate the conserved quantity and solve the equations of motion
c) Show that the following Lagrangian is equivalent to L_1 and find the total time derivative which relates them

$$L_2 = \frac{1}{2}m\left(\frac{dx}{dt}\right)^2 - q\left(\vec{E}\cdot\overrightarrow{\frac{dx}{dt}}\right)t$$

d) Calculate the conserved quantity from L_2 and compare.

4) A mass m_1 is moving frictionless on a ramp with angle α with base accelerates with constant acceleration $x_0(t)=1/2at^2$

a) Set up the constraints and the Lagrangian.

b) Find generalized coordinates and solve the Euler Lagrangian equations.

c) What is the condition that the mass leaves the ramp ?