Matt Kinsinger, MAT 372 Final

[4] Suppose that $w \in C^2(\Omega) \cap C(\overline{\Omega})$ is a solution to the given partial differential equation. Since $\Omega \subseteq \mathbb{R}^3$ is bounded, $\overline{\Omega}$ is both closed and bounded in $\mathbb{R}^3$, hence $\overline{\Omega}$ is compact. w is continuous on a compact set, thus w attains a maximum and a minimun on $\overline{\Omega}$.

- Case 1: Suppose that both the max and min of w on $\overline{\Omega}$ occur on $\partial\Omega$. Then

$$\max w = \min w = 0.$$

  So $w = 0$ on $\overline{\Omega}$.

- Case 2: Suppose that max w occurs at $x_M \in \Omega$. x_M is an interior point of Ω since Ω is open.