Matt Kinsinger, MAT 372 Final

[4] Suppose that $w \in C^2(\Omega) \cap C(\overline{\Omega})$ and $u \in C^2(\Omega) \cap C(\overline{\Omega})$ are both solutions to the given partial differential equation. Set $g = w - u$. Since $w, u \in C(\overline{\Omega})$ it follows that $g \in C(\overline{\Omega})$. Since $\Omega \subseteq \mathbb{R}^3$ is bounded, $\overline{\Omega}$ is both closed and bounded in $\mathbb{R}^3$, hence $\overline{\Omega}$ is compact. g is continuous on a compact set, thus g attains a maximum and a minimun on $\overline{\Omega}$.